Refinement of Methods for Evaluation of Near-Hypersingular Integrals in BEM Formulations

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In this paper, we present advances in singularity cancellation techniques applied to integrals in BEM formulations that are nearly hypersingular. Significant advances have been made recently in singularity cancellation techniques applied to $1/R$ type kernels [M. Khayat, D. Wilton, *IEEE Trans. Antennas and Prop.*, **53**, pp. 3180-3190, 2005], as well as to the gradients of these kernels [P. Fink, D. Wilton, and M. Khayat, *Proc. ICEAA*, pp. 861-864, Torino, Italy, 2005] on curved subdomains. In these approaches, the source triangle is divided into three tangent subtriangles with a common vertex at the normal projection of the observation point onto the source element or the extended surface containing it. The geometry of a typical tangent subtriangle and its local rectangular coordinate system with origin at the projected observation point is shown in Fig. 1.

Whereas singularity cancellation techniques for $1/R$ type kernels are now nearing maturity, the efficient handling of near-hypersingular kernels still needs attention. For example, in the gradient reference above, techniques are presented for computing the normal component of the gradient relative to the plane containing the tangent subtriangle. These techniques, summarized in the transformations in Table 1, are applied at the sub-triangle level and correspond particularly to the case in which the normal projection of the observation point lies within the boundary of the source element. They are found to be highly efficient as z approaches zero. Here, we extend the approach to cover two instances not previously addressed. First, we consider the case in which the normal projection of the observation point lies external to the source element. For such cases, we find that simple modifications to the transformations of Table 1 permit significant savings in computational cost. Second, we present techniques that permit accurate computation of the tangential components of the gradient; i.e., tangent to the plane containing the source element.

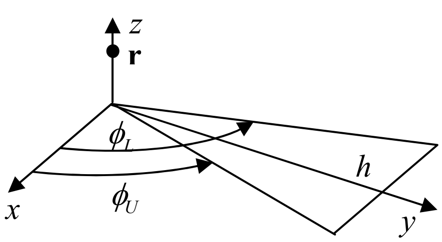

Figure 1. Subtriangle geometry.

Table 1: Summary of Gradient Transformations

	Trans.	$\mathcal{J}(u,v)$	INTEGRATION LIMITS
Radial-Angular R^2	$u=\phi$, $v=\lvert z\rvert\ln R$	$\frac{R^2}{\lvert z\rvert}$	$u_{L,U}=\phi_{L,U}$; $v_{L,U}=\lvert z\rvert\ln\lvert z\rvert,\ \frac{\lvert z\rvert}{2}\ln\left(z^2+(h/\sin u)^2\right)$
Radial-Angular R^3	$u=\phi$, $v=\frac{\lvert z\rvert}{R}$	$\frac{-R^3}{\lvert z\rvert}$	$u_{L,U}=\phi_{L,U}$; $v_{L,U}=1,\ \frac{\lvert z\rvert}{\sqrt{z^2+(h/\sin u)^2}}$