Proceedings of COMP07:
6th International Symposium on Advanced Composites
16-18 May, 2007, Corfu, Greece

COMP2007–005

SIMULATION OF FOAM DIVOT WEIGHT ON EXTERNAL TANK UTILIZING LEAST SQUARES AND NEURAL NETWORK METHODS

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ABSTRACT

Simulation of divot weight in the insulating foam, associated with the external tank of the U.S. space shuttle, has been evaluated using least squares and neural network concepts. The simulation required models based on fundamental considerations that can be used to predict under what conditions voids form, the size of the voids, and subsequent divot ejection mechanisms. The quadratic neural networks were found to be satisfactory for the simulation of foam divot weight in various tests associated with the external tank. Both linear least squares method and the nonlinear neural network predicted identical results.

1.0 APPROACH

The simulation of foam mass ejection (divot) weight is assessed using four distinct methodologies: (1) linear least squares (LLS), (2) linear neural networks (LNN), (3) nonlinear (quadratic) neural networks (QNN), and (4) linear quadratic neural networks (LQNN). The justification for using these methods is presented subsequently suffices it to say that they were selected to evaluate the effectiveness of each methodology and identify associated weaknesses and strengths. Additionally, results from one method will be correlated to those of the other methods. A good correlation among the four alternative methods will ascertain that the models established are initially correct. At the end of the process one would expect to have the capability of selecting appropriate models/methods dependent on the targeted simulation. Fundamental principles and basic features of LLS, LNN, QNN, and LQNN are briefly discussed below. Herein, it is mentioned that all the variables in the formulations are assumed to be independent.

1.1 Least Squares (LLS)

Least squares is a mathematical procedure used for finding the best-fitting curve to a given set of points. It is a widely used modeling method. The unknown parameters are estimated by minimizing the sum of the squared deviations between the data and the model. The minimization process reduces the over-determined system of equations formed by the data to a sensible system of equations. This new system of equations is then solved to obtain the parameter estimates. While LLS often give optimal estimates of the unknown parameters, it is very sensitive to the presence of unusual data points in the data used to develop the model. Outliers can skew the results of the analysis. This makes model validation critical to obtaining sound answers. A typical LLS model used in predicting foam divot weight is represented by the following equation:

$$\text{Foam divot weight} = \beta_0 + \sum_{i=1}^{n} \beta_i x_i \qquad (1.1)$$

where n is the total number of known variables (x_i) is the i^{th} variable that could influence the foam divot weight. Some of those variables are: void diameter, void height, height of foam over the void, foam surface temperature, pressure inside the void, failure time, and location of the void. β's are the coefficients evaluated by Least Squares.

1.2 Neural Networks

Neural networks are the most advanced concept in data fitting. Neural networks are powerful data modeling tools that are able to capture and represent complex data relationships. Neural networks resemble the human brain in the following two ways: (1) a neural network acquires knowledge through learning and (2) a neural network's knowledge is stored within inter-neuron connection strengths known as synaptic weights. The advantage of neural networks lies in their ability to represent both linear and non-linear relationships and in their ability to learn these relationships directly from the data being modeled. Neural networks may be thought as an in situ process of multiple linear/non-linear regression.

Neural networks are very well capable of drawing meaning from complicated, imprecise, and incomplete data. They can be used to identify patterns and detect trends that are too complex to be noticed by conventional computer techniques or humans. A trained neural network can be thought of as an "expert" in the category of information it has been given to analyze. This expert can then be used to provide projections given new situations of interest within the data that it was trained.

In the foam divot weight simulation, two models of neural networks have been used. They are: LNN and QNN. The linear and non-linear models are coefficients based. The nonlinear model was used to identify if it can provide improved simulation of the test data. The LNN has the following explicit form:

$$\text{Foam divot weight} = \beta_0 + \sum_{i=1}^{n} \beta_i x_i \qquad (1.2)$$

where n is the total number of known variables (x_i) the i^{th} variable that could influence the foam divot weight (as explained in sec. 1.1 above). β's are the coefficients evaluated by the linear neural networks. The basis function for the non-linear (Quadratic) Neural Networks has the following explicit form:

$$\text{Foam divot weight} = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij} x_i x_j \qquad (1.3)$$

Note that the β's are the coefficients calculated by the neural networks. The use of LNN or QNN is really a function of the amount of data available. The use of LNN is sufficient if the number of test data for example is equal at least to the number of variables n plus one. That is required for proper prediction of the coefficients in LNN. But if the number of data sets available would exceed the number of coefficients of a QNN model, then the use of QNN will be justified. The number of data sets and selected model could influence the accuracy of the prediction. As a result, it was decided to use the LNN and QNN models and correlate the outcome from the two models.

2.0 SIMULATION OF FOAM DIVOT IN THERMAL VACUUM TEST

The simulation results presented in this section pertain to predicting the weight of foam loss for a thermal vacuum test. The objective here is to replicate the test numerically using models from three methods: linear least squares, linear neural networks, quadratic neural networks, and multi factor interaction model. The divot weight calculation results presented in this report are based on the schematic of the physical variables depicted in figure 2.1. As shown in the figure, the void is assumed to be right on the substrate surface. The total foam thickness is basically comprised of two components: void height and foam height over the void and, therefore, it is not an independent variable. The aspect ratio L_1/L_2 indicates the type of void. For example, voids with aspect ratios $L_1/L_2 > 2.5$, are treated as slot voids. On the other hand, voids with aspect ratios $L_1/L_2 < 2.5$, are treated as cylindrical voids. For slot type voids, the critical void dimension is L_2 while L_1 is the critical dimension for cylindrical voids. The criterion for defining the type of void was provided by the RTF program based on their experience. Divot data obtained from the RTF program for a thermal vacuum test are discussed next.

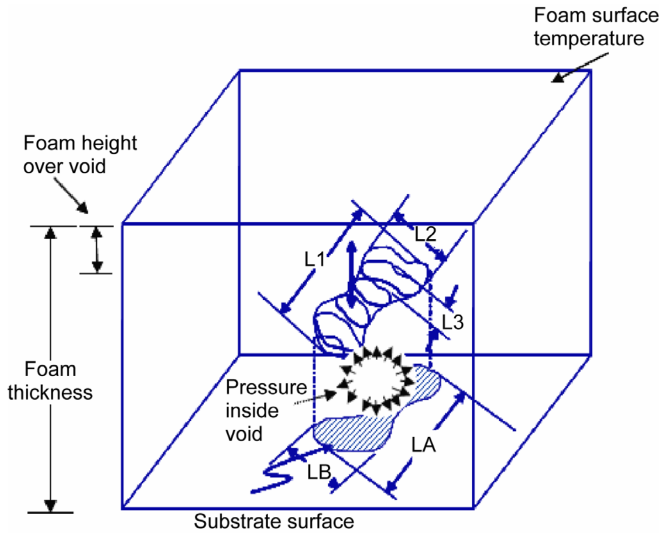

Figure 2.1. Schematic of foam physical variables that influence the divot process.

2.1 Test Data Description

Thermal vacuum testing performed by the RTF program was aimed to develop an empirically relationship for the void size which will produce a divot for regions of the tank not susceptible to cryo-ingestion and cryo-pumping type environments. According to information obtained from the program, BX 265 foam panels were cut to 1 ft^2 and machined to desired heights. For the test data supplied to GRC, notched cylindrical voids were placed at the substrate of the panels. Note that the voids are introduced into foam panels and then foam panels with voids are bonded to a substrate. During testing foam surface is heated with using quartz lamp bank and radiator plate to match the highest heating rate experienced in the flange area during flight. Tests took place in vacuum chamber to match pressure profile during flight. Pressure inside the void and time to divot and mass of divot were recorded during test. Limited test data supplied to GRC are shown in table 2.1. In the next few sections, the simulation of the divot as carried out by least squares, neural networks, and multi factor interaction model is described. Note that the divot weight measured after each test is listed in the last column to the right of the table. The lowest weight recorded for this set of data was 0.00044 lb while the highest weight was 0.145 lb. The debris allowable set by the program's Level II Review is 0.038 lb.

Table 2.1 Divot Weights From Thermal Vacuum Test (Cylindrical Voids)

Void diameter (in.)	Void height (in.)	Foam over void (in.)	Foam surface temperature (F)	Pressure inside void (psi)	Time to fail (sec)	Divot weight (lb)
1.0	0.5	0.25	183.49	12	57	0.00044
0.5	0.5	0.25	352.07	10	73	0.00022
1.0	1.0	0.25	171.85	11	52	0.00044
1.0	0.5	0.5	547.09	12	86	0.00132
0.5	0.5	0.5	645.00	12	123	0.00044
1.0	1.0	0.5	519.62	11	84	0.00154
0.5	1.0	0.5	645.00	15	108	0.00044
3.125	2.0	2.0	412.19	12.35	77	0.10318
4.125	2.0	2.0	219.86	11.5	64	0.14506

1 in. = 2.54 cm; °F=(5/9)°C; psi=6.89 Pa; lb=0.455 kgm

2.2 Simulation of Divot Weight by Least Squares, and Linear and Quadratic Neural Networks

The least squares model used to predict the foam divot weight (*W*) based on supplied thermal vacuum test data has the following explicit form:

$$W = -0.0435 + 0.0261X_1 + 0.0026X_2 + 0.028X_3 \\ -0.00007X_4 - 0.00072X_5 + 0.00062X_6 \qquad (2.1)$$

where the variables X_1 through X_7 are basically the test variables that are listed in table 2.1 and they are: void diameter VD, void height VH, foam height above the void FH, foam surface temperature at time of divoting FST, pressure inside the void PR and time at divot t. The neural network linear basis function model has the following explicit form:

$$W = -0.0435 + 0.0261X_1 + 0.0026X_2 + 0.028X_3 \\ -0.00007X_4 - 0.00072X_5 + 0.00062X_6 \qquad (2.2)$$

The neural network quadratic basic function has the following general form:

$$W = -7.673E-06 + \sum_{i=1}^{6}\sum_{j=i}^{6}\beta_{ij}x_i x_j \qquad (2.3)$$

The total number of coefficients required to obtain a solution from equation (2.3) is 38. They are listed in table 2.2. The divot weight results obtained from the linear least squares, neural network linear and quadratic are summarized in figures 2.2 through 2.8. Each figure contains plots of the divot weight as a function of the independent variable as obtained from: (a) least square, (b) linear neural networks, and (c) quadratic neural networks. Figure 2.2 shows the effect of increase in the void diameter on the divot weight. As the void diameter was perturbed, all the other independent variables were set to their respective means. Figure 2.2 indicates that the divot weight monotonically increases with the increase in the void diameter except for the LNLQ which decreases slightly. The largest variation in the divot weight was detected with the quadratic neural networks where the divot weight increased from near zero to 0.09 kg (0.223 lb). The solution calculated by the least squares showed relatively less variation as compared to nonlinear neural networks and the linear nonlinear neural networks. The results are consistent with the physical phenomenon of foam ejection. The divot weight increases at increased void diameter. However, the divot weight decreases as the void diameter increases as predicted by the NN quadratic. This effect may be attributed to insufficient data required by the quadratic NN fit. The results for the void height effect on the divot weight are presented in figure 2.3. The plots basically are used to show how one physical variable influence the divot weight when all other considered physical variables are set their respective mean. The least squares and linear neural network predicted

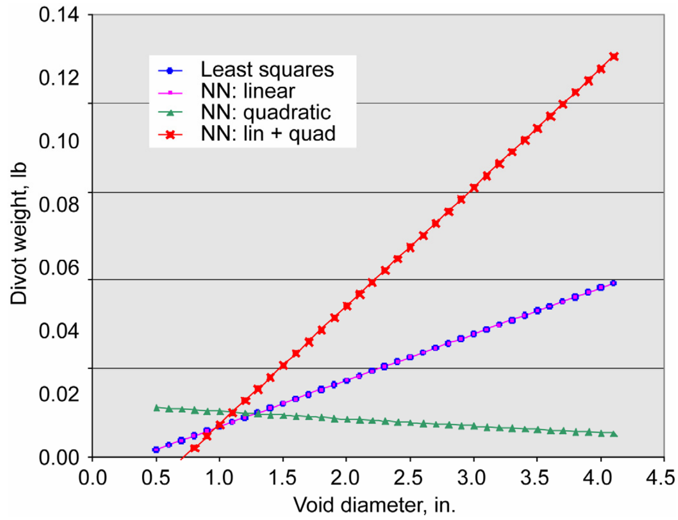

Figure 2.2. Simulation of divot weight in a thermal vacuum test as a function of void diameter as obtained from least squares and neural network. (lb=0.455 kg).

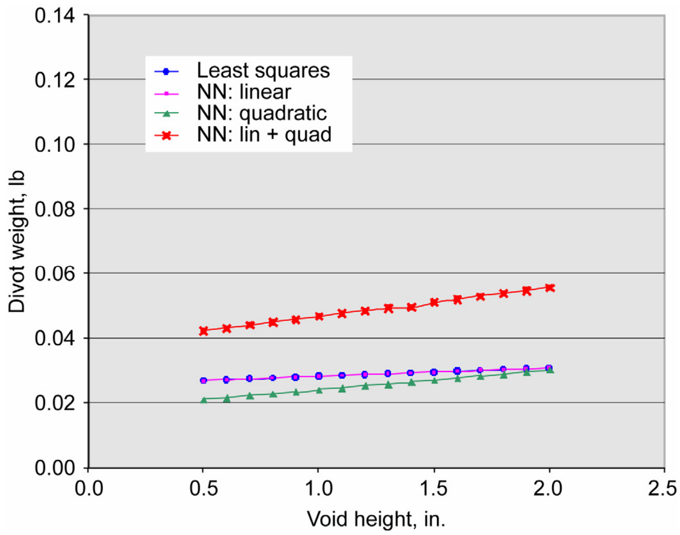

Figure 2.3. Simulation of divot weight in a thermal vacuum test as a function of void height as obtained from least squares and neural network. (lb=0.455 kg)

comparable effects for the void height that is the divot weight would increase as the void height increased. With the quadratic neural networks, the divot weight would increase the greatest as the void height is increased. A review of the test data shows a large disparity in the divot weights since 7 out of 9 samples produced divot weights under 0.0006 kg (0.00154 lb) while the remaining two samples produced divot weights above 0.04 kg (0.1 lb).

Table 2.2 Neural network coefficients beta (i,j) for quadratic basis function

Beta 0	−7.673E−06					
Beta 1,1:1,6	1.055E−04	3.112E−05	4.802E−05	−1.466E−04	1.187E−04	6.738E−04
Beta 2,2:2,6		7.017E−08	1.741E−05	3.013E−05	−6.359E−05	−6.935E−05
Beta 3,3:3,6			1.979E−05	9.482E−05	8.368E−05	4.107E−04
Beta 4,4:4,6				2.256E−06	7.692E−05	−3.606E−05
Beta 5,5:5,6					−3.342E−04	−3.456E−04
Beta 6,6						1.209E−04

The results obtained for the foam height above the void, foam surface temperature, pressure inside the void, and time to fail from least squares, linear neural networks, and quadratic neural networks are presented in figures 2.4 thru 2.7. The largest divot weight predicted by the least squares model was obtained when evaluating the effect of the foam height above the void on the divot 0.09 kg (0.233 lb). The largest divot weight predicted by the nonlinear neural networks model was obtained when evaluating the effect of time to fail on the divot 0.03 kg (0.0702 lb). Similarly, the largest divot weight predicted by the linear nonlinear neural networks model was obtained when evaluating the effect of time to fail on the divot 0.21 kg (0.47 lb). Inconsistency in the presented results is due to the fact relatively few test data were available. Next, results from a multi factor interaction model obtained during the simulation of the same test data are presented. Figure 2.4 shows the inconsistencies between the three data-fit methods. The linear least squares and the linear NN are exactly the same. The non-linear NN shows very different behavior. The NN quadratic only shows a very steep rise with foam height increase while the quadratic with linear term added shows a rather steep decrease with foam height increase. This is further evidence that non-linear NN requires a lot more data. Figure 2.3 shows that all methods decrease with foam surface temperature increase while the non-linear NN shows a steepest decrease than the other three. Figure 2.6 shows another slight inconsistency between the two non-linear NN's. Figure 2.7 shows a very interesting behavior. The linear least square and the linear NN show a rather steep increase with time to fail. The two non-linear NN's show a wide different non-linear behavior—initially a decrease bottoming out at about 60 to 70 sec and, then, a very steep increase to about 100 sec.

The collective results of the four fitting methods indicate that the NN's should only be used with a lots data for training.

The neural network linear plus quadratic basis functions have the following general form:

$$W = -5.069E-06 + \sum_{i=1}^{6} \beta_i x_i + \sum_{i=1}^{6} \sum_{j=i}^{6} \beta_{ij} x_i x_j \qquad (2.4)$$

The total number of coefficients required to obtain a solution from equation (2.4) is 28. They are listed in table 2.3.

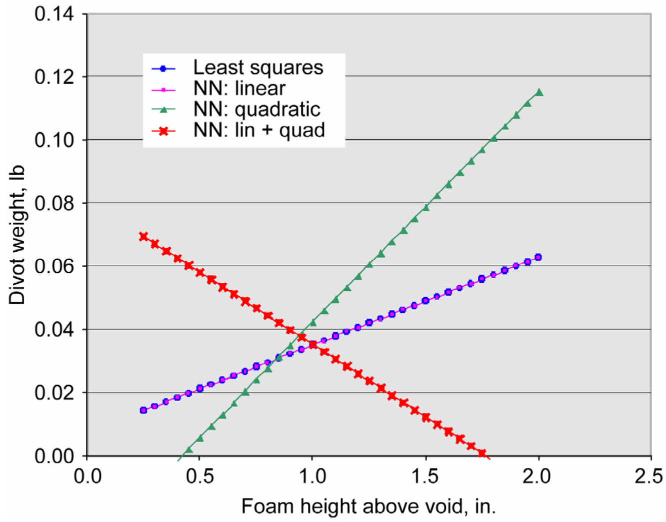

Figure 2.4. Simulation of divot weight in a thermal vacuum test as a function of foam height above void as obtained from least squares and neural network. (lb=0.455 kg)

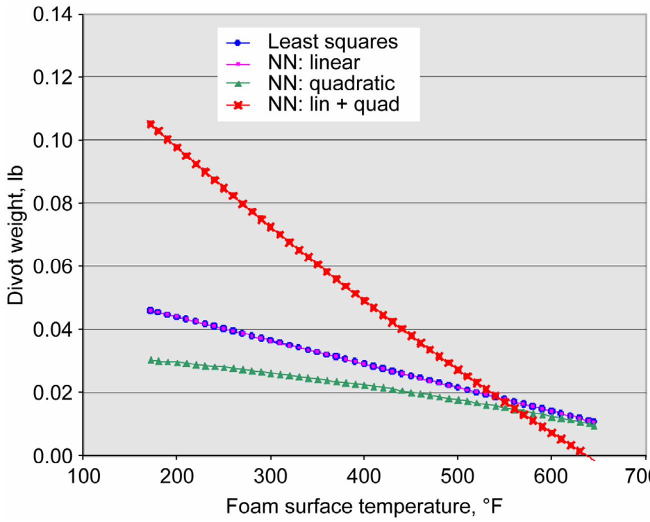

Figure 2.5. Simulation of divot weight in a thermal vacuum test as a function of foam surface temperature as obtained from least squares and neural network. (lb=0.455 kg)

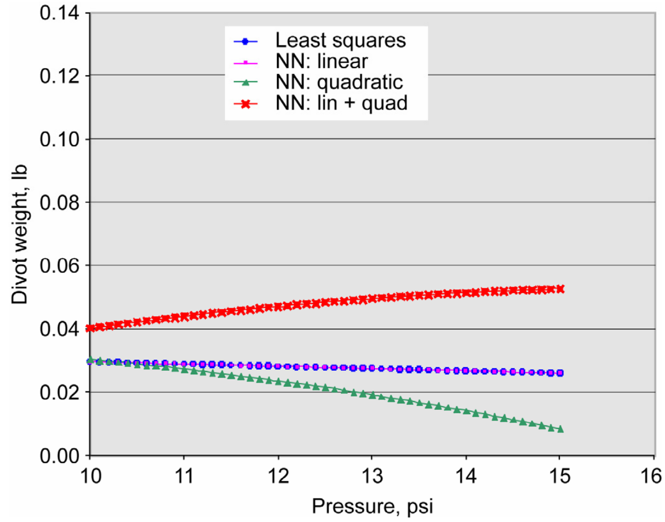

Figure 2.6. Simulation of divot weight in a thermal vacuum test as a function of pressure inside the void as obtained from least squares and neural network. (lb-0.455 kg)

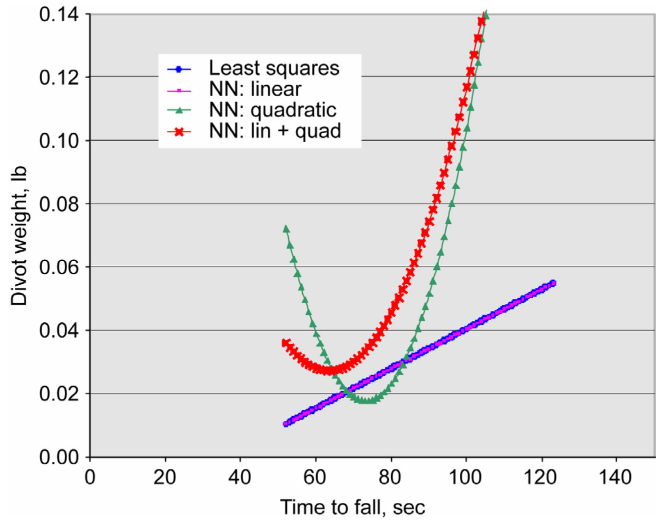

Figure 2.7. Simulation of divot weight in a thermal vacuum test as a function of time to fail as obtained from least squares and neural network. (lb=0.455 kg)

Table 2.3 Neural network coefficients beta (*i,j*) for linear plus quadratic basis functions

beta 0	−5.069E−06					
beta 1: 6	7.689E−06	−6.941E−06	5.447E−06	−2.273E−04	−4.516E−05	−9.909E−05
beta 1,1:1,6	8.424E−05	2.342E−05	3.763E−05	4.916E−05	9.246E−05	5.635E−04
beta 2,2:2,6		−2.139E−06	1.292E−05	4.211E−05	−6.652E−05	−9.453E−05
beta 3,3:3,6			1.476E−05	−1.699E−04	6.028E−05	2.887E−04
beta 4,4:4,6				8.590E−08	8.756E−05	−1.357E−05
beta 5,5:5,6					−3.142E−04	−3.192E−04
beta 6,6						6.727E−05

3.0 SIMULATION OF FOAM DIVOT IN A CRYO INGESTION TEST

As mentioned in previous sections, LMSSC/MAF conducted many tests to determine a relationship between internal voids and debris size. The team responsible for this independent assessment at NASA GRC obtained data pertaining to a cryo ingestion test which is supposed to evaluate the effect of LN2 entrapped in the void interior. This report will not discuss the details of the test but will focus on the results obtained from computationally simulating the test. The test data obtained for the cryo ingestion test are listed in table 3.1. The geometry of the void shown in figure 3.1 is also used in this test. That means all the voids faced the substrate directly. The number of test variables is four and they are: void diameter, void height, and foam above the void. The divot weight for 15 test data points are also listed in table 3.1. In this case, the lowest divot weight was 0.009 kg (0.0019 lb) and the highest divot weight was 0.0811 lb.

3.1 Simulation of Divot Weight by Least Squares, Quadratic and Linear With Quadratic Neural Networks

As is in the case of the thermal vacuum test simulation, the four models derived from least squares, linear neural network, quadratic neural network and linear plus and quadratic neural network combined were applied to simulate the cryo ingestion test. The results from the simulation of divot weight as a function of void diameter as obtained from least squares model and neural network methods are presented in figure 3.1. The relationship between the divot weight and each of the void diameter, void height, and foam over void is shown in figures 3.1, 3.2 and 3.3, respectively. The least squares fit predicted that the void weight would increase with the increase in the void diameter and the foam height above the void. However, figure 3.2 shows that all methods decrease with void height—linearly with the linear fits and non-linear with the non-linear fits. It is interesting to note that the non-linear causes decrease steeply, bottom out, and then increase rather smoothly. Figures 3.1 through 3.3 also show the results from the linear plus quadratic neural network for the divot weight as a function of the void diameter, void height, and foam height above the void. There are some similarities between the results that are obtained from least squares and the ones obtained from quadratic neural network. The quadratic neural network solution is highly non-linear compared to the one from least squares. The least squares and quadratic neural network solutions seems to be more consistent with the physical characteristics of foam devoting. Naturally, if all other factors remain unchanged, it is likely that a thicker foam above the void would not be expelled as easily as the thinner one.

One legitimate question is raised at this stage. How good are the models developed from each method? The answer to this question is provided in table 3.2. Each test data point (design vector) is plugged back into the models from the three methods. The differences between the test divot weights and the ones predicted by least squares, linear neural network and quadratic neural network are tabulated in table 3.2. The least difference between test divot weight and predicted divot weight is obtained from the quadratic neural network. The maximum error (difference between test and predicted divot weight) is about 0.0008 kg (0.0017 lb) for the quadratic plus linear neural network. Therefore, the use of the neural network quadratic plus linear form is highly recommended for simulating the test and determining an actual/accurate relationship between the physical variables and the divot weight. Tables 3.3 and 3.4 list the various coefficients used in the simulation of the divot weight using the quadratic and quadratic plus linear neural networks, respectively.

Table 3.1 Divot weights from cryo ingestion test (cylindrical voids)

Void diameter (in)	Void depth (in)	Foam over void (in)	Test divot weight (lb)
1.125	0.500	0.500	0.0019
1.625	0.500	0.500	0.0034
0.875	0.250	0.750	0.0039
1.125	0.500	1.000	0.0081
1.375	0.750	0.750	0.0051
1.875	0.750	0.750	0.0055
0.875	0.250	1.250	0.0072
2.125	0.125	2.500	0.0811
2.125	0.625	2.000	0.0470
2.125	1.125	1.500	0.0272
1.875	1.750	1.250	0.0172
1.375	1.750	1.250	0.0220
1.125	1.500	1.500	0.0182
1.125	0.100	2.100	0.0240
1.375	0.100	2.100	0.0301

1 in. = 2.54 cm; lb=6.89 Pa; 1 lb = 0.455 kg

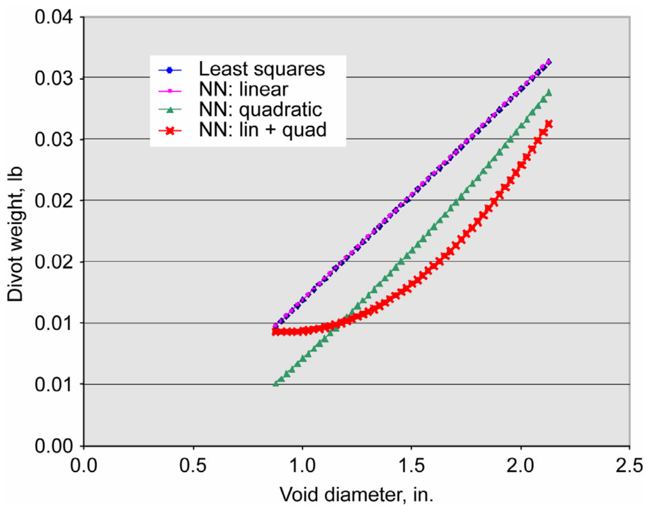

Figure 3.1. Simulation of divot weight in cryo ingestion test as a function of void diameter as obtained from least squares and neural network. (lb=0.455 kg)

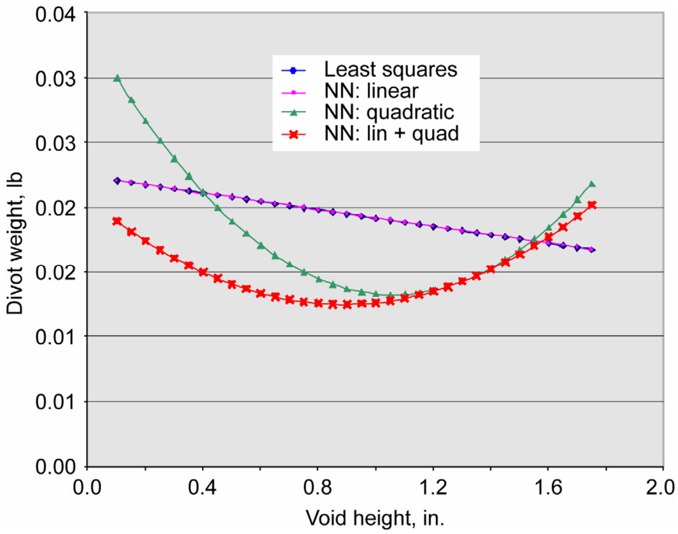

Figure 3.2. Simulation of divot weight in cryo ingestion test as a function of void height as obtained from least squares and neural network. (lb=0.455 kg)

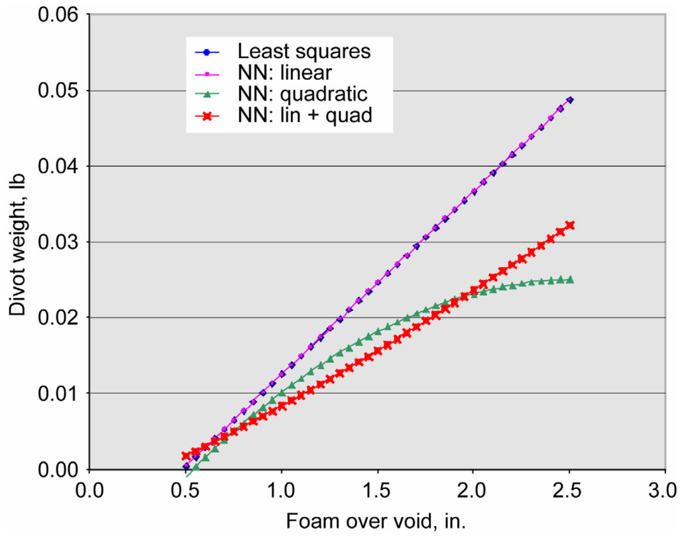

Figure 3.3. Simulation of divot weight in cryo ingestion test as a function of foam over void as obtained from least squares and neural network. (lb=0.455 kg)

Table 3.2 Validation of least squares method and neural networks (linear, quadratic and linear plus quadratic) for the simulation of divot weights in cryo ingestion test

	Least squares method		Neural network linear basis function		Neural network quadratic basis function		Neural network linear + quadratic basis function	
Divot weight (lb)	Validation data	Error	Validation data	Error	Validation data	Error	Validation data	Error
0.0019	–0.0049	0.0068	–0.0049	0.0068	0.0005	0.0014	0.0027	–0.0008
0.0034	0.0037	–0.0003	0.0037	–0.0003	0.0032	0.0002	0.0019	0.0016
0.0039	–0.0024	0.0063	–0.0024	0.0063	0.0045	–0.0006	0.0044	–0.0005
0.0081	0.0072	0.0009	0.0072	0.0009	0.0078	0.0003	0.0067	0.0014
0.0051	0.0046	0.0004	0.0046	0.0004	0.0040	0.0011	0.0051	0.0000
0.0055	0.0133	–0.0078	0.0133	–0.0078	0.0071	–0.0016	0.0073	–0.0017
0.0072	0.0097	–0.0025	0.0097	–0.0025	0.0078	–0.0006	0.0073	–0.0001
0.0811	0.0619	0.0192	0.0619	0.0192	0.0794	0.0017	0.0806	0.0005
0.0470	0.0482	–0.0012	0.0482	–0.0012	0.0488	–0.0018	0.0480	–0.0009
0.0272	0.0345	–0.0073	0.0345	–0.0073	0.0251	0.0021	0.0256	0.0016
0.0172	0.0222	–0.0050	0.0222	–0.0050	0.0181	–0.0009	0.0181	–0.0009
0.0220	0.0135	0.0085	0.0135	0.0085	0.0218	0.0002	0.0209	0.0011
0.0182	0.0160	0.0021	0.0160	0.0021	0.1812	0.0000	0.0190	–0.0009
0.0239	0.0350	–0.0111	0.0350	–0.0111	0.0215	0.0024	0.0227	0.0012
0.0301	0.0393	–0.0093	0.0393	–0.0093	0.0340	–0.0039	0.0316	–0.0016

lb=0.455 kg

The least squares model and the linear neural network model used to predict the foam divot weight (W) based on supplied cryo-ingestion test data depicted in Table 3.1 has the following explicit form:

$$W = -0.0348+0.0173X_1 - 0.0032X_2 +0.0241X_3 \qquad (3.1)$$

The quadratic neural network model used to predict the foam divot weight (W) based on supplied thermal vacuum test data depicted in table 3.1 has the following explicit form:

$$W = -3.855E-03 + \sum_{i=1}^{3} \sum_{j=i}^{3} \beta_{ij} x_i x_j \qquad (3.2)$$

The linear plus quadratic neural network combined model used to predict the foam divot weight (W) based on supplied thermal vacuum test data depicted in table 3.1 has the following explicit form:

$$W = 0.02016 + \sum_{i=1}^{3} \beta_i x_i + \sum_{i=1}^{3} \sum_{j=i}^{3} \beta_{ij} x_i x_j \qquad (3.3)$$

Table 3.3. Quadratic neural network coefficients and explicit model (cylindrical voids—cryo ingestion test)

beta 0	–3.855E–03		
beta 1,1:1,3	2.496E–03	–2.448E–02	2.196E–02
beta 2,2:2,3		1.810E–02	–1.787E–03
beta 3,3			–6.065E–03

Table 3.4. Linear plus quadratic neural network coefficients and explicit model (cylindrical voids—cryo ingestion test)

beta 0	2.016E–02		
beta 1: 3	–3.435E–02	2.296E–02	–1.215E–02
beta 1,1:1,3	1.189E–02	–2.005E–02	2.012E–02
beta 2,2:2,3		1.035E–02	–8.962E–03
beta 3,3			1.340E–03

4.0 CONCLUSIONS

This work demonstrated the availability of specialized computational methods capable of simulating the foam divot weight as performed under several test programs. The relationship between defect size and divot weight is determined by applying the three models to the test results. Based on the performed evaluation, the following conclusions are drawn:

1) The foam divot weight could be expressed as a function of the void physical dimensions such as the void diameter and the void height, though the void height was relatively insensitive in the simulation with respect to other variables such as foam thickness and foam height above the void.
2) Among the various methods used in the simulation of divot weight in various tests, the QNN seem to perform the best. The difference between predicted and test divot weights was the least with QNN.
3) The least square and linear neural network produce similar results as expected.
4) Sufficient amount of test data must to be available to make the use of neural networks more effective.
5) Confidence in the developed computational models is improved by arbitrarily using a portion of the test data to develop the model while using the remaining data for verification/ validation.