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Probabilistic Methods for Structural Reliability and Risk

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Abstract

A formal method is described to quantify structural reliability and risk in the presence of a multitude of uncertainties. The method is based on the materials behavior level where primitive variables with their respective scatters are used to describe that behavior. Computational simulation is then used to propagate those uncertainties to the structural scale where reliability and risk are usually specified. A sample case is described to illustrate the effectiveness, versatility, and maturity of the method. Typical results from this method demonstrate that the method is mature and that it can be used for future strategic projections and planning to assure better, cheaper, faster products for competitive advantages in world markets. The results also indicate that the methods are suitable for predicting remaining life in aging or deteriorating structures.

Introduction

The pursuit of achieving and retaining competitive advantages in world markets necessarily leads to proactive drives for better-cheaper-faster products to market. This becomes even more important in the high-tech sector which includes aerospace vehicles. The awareness for conservation of natural resources also leads proactively to the reliable cost effective useful life extension of existing products. In the first case, it requires very effective use of available resources. In the second case, it requires formal methods to quantify the current strength of a specific structure/component and subsequent reliable evaluation of respective remaining strength. Both cases include a multitude of uncertainties. The first case includes uncertainties in new unproven methods for design and/or manufacturing as well as uncertainties associated with lack of sufficient data of new potential material such as composites. The second case is full of uncertainties from (1) unknown assumptions and conditions of the initial design, (2) records of environmental exposure, and (3) material degradation of various factors, etc.

In scenarios of a multitude of uncertainties described above, probabilistic methods offer formal approaches to quantify those uncertainties and their subsequent effects on material behavior, on service and on attendant reliabilities and risks. The objective of this paper is to describe one probabilistic method for evaluating structural reliability and risk from material behavior to service life. That probabilistic method (1) is based on formulations that describe the physics in terms of primitive variables and respective scatter ranges, at the lowest engineering manageable scale and (2) relies on computational simulation methods to propagate those uncertainties from that elementary through all intermediate scales where metrics for structural reliability and risk are specified. The method has evolved over the past twenty years and has matured sufficiently to evaluate structural reliability and risk under various scenarios (refs. 1 to 3). The method has several unique features. The two that are most useful to present results are: (1) quantifiable reliability in terms of cumulative distribution functions and (2) sensitivity factors of the primitive variables that affect that reliability. The article first introduces the fundamental concept and computational simulation method by a simple example. This is next followed by brief description of the method and its attendant computer codes. Subsequently, one case is discussed to: (1) illustrate the versatility of the method, (2) the large amounts of information it generates, (3) the relevance of the information, and (4) recommendations that are readily inferred for design, material quality development, certification, in-service health monitoring, retirement for cause and even recycling. The description is limited to typical results obtained and their respective interpretation. References are cited for specificities.

Fundamental Concept

It is instructive to describe some fundamental concepts of probabilistic structural analysis/design by using a simple example. The simple example for that purpose is the probabilistic evaluation of the tip displacement of cantilever beam loaded at the free end as shown schematically at the top left of figure 1. The equation (deterministic—model) for predicting the tip displacement is shown under the schematic. This equation describes the physics of the response sought and includes the fundamental parameters (primitive variables—each variable requires an independent experiment) that govern the tip displacement. For example, P is the load, l is the length, E is the material stiffness, b is the width and t is the thickness. These primitive variables can also be grouped in three generic categories load (P), geometry (l, b, and t), and material (E). Experience has shown that if we make several cantilever specimens there will be a scatter of values for each of the primitive variables. The task, therefore, of probabilistic simulation is to account for the effects of that scatter on the displacement of the beam.

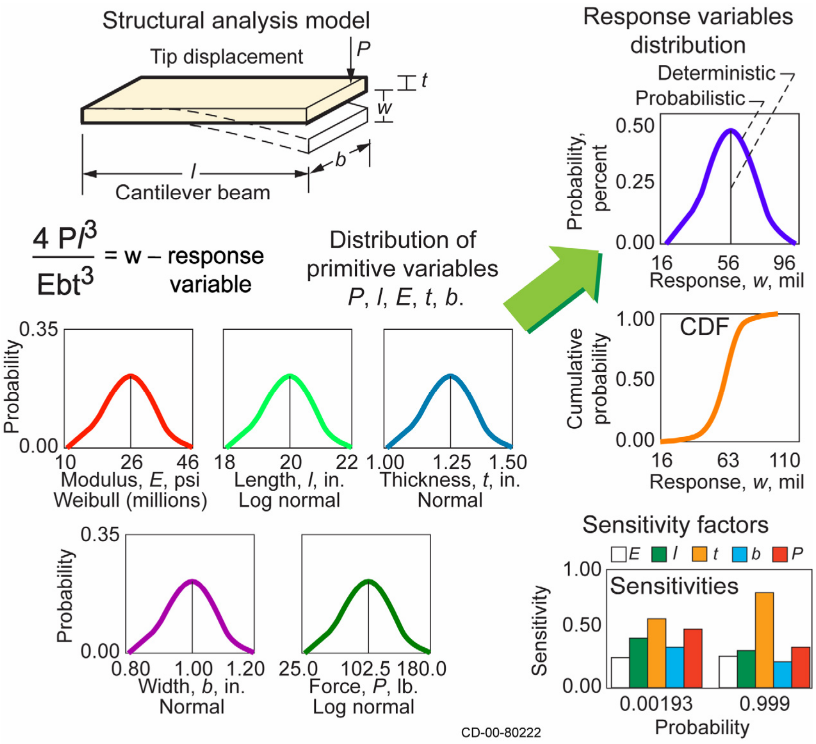


Figure 1.—Probabilistic structural analysis and response, illustrative examples.

The task is considerably simplified when we recognize that (1) the tip displacement equation is the analogue of a physical testing machine and (2) the scatter in the primitive variable, P, l, b, t, and E, can be assumed to be represented by simple and well known statistical distributions-probability density functions. (fig. 1, lower left). These distributions help us in two ways as will become evident subsequently. In order to evaluate the effects of the uncertainties of the primitive variables on the tip displacement, we proceed as follows: Step (1), we decide on the range of the scatter in each primitive variable. This range in practical cases is established from experience but for our simple example, we assume that scatter for the modulus is between 165 and 193 GPa (24 and 28 mpsi); for the length, between 48.3 and 53.4 cm (19 and 21 in.); for the width, between 2.41 and 2.67 cm (0.95 and 1.05 in.); for the thickness, 3.05 and 3.30 cm (1.20 and 1.30 in.); and for the load between 356 and 534 N (80 and 120 lb). It is important to note that the only test data we had were the mean values for the primitive variables. We assumed the range of the scatter. Note that the mean value for each primitive variable is where the vertical line, drawn from the peak of the respective distribution, intersects the horizontal line. Step (2), for each primitive variable in the equation we select randomly a value from within its respective scatter. Having the simple statistical distributions allows us to make non-biased random selections. For example, the values selected randomly can be: 176 GPa (25.5 mpsi) for E, 52.8 cm (20.0 in.) for l, 2.51 cm (0.99 in.) for b, 3.23 cm (1.27 in.) for t and 512 N (115 lb) for P. Step (3), we substitute these values sin the equation and we get 0.193 cm (0.08 in.) for the tip displacement. Step (4), repeat Step 3 for different sets of primitive variable values until sufficient data have been accumulated to plot the probability distribution graph (fig. 1, center right). For example, the mean value will be close to 0.165 cm (0.065 in.). There is about 95 percent of 100 will be less than 0.193 cm (0.08 in.) calculated in Step 3.

When the data is generated in Step 4, as just described, it is called Direct Monte Carlo Simulation and generally requires a large number of simulations. Methods/algorithms have been developed to generate the two probability graphs for the displacement with a relative few number of simulations. One such method is known as the Fast Probability Integration (FPI) (ref. 3). That method was used to generate the probability graphs (fig. 1). Application of FPI requires input of mean value, scatter range and probability density function for each participating primitive variable. As was already mentioned, the probabilistic simulation can be performed with known mean values and judiciously assumed probability density distributions, and scatter ranges for the primitive variables.

A byproduct of the FPI is the sensitivity factors (fig. 1, lower right). These factors quantify and order the sensitivity of the cumulative distribution function of the response variable to the uncertainty (scatter range) in the primitive variables. For our simple example, the load (primitive variable) has about the same effect on the tip displacement (response variable) as the geometry parameters (primitive variables) for low probability (less than 1 in 1000) while the thickness (primitive variable) dominates at high probabilities (greater than 999 to 1000). More about sensitivities in latter sections. For structural components/systems the above procedure is generalized as follows:

Step 1: Develop or use a deterministic model of the entire component/system with its boundary and load conditions and expected environmental conditions. In practical structural situations this would be mostly a finite element model. Step 2: Identify the primitive variables in the deterministic model. These will include material properties, fabrication process variables, structural parameters, loads, (including environment), boundary conditions, etc. In the case of composite structures, use integrated composite mechanics to predict the composite properties starting with micromechanics and accounting for both fabrication and environmental effects. Step 3: Obtain/assume mean values, scatter range and probabilistic distribution for each primitive variable. Step 4: Perturb each primitive variable on either side of their respective means by a reasonable small amount usually up to about 10 percent. Anything greater than 20 percent may be more of a shift or even multi-modal instead of a reasonable scatter. Step 5: Conduct deterministic analyses with the values selected in Step 4. Step 6: Repeat Steps 4 and 5 several times to generate sufficient information for FPI use. Step 7: Use FPI to generate the probability distribution functions for the desired response, displacement, stress, frequency, etc., as well as respective sensitivities at select probability levels.

The above generalized procedure is practical through computer codes as will be described subsequently. It is applicable to practically all disciplines as well as it is to structures described herein.

Probabilistic Simulation of Component/System Reliability

There are three essential parts in evaluating component/system reliability. These are: (1) probabilistic simulation of the loading conditions, (2) probabilistic simulation of the structural component including support, and (3) probabilistic simulation of the material(s) behavior. Each of these parts must be defined by inputs of its respective deterministic model, primitive variables and their attendant scatter. The probabilistic simulation proceeds to evaluate a specific response and its attendant scatter. The evaluated response is then compared with the corresponding resistance to assess the probability of failure which can be used subsequently to evaluate component/system reliability and risk. A conceptual schematic of the procedure is shown in figure 2. Shown in the figure are: (1) the three essential parts of component/system reliability simulation, (2) the structural response obtained, (3) the resistance evaluated, (4) probable damage (overlap of response scatter with resistance scatter), (5) info passed on for reliability and risk assessments, and (6) the institutions that participated to develop the requisite formalism of the concept and to implement it into an operational computational procedure. A block diagram of the computer code logic is shown in figure 3.

The schematics in figures 2 and 3 succinctly summarize probabilistic structural performance assessment. The concept is relatively straight forward and perhaps appears simple. However, implementation into a workable computer code requires knowledge of: (1) advanced structural mechanics, (2) efficient probabilistic algorithms, (3) material behavior, and (4) proficient and subtle computer programming

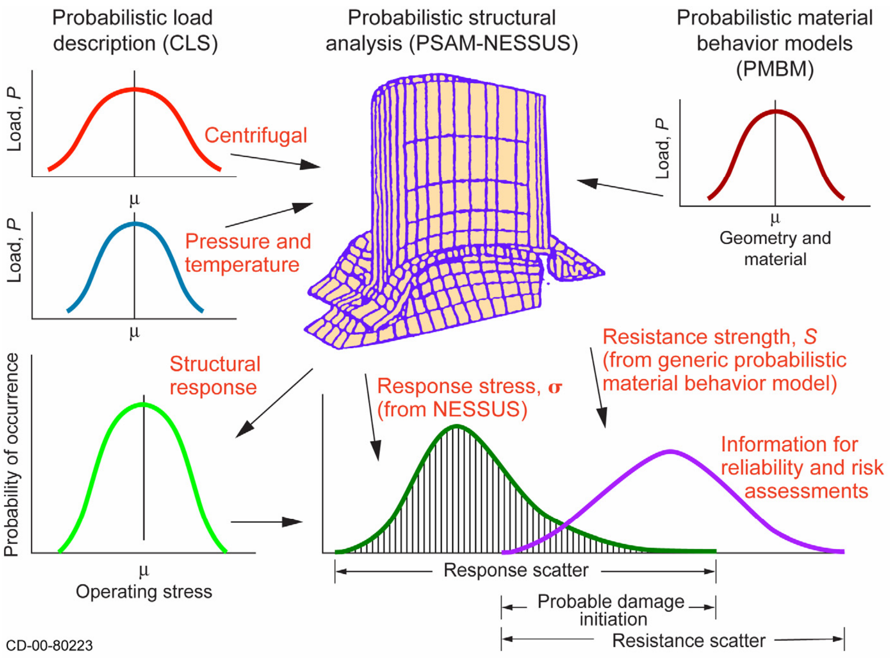


Figure 2.—Conceptual schematic of probabilistic structure and component reliability.

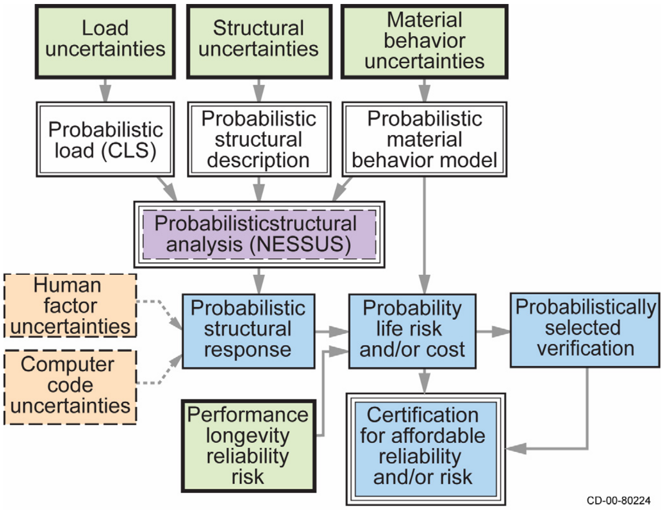


Figure 3.—Probabilistic structural simulation for assured certification.

techniques. Note in figure 3 that (1) uncertainties in the human factor and in the computer code can also be included, (2) inputs for required performance, component/system longevity and acceptable reliability and risk must be provided, and (3) the simulation provides information to probabilistically select verification tests to assure component/system certification with an acceptable reliability and affordable risk.

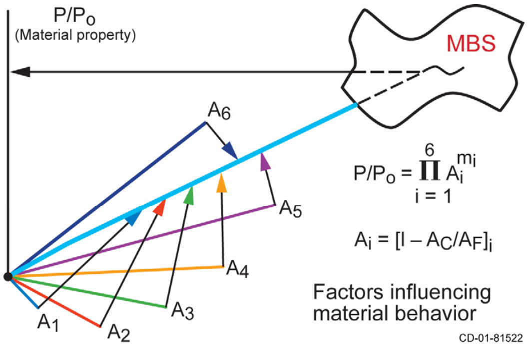


Figure 4.—Conceptual schematic of material behavior through a multi-factor interaction model (MFIM).

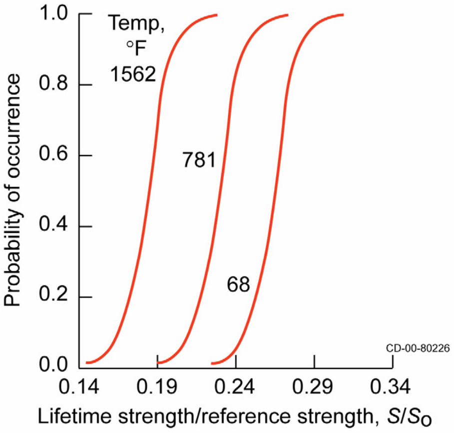


Figure 5.—PMBM-simulated lifetime strength for nickel-based superalloy subjected to 3162 stress cycles and 100 hr of creep.

Probabilistic Simulation of Material Behavior

Probabilistic simulation of material behavior is an open-ended, in-house development and so far as the author knows is the only one of its kind. Since the impetus for this article is materials based life prediction, it is appropriate to describe the probabilistic material models (PMBM) used in the simulation in some detail. The deterministic model evolved during the course of research for high temperature metal matrix composites (ref. 4). Implementing the deterministic model for probabilistic simulations evolved from a research grant with the objective to formally describe uncertainties in material behavior (ref. 5) for space shuttle main engine components. Consequently the model is based on the rather self-evident axiom that: “each material characteristic property, observed by conventional testing, constitutes a multi-dimensional surface.” That surface is described by an attendant multi-dimensional vector where each component of the vector represents one observed, or hypothetical, effect on that material characteristic property. The surface is represented by respective multi-factor model of product form, figure 4. The product form is assumed to conveniently represent manual interactions among the various factors. Each factor consists of three different variable/parameters as follows: (1) the factors terminal or final value –AF, (2) the factors current value –AC , and (3) and exponent –m. The exponent is selected to represent continuous monatomic behavior so that the factor equals unity when the current value equals the final value and approaches either zero or infinity (depending on the factors specific behavior) when the current value approached the final value.

Probabilistic results from the model (ref. 4) are shown in figure 5. Shown in figure 5 are cumulative distribution function curves for lifetime strengths at three different temperatures. The curves shift to the left and their scatter range increases with increasing temperature as physically would be expected. A very important observation from these curves is that the MFIM can be used in conjunction with selective testing to substantially reduce the number of tests and amount of time required to characterize material behavior in complex environments. It can readily be deduced that the MFIM is not restricted to the use just described (ref. 5).

Demonstration Component

A space shuttle main engine high pressure blade will be used as a demonstration case of probabilistic structural analysis/design. These blades are in the liquid hydrogen pump. They are relatively small, rotate at about 40,000 revolutions per minute and are subjected cyclically to very cold and very high temperature (thermomechanical fatigue). Schematics of the blade airfoil with its respective operating loading conditions and finite element model as shown in figure 6. The blade has relatively steep span-wise thermal and pressure gradients.

The cyclic temperature and load effects on the blade materials are simulated by the multi-factor interaction model (MFIM) described previously. The specific values for the various factors used are listed in table 1. Note that four factors were sufficient for that simulation. These were as follows: (1) the temperature dependence factor with an exponent of 1/2; (2) the stress dependent factor with exponent n; (3) the pressure cyclic load factor with exponent p; and (4) the thermal cyclic load factor with exponent g. The temperature effects exponent was assumed to be a constant based on previous studies while the exponent's n, p, and q were assumed to have scatter as shown in table 1.

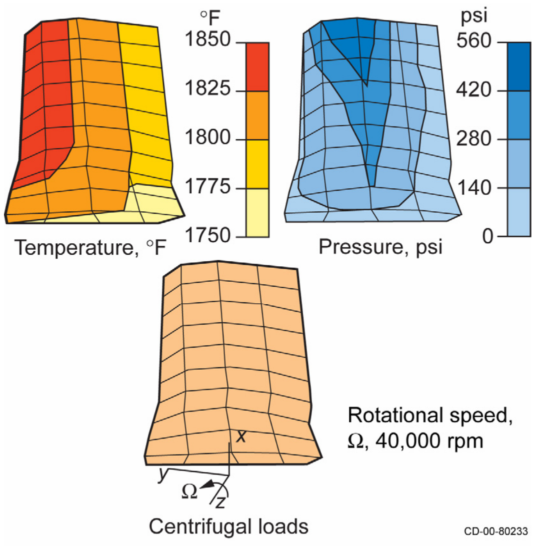
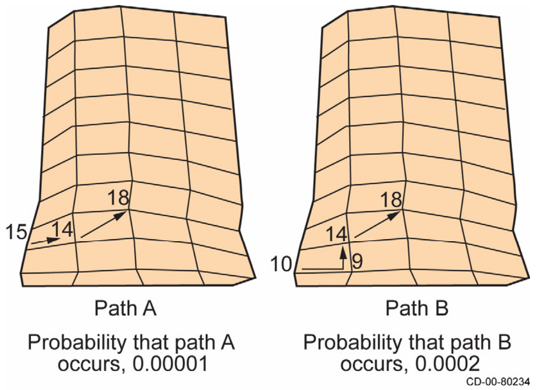


Figure 6.—Space shuttle main engine (SSME) blade finite element model with thermomechanical loads.

TABLE 1.—SPACE SHUTTLE MAIN ENGINE: MULTI-FACTOR INTERACTION MODEL VALUES USED IN PROBABILISTIC SIMULATION OF THE MATERIAL BEHAVIOR

Variable	Distribution type	Mean	Standard (Value)	Deviation % of mean
T_F	Normal	2750 °F	51.4 °F	2.0
T_0	Normal	68 °F	2.04 °F	3.0
S_F	Normal	212.0 ksi	10.6 ksi	5.0
σ_0	Constant	0	0	0
N_{MF}	Lognormal	10^8	5×10^6	5.0
N_{M0}	Lognormal	10^3	50	5.0
n	Normal	0.25		3.0
p	Normal	0.25		3.0
q	Normal	0.25		3.0



Path A

Probability that path A occurs, 0.00001

Path B

Probability that path B occurs, 0.0002

Figure 7.—Structural reliability of space shuttle main engine blade for 100,000 fatigue cycles and probable damage propagation paths to structural fracture.

The damage propagation path caused by 100,000 fatigue cycles are shown in figure 7 for two probability levels (1/100,000 and 2/10,000). Obviously the path with the highest probability will occur first. It is noted that several other paths are probable with probability levels between the two shown in figure 7. However, none was found with a higher probability than (2/10,000). The durability or damage tolerance of the blade in its operating environment can be simulated by using structural fracture (ref. 6). Results for the strain-energy release rate versus damage state are plotted in figure 8. Two major points are worth noting in figure 8: (1) the damage is stable and progresses rather slowly up to damage state 3; and (2) the damage progression increases very rapidly from damage state 3 to damage state 4. The plot in this figure displays several important aspects of structurally durability and/or damage tolerance: (1) the blade is damage tolerant up to damage state 3; (2) with continuing operation the blade will fracture (disintegrate) just a trifle past damage state 4; (3) the safe design of the blade with 100 percent reliability is up to damage state 2; (4) the blade should be inspected for damage state 1 and damage state 2; (5) at damage state 2, the blade must be replaced (retired for cause) to assure safe operation of

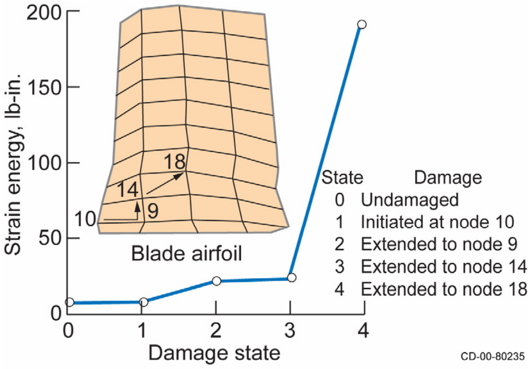


Figure 8.—Damage tolerance of space shuttle main engine blade along most probable progressive damage path leading to structural fracture.

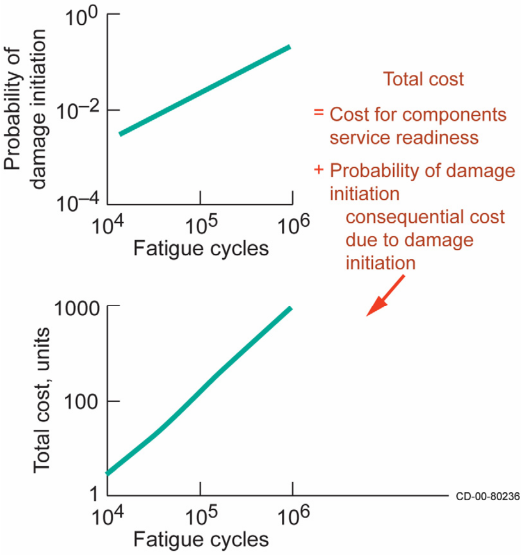


Figure 9.—Space shuttle main engine blade, probabilistic risk-cost assessment.

the SSME; (5) in-service health based on monitored changes in select blade responses (changes in vibration frequencies and vibration mode shapes for example) in order to infer damage state if blade inspection is costly and time consuming.

The important message from the afore discussion is that an abundance of information is generated by probabilistic computational simulation that can be judicially used from conceptual design to retirement for cause (from cradle to grave). Another important message is that comparable plots can be made for other responses, for example, blade material degradation due to oxidation.

The information from figure 8 can be combined with costs for fabrication and costs (penalties) for failure. The results are shown in figure 9 as log/log plots for probability of damage initiation versus number of cycles and for total cost versus fatigue cycles. Note that the cost increases very rapidly with fatigue cycles higher than 10,000. Interestingly the information in figure 9 is really of the cascading type because it can also be used to generate information for benefits accrued by improving material strength as shown in figure 10, or controlling the quality of processing as shown in figure 11. Costs to improve structural reliability by decreasing scatter are more beneficial than costs to increase strength for comparable probabilities.

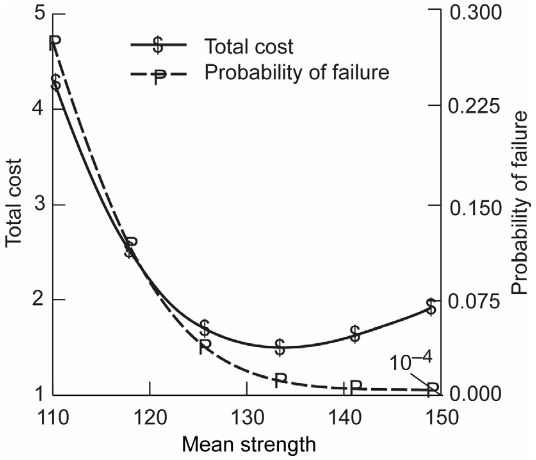


Figure 10.—Cost-reliability tradeoffs for improved material mean strength.

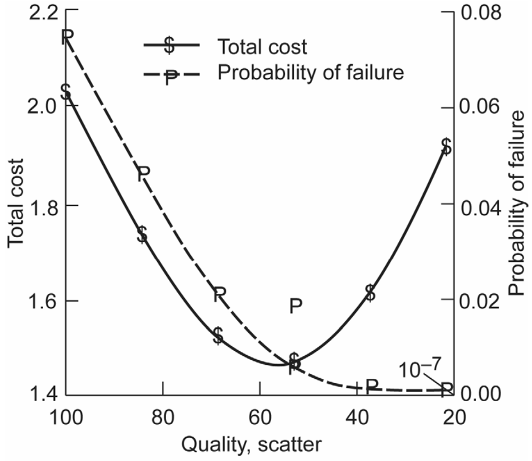


Figure 11.—Cost-reliability tradeoffs for controlling material strength scatter.

Concluding Remarks

Description of probabilistic methods for structural reliability and risk from materials to service environments leads to the following remarks: (1) Probabilistic methods via computational simulation are mature and credible to be adapted throughout the structural design practice. They provide quantifiable information that can be used to reduce costs product development, certification and reliability. (2) These methods constitute a “virtual” statistical desk top laboratory applicable at all stages and for all aspects of the design, material selection and qualification, development, certification and service life cycles. (3) Probabilistic methods rely on computational simulation results for decision making while statistics methods rely on relatively scarce available experimental data and inferred information there from. (4) Probabilistic methods can be used for future strategic projections and planning to assure better, cheaper, faster products for competitive advantages with acceptable reliability and quantifiable risk as well as to reliably evaluate the remaining life of existing products.

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14. ABSTRACT

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