SIMULATION OF DELAMINATION UNDER HIGH CYCLE FATIGUE IN COMPOSITE MATERIALS USING COHESIVE MODELS

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ABSTRACT

A new thermodynamically consistent damage model is proposed for the simulation of high-cycle fatigue crack growth. The basis for the formulation is an interfacial degradation law that links Fracture Mechanics and Damage Mechanics to relate the evolution of the damage variable, d, with the crack growth rate da/dN. The damage state is a function of the loading conditions (R and ΔG) as well as the experimentally-determined crack growth rates for the material. The formulation ensures that the experimental results can be reproduced by the analysis without the need of additional adjustment parameters.

1. INTRODUCTION

Mechanical fatigue, especially high-cycle fatigue, is a common cause of failure of aerospace structures. In laminated composite materials, the fatigue process involves several damage mechanisms that result in the degradation of the structure. One of the most important fatigue damage mechanisms is interlaminar damage (delamination).

There are two basic approaches for the analysis of delamination under fatigue: Fracture Mechanics, which relates the fatigue crack growth rate as a function of the energy release rate and mode ratio; and Damage Mechanics, in which the concept of a cohesive zone [1,2] is used to establish damage evolution as a function of the number of cycles.

In a degradation process involving high cycle-fatigue, damage evolution can be obtained as the sum of the damage caused by static or quasi-static loads and the damage that results from the cyclic loads. The damage evolution produced by cyclic loads is usually formulated as a function of the number of cycles and strains (or displacement jumps) [3-5], where a damage evolution law expressed in terms of the number of cycles is established a priori. However, the damage evolution law must be expressed as a function of several parameters that have to be adjusted through a trial-and-error calibration of the whole numerical model. In this paper, an alternative approach is proposed whereby the evolution of damage is based on linking Fracture Mechanics and Damage Mechanics, and relating the evolution of the damage variable, d, with the crack growth rate.

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The present model is implemented by means of a user-written element in ABAQUS [6] by adding the damage evolution law formulated in the decohesion element previously developed by the authors [7,8].

2. CONSITUTIVE MODEL FOR QUASI-STATIC LOADS

The constitutive model used in this paper for quasi-static loads uses a bilinear constitutive equation that relates surface tractions, τ, to displacement jumps, Δ, as shown in Figure 1. While the interface is undamaged, a high interface stiffness, K, ensures a stiff connection between the two adjacent layers. Interface damage initiates when the displacement jump norm reaches the onset displacement, Δ^0, and the interface is considered fractured when the displacement jump norm exceeds the final displacement, Δ^f. The energy dissipated during the damage evolution is called Ξ, and the area under the traction-displacement jump law is equal to the critical energy release rate G_c. The critical displacement jumps Δ^0 and Δ^f are functions of the mode mixity and details of the formulation can be found in [8].
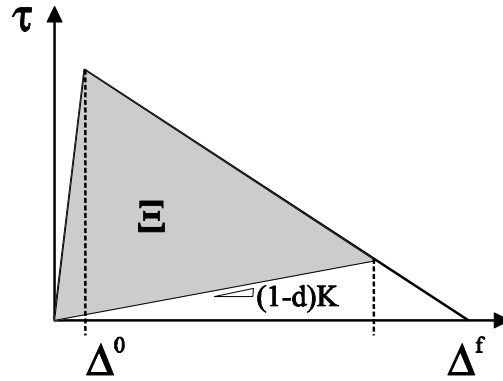

Figure 1. Bilinear constitutive law used for quasi-static loads.

3. CONSITUTIVE MODEL FOR HIGH CYCLE FATIGUE

The damage evolution in a degradation process involving high-cycle fatigue can be considered as the sum of the damage sustained from quasi-static loads and the damage sustained from cyclic loads:

$$\dot{d} = \dot{d}_{static} + \dot{d}_{cyclic}$$

(1)

In the framework of the Damage Mechanics, the damage evolution that results from cyclic loads can be formulated as a function of the number or cycles, usually as a function of strains (or displacement jumps) [3-5], where a damage evolution law is established a priori as a function of the number of cycles. The damage evolution law is a function of several parameters that have to be adjusted calibrating the numerical model with experimental results, usually by trial and error. However, in this paper, the evolution of the damage evolution law is formulated using Fracture Mechanics and creating a link between Fracture Mechanics and Damage Mechanics to relate the evolution of the damage variable, d, with the crack growth rate, da/dN. The evolution of the damage variable is related to the evolution of the crack surface as follows:

$$\frac{\partial d}{\partial N} = \frac{\partial d}{\partial A_d} \frac{\partial A_d}{\partial N} \qquad (2)$$

where A_d is the damaged area, and $\frac{\partial A_d}{\partial N}$ is the growth rate of the damaged area. While the second term in the right hand side of equation (2) must be characterized experimentally, the first term $\frac{\partial d}{\partial A_d}$ can be obtained from either a Damage Mechanics approach or a Fracture Mechanics approach, as described in the following sections.

3.1- Determination of $\frac{\partial d}{\partial A_d}$ using a Damage Mechanics approach

In the framework of the Damage Mechanics, the damage variable, d, can be defined as the ratio between the damage area and the total area, i.e.,

$$d = \frac{A_d}{A} \qquad (3)$$

The damaged area is zero when no damage is present in the process, $A_d = 0$, while $A_d = A$ when the surface is completely damaged. Since the total area is constant, the variation of the damaged variable with respect to the damaged area can be written as:

$$\frac{\partial d}{\partial A_d} = \frac{1}{A} \qquad (4)$$

3.2- Determination of $\frac{\partial d}{\partial A_d}$ using a Fracture Mechanics approach

In the framework of the Fracture Mechanics, the fraction of the damaged area, A_d, with respect to the total area, A, can be written as a function of the dissipated energy:

$$\frac{A_d}{A} = \frac{\Xi}{G_c} \qquad (5)$$

where Ξ is the fraction of the energy per surface unit dissipated during the damage process, i.e., the area under the cohesive law for the current damage threshold, and G_c is the critical energy release rate (see Figure 1).

Assuming no change between modes, G_c is constant, while Ξ is a function of the cohesive law used and the current damage threshold. Using this approach, the variation of the damage variable with the damaged area can be written as:

$$\frac{\partial \mathbf{d}}{\partial A_{\mathbf{d}}} = \frac{G_c}{A} \frac{\partial \mathbf{d}}{\partial \Xi} \qquad (7)$$

The evolution of the energy dissipation with the damage evolution is obtained from the equations of the constitutive law used. Using a bilinear constitutive law:

$$\frac{\partial \Xi}{\partial \mathrm{d}} = G_c \frac{\Delta^\rho \left(\Delta^f - \Delta^\rho \right)}{\left(\Delta^f \left(1 - \mathrm{d} \right) + \mathrm{d} \Delta^\rho \right)^2} \tag{8}$$

Using equation (8), equation (7) can be written as:

$$\frac{\partial \mathrm{d}}{\partial A_{\mathrm{d}}} = \frac{1}{A} \frac{\left(\Delta^f \left(1 - \mathrm{d} \right) + \mathrm{d} \Delta^\rho \right)^2}{\Delta^\rho \left(\Delta^f - \Delta^\rho \right)} \tag{9}$$

3.3- Determination of the damaged area growth rate as a function of the number of cycles

In this section, we establish the dependence of the damaged area growth rate, $\dfrac{\partial A_{\mathrm{d}}}{\partial N}$ on the surface crack growth rate, $\dfrac{\partial A}{\partial N}$. For a specimen with just one crack front, the crack growth rate is equal to the sum of the damaged surface growth rates of all elements in the cohesive zone. In the other regions of the specimen, there is no possibility of new surface generation.

$$\frac{\partial A}{\partial N} = \sum_{e \in A_{CZ}} \frac{\partial A_{\mathrm{d}}^e}{\partial N} \tag{10}$$

Using the simplification that $\dfrac{\partial A_{\mathrm{d}}^e}{\partial N}$ is constant over the cohesive zone, the previous equation can be written as:

$$\frac{\partial A}{\partial N} = \sum_{e \in A_{CZ}} \frac{\partial A_{\mathrm{d}}^e}{\partial N} = \frac{A_{CZ}}{A} \frac{\partial A_{\mathrm{d}}}{\partial N} \tag{11}$$

where the ratio $\dfrac{A_{CZ}}{A}$ represents the number of areas in which the cohesive zone have been divided. In a finite element environment, this ratio represents the number of elements that span the cohesive zone. Rearranging terms in equation (11), the surface damage growth rate can be written as:

$$\frac{\partial A_{\mathrm{d}}}{\partial N} = \frac{A}{A_{CZ}} \frac{\partial A}{\partial N} \tag{12}$$

3.4- Evolution of the damage variable under cyclic loading

Replacing equation (12) and equation (4) (based on Damage Mechanics) in equation (2) the evolution of the damage variable as a function of the number of cycles can be written as:

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$$\frac{\partial \mathbf{d}}{\partial N} = \frac{1}{A_{CZ}} \frac{\partial A}{\partial N} \tag{13}$$

Likewise, using the approach based on Fracture Mechanics, equation (7), the evolution of the damage variable as a function of the number of cycles can be written as:

$$\frac{\partial \mathbf{d}}{\partial N} = \frac{G_c}{A_{CZ}} \frac{\partial \mathbf{d}}{\partial \Xi} \frac{\partial A}{\partial N} \tag{14}$$

where G_c depends on the material used and loading mode, and $\frac{\partial \mathbf{d}}{\partial \Xi}$ depends on the cohesive law used in the formulation of the surface traction-displacement jump relation.

The area of the cohesive zone can be computed intensively using Rice's model [9]:

$$A_{CZ} = b \frac{9\pi}{32} \frac{E_3 G}{(\tau^o)^2} \tag{15}$$

where b is the width of the specimen, G is the energy release rate, E_3 is the Young's modulus of the bulk material in the direction perpendicular to the crack plane, and τ^o is the interfacial strength.

3.4- Crack growth rate

The surface crack growth rate under fatigue loading, $\frac{\partial A}{\partial N}$, is a load and material-dependent characteristic that has been widely studied, and models such as the Paris law have been developed to represent it. For instance,

$$\frac{\partial A}{\partial N} = C \left(\frac{\Delta G}{G_c} \right)^m \tag{16}$$

where C and m are parameters that depend on the mode ratio and must be determined experimentally. ΔG, is the cyclic variation in the energy release rate, which can be computed using the constitutive law of the interface

$$G = \int_0^\Delta \tau(\Delta) d\Delta \tag{17}$$

for the maximum and minimum displacement jump over the cycle (see Figure 2):

$$\Delta G = G_{max} - G_{min} \tag{18}$$

Defining the reversibility factor, R, the relation between the minimum and the maximum displacement is:

$$\Delta^{min} = R\Delta^{max} \tag{19}$$

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and the variation of the energy release rate can be written as:

$$\Delta G = \frac{K \Delta^0 \Delta^{max} \left(\Delta^f - \Delta^{max} \frac{1+R}{2} \right)}{\Delta^f - \Delta^0} (1 - R)$$
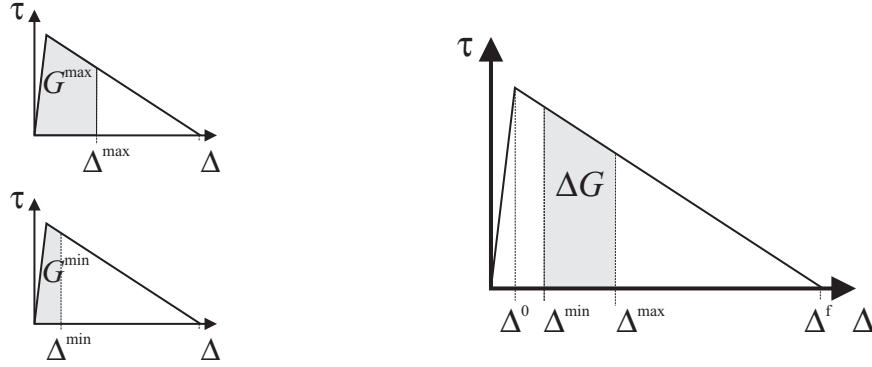
(20)

Figure 2. Variation of the energy release rate.

4. RESULTS AND DISCUSION

The present model is implemented by adding the fatigue component of damage to the damage evolution law of a user-written decohesion element previously developed by the authors [7,8]. To verify the constitutive model, several single-element tests were performed. The evolution of the interface traction in the constitutive equation for a high cycle fatigue test subjected to displacement control is shown in Figure 3.
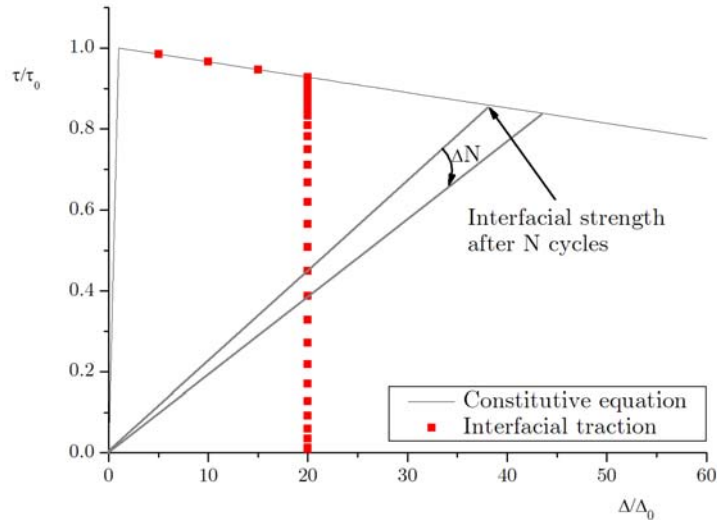

Figure 3. Evolution of the interface traction in the constitutive equation for a displacement jump controlled high cycle fatigue test.

It can be observed that fatigue loading results in a reduction of the stiffness, and in a reduction of the interfacial traction and interfacial strength. The evolution of the

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interface traction and the interfacial strength with the number of cycles is shown in Figure 4. The shape of the obtained curves is similar to the widely-used S-N curves used in the design for fatigue strength.

Several tests were conducted to simulate the surface crack growth velocity under mode I loading for different levels of the energy release rate. A description of the experimental procedure that was simulated is reported by Asp et al. [10]. The boundary conditions used in the simulation are the same to those presented by Robinson et al. [4]. The results obtained from the simulations are shown in Figure 5 against the experimental data. A better agreement between simulated and experimental data is observed when the Fracture Mechanics approach is used instead of the Damage Mechanics approach.
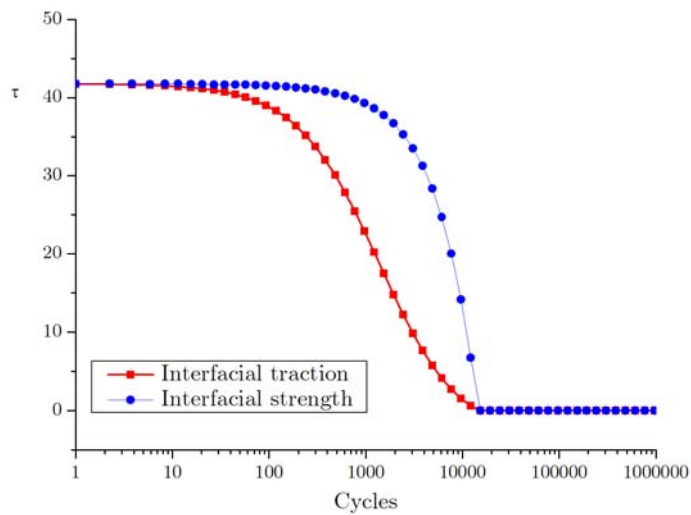

Figure 4. Evolution of the interface traction and the maximum interface strength as a function of the number of cycles for a displacement jump controlled high cycle fatigue test.
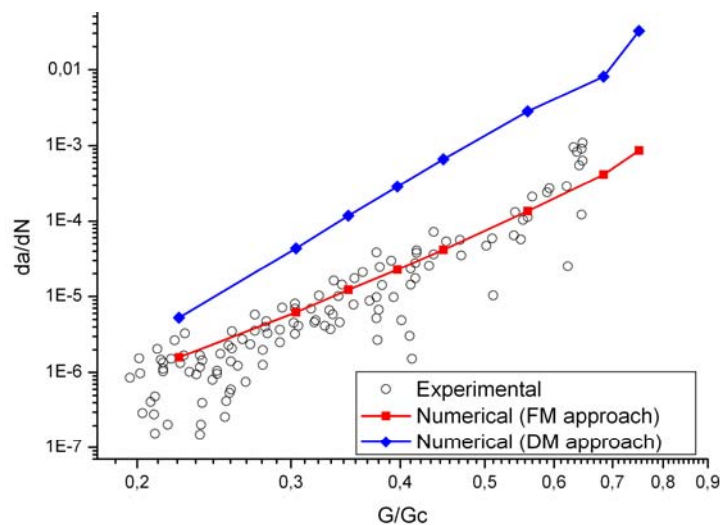

Figure 5. Comparison of the experimental data with the crack growth rate obtained from the numerical simulation for a Mode I DCB test.

5. CONCLUSIONS

A thermodynamically consistent damage model for high-cycle fatigue delamination was developed. The evolution of the damage variable was derived by linking Fracture Mechanics and Damage Mechanics to relate damage evolution to crack growth rates. The damage evolution laws for cyclic fatigue were combined with the law of damage evolution for quasi-static loads within a decohesion element previously developed by the authors. The model was validated with single-element numerical tests as well as a simulation of a DCB problem. The model was able to reproduce the test data without the need of additional adjustment parameters that are typically used in other fatigue growth models.

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