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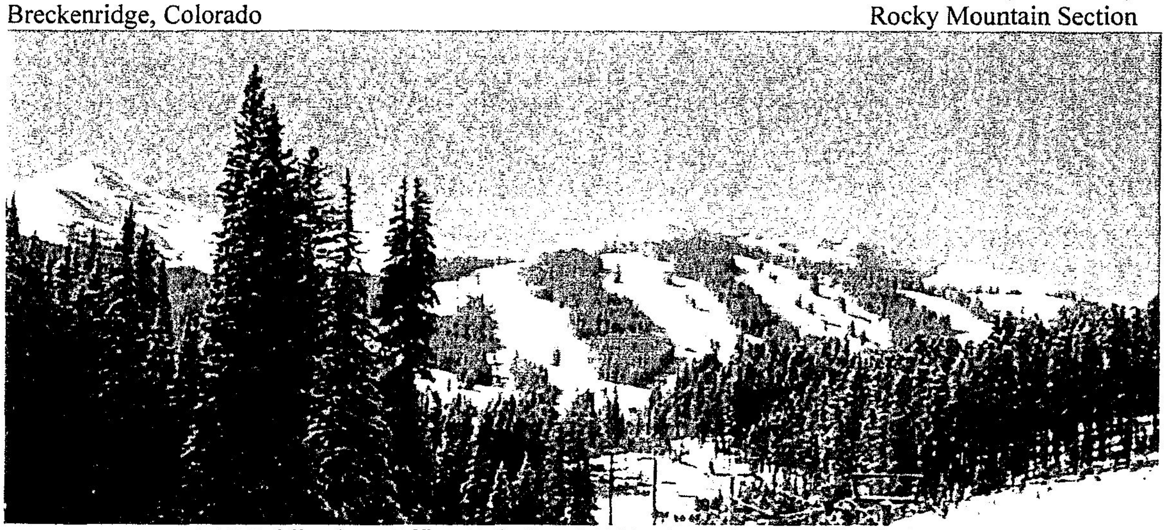

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IMPLEMENTATION OF A COLLISION PROBABILITY PREDICTION TECHNIQUE FOR CONSTELLATION MANEUVER PLANNING

Marco A. Concha*

On March 22, 2006, the Space Technology 5 (ST5) constellation spacecraft were successfully delivered to orbit by a Pegasus XL launch vehicle. An unexpected relative motion experienced by the constellation after orbit insertion brought about a problem. Soon after launch the observed relative position of the inert rocket body was between the leading and the middle spacecraft within the constellation. The successful planning and execution of an orbit maneuver that would create a fly-by of the rocket body was required to establish the formation. This maneuver would create a close approach that needed to conform to pre-defined collision probability requirements. On April 21, 2006, the ST5 "155" spacecraft performed a large orbit maneuver and successfully passed the inert Pegasus 3rd Stage Rocket Body on April 30, 2006 15:20 UTC at a distance of 2.55 km with a Probability of Collision of less than 1.0E-06.

This paper will outline the technique that was implemented to establish the safe planning and execution of the fly-by maneuver. The method makes use of Gaussian distribution models of state covariance to determine underlying probabilities of collision that arise under low velocity encounters. Specific numerical examples used for this analysis are discussed in detail. The mechanics of this technique are explained to foster deeper understanding of the concepts presented and to improve existing processes for use in future constellation maneuver planning.

INTRODUCTION

The Space Technology 5 (ST5) mission is a project within the New Millennium Program, designed and built by the National Aeronautics and Space Administration's Goddard Space Flight Center (GSFC) in Greenbelt, Maryland. ST5 was launched aboard a Pegasus XL launch vehicle from the Vandenberg Air Force Base Western Range on March 22, 2006. The mission payload consisted of 3 independent 25 kg spacecraft and the Pegasus Support Structure (PSS). The PSS, mounted to the rocket's 3rd stage, secured the spacecraft during launch and ascent and performed the release sequence to separate the spacecraft from the rocket body. All spacecraft were inserted into orbits that conformed to mission requirements. However, soon after launch the observed relative

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position of the inert rocket body was between the leading and the middle spacecraft within the constellation, instead of moving away from the entire constellation.

ST5 MISSION

The mission was designed for 90 days in duration from initial launch and early orbit checkout. Designed to be a "string-of-pearls" formation within an eccentric orbit regime, the three ST5 spacecraft, dubbed 155, 094, and 224[†], had as their primary mission to demonstrate and validate several small satellite technologies. All three spacecraft were delivered in orbits close to 301 x 4570 km at a sun-synchronous inclination of 105.6°. The science objectives of the original formation plan called for large in-track separations that were designed to allow all three spacecraft to perform in-situ science measurements simultaneously within a region of interest. In particular, the design of the formation was to place one of the lead or trailing spacecraft in a small relative separation along track while the other lead/trail spacecraft would be in a large relative separation. This arrangement was designed to capture science events both inside and outside of a magnetospheric current sheet at high latitudes.

DEPLOYMENT

Figure 1 illustrates the configuration of the PSS, the ST5 spacecraft, and the Pegasus 3rd stage assembly. An artifact of an earlier design meant to accommodate a larger launch vehicle, the release mechanisms were configured on the PSS such that the spacecraft were released normal to the longitudinal axis of the launch vehicle. The launch vehicle was aligned prior to separation so that the spin axis of each spacecraft upon release would be pointed to the north ecliptic pole. This scheme obviated a sun acquisition maneuver intended to preserve the relative formation design. The order of release of the spacecraft was 155, 094, 224. The release sequence was automated in time, keyed off of the on-board determination of the Pegasus Stage 3 Burn Out (S3BO) of the solid motor[1].

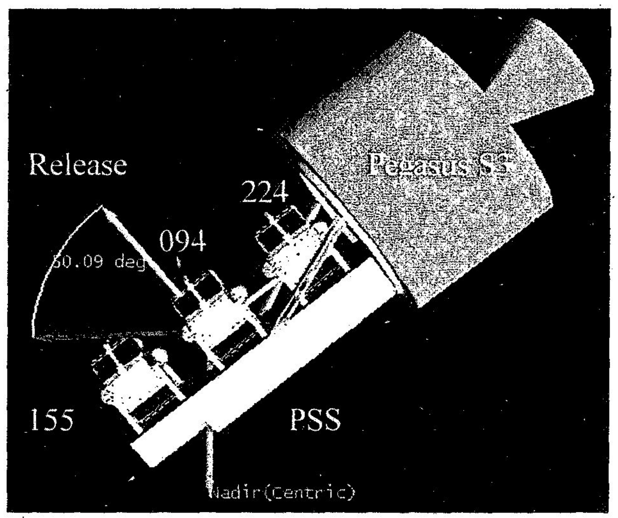


Figure 1 ST5 Stowed Configuration

[†] The naming convention used in the GSFC Mission Operations Center for the ST5 spacecraft. The convention is a decimal conversion of the unique transponder ID's, chosen such that a single event upset resulting in a bit-flip on the ID code transmitted to ground is unambiguous.

The design of the release sequence involved modeling the separation dynamics, including the reaction forces due to the releases and the Pegasus control response to those releases. Spacecraft releases were designed such that each spacecraft would separate passively from each other, preventing close approach due to one spacecraft "catching up" to and crossing paths with a spacecraft with a larger orbit period. Even the spring pre-loads within the release mechanisms were set to maximize the small differences in release delta-v. Nonetheless, the first look at telemetry for each release event immediately following launch determined that the formation order was not as predicted from simulation.

A key assumption for the simulation and analysis of the release dynamics by Orbital Sciences Corporation and GSFC was that the residual thrust present at the beginning of the release sequence was negligible, based on flight data for this motor. Pegasus flight data prior to the ST5 mission had been limited to 200 seconds past the S3BO thrust level determination and all pre-flight analysis predicted a zero thrust condition past S3BO+200 seconds.[2] The post flight report from Orbital determined that the residual thrust profile for the ST5 Pegasus flight extended 600 seconds past S3BO[1].

After the release of the third ST5 spacecraft, the Pegasus 3rd stage performed a Collision and Contamination Avoidance Maneuver (CCAM), which implements a "crab-walk" technique that alternates RCS firings in orthogonal directions such that the separation velocity from the payload is increased. The procedure has two objectives-the depletion of the remaining propellant aboard the 3rd stage and the safe application of delta-v promoting separation from the payload. With respect to the second and third spacecraft releases, the CCAM was successful. Unfortunately the residual thrust present after the first spacecraft release added unanticipated energy to the Pegasus stack. Spacecraft 155 achieved a separation rate from the reference spacecraft 094 that was twice as large as the worst case expected, and the Pegasus CCAM could not overcome it.

Figure 2 shows the relative formation of the ST5 spacecraft on April 6, 2006. The labels are the The NORAD catalog identification numbers correspond to the rise order of the ST5 constellation: 155 (28981), 094 (28980), and 224 (28982). The NORAD object

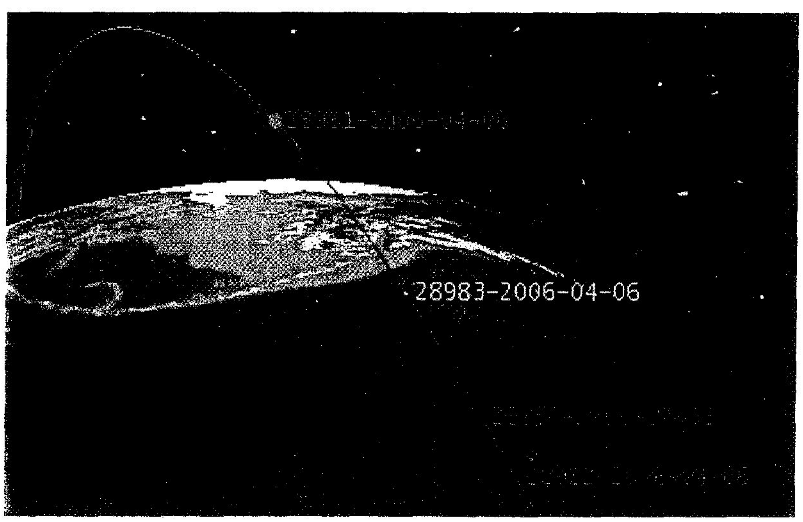


Figure 2
ST5 Constellation + Rocket Body, April 6, 2006

28983 was determined to be the inert rocket body, having a radar cross section area that was an order of magnitude greater than the other bodies. The rocket body and the 155 spacecraft were moving away from the reference spacecraft at a rate of 200 kilometers per day.

COLLISION PROBABILITY METHOD

The literature on collision avoidance provides various methods to consider in calculating the probability of collision, P_C. Chan[3] provides a thorough discussion on collision probability estimation and has defined many standard conventions in use. Patera[4] develops a method for calculating P_C for non-linear relative motion during low velocity encounters. Slater and others[5] provide an excellent study of formation flying collision probability, with emphasis on developing minimum delta-v collision avoidance maneuvers. Carpenter[6] has proposed upper bound approximations of P_C for preliminary design. While many of the methods mentioned introduce an attractive reduction in calculation, the expediency of the maneuver planning led the author to implement the most direct and basic method as outlined in Carpenter and others.

Development of the collision probability prediction began much prior to launch during formulation of the mission, when various formation concepts were traded against one another. Although not in the baseline mission plan, the concept of demonstrating a "fly-by" maneuver of one spacecraft relative to another was foreseen as a possible experiment for an extended mission plan or end of mission plan. This anticipation was fortuitous in fact, when the need to establish the safety of the "fly-by" became suddenly necessary right after launch.

The computation technique generally follows the convention prescribed by Carpenter. The calculation of P_C for any two bodies requires the trajectory, error covariance, and physical hard body radius of each object. At each time step over the encounter period, the error covariances are combined at the inert body, while the hard body radii are combined at the maneuvering body to form a spherical avoidance region. The relative state is calculated and transformed into a local coordinate frame. See Carpenter for more detail.

$$X = X_{inert} - X_{maneuver} \tag{1}$$

By combining the covariance matricies at the inert body, a probability density distribution can be characterized at its center. For combined error covariance matrix, P, and relative state vector, X, the probability density function may be expressed as:

$$pdf(X;t) = \frac{1}{\sqrt{(2\pi)^3|P|}} e^{-\frac{1}{2}X^T P^{-1} X}, \tag{2}$$

where $|P|$ denotes the determinant of combined covariance matrix P. The spherical volume, V, of the avoidance region can be integrated directly within the three dimensional probability density distribution to calculate a probability mass corresponding to the probability of collision, P_C, at time t.

$$P_C = \iiint_V pdf(X;t)dxdydz \tag{3}$$

This process is illustrated by the flowchart in Figure 3. X1, X2, X3 in Figure 3 represent the state information. P1, P2, P3 in Figure 3 represent the covariance information used in the process.

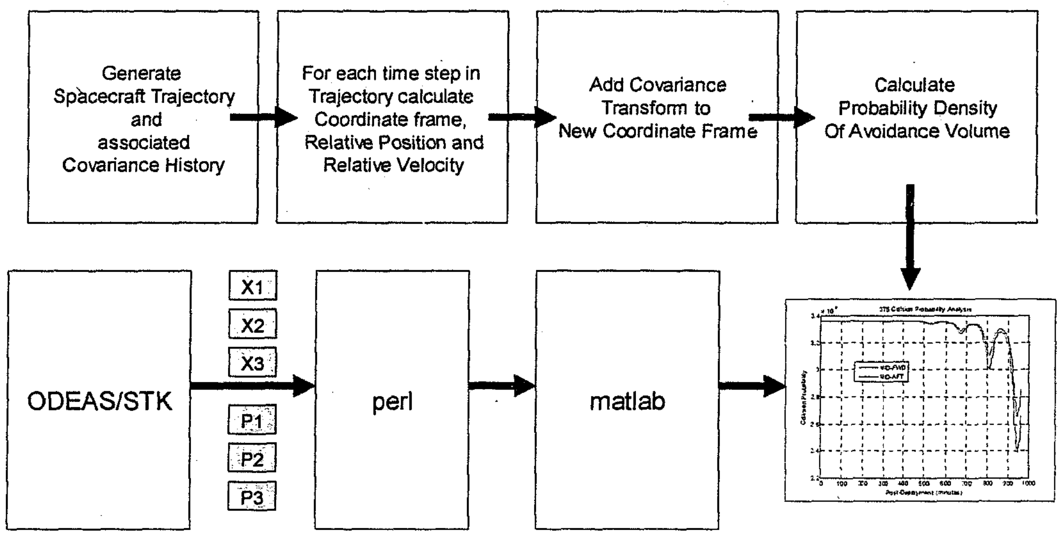


Figure 3 Process for Calculating P_c

The calculation of collision probability is performed at small time steps over the determined encounter period. The typical ST5 calculation used a time step of 5 seconds over a 12 hour encounter period, for 8,640 time step evaluations. The rationale for such a large step count was to cover multiple close approach encounters over the course of several orbit periods. This was due mostly to the method for determining the encounter period.

While the trajectories of the two bodies provide a time of closest approach (TCA), the uncertainty associated with error covariance make the TCA non-deterministic. Thus, establishing an encounter period as a span of time about the nominal TCA is necessary. For this analysis, the determination of encounter period was a very simple process. The latest orbit determination solutions provided the initial states for propagation. An initial covariance was input. Then both the trajectories and covariance were propagated forward in the maneuvering spacecraft frame. Visualization of a 0.95 probability covariance ellipsoid was enabled using STK/VO. In this manner the predictive orientation and shape of the 0.95 probability ellipsoid could be inspected to the time when it intersected the maneuvering body. The encounter period was set to begin at this epoch. Figures 4, 5, and 6 illustrate the rotating and expanding covariance ellipsoid as it approaches the avoidance region at the center of the picture. In the figures below, the maneuvering spacecraft, 155, is in the center of the relative motion path. Each loop in the relative path represents one orbit revolution. The covariance ellipsoid is centered on the inert rocket body. The time between Figure 4 and Figure 6 represent 1.5 orbit revolutions.

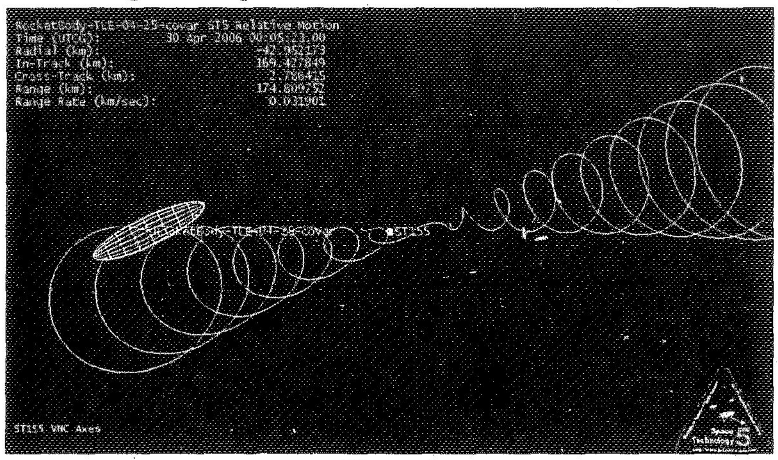


Figure 4
Encounter with Rocket Body 00:05:23

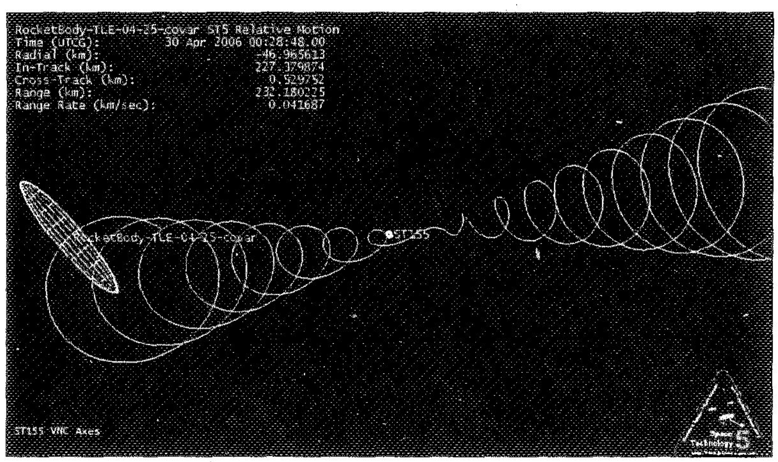


Figure 5
Encounter with Rocket Body 00:28:48

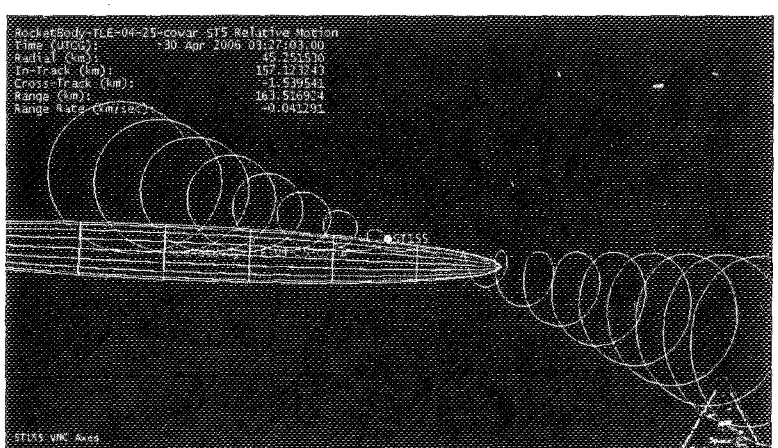


Figure 6
Encounter with Rocket Body 03:27:03

The estimation of the covariance input for this method came from a comparison of the NORAD two line element (TLE) set to the orbit determination solutions produced by the Flight Dynamics Facility at GSFC for the 155 spacecraft. An assumption was made that the accuracy of the rocket body TLE was comparable to that of the ST5 TLE, as described below.

A 155 TLE was propagated using the SGP4 propagator and compared to a definitive ephemeris for the 155 spacecraft[‡] over the same time span. For a four day predictive comparison, the maximum error from the definitive ephemeris was cataloged. This was done for 4 separate epoch TLE and definitive ephemeris comparison sets. From these values the maximum difference component observed was assigned a value of 2σ, as an estimate of the confidence of knowing the covariance from 4 comparative samples. The results are listed in Table 1. The process was repeated for predictive 3, 2, and 1 day propagations in anticipation of needing various prediction spans prior to the close approach. For modeling the maneuvers, the initial conditions also added velocity covariance term values σ^2 = 1.0E-9 km^2/sec^2. All cross terms were set to zero for the initial covariance matrix. Thus, the a-priori error covariance matrix of the rocket body in the radial, along track, cross-track frame, km^2, km^2/sec^2 was determined to be

$$P(t_0) = \begin{array}{c} \begin{array}{cccccc} R & C & L & VR & VC & VL \end{array} \\ \begin{bmatrix} 12.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 256 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0E-9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0E-9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0E-9 \end{bmatrix} \end{array}. \quad (4)$$

Table 1 Comparison of 155 OD Solution to TLE

155 Definitive Ephem	Max Diff from TLE Radial (km)	Max Diff from TLE Cross-track (km)	Max Diff from TLE Along-Track (km)
092-096	2.2	1.5	6.0
093-097	5.0	1.8	18.0
094-098	7.0	2.0	32.0
096-100	5.0	1.7	21.0
MAX = 2σ	**7.0**	**2.0**	**32.0**
σ^2 (km^2)	**12.3**	**1.0**	**256**

‡ At the time the 155 spacecraft had acquired the most tracking passes and had the best definitive orbit determination solution.

ROCKET BODY FLYBY

One of the mission requirements levied on the separation scheme and the maneuver planning for formation operations was that the P_C could not exceed 1.0 E-06. This level of safety was established in response to project concerns of what constituted "far away" or "too close" for formation design. In order to establish the formation as originally planned the 155 spacecraft would need to "fly by" the rocket body, while also adhering to the P_C limit. Planning began in earnest on April 10, 2006 in the ST5 Mission Operations Center.

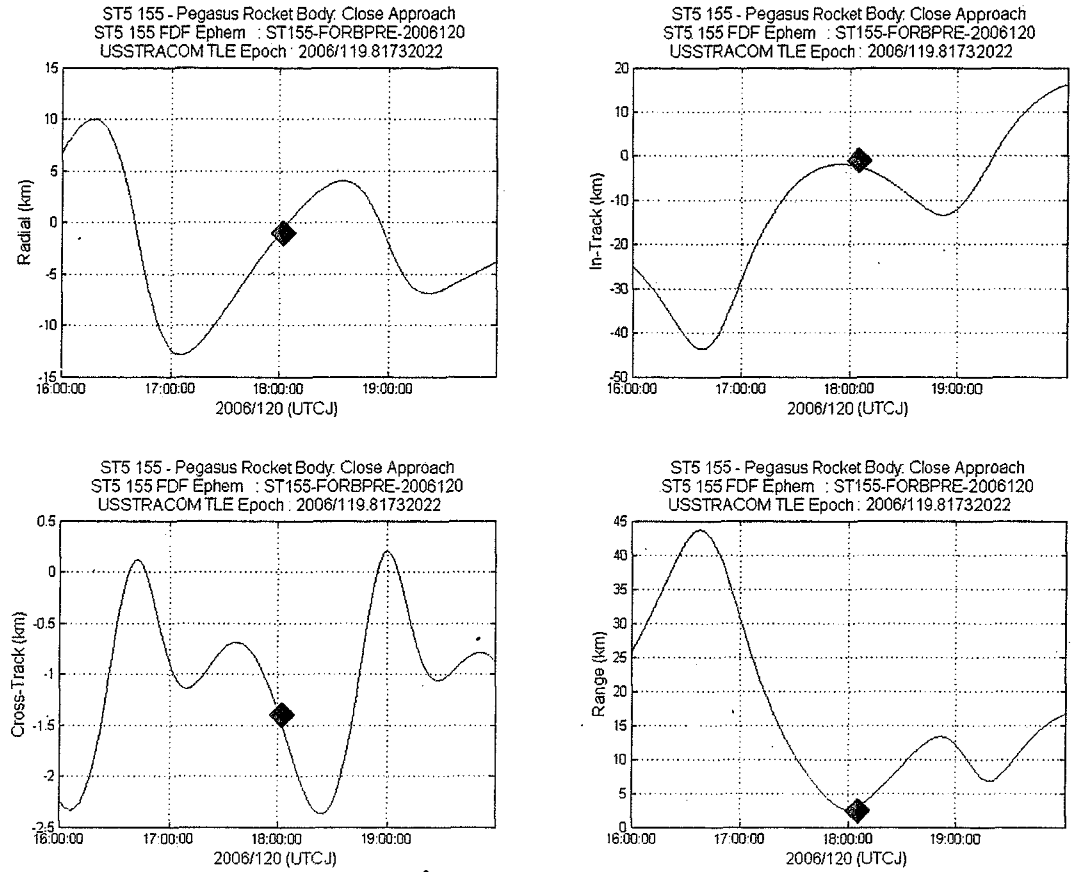
The orbit phasing maneuver performed by the 155 spacecraft, 155 ORB2, was performed on Friday April 21, 2006 at 15:22:42 UTC. The delta-v imparted was 0.692 m/s[§]. The TCA was predicted to be April 30, 2006 at 15:40 UTC. The time between planning of the maneuver and the encounter period was 9 days. It was soon discovered that the propagated covariance would have grown to such a size as to render a P_C estimate useless. Thus, the strategy to execute the fly-by would be to perform the maneuver, and monitor and assess the fly-by conditions as TCA grew near.

After the maneuver was executed, the post maneuver motion was monitored beginning at 4 days prior to TCA. It should be noted that the large covariance growth expected also affected the predictive accuracy of the TCA. Approximately 1 day before the expected TCA, the prediction changed by 137 minutes, a shift of one orbit period. At one week from TCA, the close approach estimate would change by up to 2 km per update. After 4 days from TCA, the close approach converged and did not change more than 1 km per update. The definitive estimate of the close approach distance was 2.55 km. The definitive TCA was estimated to be April 30, 2006 at 18:00:45 UTC. Figure 7 plots the definitive relative motion of the encounter.

The computed collision probabilities were negligible, as shown in Figure 8, where P_C=0 for the encounter period. A safety factor of 10 was applied to the combined body radius for an overall avoidance radius of 0.11 km. The close approach distance was determined to be 2.55 km. The a-priori spacecraft covariance P_{155} was estimated at 4 days before the encounter. The covariance was determined to be

$$P_{155}(t_0) = \begin{bmatrix} 1.47E-3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.70E-3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.96E-2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0E-9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0E-9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0E-9 \end{bmatrix}. \tag{5}$$

[§] ST5 spacecraft did not slew the spin axis prior to performing an orbit maneuver, therefore the delta-v along the velocity direction was only a fraction of the total delta-v.



= Close Approach

Figure 7
Rocket Body Fly-by Relative Motion

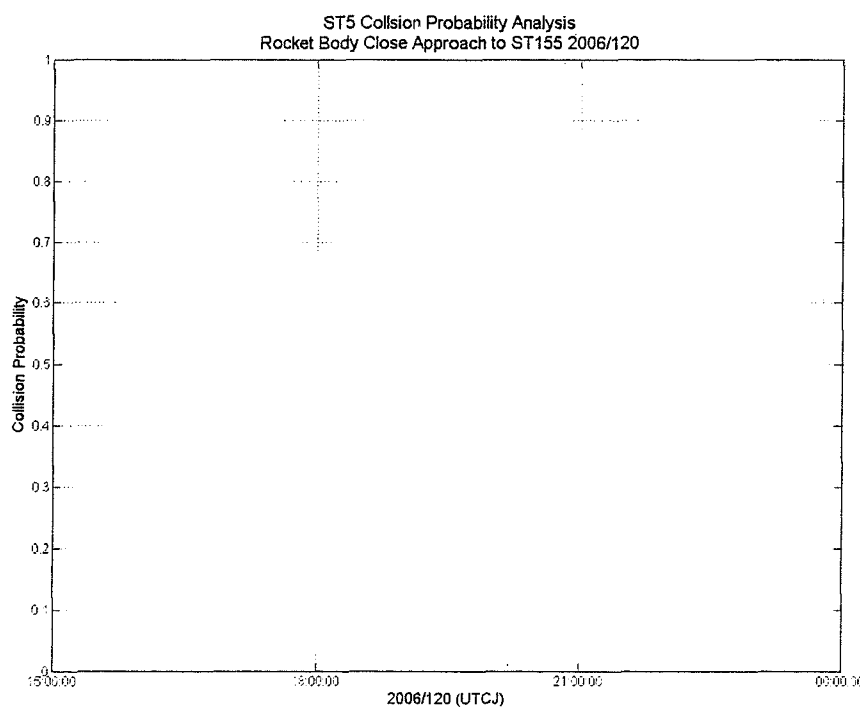


Figure 8
Rocket Body Fly-by Collision Probability

END OF MISSION MANEUVERS

Spacecraft 155 successfully passed the rocket body on April 30, 2006. While the encounter calculation of P_C gave no insight as to how sensitively the maneuver planning could be affected by it, a second opportunity to perform the analysis presented itself for the end of mission plan. Within the ST5 constellation, the middle spacecraft, 094, was used as reference for the along track separation distances that defined the formation. The lead spacecraft performed the bulk of the maneuvers to establish the formation. As a result, at the end of the mission, the other spacecraft had the largest unused delta-v capacity in need of depletion. This presented a challenge to the operations team, faced with inducing another fly-by condition or breaking up the end of mission plan into many small maneuvers and monitoring the formation. The decision was made to induce as fast a fly-by as possible, to decrease the encounter time and to decrease the collision risk.

As part of its decommissioning, spacecraft 224 performed a series of maneuvers which increased the cross-track and radial separation relative to 094 and 155, induced a large along-track separation rate, and depleted the tanks while lowering perigee. Spacecraft 094 performed a series of maneuvers designed to increase radial and cross-track separation relative to 155, and deplete the tanks while lowering perigee. Spacecraft 094 end of mission maneuvers did not induce an encounter with either 155 or 224 by design.

For the analysis of the 224 fly-by of 094 and 155, the same procedures were followed as the rocket body fly-by. The avoidance region had a combined radius of 20 meters. In applying the same initial covariance as used in the rocket body fly-by, the following maximum Pc values were calculated for the 224 fly-bys of 155 and 094:

Table 2 Fly-by Results 224 End of Mission Maneuvers

	224-094 Fly-by	224-155 Fly-by
Maximum P_C	3.8E-8	2.6E-7
TCA (UTC)	June 26, 2006 11:52:30	June 26, 2006 17:09:55
Close Approach (km)	9.1	5.5

Figures 9 and 10 show the P_C history over the encounter period and the relative range history for the 224-094 fly-by. Figures 11 and 12 show the P_C history over the encounter period and the relative range history for the 224-155 fly-by. The ST5 constellation was successfully decommissioned on June 30, 2006. The last tracking and telemetry was received from the 155 spacecraft on June 30, 2006 at 17:30 UTC. All three spacecraft were operating nominally and no evidence of collision was detected throughout the mission.

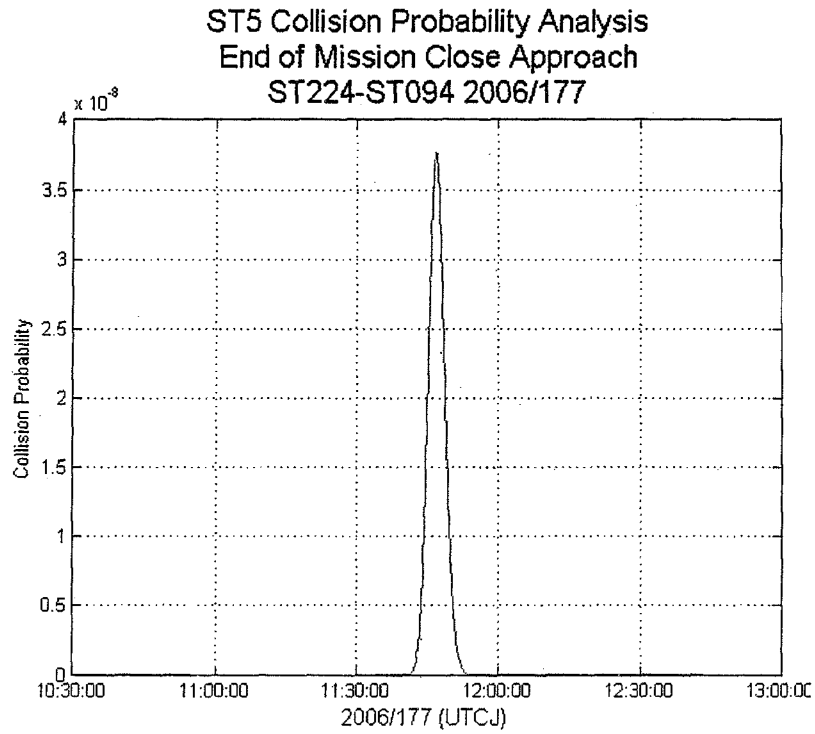


Figure 9
224 Fly-by 094 Collision Probability

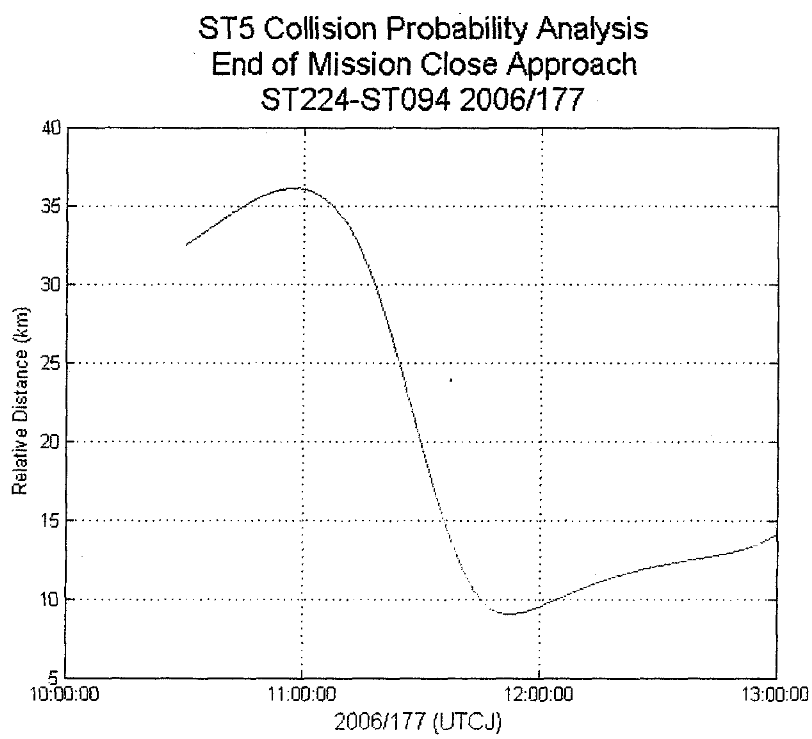


Figure 10
224 Fly-by 094 Relative Distance

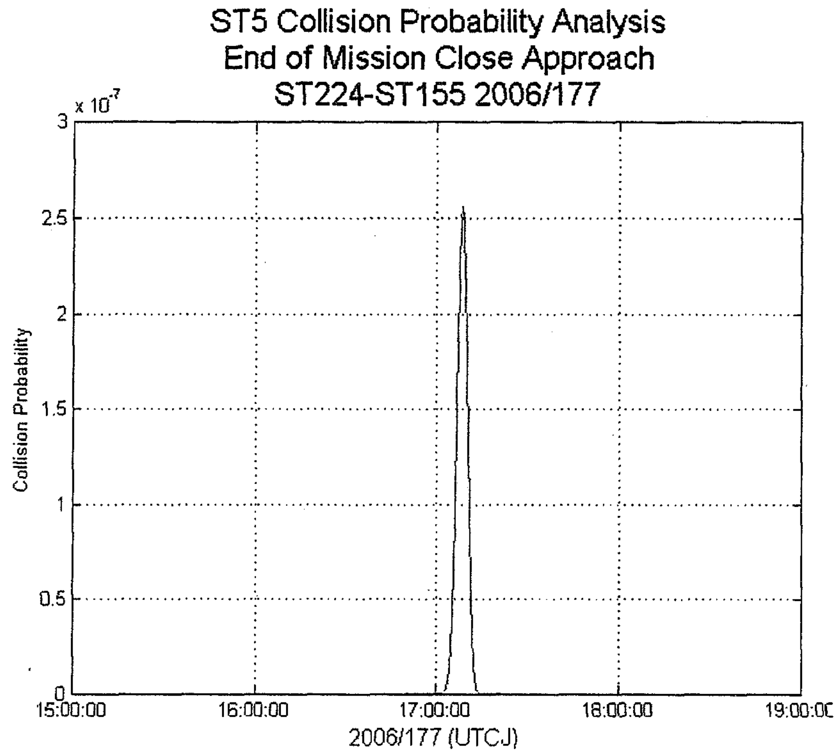


Figure 11
224 Fly-by 155 Collision Probability

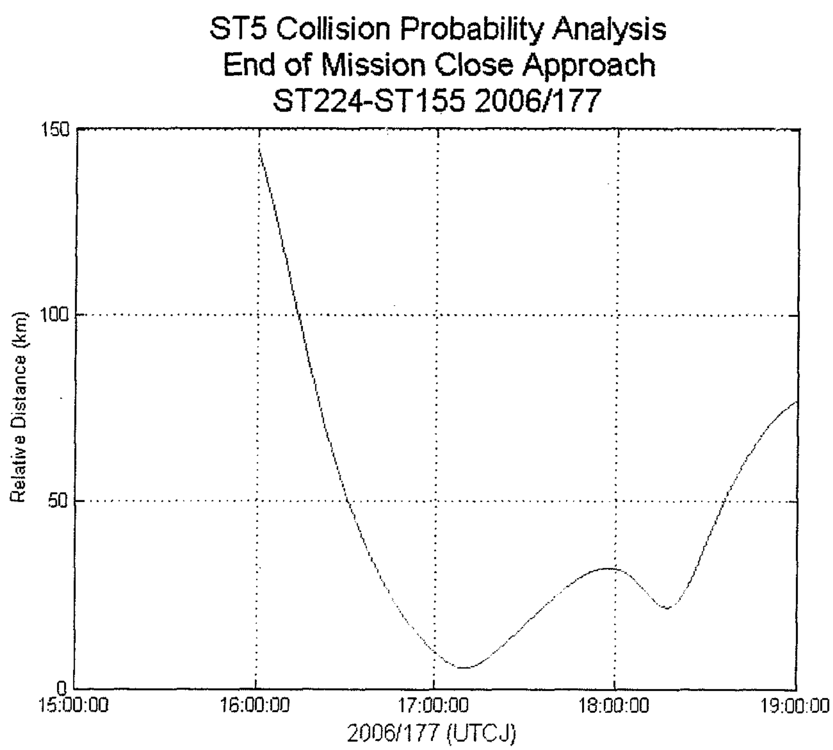


Figure 12
224 Fly-by 094 Relative Distance

CONCLUSIONS

The techniques outlined in this paper have demonstrated how a calculation of collision probability can be performed in an operational setting to determine the safety of a maneuver in a constellation formation. ST5 spacecraft successfully performed 3 fly-by maneuvers of bodies in close formation, two times with other operational spacecraft, and once with an inert rocket body. Further investigation can lead to more computationally efficient methods to implement in support of operational missions. Undoubtedly, improved understanding of the initial conditions input into the calculation would provide better insight in maneuver planning for collision avoidance.

ACKNOWLEDGEMENTS

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