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ON THE DETERMINATION OF POISSON STATISTICS FOR HAYSTACK RADAR OBSERVATIONS OF ORBITAL DEBRIS

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ABSTRACT

A convenient and powerful method is used to determine if radar detections of orbital debris are observed according to Poisson statistics. This is done by analyzing the time interval between detection events. For Poisson statistics, the probability distribution of the time interval between events is shown to be an exponential distribution. This distribution is a special case of the Erlang distribution that is used in estimating traffic loads on telecommunication networks. Poisson statistics form the basis of many orbital debris models but the statistical basis of these models has not been clearly demonstrated empirically until now. Interestingly, during the fiscal year 2003 observations with the Haystack radar in a fixed staring mode, there are no statistically significant deviations observed from that expected with Poisson statistics, either independent or dependent of altitude or inclination. One would potentially expect some significant clustering of events in time as a result of satellite breakups, but the presence of Poisson statistics indicates that such debris disperse rapidly with respect to Haystack's very narrow radar beam. An exception to Poisson statistics is observed in the months following the intentional breakup of the Fengyun satellite in January 2007.

INTRODUCTION

Poisson statistics form the basis of the statistical mechanics of several orbital debris models.[1,2] Orbital debris models are critical for assessing the risk to national and international space assets. Specifically, orbital debris models are used for risk analysis of space operations, shielding design of spacecraft for protection from impacts with space debris, debris mitigation studies and policies, and long-term projections for future population growth of space debris. However, the orbital debris environment is continually evolving since there are new debris sources and debris loss mechanisms that are dependent on the dynamic space environment.[3] Radar observations by NASA using the Massachusetts Institute of Technology Lincoln Laboratory (MIT/LL) Long Range Imaging Radar indicate significant clustering of debris in altitude and inclination. It is therefore unclear whether radar observations of debris obey Poisson statistics or if debris pass through the radar in distinct swarms, *i.e.*, if event observations cluster in time. It is also unclear how quickly a debris population randomizes following a satellite breakup such that radar observations of debris clustering are possible relative to the altitude- and inclination-dependent background debris population.

The determination of the type of statistics that govern a system is crucial for an understanding of the underlying statistical and physical mechanisms. Many systems are assumed to obey Poisson statistics and this is correct if the event rate is low and the events are uncorrelated. Poisson statistics describe a vast array of phenomena among various branches of diverse fields such as physics (particularly nuclear physics), engineering, finance, social sciences, biology, and botany. Poisson statistics are also used in disparate, yet important, fields of engineering such as modeling traffic loads on telecommunication networks. The statistics of telephone traffic engineering and queuing theory was pioneered by A.K. Erlang and his analysis is frequently used in the field of stochastic processes.[4] The statistical techniques commonly used in telecommunication traffic engineering are directly applicable to the statistics of orbital debris detections. The key to this analysis is the wait time between occurrences of telephone call events or in this case, the time between debris detections. The time interval between events provides a convenient and powerful method for determining if an event obeys Poisson statistics. The analysis is considerably simplified if the event durations are very small compared to the average time interval between events. As will be shown, this simplification is possible for radar observations of orbital debris.

Statistically significant deviations from Poisson statistics indicate many possible scenarios, none of which are mutually exclusive. A non-Poisson process could indicate that the system may be far from equilibrium, that some physical correlation between events exist, or that there is an underlying non-Poisson statistical mechanism. Such deviations have been observed in several notable discoveries in astronomy, nuclear physics, and atomic physics.[5, 6]

This paper will start with a discussion of the analytic expression for the probability density distribution of time intervals for a Poisson process. Following this, several fitting methods will be investigated. A description of the radar observations and the data collection methodology will be given. Lastly, an analysis of the time interval distributions for orbital debris will be examined.

ANALYTIC EXPRESSION FOR THE TIME INTERVAL BETWEEN EVENTS FOR A POISSON PROCESS

To determine if a process obeys the simple Poisson statistics model, consider the time interval between subsequent events. If m is the mean number of counts observed in an interval, then the probability of n counts in the interval is

$$p_m(n) = e^{-m}\frac{m^n}{n!} \qquad [1]$$

In particular, the probability of observing no detections in the interval is

$$p_m(0) = e^{-m} \qquad [2]$$

Introducing the average event rate μ, where

$$m = \mu t \qquad [3]$$

then the probability that no counts are observed in a time interval t is

$$p_{\mu t}(0) = e^{-\mu t} \qquad [4]$$

The probability that at least one count is observed in Δt is

$$\Delta p = \left(1 - p_{\mu\Delta t}(0)\right) = \left(1 - e^{-\mu\Delta t}\right) \approx \mu\Delta t \qquad [5]$$

Therefore the probability of no counts in interval t and at least one count in Δt is then

$$f(t)\Delta t = p_{\mu t}(0)\left(1 - p_{\mu\Delta t}(0)\right) = e^{-\mu t}(1 - e^{-\mu\Delta t}) \qquad [6]$$

The normalized interval probability distribution then reduces to [6]

$$f(t)dt = \mu e^{-\mu t}dt \qquad [7]$$

For comparison, the Erlang distribution is

$$f(t)dt = \frac{\mu^k t^{k-1} e^{-\mu t}}{(k-1)!} dt \qquad [8]$$

which equates to Equation 2 for k=1. The Erlang distribution is a special case of the Gamma probability density function. The Gamma function plays a fundamental role in probability and statistics.

The rate term μ may be estimated by the technique of maximum likelihood. The joint probability is

$$P = \prod_{i=1}^{N_0} \mu e^{-\mu t_i} \qquad [9]$$

where N_0 is the total number of counts.

The log-likelihood is then

$$L = \sum_{i=1}^{N_0} \ln(\mu e^{-\mu t_i}) = N_0 \ln(\mu) - \sum_{i=1}^{N_0} \mu t_i \qquad [10]$$

Maximizing the log-likelihood results in

$$\mu = \frac{N_0}{\sum_{i=1}^{N_0} t_i} \equiv \frac{N_0}{T_0} \qquad [11]$$

This is equivalent to considering the mean time $\bar{t}$ between events to estimate μ.

$$\bar{t} = \int_0^\infty \mu t e^{-\mu t} dt = \frac{1}{\mu} \qquad [12]$$

Histograms provide a convenient method for visualizing and analyzing the time interval data. Excessive histogram counts above the predicted distribution of counts for the smallest time intervals can be an indicator of debris swarms.

This outlier data can significantly affect the estimate of the rate μ. The rate of a Poisson-like background μ is desired since deviations from the background rate determine how numerous the debris swarm is. Equation 10 is not appropriate to use when outliers have been systematically removed. The method of maximum likelihood in equation 9 also fails when outlier time bins have been excluded. For example, consider removing the first bin. The log likelihood reduces μ to

$$\mu = \frac{N_0 - N}{T_0 - T} \qquad [13]$$

where N is the number of counts in the excluded bin and T is the time corresponding to the first bin. Since N is very large and T is small, the predicted rate μ drops significantly. The same issue occurs for any systematic exclusion of data.

A chi-squared minimization of data to an exponential distribution provides a convenient method to fit the data when outlier bins need to be excluded.

$$\chi^2 = \sum_{i=1}^{N_{bins}} \frac{(x_i - \Delta N_i)^2}{\sigma_i^2} \qquad [14]$$

Where

$$x_i = N_o \mu e^{-\mu t_i} \Delta t \qquad [15]$$

$$t_i = i \Delta t \qquad [16]$$

$$\sigma_i = \sqrt{\Delta N_i} \qquad [17]$$

The term N_{bins} is the total number of bins. As shown in Figure 1, the width of the time bins is Δt. The term ΔN_i is the numbers of counts in bin i which corresponds to time intervals between t and $t + \Delta t$.

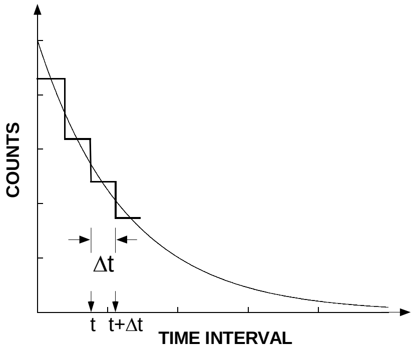

Fig. 1. A histogram of the distribution of time intervals between orbital debris observations and an exponential fit of the histogram. A Poisson process is predicted to result in an exponential distribution.

DESCRIPTION OF THE ORBITAL DEBRIS DATA COLLECTION

To minimize the effects of lurking statistical variables, the statistical analysis of any data must be combined with an analysis of the detectors and the methodology of the data collection. Orbital debris is often observed by statistical sampling. Specifically, the radar is operated in a staring, or "beam park," mode in which the antenna is pointed at a chosen elevation and azimuth and remains there while debris objects randomly pass through the field-of-view. This operational mode provides a fixed detection volume important to the measurement of the debris flux, or number of objects detected per unit area per unit time.

NASA's primary source of debris data is from the MIT/LL Long Range Imaging Radar, frequently referred to as Haystack. NASA annually obtains anywhere from 500 to 900 hours of debris observation time by frequent periodic sampling. Haystack is a monopulse tracking radar with very high sensitivity. A monopulse radar can determine the position of the detection within the beam. This allows an accurate average radar cross section to be calculated as well as determining the path through the radar beam. Powerful and highly sensitive radars are required for orbital debris observations since the observation time for a single debris piece is generally limited to less than 1 second and most debris is small (<10 cm), returning signals with low signal-to-noise ratio (SNR). To detect debris, a pulsed continuous wave single frequency waveform is used. The Haystack radar consists of a 36.6 m parabolic dish operating in a Cassegrain configuration. Its (one-way) half-power beamwidth is 0.058 degrees (209 arcseconds), very narrow compared to most radars. A detailed description of the radar is given in reference 7.

The Haystack radar count rate is typically approximately 10 counts per hour for debris 5 mm and larger in low Earth orbit. It is important to note that the average time between detections is typically 300 to 400 seconds and the duration of each detection is typically about 0.5 seconds.

The radar data acquisition system has been programmed to record data in a buffer that is saved only when the integrated signal exceeds a predetermined threshold above system noise. This is done so that debris observations can be performed without using an impractical amount of recording medium. When recording data, the recording threshold is set intentionally lower than allowed in subsequent processing to ensure that no usable data are missed. Moreover, several pulses before and after a declared detection event are recorded to ensure any useful data are not missed. Generally, time blocks of recorded data have a duration from 15 to 90 minutes with potentially significant time gaps between data recording sessions.

Since at least two events are required for the time-between-events statistical technique, windowing on a particular region of altitude and inclination significantly reduces the available events for the Poisson analysis. This is particularly true since each time block may only have about 10 detections. This could be alleviated somewhat by improving the statistical analysis by incorporating the time between the start of data recording (which is essentially random) and the time to an event to improve the statistics.

RESULTS

Shown in Figure 2 is the altitude versus Doppler inclination for orbital debris measured by

Haystack for fiscal year 2003.[6] Orbital inclination is estimated from range rate information by assuming circular orbits. This is referred to as the Doppler inclination. Depending on where in the orbit an object is detected, the Doppler inclination may be invalid for objects with appreciable eccentricity. While it is impossible to say whether an individual detection is in a particular family because of the uncertainty in its eccentricity, the data show detections clearly clustering in altitude and inclination. The circular orbit assumption is reasonable since many satellites between 300 km to 1500 km are launched into near circular orbits.

Inclination measurements are restricted to approximately 40 to 140 degrees since the radar is pointed East at an elevation of 75 degrees and the radar is located at 42.6 degrees northern latitude. There are several debris families evident in the figure that roughly represent regions of active satellites. This is known by comparing debris populations with catalogued satellites.

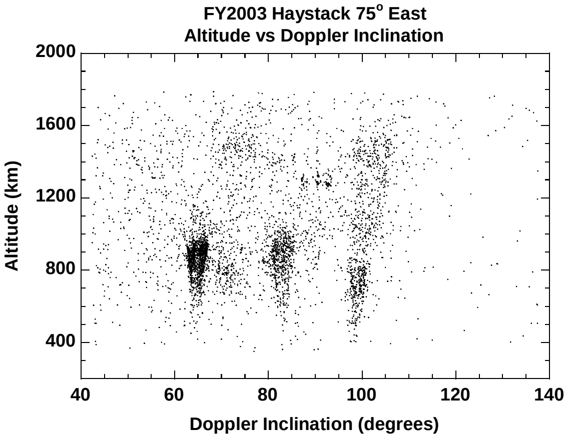

Fig. 2. The altitude versus Doppler inclination for all debris.

A histogram of the time interval between detections for all the inclinations and altitude available to the radar during observations is shown in Figure 3. An exponential fit of the data indicates a near perfect agreement. Three sigma error bars on the fit are included. There are no statistically significant variations of the data outside of the estimated three sigma errors.

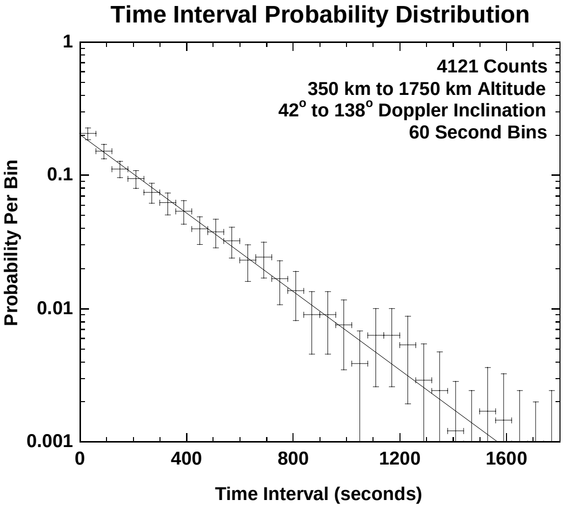

Fig. 3. The normalized probability per 60-second time interval bin without restricting detections in altitude or Doppler inclination.

Now consider the time interval distributions of debris detections for three distinct debris "families." Even though these debris families represent many individual breakups, it is a priori unknown if each family as a whole behaves according to Poisson statistics. The first debris family under consideration has an altitude between 700 to 1000 km and a Doppler inclination between 62 to 68 degrees. Debris family two has an altitude between 700 to 1050 km and a Doppler inclination between 80 to 86 degrees. Debris family three has an altitude between 600 to 1200 km and a Doppler inclination between 96 to 104 degrees.

The Poisson rates among the three families are different. This is to be expected since the altitude and range window is different for each family. Additionally, each family has a different population quantity. From the exponential fits of the three regions, as shown in Figures 4, 5, and 6, the debris environment appears to obey Poisson statistics and there are no statistically significant debris clusters observed. The final results are not very sensitive to the time interval bin size. A debris cloud would be observed as excessive counts for small time intervals in the histogram. The presence of Poisson statistics in this analysis does not preclude the existence of debris clouds. Poor statistics may make debris clouds unobservable. Additionally, debris clouds may be difficult to discern from a significant background debris population that has already dispersed to the point of appearing Poisson-like.

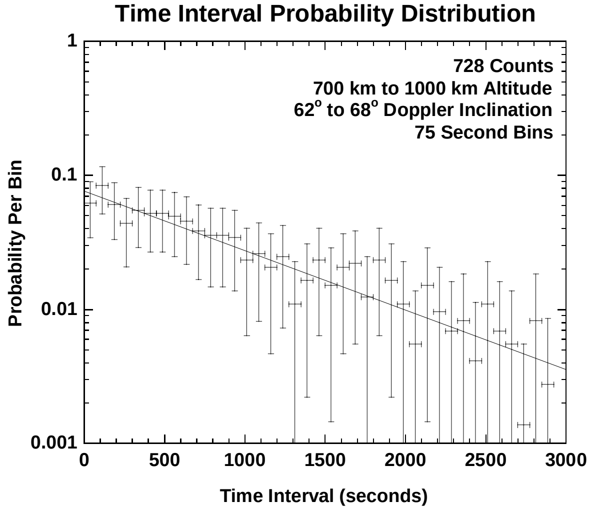

Fig. 4. The normalized probability per 75-second time interval bin for detections between 700 to 1000 km and Doppler inclination between 62 and 68 degrees.

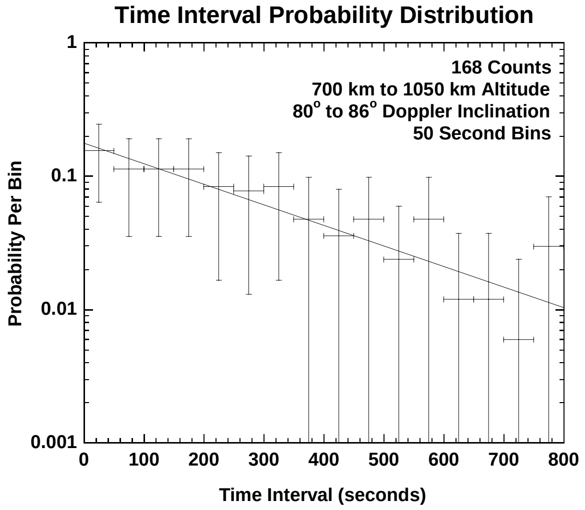

Fig. 5. The normalized probability per 50-second time interval bin for detections between 700 to 1050 km and Doppler inclination between 80 and 86 degrees.

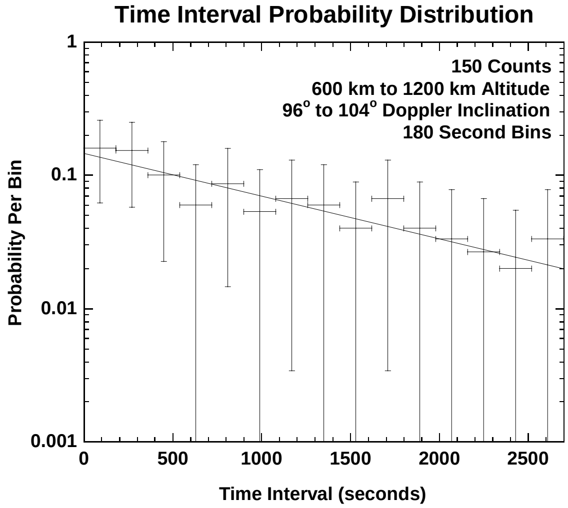

Fig. 6. The normalized probability per 75-second time interval bin for detections between 600 to 1200 km and Doppler inclination between 96 and 104 degrees.

The intentional break-up of the Fengyun-1C spacecraft on 11 January 2007 provides a unique opportunity to observe a significant debris swarm. This satellite was in a nearly circular, sun-synchronous orbit with a mean altitude of approximately 850 km and an inclination of 98.8 degrees. This break-up was caused by a hypervelocity collision with a ballistic object. It created the most severe artificial debris cloud in Earth orbit since the beginning of space exploration. More than 2000 debris on the order of 10 cm or greater in size have been identified by the U.S. Space Surveillance Network.

The Haystack radar collected significant amounts of radar staring debris data in the months following the breakup. Even without making cuts on altitude and inclination, the time interval Poisson analysis indicates swarms of debris are occurring at the short time intervals as shown in Figure 7. The fit was done by excluding the first two time bins. The mean of the data in the first bin is 14 standard deviations from the fit. The results are statistically significant even when the fit is done without any data exclusion. The presence of non-Poisson statistics can significantly complicate the risk analysis associated with space operations and the protection of space assets.

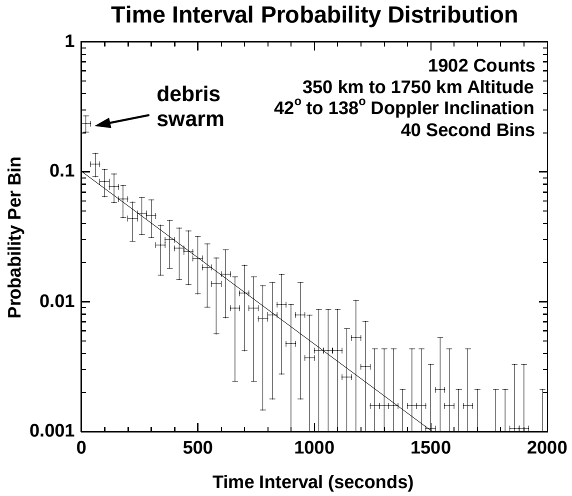

Fig. 7. The normalized probability per 75-second time interval bin for detections between 600 to 1200 km and Doppler inclination between 96 and 104 degrees.

CONCLUDING REMARKS

The time interval between debris detections is a convenient technique to determine if Poisson statistics apply to radar detections of orbital debris. The derived time interval distribution is a falling exponential curve with increasing time interval. The Haystack fiscal year 2003 radar data indicate that the debris population appears Poisson-like to the radar independent of altitude and inclination. One would expect debris rings to be observed for small windows of altitude and inclination, but no statistically significant deviations from Poisson-like behavior are observed. The debris environment appears to rapidly disperse. Debris rings may also be shrouded by a much larger population of already randomized debris.

Poor statistics of some of the data reveal that the changes to the statistical techniques and increasing the data collection durations should be investigated. Since the count rate is approximately 10 counts per hour and typically less than one hour of data is recorded (with a potentially significant pause before resuming recording), very few counts appear in the same region of altitude and inclination during a data recording session. For analyzing regions of similar altitude and inclination, at least two events are required for the time-between-events statistical technique. A statistical analysis that incorporates the time between the start of data recording and the time to an event could significantly improve the statistics since only one event is needed.

Analysis of the fiscal year 2007 radar data in the months following the Fengyun breakup indicate the presence of a significant debris ring. It will be interesting to observe how long it takes for this ring to randomize to the point of being indistinguishable from the background debris population. The occurrence of the intentional Fengyun breakup and the continuing presence of unintentional satellite breakups indicate the debris environment is dynamic and can change rapidly. Therefore, continued monitoring, or at least, frequent, periodic sampling of the debris environment to sizes below 1 cm should be continued.

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