

# Safety and Performance Analysis of the Non-Radar Oceanic/Remote Airspace In-Trail Procedure 

Victor A. Carreño<br>Langley Research Center, Hampton, Virginia<br>Cesar A. Muñoz<br>National Institute of Aerospace, Hampton, Virginia

Since its founding, NASA has been dedicated to the advancement of aeronautics and space science. The NASA Scientific and Technical Information (STI) Program Office plays a key part in helping NASA maintain this important role.

The NASA STI Program Office is operated by Langley Research Center, the lead center for NASA's scientific and technical information. The NASA STI Program Office provides access to the NASA STI Database, the largest collection of aeronautical and space science STI in the world. The Program Office is also NASA's institutional mechanism for disseminating the results of its research and development activities. These results are published by NASA in the NASA STI Report Series, which includes the following report types:

- TECHNICAL PUBLICATION. Reports of completed research or a major significant phase of research that present the results of NASA programs and include extensive data or theoretical analysis. Includes compilations of significant scientific and technical data and information deemed to be of continuing reference value. NASA counterpart of peerreviewed formal professional papers, but having less stringent limitations on manuscript length and extent of graphic presentations.
- TECHNICAL MEMORANDUM. Scientific and technical findings that are preliminary or of specialized interest, e.g., quick release reports, working papers, and bibliographies that contain minimal annotation. Does not contain extensive analysis.
- CONTRACTOR REPORT. Scientific and technical findings by NASA-sponsored contractors and grantees.
- CONFERENCE PUBLICATION. Collected papers from scientific and technical conferences, symposia, seminars, or other meetings sponsored or co-sponsored by NASA.
- SPECIAL PUBLICATION. Scientific, technical, or historical information from NASA programs, projects, and missions, often concerned with subjects having substantial public interest.
- TECHNICAL TRANSLATION. Englishlanguage translations of foreign scientific and technical material pertinent to NASA's mission.

Specialized services that complement the STI Program Office’s diverse offerings include creating custom thesauri, building customized databases, organizing and publishing research results ... even providing videos.

For more information about the NASA STI Program Office, see the following:

- Access the NASA STI Program Home Page at http://www.sti.nasa.gov
- E-mail your question via the Internet to help@sti.nasa.gov
- Fax your question to the NASA STI Help Desk at (301) 621-0134
- Phone the NASA STI Help Desk at (301) 621-0390
- Write to:

NASA STI Help Desk
NASA Center for AeroSpace Information
7115 Standard Drive
Hanover, MD 21076-1320


# Safety and Performance Analysis of the Non-Radar Oceanic/Remote Airspace In-Trail Procedure 

Victor A. Carreño<br>Langley Research Center, Hampton, Virginia<br>Cesar A. Muñoz<br>National Institute of Aerospace, Hampton, Virginia

National Aeronautics and
Space Administration
Langley Research Center
Hampton, Virginia 23681-2199

Available from:

# Safety and Performance Analysis of the Non-Radar Oceanic/Remote Airspace In-Trail Procedure 

Victor Carreño<br>NASA Langley Research Center, Hampton Virginia 23681<br>Cesar Muñoz<br>National Institute of Aerospace, Hampton Virginia 23681


#### Abstract

This document presents a safety and performance analysis of the nominal case for the In-Trail Procedure (ITP) in a non-radar oceanic/remote airspace. The analysis estimates the risk of collision between the aircraft performing the ITP and a reference aircraft. The risk of collision is only estimated for the ITP maneuver and it is based on nominal operating conditions. The analysis does not consider human error, communication error conditions, or the normal risk of flight present in current operations. The hazards associated with human error and communication errors are evaluated in an Operational Hazards Analysis presented elsewhere.


## Acknowledgments

The authors would like to thank John Brown for suggesting the directional gradient wind scenario. The authors would also like to thank Bart Obbink and the Requirements Focus Group group for comments and suggestions.

## 1 Definitions

Ground Speed Differential. "The speed difference over the ground between the ITP aircraft and the Potentially Blocking Aircraft along each aircraft's track. This measurement would use a technique similar to the Doc 4444 - PANS-ATM longitudinal separation procedure using DME (Doc 4444 - PANS-ATM section 5.4.2.3) to determine the Ground Speed Differential." [2] A Positive Ground Speed Differential signifies that the ITP Aircraft and the Reference Aircraft are closing on each other (the distance between aircraft is being reduced).

ITP Aircraft. "An aircraft that is fully qualified (from an equipment, operator, and flight crew qualification standpoint) to conduct an ITP and whose flight crew is considering a change of flight level." [2]

Potentially Blocking Aircraft. "Aircraft at the Intervening Flight Level whose ADS-B report data are available to the ITP aircraft. A Potentially Blocking aircraft that is less than the standard longitudinal separation minimum will prevent an aircraft from climbing through the intervening Flight level under normal operating conditions (without an ITP procedure)." [2] See Figure 1.

Reference Aircraft. One or two Potentially Blocking Aircraft that meet the ITP criteria and that will be identified to ATC by the ITP Aircraft as part of the ITP clearance request.

ADS-B. Automated Dependent Surveillance - Broadcast.

NUC. Navigation Uncertainty Category [3, 5].
NAC. Navigation Accuracy Category [4, 6].
NIC. Navigation Integrity Category [4, 6].
SIL. Surveillance Integrity Level [4, 6].

## 2 Introduction

The objective of this safety and performance analysis is to evaluate the risk of collision for the proposed procedure as a function of the system performance requirements. It is based on a probabilistic event tree analysis. The analysis evaluates the soundness of the proposed procedure from the design point of view. It estimates the probability of a collision under nominal conditions. Nominal conditions do not include human error, communication errors, degradation in engine and airframe performance, etc. The factors considered in the analysis are error in surveillance (position and velocity), altitude error, winds, geometry and see and avoid capabilities. The calculations of collision risk do not take into account collision avoidance equipment on board and other factors discussed in Section 5.1. Therefore, collision avoidance equipment such as TCAS and other considerations could further reduce the collision probability.

The analysis performs a parametric study of surveillance accuracy versus collision probability. Given a target collision risk, the minimum required surveillance standard can be determined. An Operational Safety Analysis which includes an Operational Hazards Analysis has been performed for the In-Trail Procedure and is reported in [1].

## 3 ITP Description

The In-Trail Procedure (ITP) has been developed to enable aircraft that desire flight level changes in oceanic and remote airspace to achieve these changes on a more frequent basis, thus improving flight efficiency, comfort, and safety. Under normal non-radar oceanic operations, aircraft typically fly using time separation. The North Atlantic Organized Track System (NAT OTS), for example, uses a 10 minutes (time) separation at every point along the flight level track. This results in a longitudinal separation of approximately 80 nautical miles at 0.83 Mach.

Figure 1 shows an example of a flight level change request which will be denied in today's operations. In this example, the requesting aircraft would like to perform a climb from FL340 to FL360. A blocking aircraft at the intervening flight level FL350 is less than the minimum in-track (time) separation and prevents the request from being granted.


Figure 1. Blocking Aircraft Preventing Climb.
This climb will be permitted under the In-Trail Procedure given that the FL360 is open and the requesting and blocking aircraft meet the ITP criteria. When a Blocking aircraft is considered qualified for ITP, it is called a Reference aircraft. The ITP also permits descents behind a Reference aircraft climbs/descents in front of a Reference aircraft, and climbs/descents between two Reference aircraft. The definition and
detailed description of the In-Trail Procedure including criteria, procedure phases and flow diagrams, communication requirements, roles and responsibilities, operational environment, etc., can be found in [2]. A high level summary of the ITP criteria is as follows:

- Initiation range of no less than 15 NM when positive ground speed differential is 20 knots or less
- Initiation range of no less than 20 NM when positive ground speed differential is 30 knots or less
- Requesting (ITP) aircraft capable of $300 \mathrm{ft} / \mathrm{min}$ minimum climb/descent rate
- Reference aircraft not maneuvering and not expected to maneuver during ITP
- Requested flight level open using current non-ITP separation standards
- Positive (closing) Mach difference between ITP and Reference aircraft of no more than 0.03 Mach.
- Maximum altitude change of 4000 feet


## 4 Probabilistic Model (Event) Tree

The risk of collision of an In-trail Procedure is estimated using a probabilistic model tree, Figure 2.


Figure 2. Event Tree for Collision Probability.

The probability is calculated assuming that the criteria for the ITP are met. Non-normal conditions such as an aircraft performing an ITP maneuver when not given a clearance or not being qualified for the maneuver are not included in this analysis. A Hazard Analysis has been conducted to cover non-normal conditions and is reported in [1].
There are 6 nodes in the tree from which the collision probability is derived. Node 1 is based on 5 factors which are combined to calculate the probability of proximity. Node 4 combines the probability of an undetected surveillance error due to failure with winds, altitude and track intercept error. Nodes $2,3,5$, and 6 are event probabilities.

### 4.1 Node 1. Probability of ITP aircraft within One Mile Segment of Reference Aircraft

Node 1 estimates the probability that the ITP aircraft crosses a one mile segment of track, which the Reference aircraft is occupying, during its climb/descent, Figure 3. The Reference aircraft track is arbitrarily divided into 1 mile segments and the probability that the ITP aircraft crosses a segment is estimated. This method was chosen to facilitate Monte Carlo simulations. However, as the simulations were performed with higher surveillance accuracy, the number of incidents in which the ITP aircraft crossed the Reference aircraft segment decreased to zero. This required larger data samples to be able to observe this event. It became evident that very small probabilities will require very large sets of data samples which exceeded the time and computational capabilities. Therefore, simulation is not a feasible method to calculate these probabilities.


Figure 3. Reference Aircraft in One Mile Segment.
A probabilistic analysis was then developed which uses the probability density functions of the five sources. The probability of the ITP aircraft being inside the Reference aircraft one mile segment is calculated using the five factors: position and velocity surveillance error, altitude error, winds, and track intercept error. The calculation follows a probabilistic method for random variables. The variance of each distribution is estimated for all the contributing factors. For factors that are the product of two random variables, the method described in Appendix B is used. The variance of all the factors is the sum of the variances [12].

$$
\begin{equation*}
\sigma^{2}=\sigma_{P I T P}^{2}+\sigma_{V I T P}^{2}+\sigma_{P R e f}^{2}+\sigma_{V R e f}^{2}+2 \times\left(\sigma_{a l}^{2}+\sigma_{a 2}^{2}+\sigma_{a 3}^{2}\right)+\sigma_{w}^{2}+\sigma_{t r k}^{2} \tag{1}
\end{equation*}
$$

where,
$\sigma_{\text {PITP }}$ and $\sigma_{\text {PRef }}$ are the standard deviations of the ITP and Reference aircraft position error and are given in section 4.1.1,
$\sigma_{V I T P}$ and $\sigma_{V \text { Ref }}$ are the standard deviations of the longitudinal error distributions caused by the ITP and Reference aircraft longitudinal velocity error and the time to climb/descend. These standard deviations are calculated in section 4.1.1,
$\sigma_{a 1}, \sigma_{a 2}$ and $\sigma_{a 3}$ are the standard deviations of the longitudinal error distributions for both the ITP and Reference aircraft caused by the combination of altitude error and by ground speed, wind, and velocity error. These standard deviations are calculated in section 4.1.2,
$\sigma_{w}$ is the standard deviation of the longitudinal error distribution caused by wind and the time to climb/descend. This standard deviation is calculated in section 4.1.3,
$\sigma_{t r k}$ is the standard deviation of the longitudinal error distribution caused by Flight Technical Error and the relative angle between the ITP and Reference aircraft tracks. This standard deviation is calculated is section 4.1.4.

The ITP aircraft is assumed to start the maneuver 15 nautical miles from the Reference aircraft which is worst case initiation. The probability of being in the Reference aircraft one mile segment is then the probability that the ITP aircraft is from 14.5 to 15.5 nautical miles of the starting position. This probability is given by the difference of the cumulative distribution functions,

$$
\begin{align*}
F(15.5)-F(14.5) & =\int_{-\infty}^{15.5} f(t) d t-\int_{-\infty}^{14.5} f(t) d t  \tag{2}\\
& =\left(1 / 2+1 / 2 \operatorname{erf}\left(\frac{15.5-\mu}{\sigma \sqrt{2}}\right)\right)-\left(1 / 2+1 / 2 \operatorname{erf}\left(\frac{14.5-\mu}{\sigma \sqrt{2}}\right)\right) \tag{3}
\end{align*}
$$

Where $f(t)$ is the probability density function based on the variance of the contributing factors, $\operatorname{erf}()$ is the error function used to calculate the cumulative distribution function $F(x), \sigma$ is the standard deviation of the density function which is equal to the positive square root of the variance as given by equation 1 , and
$\mu$ is the expected value of the distribution. Some of the calculations were validated using the initial Monte Carlo parametric simulations. Table 1 shows probabilities for values of position and velocity errors represented by the Navigation Accuracy Category, NAC (described in section 4.1.1). The probability is calculated for both the ITP and Reference aircraft having Navigation Accuracy Category for position, $\mathrm{NAC}_{\mathrm{P}}$, and Navigation Accuracy Category for velocity, $\mathrm{NAC}_{\mathrm{V}}$, values as shown on the table. The top numbers are calculated using equation 3 and the bottom numbers (in parenthesis) are calculated using Monte Carlo simulation. The number of samples for the Monte Carlo simulation is limited to 1 x $10^{11}$ due to run time durations. Hence, when the simulation produces zero incidents of ITP inside the one mile segment, this is represented in the table as $\left(<10^{-11}\right)$.

| NAC $\boldsymbol{p}$ | NAC $\boldsymbol{v}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 |
| 1 | 0.0123 | 0.0118 | 0.0118 | 0.0118 |
|  | $(0.0124)$ | $(0.0119)$ | $(0.0118)$ | $(0.0118)$ |
|  | $1.2 \times 10^{-4}$ | $2.6 \times 10^{-5}$ | $2.2 \times 10^{-5}$ | $2.2 \times 10^{-5}$ |
|  | $\left(1.2 \times 10^{-4}\right)$ | $\left(2.4 \times 10^{-5}\right)$ | $\left(2.1 \times 10^{-5}\right)$ | $\left(2.3 \times 10^{-5}\right)$ |
| 3 | $5.8 \times 10^{-8}$ | $2.0 \times 10^{-11}$ | $4.7 \times 10^{-12}$ | $3.9 \times 10^{-12}$ |
|  | $\left(9.0 \times 10^{-8}\right)$ | $\left(1.0 \times 10^{-11}\right)$ |  |  |


| NAC $\boldsymbol{p}$ | NAC $\boldsymbol{v}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 |
| 4 | $9.5 \times 10^{-10}$ <br> $\left(9.9 \times 10^{-10}\right)$ | $1.6 \times 10^{-20}$ <br> $\left(<10^{-11}\right)$ | $6.1 \times 10^{-23}$ | $2.9 \times 10^{-23}$ |
|  | $1.1 \times 10^{-10}$ |  |  |  |
|  | $4.3 \times 10^{-26}$ <br> $\left(<10^{-11}\right)$ | $2.9 \times 10^{-30}$ | $7.8 \times 10^{-31}$ |  |
| 6 | $6.6 \times 10^{-11}$ | $1.0 \times 10^{-27}$ <br> $\left(3.0 \times 10^{-11}\right)$ | $1.6 \times 10^{-32}$ | $\left.3.5 \times 10^{-33}\right)$ |
| 7 | $5.0 \times 10^{-11}$ | $1.3 \times 10^{-28}$ | $8.7 \times 10^{-34}$ | $1.7 \times 10^{-34}$ |
| 8 | $4.9 \times 10^{-11}$ | $1.0 \times 10^{-28}$ | $6.5 \times 10^{-34}$ | $1.2 \times 10^{-34}$ |
| 9 | $4.9 \times 10^{-11}$ | $9.8 \times 10^{-29}$ | $6.0 \times 10^{-34}$ | $1.1 \times 10^{-34}$ |
| 10 | $4.9 \times 10^{-11}$ | $9.7 \times 10^{-29}$ | $5.9 \times 10^{-34}$ | $1.1 \times 10^{-34}$ |
| 11 | $4.9 \times 10^{-11}$ | $9.7 \times 10^{-29}$ | $5.9 \times 10^{-34}$ | $1.1 \times 10^{-34}$ |

Table 1. Probability of ITP Aircraft within One Nautical Mile Segment; Surveillance Accuracy.

The values in Table 1 are for an altitude error of 75 feet ( $95 \%$ ), a wind gradient standard deviation of 16.03 knots, 45 degrees merging tracks, track intercept error with FTE of 2 NM ( $95 \%$ ), a positive delta Mach of 0.03, 2000 feet altitude difference between the ITP and Reference aircraft, and 300 feet $/$ minute climb/descent rate. Table 1 is given for NAC values. For aircraft with ADS-B equipment which transmit NUC, the probability of ITP aircraft to the 1 NM segment should be correlated to Table 1. As specified in RTCA DO-260A [6], a NUC ${ }_{P}$ value of 5 corresponds to a NAC ${ }_{P}$ of 6 , NIC of 6 and SIL of 2. Values of $\mathrm{NUC}_{\mathrm{R}}$ and $\mathrm{NAC}_{\mathrm{V}}$ correspond directly to each other. The next sections describe the surveillance error, altitude error, wind factor, and track intercept error.

### 4.1.1 Surveillance Error

The ITP application uses ADS-B (Automated Dependant Surveillance-Broadcast) to perform the criteria check regarding range and ground speed differential. The accuracy and integrity of the ADS-B information broadcast from the Reference aircraft as well as the ITP aircraft own position and velocity data will be factors in the collision risk. The accuracy and integrity of ADS-B information are defined in the RTCA DO-242, DO-242A, DO260 and DO260A documents [3,4,5,6]. RTCA DO242 and DO260 documents quantify the surveillance error by means of a Navigation Uncertainty Category (NUC) parameter which is broadcast as part of the surveillance data message. The position and velocity (rate) errors are characterized by $\mathrm{NUC}_{p}$ and $\mathrm{NUC}_{\mathrm{R}}$, respectively. The value for the NUC parameters are obtained from the surveillance equipment which might provide a Horizontal Figure of Merit (HFOM) or Horizontal Protection Level (HPL). How the NUC parameters are derived is described in details in the RTCA documents.

RTCA DO-242A and DO260A define four parameters: NACp, NACv, NIC, and SIL. These parameters correspond to Navigation Accuracy Category for position and velocity, Navigation Integrity Category, and Surveillance Integrity Level. DO260A defines how to interpret messages from equipment which has been designed to the DO260 standard. Hence, NAC, NIC, and SIL parameters will be inferred from a NUC parameter. DO260A also defines how equipment designed to the DO260 standard interprets NAC, NIC, and SIL. Hence, a NUC parameter will be extracted from NAC, NIC, and SIL. The values shown in Table 1 give probabilities as a function of NACP and NACv. However, the results can be converted to the

NUCP and $N_{\text {P }}$ parameters using the tables in the next section and the equivalence conversion defined in RTCA DO-260A.

## Surveillance Position Error

The NUCp and NACp define the horizontal accuracy of the position reported (and vertical for NUC 8, 9 and NAC $9,10,11$ ). Tables 2 and 3 show the $95 \%$ horizontal and vertical accuracy bounds for values of NUC and NAC, respectively.

| NUC $_{\boldsymbol{P}}$ | Horizontal <br> Error (95\%) | Vertical Error <br> (95\%) |
| :---: | :--- | :--- |
| 0 | Unknown | Unknown |
| 1 | $<10 \mathrm{NM}$ | Barometric Altitude |
| 2 | $<5 \mathrm{NM}$ | Barometric Altitude |
| 3 | $<1 \mathrm{NM}$ | Barometric Altitude |
| 4 | $<0.5 \mathrm{NM}$ | Barometric Altitude |
| 5 | $<0.25 \mathrm{NM}$ | Barometric Altitude |
| 6 | $<0.1 \mathrm{NM}$ | Barometric Altitude |
| 7 | $<0.05 \mathrm{NM}$ | Barometric Altitude |
| 8 | $<10 \mathrm{~m}$ | $<15 \mathrm{~m}$ |
| 9 | $<3 \mathrm{~m}$ | $<4 \mathrm{~m}$ |
| $10-15$ | TBD | TBD |

Table 2. Navigation Uncertainty Category for Position (NUC $\mathbf{P}_{\mathbf{P}}$ ), Reference [4].

| $N A C_{P}$ | 95\% Horizontal and Vertical Accuracy Bounds |
| :---: | :---: |
| 0 | EPU $\geq 18.52 \mathrm{~km}$ (10 NM) |
| 1 | EPU < 18.83 km (10 NM) |
| 2 | EPU < 7.408 km (4 NM) |
| 3 | EPU < 3.704 km (2 NM) |
| 4 | EPU < 1852 m (1 NM) |
| 5 | EPU < 926m (0.5 NM) |
| 6 | EPU < 555.6 m (0.3 NM) |
| 7 | EPU < 185.2 m (0.1 NM) |
| 8 | EPU < 92.6m (0.05 NM) |
| 9 | EPU < 30 m and VEPU < 45 m |
| 10 | EPU < 10 m and VEPU < 15 m |
| 11 | EPU < 3 m and VEPU < 4 m |
| 12-15 | Reserved |

Table 3. Navigation Accuracy Category for Position (NACP), Reference [6].

The Estimated Position Uncertainty (EPU) is the $95 \%$ accuracy bound on horizontal position. The Vertical Estimated Position Uncertainty (VEPU) is the $95 \%$ accuracy bound on the vertical position (geometric altitude). The $95 \%$ accuracy bounds represents the probability that the aircraft will be within the stated bound with a 0.95 probability. This bound corresponds approximately to the 2 -sigma deviation in a Normal (Gaussian) distribution. Figure 4 shows the accuracy bound and distribution for the position uncertainty.


Figure 4. Position Uncertainty for 95\% Accuracy Bound and Probability Distribution Function.

For example, an aircraft reporting a position with a NACp value of 5 will be within 0.5 NM of the reported position $95 \%$ of the time. Following a Normal distribution as shown in Figure 4, this aircraft reporting a $\mathrm{NACP}_{\mathrm{P}}$ value of 5 will be within 0.75 NM of the reported position $99.7 \%$ of the time. The standard deviation used in equation 1 is obtained from the Accuracy Bound (the 0.95 -quantile),

$$
\begin{equation*}
\sigma_{\text {PITP }}=\sigma_{P R e f}=\frac{\text { AccuracyBound }}{1.96} \mathrm{NM} \tag{4}
\end{equation*}
$$

## Surveillance Velocity Vector Error

Velocity error is characterized by the $\mathrm{NUC}_{\mathrm{R}}$ and NACv parameters. Table 4 shows the horizontal and vertical velocity figures of merit for the $\mathrm{NUC}_{\mathrm{R}}$ and NACv value encoding.

| $\begin{gathered} N U C_{R} \\ N A C_{V} \end{gathered}$ | $\mathrm{HFOM}_{R}$ value <br> (95\% accuracy) |  | VFOM $_{R}$ value <br> (95\% accuracy) |
| :---: | :---: | :---: | :---: |
| 0 | $\mathrm{HFOM}_{\mathrm{R}} \geq 10 \mathrm{~m} / \mathrm{s}$ or unknown | AND | $\mathrm{VFOM}_{\mathrm{R}} \geq 15.24 \mathrm{~m} / \mathrm{s}(50 \mathrm{fps})$ or unknown |
| 1 | $\mathrm{HFOM}_{\mathrm{R}}<10 \mathrm{~m} / \mathrm{s}$ | AND | $\mathrm{VFOM}_{\mathrm{R}}<15.24 \mathrm{~m} / \mathrm{s}(50 \mathrm{fps})$ |
| 2 | $\mathrm{HFOM}_{\mathrm{R}}<3 \mathrm{~m} / \mathrm{s}$ | AND | $\mathrm{VFOM}_{\mathrm{R}}<4.57 \mathrm{~m} / \mathrm{s}(15 \mathrm{fps})$ |
| 3 | $\mathrm{HFOM}_{\mathrm{R}}<1 \mathrm{~m} / \mathrm{s}$ | AND | $\mathrm{VFOM}_{\mathrm{R}}<1.52 \mathrm{~m} / \mathrm{s}(5 \mathrm{fps})$ |
| 4 | $\mathrm{HFOM}_{\mathrm{R}}<0.3 \mathrm{~m} / \mathrm{s}$ | AND | $\mathrm{VFOM}_{\mathrm{R}}<0.46 \mathrm{~m} / \mathrm{s}(1.5 \mathrm{fps})$ |

Table 4. Navigation Uncertainty and Navigation Accuracy Category for Velocity ( $\mathbf{N U C}_{\mathbf{R}}, \mathrm{NAC}_{\mathrm{V}}$ ).
The velocity error used in the calculation of collision probability refers to the velocity reported by the surveillance equipment on-board. The Mach difference criterion imposed on the procedure is based on the assigned Mach numbers given to the ITP and Reference aircraft by the oceanic/remote ATC. The collision probability calculation does not take into account the possibility that the aircraft are flying at a Mach number other than that assigned by ATC. The standard deviation used in equation 1 is obtained from the velocity Accuracy Bound (the 0.95 -quantile) and the time to altitude,

$$
\begin{equation*}
\sigma_{V I T P}=\sigma_{V R e f}=\frac{3600 \mathrm{sec} / \mathrm{hour} \times H F O M_{R} \mathrm{~m} / \mathrm{s}}{1.96 \times 1852 \mathrm{~m} / \mathrm{NM}} \times t_{a l t} \mathrm{NM} \tag{4}
\end{equation*}
$$

where,

$$
\begin{equation*}
t_{\text {alt }}=\frac{2000 \mathrm{ft}}{300 \mathrm{ft} / \mathrm{min} \times 60 \mathrm{~min} / \mathrm{hour}} \text { hours } \tag{5}
\end{equation*}
$$

### 4.1.2 Altitude Error

Altitude error is estimated from standard barometer error models. The altitude error is assumed to follow a Normal distribution with plus or minus 75 feet ( 22.86 meters) $95 \%$ of the time. The altitude error contributes to the longitudinal error by increasing or decreasing the time to altitude during a climb/descent. The altitude error is multiplied by closure rate, wind component, and velocity error to obtain longitudinal distance. The standard deviation used in equation 1 for the first component of altitude error is,

$$
\begin{equation*}
\sigma_{a l}=\frac{75 \mathrm{ft}}{1.96 \times 300 \mathrm{ft} / \min \times 60 \text { min } / \text { hour }} \times g s \mathrm{NM} \tag{6}
\end{equation*}
$$

where $g s$ is the relative ground speed between the two aircraft. Because $\sigma_{a 2}$ and $\sigma_{a 3}$ are the product of two random variables, their standard deviations are numerically calculated using equation B1 of Appendix B.

### 4.1.3 Wind

Two conditions are considered when determining the effect of wind on distance reduction between the ITP and Reference aircraft. The first condition is when the relative trajectory angle between the aircraft is approximately zero. The second condition is when the relative trajectory angle between the aircraft is greater than zero and less than 45 degrees.

## Wind, Zero Relative Angle Between Aircraft

The ITP criteria bounds the ground speed difference and Mach number difference between the ITP and Reference aircraft. This in turn constrains the wind difference between the ITP and Reference aircraft levels. The ground speed criterion limits the ground speed difference (for 15 NM separation) to 20 knots
or less:

$$
\begin{equation*}
v g_{I T P}-v g_{R e f} \leq 20 \tag{6}
\end{equation*}
$$

The Mach criteria limits the Mach difference to 0.03 Mach:

$$
\begin{equation*}
v m_{\text {ITP }}-v m_{\text {Ref }}=\Delta \text { Mach } \leq 0.03 \tag{7}
\end{equation*}
$$

where $v g_{I T P} v g_{\text {Ref }} v m_{I T P}$ and $v m_{\text {Ref }}$ are the ground speeds and Mach speeds of the ITP and Reference aircraft. Approximating the speed of sound to 576.6 knots $^{1}$, it is possible to relate the ground speed, Mach speed and wind speed:

$$
\begin{align*}
& W F L_{I T P}+v m_{I T P} \times 576.6=v g_{I T P}  \tag{8}\\
& W F L_{\text {Ref }}+v m_{I T P} \times 576.6=v g_{\text {Ref }} \tag{9}
\end{align*}
$$

where $W F L_{I T P}$ and $W F L_{\text {Ref }}$ are the wind speeds at the ITP and Reference aircraft flight levels, respectively. Substituting for $v g_{I T P}$ and $v g_{\text {Ref }}$ into the ground speed criterion, equation 6 , and using the Mach criterion, equation 7, yields:

$$
\begin{equation*}
W F L_{I T P}-W F L_{R e f}+\Delta \text { Mach } \times 576.6 \leq 20 \text { knots } \tag{10}
\end{equation*}
$$

Figure 5 shows a graph of possible wind speeds and Mach numbers to satisfy equation 10, with the plane satisfying the equality and the darker shaded volume the inequality.


Figure 5. Wind and Mach Delta to Satisfy Ground Speed and Mach Criteria for Track Angle 0.

[^0]Equation 10 represents the requirements for a climb/descend for separations of 15 NM or more and maximum 20 knots ground speed differential. The ITP is also defined for a 20 NM separation and a maximum of 30 knots ground speed difference. A similar equation must be satisfied for this case.
Figure 6 is a two dimensional view of the wind and Mach conditions to satisfy ground speed and Mach criteria. The graph of Figure 6 combines the wind speeds at the ITP and Reference aircraft flight levels into a relative wind speed. Conditions on the line and to the left and below the line, depicted by the shaded area, satisfy the criteria.


Figure 6. Relative Wind and Mach Delta to Satisfy Ground Speed and Mach Criteria for 0 Angle.

Winds and Mach numbers can contribute to reducing the distance between the ITP aircraft and the Reference aircraft. Because of the ground speed and Mach requirements, the relative wind speed at the ITP and Reference aircraft flight levels is limited by equation 10. In addition to the reduction in distance permitted by equation 10, there could be additional reduction in distance due to winds between the two levels. Whether or not the wind between levels contributes to additional reduction in distance depends on the wind profile. Figure 7 shows positive and negative wind gradients on the left and right side of the figure, respectively.


Figure 7. Positive and Negative Wind Gradient Between Flight Levels.

When a positive wind gradient is encountered as the left example of Figure 7, the close-in speed is limited by the Mach criterion, equation 7. The maximum close-in speed at levels between the ITP aircraft and the Reference aircraft will occur at FL(Ref), the Reference aircraft flight level. The maximum closein speed will be 17.3 knots $^{2}$ at $\mathrm{FL}($ Ref). Therefore, the close-in speed will be 17.3 knots or less everywhere between FL(ITP) and FL(Ref). The positive wind gradient case includes the case of zero wind gradient.

When a negative wind gradient is encountered as the right example of Figure 7, the close-in speed could be limited by the Mach criterion or the ground speed criterion, depending on the magnitude of the wind speed differential. In the worst case condition, the close-in speed will be no more than 20 knots at FL(ITP) and decreases to no more than 17.3 knots at FL(Ref).

The positive, zero, and negative wind gradients maximum close-in speeds apply similarly to in-trail climbs and descents as well as to lead climbs and descents. For the condition where a maximum of 30 knots and 20 nautical miles separation is used, similar constraints will occur for positive and negative wind gradients. For a positive or zero wind gradient, the close-in speed is limited by the Mach criterion to 17.3 knots. For a negative wind gradient, the close-in speed could be a maximum of 30 knots at the ITP level and decrease to a maximum of 17.3 knots at the Reference aircraft level.

Figure 8 shows an example where the winds between levels further reduces the separation between aircraft. This example produces the greatest reduction in distance due to winds and Mach numbers.


Figure 8. Wind Gradient Between ITP and Reference Aircraft Levels.
The chevron figure between levels depict the wind velocity gradient. The worst case scenario is

[^1]considered in this example and in the analysis for calculating collision probability. In this example, the winds at the ITP and Reference aircraft levels are such that it meets the ITP requirement. The wind between levels is such that it contributes to a further reduction of separation as the ITP aircraft performs the maneuver. The wind gradient magnitude is obtained from observations of oceanic winds above FL260 [7]. The data gathered over a 7 month period considers along-track wind differences between 2,000 and 4,000 feet vertical separations.
The case used for the ITP safety analysis is worst case and extreme in two ways: First, the maximum wind differential in the ITP analysis occurs at 1,000 feet vertical separation. Hence, the average wind differential magnitudes will likely be less than for 2,000 and 4,000 feet vertical separations in real life situations; Second, it is improbable to encounter wind currents that will be approximately the same at levels 2,000 feet apart and significantly higher between the levels as depicted in Figure 8. The observations reported in [7] are for wind gradients as shown in Figure 7, in which the wind increases or decreases from one level to the other.

The worst case scenario wind model, used in the analysis and depicted in Figure 8, will allow the following close-in rates: at the ITP aircraft level, the close-in speed will be a maximum of 20 knots for the 15 nautical mile or more and 30 knots for the 20 nautical miles or more range; the close-in speed will increase to 20 knots ( 30 knots) + maximum gradient at the middle of the climb (descent); the close-in speed will decrease to a maximum of 17.3 at the Reference aircraft level. The wind speed distribution used for the calculations is a normal distribution with a 16.03 knots standard deviation. The standard deviation used in equation 1 for the wind component when the angle is approximately zero is,

$$
\begin{equation*}
\sigma_{w}=\frac{16.03}{2} \times t_{a l t} \mathrm{NM} \tag{11}
\end{equation*}
$$

where $t_{\text {alt }}$ is the time to altitude as defined by equation 5 .

## Wind, Relative Trajectory Angle Between Aircraft Greater than $\mathbf{0}$ up to $\mathbf{4 5}$ degrees.

The second condition considered for distance reduction due to wind is the case when the ITP and Reference aircraft have a relative trajectory difference greater than 0 and less than 45 degrees. This case considers a wind model in which the magnitude of the wind is the same from the ITP aircraft flight level to the Reference aircraft flight level but the direction of the wind changes from the two flight levels. The example of Figure 9 shows a wind speed of 120 knots out of the southeast at flight level FL340 changing to 120 knots out of the south at FL360.


Figure 9. Change in ITP Aircraft Ground Speed due to Wind Direction Difference at Flight Levels.

For this example, the ground speed of the ITP aircraft is 382 knots at FL 340 and 366 knots for the Reference aircraft at FL 360. The geometry and conditions satisfy the ITP criteria. When the ITP aircraft climbs to the Reference aircraft flight level (FL 360), its ground speed increases to 455 knots. This produces a ground speed difference of 89 knots. This example geometry and wind direction shift represents approximately the worst case for the given wind speed. A further wind shift from the east at FL340 will allow the reference aircraft to have a higher Mach speed, up to the maximum Mach speed differential, producing a ground speed difference of 95 knots.

Assuming a continuous direction change between flight levels, 120 knot winds, and 300 feet/minute climb rate, the worst case will produce an erosion in distance between the aircraft of 3.2 nautical miles for 1000 feet, 6.4 nautical miles for 2000 feet, and 9.6 nautical miles for 3000 feet. That is, for the worst case and not considering any other factor, the ITP aircraft will cross 5.4 nautical miles from the Reference aircraft.

There is a relatively high probability of a 45 degree wind shift between altitudes for wind speeds in the order of 15 to 20 knots. Hourly observations by a Radar Wind Profiler Radio Acoustic Sounding System located in Whitewater, Kansas [14] shows wind shift of 45 degrees between 3000 feet altitude in about 1 in 10 observations. However, a wind direction shift of 45 degrees with a wind speed magnitude of 120 knots (example of Figure 9) will cause an along track difference of 85 knots. Reference 7 reports an along track difference in the 80 to 85 knots range of only 1 in 28000 observations for 4000 feet altitude difference. For an altitude difference of 2000 feet, an along track wind difference of more than 75 knots was never observed for 70000 reports. The probability density function of 45 degrees wind shifts is estimated based on the upper bound imposed by the north Atlantic wind observations. The standard deviation used in equation 1 for the wind component when the angle is larger than zero and less than 45 is,

$$
\begin{equation*}
\sigma_{w}=\frac{\frac{16.03}{2} \times \sin (\theta) \times \sin (\gamma)}{(\sin (45))^{2}} \times t_{a l t} \mathrm{NM} \tag{12}
\end{equation*}
$$

where $\theta$ is the track angle between the ITP and Reference aircraft, $\gamma$ is the wind shift angle, and $t_{a l t}$ is the time to altitude as defined by equation 5. This equation is only valid for $0<\theta \leq 45^{\circ}$ and $0<\gamma \leq 45^{\circ}$.

### 4.1.4 Track Intercept Error

The initial separation required for initiation of an ITP could be eroded due to track deviation when an ITP is performed at converging tracks. Figure 10 shows a top view of merging tracks and the calculation of range between the Potentially Blocking aircraft and the ITP aircraft.


Figure 10. Merging Tracks.

The maximum track intercept angle $\theta$ to perform an ITP at merging tracks is 45 degrees. The along track range ${ }^{3}$ between the Reference aircraft and the ITP aircraft is given by:

$$
\text { along track range }=D_{I T P}-D_{\text {Ref }}
$$

When the Reference aircraft deviates from its nominal track, an increase or reduction of range will result at the merging point. The track of an aircraft is defined by the error model shown in Figure 11.


Figure 11. Track Error Model.

[^2]The desired path is the physical track on which the aircraft is expected to fly. The computed path is the one calculated by the Flight Management Computer and includes numerical error. Flight Technical Error is the ability of the aircraft to follow the computed path. The estimated position is the surveillance equipment current estimated aircraft location.
For the ITP initiation distance criterion, the along track range between the ITP and Reference aircraft is calculated between the estimated paths of the aircraft and not the desired paths. Therefore, the deviation taken into account to calculate track intercept error is the Flight Technical Error. The difference between the estimated and actual position is taken into account by the surveillance error factor calculated in Section 4.1.1. The expected Flight Technical Error (FTE) is given in Table 5 and was obtained from reference [15].

| Flight Phase | Manual | Flight Director | Autopilot |
| :---: | :---: | :---: | :---: |
| Oceanic | 2.0 NM | 0.50 NM | 0.25 NM |
| En-route | 1.0 | 0.50 | 0.25 |
| Terminal | 1.0 | 0.50 | 0.25 |
| Approach | 0.50 | 0.25 | 0.125 |

Table 5. Expected Flight Technical Error, 95\%.
Reference [15] also shows the results of a study conducted for large and small transport aircraft in which the Flight Technical Error was measured. These results are shown in Table 6.

| Flight Phase | Manual Flight with <br> Map Display | LNAV with Flight <br> Director Coupled | LNAV with Autopilot <br> Coupled |
| :---: | :---: | :---: | :---: |
| En-route | $0.502-0.918 \mathrm{NM}$ | $0.111-0.232 \mathrm{NM}$ | $0.055-0.109 \mathrm{NM}$ |

Table 6. Measured Flight Technical Error on Small and Large Transport Aircraft.
Although it is expected that an aircraft flying on a remote/oceanic route will have the autopilot engaged, the worst case is assumed with a Flight Technical Error under manual flight. A Flight Technical Error value of $2 \mathrm{NM}(95 \%)$ is used for the calculation of track intercept error.

Figure 12 shows the geometrical variables used in the calculation of separation erosion due to track intercept error. It is assumed that after range calculation, the Reference aircraft follows the E track instead of the N track.


Figure 12. Geometry for Calculation of Distance Erosion due to Track Error.

The Reference aircraft will intercept the merging track at point P1 instead of P0. However, the Reference aircraft will arrive at point P 1 sooner than it would have arrived at P 0 following track N . The Reference aircraft will further fly to point P2 before the ITP aircraft reaches the estimated range distance to P0. The erosion of separation is the resulting segment $S$. The value of segment $S$ is given by:

$$
\begin{equation*}
S=E+M-N \tag{11}
\end{equation*}
$$

where

$$
\begin{align*}
& E^{2}=F T E^{2}+N 2^{2}  \tag{14}\\
& N 2=N-N 1  \tag{15}\\
& N 1=\frac{F T E}{\tan (\theta)}  \tag{16}\\
& M=\frac{F T E}{\sin (\theta)} \tag{17}
\end{align*}
$$

and

$$
\begin{equation*}
S=\sqrt{F T E^{2}+\left(N-\frac{F T E}{\tan (\theta)}\right)^{2}}+\frac{F T E}{\sin (\theta)}-N \tag{18}
\end{equation*}
$$

This geometric equation makes operational sense when the value of N 1 does not exceed the value of N and when the value of N is not too small compared to the FTE. Figure 13 gives examples for cases when N 1 is larger than N and FTE is comparable to N .


Figure 13. Examples of Scenarios where the above Equations should not be Applicable.

In order for the aircraft to be at its maximum FTE deviation, extremely improbable maneuvers would have to be performed. In the first example, where N1 > N, the Reference aircraft is at a distance N from point P0. It must turn more than 90 degrees to reach point P1. The trajectory is depicted as the dark line segments. The second example shows a value N of 3 nautical miles, an FTE of 2 and an angle of 45 degrees. This example also produces a large angle turn from the nominal trajectory. Since these are not probabilistically credible maneuvers, the values of N, FTE and $\theta$ are bounded in the calculations of separation erosion due to track deviation error. Figures 14, 15, and 16 give values of separation erosion as a function of N, FTE and $\theta$, respectively. For values of $\mathrm{N}=30, \mathrm{FTE}=2 / 1.96, \theta=45$ degrees and using equation 18 , the standard deviation for track error used in equation 1 is,


Figure 14. Separation Erosion as function of Reference Aircraft Distance to Merge Point.


Figure 15. Separation Erosion as function of FTE.


Figure 16. Separation Erosion as a function of Merge Angle.

### 4.2 Node 4. Undetected Surveillance Failure

RTCA DO260 and DO260A specify how often an ADS-B equipment can transmit data which is outside of a containment bound without being detected. This is represented by the Navigation Integrity Category, NIC, and Surveillance Integrity Level, SIL, parameters. ADS-B equipment designed to the DO-260 standard will not transmit NIC and SIL parameters. For these equipment, an integrity category will be inferred from the NUC parameter.
The NIC parameter defines the containment bound. The SIL parameter defines the probability that the reported position is outside the containment bound due to a surveillance failure and this condition is not detected. Tables 7 and 8 give the values of NIC and SIL for containment bounds and probability of exceeding the bounds without detection.

| NIC | Containment Radius, Rc, and <br> Vertical Protection Limit, VPL |
| :---: | :--- |
| 0 | $\mathrm{Rc} \geq 20 \mathrm{NM}$ |
| 1 | $\mathrm{Rc}<20 \mathrm{NM}$ |
| 2 | $\mathrm{Rc}<8 \mathrm{NM}$ |
| 3 | $\mathrm{Rc}<4 \mathrm{NM}$ |
| 4 | $\mathrm{Rc}<2 \mathrm{NM}$ |
| 5 | $\mathrm{Rc}<1 \mathrm{NM}$ |
| 6 | $\mathrm{Rc}<0.6 \mathrm{NM}$ |
| 7 | $\mathrm{Rc}<0.2 \mathrm{NM}$ |
| 8 | $\mathrm{Rc}<0.1 \mathrm{NM}$ |
| 9 | $\mathrm{Rc}<75 \mathrm{~m}$ and $\mathrm{VPL}<112 \mathrm{~m}$ |
| 10 | $\mathrm{Rc}<25 \mathrm{~m}$ and $\mathrm{VPL}<37.5 \mathrm{~m}$ |
| 11 | $\mathrm{Rc}<7.5 \mathrm{~m}$ and $\mathrm{VPL}<11 \mathrm{~m}$ |

Table 7. Navigation Integrity Category, NIC, and Protection Bounds.

| SIL | Probability of Exceeding the Rc Integrity <br> Containment Radius Without Detection |
| :---: | :--- |
| 0 | Unknown |
| 1 | $1 \times 10^{-3}$ per flight hour or per operation |
| 2 | $1 \times 10^{-5}$ per flight hour or per operation |
| 3 | $1 \times 10^{-7}$ per flight hour or per operation |

Table 8. Surveillance Integrity Level and Probability of Exceeding the Protection Bound without Detection.

Node 4 estimates the probability that the ITP aircraft crosses a one mile segment of track which the Reference aircraft is occupying when there are undetected surveillance errors due to failure. There are four cases to consider when there is a surveillance failure: the surveillance failure is detected and the
aircraft position is inside or outside the containment bounds; the surveillance failure is undetected and the aircraft position is inside or outside the containment bounds. When the surveillance failure is detected, the criteria for ITP will not be met and the procedure shall not be performed. Node 4 considers the last two cases: the surveillance failure is undetected and the aircraft is inside the containment bound; the surveillance failure is undetected and the aircraft is outside the containment bound.

Figure 17 shows an example that illustrates a distribution without failure and a surveillance failure distribution with reported positions inside and outside the containment bound.


Figure 17. Failure Condition Distribution Example.
The probability that an error goes undetected by integrity monitoring decreases as the reported position moves further away from the actual position. The horizontal containment radius is defined as the largest distance from the actual location such that the integrity monitoring fails to detect with a probability less than or equal to the value represented by SIL. The calculation of Node 4 is performed by setting the mean of the failure condition distribution over the containment bound with a standard deviation of 0.255 nautical miles. An approximation is used for the distribution inside the containment bounds to be uniform. The case inside the containment bounds is multiplied by the probability of GPS satellite failure which is conservatively set to $10^{-4}$ from reference [16]. The case outside the containment bound is multiplied by the probability represented by SIL.
The analysis is similar to that performed for Node 1 and the undetected position error due to failure is one of the factors in determining reduction in distance to the Reference aircraft. The values in Table 9 are for an altitude error of 75 feet ( $95 \%$ ), a wind gradient standard deviation of 16.03 knots, 45 degrees merging tracks, a track intercept error equivalent to a FTE of 2, a NAC ${ }_{\mathrm{V}}$ of 2, a positive delta Mach of 0.03 , a 2000 feet altitude difference, and 300 feet/minute climb/descent rate.

| NIC | SIL |  |  |
| :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 |
| 1 | $1.1 \times 10^{-5}$ | $4.7 \times 10^{-7}$ | $3.7 \times 10^{-7}$ |
| 2 | $4.3 \times 10^{-8}$ | $1.6 \times 10^{-9}$ | $1.2 \times 10^{-9}$ |
| 3 | $3.0 \times 10^{-16}$ | $5.8 \times 10^{-18}$ | $2.9 \times 10^{-18}$ |
| 4 | $1.5 \times 10^{-22}$ | $2.1 \times 10^{-24}$ | $6.9 \times 10^{-25}$ |
| 5 | $3.3 \times 10^{-26}$ | $4.3 \times 10^{-28}$ | $1.0 \times 10^{-28}$ |
| 6 | $9.4 \times 10^{-28}$ | $1.2 \times 10^{-29}$ | $2.5 \times 10^{-30}$ |


| NIC | SIL |  |  |
| :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 |
| 7 | $2.4 \times 10^{-29}$ | $3.1 \times 10^{-31}$ | $7.6 \times 10^{-32}$ |
| 8 | $1.0 \times 10^{-29}$ | $1.4 \times 10^{-31}$ | $4.2 \times 10^{-32}$ |
| 9 | $7.6 \times 10^{-30}$ | $1.1 \times 10^{-31}$ | $3.4 \times 10^{-32}$ |
| 10 | $7.1 \times 10^{-30}$ | $1.0 \times 10^{-31}$ | $3.3 \times 10^{-32}$ |
| 11 | $7.0 \times 10^{-30}$ | $1.0 \times 10^{-31}$ | $3.3 \times 10^{-32}$ |

Table 9. Probability of ITP Aircraft inside One Mile Segment; Undetected Position Error.

### 4.3 Node 2. Conditional Collision Trajectory

This node is the probability that the trajectories are collision trajectories, given that the ITP aircraft will pass inside the one mile segment of the Reference aircraft. Because of the 0.03 Mach restriction, the maximum relative speed between the ITP and Reference aircraft is 17.3 knots when they are at or near the same altitude. Wind has an insignificant or no effect on the relative speed between the aircraft because wind gradient is zero (or near zero) at co-altitude. Figure 18 shows possible locations of the ITP aircraft with the ITP aircraft climbing and gaining on the Reference aircraft.


Figure 18. Geometry for Collision Trajectory.

The aircraft are modeled by a rectangular box as depicted in the figure. If any part of the box of the ITP aircraft falls inside the dashed lines, then the aircraft will be in a collision trajectory.

A vertical speed of $300 \mathrm{ft} /$ minute and a relative horizontal speed of 17.3 knots produces an angle $\theta$ of

$$
\begin{align*}
\theta & =\arctan (2.96 / 17.3)  \tag{20}\\
& =9.72^{\circ} \tag{21}
\end{align*}
$$

The length of the aircraft is represented by $l$ and its height by $h$. The collision trajectory length is given by

$$
\begin{equation*}
c t=l+x l+x 2+x 3 \tag{22}
\end{equation*}
$$

where

$$
\begin{equation*}
x l=\frac{h}{\tan (\theta)} \tag{23}
\end{equation*}
$$

$$
\begin{align*}
& x 2 \approx \frac{h}{\sin (\theta)}  \tag{24}\\
& x 3=l \cos (\alpha)-\frac{l \sin (\alpha)}{\tan (\theta)} \tag{25}
\end{align*}
$$

The probability of a collision trajectory given a one mile proximity is

$$
\begin{align*}
P(c t \mid 1 \mathrm{NM}) & =\frac{c t}{1 N M}  \tag{26}\\
& =\frac{l+\frac{h}{\tan (\theta)}+\frac{h}{\sin (\theta)}+l \cos (\alpha)-\frac{l \sin (\alpha)}{\tan (\theta)}}{1 N M} \tag{27}
\end{align*}
$$

As the vertical speed increases, the angle $\theta$, equation 20 , increases, and the angle of attack ${ }^{4} \alpha$ of the aircraft increases. Increasing $\theta$ and $\alpha$ decreases $c t$ and the probability of a collision trajectory, equations 26 and 27. Also, smaller Mach number differentials will increase the angle $\theta$ and reduce the probability of a collision trajectory. Therefore, the $300 \mathrm{ft} /$ minute climb and 0.03 Mach closing produce the worst probability of collision trajectory. Higher climb rates and lower Mach closing speeds reduce the probability. Using the values $300 \mathrm{ft} /$ minute, 1.0 degree angle $\alpha$, delta Mach of 0.03 , aircraft height $h$ of 65 feet, aircraft length $l$ of 200 feet, and 6076 feet in one nautical mile,

$$
\begin{equation*}
P(c t \mid 1 \mathrm{NM})=943.1 / 6076=0.1552 \tag{28}
\end{equation*}
$$

A reduction in distance to within the one nautical mile segment of the Reference aircraft will have a large horizontal surveillance position error. When a large horizontal longitudinal error is experienced by the surveillance equipment, it is very likely that a lateral position error is also present. This lateral position error will further reduce the probability of a collision trajectory because the error will cause the aircraft to fly offset of its center track. However, the worst case is assumed in which a large longitudinal error exists but no lateral error is present.

### 4.4 Node 3. No Appropriate See and Avoid Actions.

A crew operating an aircraft, regardless of whether an operation is conducted under instrument flight rules or visual flight rules, should perform the function of see and avoid to prevent a collision. This requirement is mandated in the USA National Airspace System under Chapter 14 of the Code of Federal Regulations, Part 91.113. A flight crew of an aircraft which is in a collision trajectory can successfully implement the see and avoid functionality if it can visually acquire the traffic in sufficient time to perform an avoidance maneuver.

There are several factors that affect the probability of visually acquiring a traffic aircraft by the own ship (or the own ship being visually acquired by the traffic). The geometry of the encounter is one of such factors. Figure 21 in Section 4.6 depict possible geometries for which visual acquisition is not possible.

A visual acquisition program, Visual 3D, was used to determine the probability that an aircraft, which is in a collision course with a traffic aircraft, will see the traffic in sufficient time to avoid a collision [8,9]. The time required to avoid a collision after visually acquiring a target has been estimated to be on the order of 12 seconds [10]. The visual acquisition program uses the following parameters to estimate the probability of seeing the traffic:

[^3]Type of aircraft, traffic
True air speed, own
True air speed, traffic
Crossing angle
Climb rate, own
Climb rate, traffic
Visual range

Resolution limit
Field of view, H up
Field of view, H down
Field of view, left
Field of view, right
Search, pilot 1
Search, pilot 2

Beta 0 , search effectiveness
Beta 1, search effectiveness
Tau 1, beta transition, time
Tau 2, Pacq, time
D1, beta transition, range
D2, Pacq, range

The probability of visually acquiring the traffic aircraft during an ITP maneuver with a 10 NM visual range is estimated to be near one for the geometry conditions described in Section 4.3. Figure 19 shows an aircraft at approximately 1 NM and 1000 feet above the own ship (photo courtesy of Rick Shay).


Figure 19. Aircraft on Track alpha of North Atlantic Track System.

At a closing rate of 17.3 knots and a distance of 1NM, it will take 208 seconds before a collision results. This will greatly exceed the 12 second collision avoidance time. The other critical factor in visually avoiding a mid air collision is weather conditions. The probability of having IMC or VMC is obtained from [11] and it is 0.2 and 0.8 , respectively. The dominant factor in visually acquiring or not acquiring the traffic, for the ITP geometries and speeds, is the weather conditions. Therefore, it is assumed that in good visibility the crew will be able to see and avoid the traffic with probability 1 and in bad visibility it will see and avoid with probability 0 . Node 3 is given the value 0.2 for no see and avoid.

### 4.5 Node 5. Geometry for Undetected Surveillance Error.

Values of NIC of 2 or greater produces containment bounds of less than 8 nautical miles. The geometries are similar to that of Node 2. The minimum climb rate and maximum Mach closing rate are used as the worst case parameters. The value for Node 5 under these assumptions is 0.1552 .
For values of NIC of 1 or zero the undetected surveillance error could produce geometries in which the Reference aircraft could be behind, above or in front of the ITP aircraft. The collision trajectory could be a positive relative speed, a negative relative speed, or a zero relative speed. The positive relative speed between the ITP aircraft and the Reference aircraft produces a geometry as depicted in Figure 18. A negative relative speed will have a geometry similar to that of Figure 18, but with the Reference aircraft trailing the ITP aircraft. However, since there are no limits to the negative relative Mach number between the aircraft, the angle could be smaller than that calculated in Section 4.3. Figure 20 shows an example in which the Reference aircraft's position has a large longitudinal error and its actual location is behind the ITP aircraft instead of ahead as reported by the surveillance equipment.


Figure 20. Geometry for Undetected Surveillance Error.
A negative relative Mach number of 0.10 is used for the calculation of probability of collision trajectory for a geometry as illustrated in Figure 20. This is a rather large speed difference for aircraft flying in the oceanic environment at approximately the same altitude.

$$
\begin{align*}
\theta & =\arctan \left(\frac{2.96}{0.1 \times 576.6}\right)  \tag{29}\\
& =2.94^{\circ} \tag{30}
\end{align*}
$$

The collision trajectory length is given by

$$
\begin{equation*}
c t=l+x 1+x 2+x 3 \tag{31}
\end{equation*}
$$

where

$$
\begin{align*}
& x 1=\frac{h}{\tan (\theta)}  \tag{32}\\
& x 2=l \cos (\theta) \tag{33}
\end{align*}
$$

$$
\begin{equation*}
x 3 \approx \frac{h+l \sin (\alpha)}{\tan (\theta)} \tag{34}
\end{equation*}
$$

and
$h, l$ are the height and length of the aircraft.

The probability of a collision trajectory given a one mile proximity is

$$
\begin{align*}
P(c t \mid 1 \mathrm{NM}) & =\frac{c t}{1 N M}  \tag{35}\\
& =\frac{l+\frac{h}{\tan (\theta)}+l \cos (\alpha)+\frac{h+l \sin (\alpha)}{\tan (\theta)}}{1 N M} \tag{36}
\end{align*}
$$

Using the values $300 \mathrm{ft} /$ minute, 1.0 degree angle $\alpha$, delta Mach of 0.10 , aircraft height $h$ of 65 feet, aircraft length $l$ of 200 feet, and 6076 feet in one nautical mile,

$$
\begin{equation*}
P(c t \mid 1 \mathrm{NM})=2999.2 / 6076=0.4936 \tag{37}
\end{equation*}
$$

The probability of a collision trajectory given ITP aircraft within one mile segment will depend on whether the Reference aircraft is in front of the ITP aircraft or trailing the ITP aircraft. The value 0.1552 is used when the Reference aircraft is in front of the ITP aircraft as calculated in Section 4.3. The value 0.4936 is used when the Reference aircraft is trailing the ITP aircraft as calculated in this section. The probability of the Reference aircraft leading or trailing the ITP aircraft is dependent on the values of NIC and SIL. Therefore, The collision trajectory probability is weighed with the values of NIC and SIL to obtain collision trajectory probability. For the case NIC $=1$, the Reference aircraft has a $15 / 29$ probability of trailing the ITP and $14 / 29$ of leading the ITP aircraft. For this value of NIC,

$$
\begin{equation*}
P(c t \mid 1 \mathrm{NM})=0.4936 \times 15 / 29+0.1552 \times 14 / 29=0.3302 \tag{38}
\end{equation*}
$$

### 4.6 Node 6. No Appropriate See and Avoid Actions.

Visual acquisition for undetected position error depends on the geometry of the encounter. In contrast to the geometry of visual acquisition for Node 3, an undetected position error could have the ITP aircraft below (or above), the reference aircraft. The most severe case for visual acquisition is when the ITP aircraft is immediately below (or above) the Reference aircraft with near zero relative horizontal velocity. In this case, neither the ITP nor the Reference aircraft will see each other before a collision. Although a zero relative velocity is the worst scenario when the aircraft are horizontally co-located, zero relative velocity will produce extremely low probabilities of getting close to the Reference aircraft for Node 1 and Node 4.

The three geometry conditions are considered in calculating probability of no visual for a uniform distribution. The vertical field of regard of a commercial passenger aircraft is approximately -75 degrees to 60 degrees as measured from the horizontal [8]. Aircraft encounters in which the relative trajectory of the aircraft is outside these limits will result in no visual acquisition, Figure 21.


Figure 21. Range for No Visual due to Geometry.

The length for the No Visual range is:

$$
\begin{equation*}
\text { No Visual range }=l+\frac{F L(N+1)-F L(N)}{\tan (\phi)}+\frac{F L(N+1)-F L(N)}{\tan (\psi+\alpha)} \tag{39}
\end{equation*}
$$

where $l$ is the length of the aircraft and $\phi$ and $\psi$ are the field of regard from the cockpit.
A flight level difference of 1000 feet gives a No Visual range of 1022 feet. A 2000 feet flight level difference give a No Visual range of 1844 feet. The probability of No Visual is then the probability of the ITP aircraft in the no visual range or IMC conditions:

$$
\begin{align*}
\text { Node } 6, \text { No Visual } & =1844 / 6076+0.2-1844 / 6076 \times 0.2  \tag{40}\\
& =0.4428 \tag{41}
\end{align*}
$$

Because the probabilities of nodes 1,4 and 6 are interdependent, the value calculated above will be used when a near zero relative horizontal velocity is used. When other relative velocities are used, the value calculated for Node 3 will be used.

### 4.7 Other Considerations Not Included in the Collision Probability Calculations

There are other considerations that have not been used in the calculation of collision probability. All of these considerations, as discussed below, further reduce the collision probability. These include:

1. Track offset
2. Track offset due to error
3. On-board collision avoidance systems
4. Error data latency
5. Common mode error

Track offset is the procedure of flying 0,1 , or 2 nautical miles offset from the center of the track. This procedure not only reduces the collision probability during an ITP maneuver but also during regular operations. When using track offset in the calculation of collision probability, the probability will be $1 / 3$ that of operations without track offset.

An aircraft reporting a location which has a significant deviation from its actual location will likely be navigating with a comparable error off of its intended track. The larger the positional error, the more likely that it will be off of its track. This track offset due to error will reduce the probability of collision when an aircraft is preforming an ITP.

On-board collision avoidance systems such as TCAS II will further reduce the collision probability by providing traffic and resolution advisories. An on-board collision avoidance system will have a greater impact during IMC and conditions where the geometry prevents visual acquisition.
Data error latency can also have an impact on the probability of collision. The ITP requires that the flight crew assesses the distance and relative ground speed between the aircraft before a request is made and again before initiating the maneuver. A large surveillance error which is undetected by the integrity monitoring equipment might be detected during the ITP criteria reassessment if the duration of the error is less than the request, clearance, and reassessment time.
Common mode error will reduce the contribution of position and velocity surveillance error to the collision probability. When the ITP and Reference aircraft are receiving surveillance data from the same source such as GNSS/GPS, bias errors will likely affect both aircraft and the relative position error will be minimized.

## 5 Results Summary and Conclusion

### 5.1 Results Summary

The probability of collision due to an In-Trail Procedure is calculated taking into account several factors. These factors are shown in the probability tree of Figure 2. The two branches of the tree can be calculated assuming different parameters of NAC, NIC, and SIL.
Selecting the values $\mathrm{NAC}_{\mathrm{P}}=6, \mathrm{NAC}_{\mathrm{V}}=2, \mathrm{NIC}=5, \mathrm{SIL}=2$, track intercept of 45 degrees, FTE 2 NM, delta Mach $=0.03,2000$ feet altitude difference, and $300 \mathrm{ft} / \mathrm{min}$ climb $/$ descent rate produces the collision probability as shown in the tree of Figure 22. These values of NAC, NIC and SIL correspond to a NUC ${ }_{P}$ value of 5 and $\mathrm{NUC}_{\mathrm{R}}$ value of 2 . Note that Nodes 1-6 will have different values if different parameters of accuracy, integrity, track angle, FTE, delta Mach, vertical difference and climb rate are chosen.


Figure 22. Representative Collision Probability for parameter values of $\mathbf{N A C}_{\mathbf{P}}=\mathbf{6}, \mathbf{N A C}_{\mathbf{V}}=\mathbf{2}$, NIC = 5, SIL = 2, 45 Degrees Track Intercept with FTE 2 NM, 2000 feet altitude difference, and 300 feet/minute climb/descent rate.

A target level of safety (TLS) in a per-flight-hour basis can be determined by estimating the average number of In-Trail Procedures which will be performed in a typical flight and normalizing for a per-flight -hour basis. The method is defined in reference [13]. The probability of collision can be reduced to meet the desired target level of safety by imposing more stringent requirements on the surveillance equipment and/or tighter ITP requirements. Also, requirements could be relaxed if more relaxed requirements can meet the TLS.

### 5.2 Conclusion

The nominal procedure for ITP is inherently safe with regard to collision risk. For the parameters shown in Figure 22, the collision risk is estimated at $4.47 \times 10^{-29}$ per ITP operation for the nominal non-failure condition. A hazard assessment must be performed to determine if the ITP procedure meets the Target Level of Safety in the presence of failures and operational errors. A target level of safety of $5 \times 10^{-9}$ fatal accidents per dimension per flight hour is specified by ICAO as stated in reference [17]. This target level of safety must be met when considering all contributing factors for Air Traffic Services, not just ITP. Depending on the allocation of safety to hazards, the target level of safety for the ITP might be achieved even when relaxing some of the criteria. This is shown in Appendix A. For example, a positive delta Mach number of 0.07 and 3000 feet altitude difference will meet a nominal non-failure case collision risk objective of $1 \times 10^{-9}$.

## 6 References

[1] Requirement Focus Group; Operational Safety Assessment (OSA) In-Trail Procedure (ITP), February, 2006.
[2] Requirement Focus Group; Operational Services and Environment Definition In-Trail Procedure, October, 2005
[3] Minimum Aviation System Performance Standards For Automated Dependent Surveillance Broadcast (ADS-B), RTCA/DO-242.
[4] Minimum Aviation System Performance Standards For Automated Dependent Surveillance Broadcast (ADS-B), RTCA/DO-242A, June 2002.
[5] Minimum Operational Performance Standards for 1090 MHz Automatic Dependent Surveillance Broadcast (ADS-B), RTCA/DO-260, September 2000.
[6] Minimum Operational Performance Standards for 1090 MHz Extended Squitter Automatic Dependent Surveillance - Broadcast (ADS-B) and Traffic Information Services - Broadcast (TIS-B), RTCA/DO-260A, April 2003.
[7] Sherry, J.; A Determination of Oceanic Winds Aloft Data Error and Its Impact on Aircraft Position Uncertainty, The MITRE Corporation.
[8] V. Carreno; Sense and Avoid Safety Analysis for Remotely Operated Unmanned Aircraft in the National Airspace System, February 2006.
[9] V. Carreno; The Visual 3D Users Manual, in preparation.
[10] Defining the Phrase: "Equivalent Level of Safety, Comparable to See-and-Avoid Requirements for Manned Aircraft", NASA Access 5, Work Package 2, Cooperative Collision Avoidance team, July 2004.
[11] A. Zeitlin; TCAS Oceanic In-Trail Climb Procedure Safety Analysis, MITRE MTR 95W0000096, August, 1995.
[12] George Canavos; Applied Probability and Statistical Methods, Little, Brown and Company, 1984.
[13] FAA Advisory Circular AC 23.1309 -1C Appendix C. FAA Advisory Circular AC 25.1309-1B Appendix 3.
[14] Atmospheric Boundary Layer Experiment. http://www.atmos.anl.gov/ABLE/
[15] Dave Nakamura; General Information on the Functional and Technical Aspects of Required Navigational Performance (RNP) Area Navigation (RNAV) and Applications, CNS/ATM Technical Requirements and Standards, FMS RNAV Workshop, February 2000.
[16] S.R. Jones; ADS-B Surveillance Quality Indicators: Their Relationship to System Operational Capabilities and Aircraft Separation Standards, Air Traffic Control Quarterly, Vol. 11(3) pp 225-250, 2003.
[17] ICAO, Annex 11, Air Traffic Services, Thirteenth Edition, July 2001.

## Appendix A

This appendix contains comparisons of collision probabilities for different values of delta Mach numbers and altitude differences between ITP and Reference aircraft. The results are presented first, followed by the analysis for each node. Nodes 3 and 6 do not change for Mach and altitude differences.

The Mach initiation criterion requires that the ITP and Reference aircraft have a bounded positive delta Mach. That is, the closure rate of the two aircraft due to their Mach speed is limited by the Mach criterion. Table A. 1 shows the probability of collision for the ITP maneuver as a function of delta Mach and altitude difference for $\mathrm{NAC}_{\mathrm{P}}=6, \mathrm{NAC}_{\mathrm{V}}=2, \mathrm{NIC}=5, \mathrm{SIL}=2$, track intercept of 45 degrees, FTE 2 NM , and $300 \mathrm{ft} / \mathrm{min}$ climb/descent rate.

| P (collision) | Delta Mach |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | 0.03 | 0.05 | 0.07 | 0.10 |
| 2000 feet altitude <br> difference | $4.47 \times 10^{-29}$ | $7.32 \times 10^{-27}$ | $3.45 \times 10^{-24}$ | $2.75 \times 10^{-19}$ |
| 3000 feet altitude <br> difference | $9.89 \times 10^{-14}$ | $4.35 \times 10^{-12}$ | $2.79 \times 10^{-10}$ | $1.9 \times 10^{-8}$ |

Table A.1. Probability of Collision for Positive Delta Mach.

## Node 1

The probability of passing inside the Reference aircraft one mile segment is calculated for different delta Mach numbers and altitude difference for $\mathrm{NAC}_{\mathrm{P}}=6, \mathrm{NAC}_{\mathrm{V}}=2, \mathrm{NIC}=5, \mathrm{SIL}=2$, track intercept of 45 degrees, FTE 2 NM, and $300 \mathrm{ft} / \mathrm{min}$ climb/descent rate.

| Probability of one <br> mile segment | Delta Mach |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | 0.03 | 0.05 | 0.07 | 0.10 |
| 2000 feet altitude <br> difference | $1.01 \times 10^{-27}$ | $1.36 \times 10^{-25}$ | $4.21 \times 10^{-23}$ | $2.62 \times 10^{-19}$ |
| 3000 feet altitude <br> difference | $3.18 \times 10^{-12}$ | $8.06 \times 10^{-11}$ | $3.99 \times 10^{-9}$ | $2.02 \times 10^{-7}$ |

Table A.2. Probability of ITP inside One Mile Segment; Surveillance Accuracy.

## Node 2

A vertical speed of $300 \mathrm{ft} /$ minute and a relative horizontal speed of 28.83 knots (Mach 0.05 at 35 K feet) produces an angle $\theta$ of

$$
\begin{align*}
\theta & =\arctan (2.96 / 28.83)  \tag{A1}\\
& =5.86^{\circ} \tag{A2}
\end{align*}
$$

A vertical speed of $300 \mathrm{ft} /$ minute and a relative horizontal speed of 40.36 knots (Mach 0.07 at 35 K feet) produces an angle $\theta$ of

$$
\begin{align*}
\theta & =\arctan (2.96 / 40.36)  \tag{A3}\\
& =4.19^{\circ} \tag{A4}
\end{align*}
$$

A vertical speed of $300 \mathrm{ft} /$ minute and a relative horizontal speed of 57.66 knots (Mach 0.1 at 35 K feet) produces an angle $\theta$ of

$$
\begin{align*}
\theta & =\arctan (2.96 / 57.66)  \tag{A5}\\
& =2.94^{\circ} \tag{A6}
\end{align*}
$$

The collision trajectory length is given by

$$
\begin{equation*}
c t=l+x 1+x 2+x 3 \tag{A7}
\end{equation*}
$$

where

$$
\begin{align*}
& x l=\frac{h}{\tan (\theta)}  \tag{A8}\\
& x 2 \approx \frac{h}{\sin (\theta)}  \tag{A9}\\
& x 3=l \cos (\alpha)-\frac{l \sin (\alpha)}{\tan (\theta)} \tag{A10}
\end{align*}
$$

and
$h, l$ are the height and length of the aircraft.
The probability of a collision trajectory given a one mile proximity is

$$
\begin{align*}
P(c t \mid 1 \mathrm{NM}) & =\frac{c t}{1 N M}  \tag{A11}\\
& =\frac{l+\frac{h}{\tan (\theta)}+\frac{h}{\sin (\theta)}+l \cos (\alpha)-\frac{l \sin (\alpha)}{\tan (\theta)}}{1 N M} \tag{A12}
\end{align*}
$$

Using the values $300 \mathrm{ft} /$ minute, $\alpha$ of 1.0 degree, aircraft height $h$ of 65 feet, aircraft length $l$ of 200 feet, delta Mach numbers of $0.05,0.07$, and 0.1 , and 6076 feet per nautical mile,

$$
\begin{align*}
& P(c t|1 \mathrm{NM}| 0.05)=1635.9 / 6076=0.2692  \tag{A13}\\
& P(c t|1 \mathrm{NM}| 0.07)=2129.2 / 6076=0.3504  \tag{A14}\\
& P(c t|1 \mathrm{NM}| 0.10)=2864.9 / 6076=0.4715 \tag{A15}
\end{align*}
$$

## Node 4

Probability of passing inside the Reference aircraft one mile segment using undetected surveillance error due to failure as a factor. The probability is calculated for different delta Mach numbers and altitude difference for $\mathrm{NAC}_{\mathrm{P}}=6, \mathrm{NAC}_{\mathrm{V}}=2, \mathrm{NIC}=5, \mathrm{SIL}=2$, track intercept of 45 degrees, FTE 2 NM, and 300 $\mathrm{ft} / \mathrm{min}$ climb/descent rate.

| Probability of one <br> mile segment for <br> undetected error <br> due to failure | 0.03 | 0.05 | 0.07 | 0.10 |
| :--- | :---: | :---: | :---: | :---: |
| Delta Mach <br> difference | $4.31 \times 10^{-28}$ | $3.28 \times 10^{-26}$ | $7.22 \times 10^{-24}$ | $1.33 \times 10^{-20}$ |
| 3000 feet altitude <br> difference | $6.13 \times 10^{-15}$ | $1.19 \times 10^{-13}$ | $1.60 \times 10^{-12}$ | $1.38 \times 10^{-10}$ |

Table A.2. Probability of ITP inside One Mile Segment; Undetected Surveillance Error.

## Node 5

The collision trajectory probability for undetected surveillance error due to failure is calculated following the method in Section 4.2. For the delta Mach numbers $0.05,0.07$ and 0.10 ,

$$
\begin{align*}
& P(c t|1 \mathrm{NM}| 0.05)=0.4936 \times 15 / 29+0.2692 \times 14 / 29=0.3853  \tag{A16}\\
& P(c t|1 \mathrm{NM}| 0.07)=0.4936 \times 15 / 29+0.3504 \times 14 / 29=0.4245  \tag{A17}\\
& P(c t|1 \mathrm{NM}| 0.10)=0.4936 \times 15 / 29+0.4715 \times 14 / 29=0.4829 \tag{A18}
\end{align*}
$$

## Appendix B

## Calculation of the variance of the product of two random variables

There are terms in the calculation of reduction-in-distance that are the product of two normally distributed random variables. The terms are:
(altitude error/climb rate) x (wind)
(altitude error/climb rate) x (velocity error)
To account for these factors, the variances of their distributions are needed. The variances can be calculated from the probability density functions. However, a closed form of the probability density function of the product of two random variables is infeasible [1]. Therefore, the variance of the distribution is numerically calculated using the formula,

$$
\begin{equation*}
s^{2}=\frac{\sum_{i=1}^{n} x_{i}^{2}-\frac{\left(\sum_{i=1}^{n} x_{i}\right)^{2}}{n}}{n-1} \tag{B1}
\end{equation*}
$$

where $s^{2}$ is the variance, $x_{i}$ is a sample or outcome, and $n$ is the number of samples.

## References

[1] A. Glen, L. Leemis, J. Drew; Computing the distribution of the product of two continuous random variables, in Computational Statistics and Data Analysis, Elsevier, July 2002.



[^0]:    ${ }^{1}$ Using International Standard Atmosphere at 35 K feet.

[^1]:    ${ }^{2} 17.3$ knots is the Mach difference times the speed of sound at altitude; $0.03 \times 576.6$ knots.

[^2]:    ${ }^{3}$ The along track range is called "ITP distance" in the OSED document, reference [2].

[^3]:    ${ }^{4}$ The angle of attack of an aircraft is the angle between the aircraft's horizontal axis and the horizontal plane. The angle of attack of an aircraft and its climb rate are proportional. Is is assumed that an aircraft with a $300 \mathrm{ft} / \mathrm{min}$ climb rate will have approximately a 1 degree angle of attack.

