Strip Yield Model Numerical Application to Different Geometries and Loading Conditions

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1. Abstract:

A new numerical method based on the strip-yield analysis approach was developed for calculating the Crack Tip Opening Displacement (CTOD). This approach can be applied for different crack configurations having infinite and finite geometries, and arbitrary applied loading conditions. The new technique adapts the boundary element / dislocation density method to obtain crack-face opening displacements at any point on a crack, and succeeds by obtaining requisite values as a series of definite integrals, the functional parts of each being evaluated exactly in a closed form.

Keywords: Crack Tip Opening Displacement; Strip Yield model; Dugdale

2. Introduction:

The strip-yield model was introduced by Dugdale [1] in 1960. It was first applied to a through crack in an infinite plate. In [1] it was assumed that plastic deformation is concentrated in a strip in front of the crack. It was also assumed that the material behavior is elastic-perfectly plastic, and that a state of plane stress exists. The strip-yield model replaces the elastic-plastic problem with an elastic one (and additional boundary conditions) that can be solved analytically. The advantage of using a strip-yield model is that the stress and deformation can be obtained by superposition of two linear elastic solutions and the requirement that the stress singularity at the crack tip vanishes. The strip-yield model is an approximation and not exact, but for some important applications, it is an efficient method for deriving useful quantitative information.

Finite Element Analysis (FEA) is a very common method for determining CTOD [2]. The 45-degree intercept proposed first by Rice [3] to be the opening distance where 45 ° lines from where the crack tip intercept the crack faces, is commonly used in Finite Element Analysis to infer CTOD measurements. Some researchers however have used the first node behind the crack tip to compute the crack opening levels [4, 5, 6]. Other investigations have used the second node behind

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the crack tip [6, 7]. Both 2D and 3D analyses tend to over predict crack extension in the early stages of stable tearing. But 3D analysis does a better job accurately matching the crack extension behavior for the remainder of the test [8, 9]. The method involves intensive computations and powerful computers to be used. FEA is also very time consuming when handling variable amplitudes, and large number of cycles

Other methods can be used to measure CTOD experimentally. These include Optical Microscopy (OM), Digital Image correlation, and the delta method [10]. The main difficulties in the application of CTOD are the measurement of the critical CTOD in fracture tests. The direct microscopic measurement of a critical CTOD seems extremely difficult since the critical condition is defined not at a stationary moment but at a passing moment. Also the CTOD increment at a current crack tip of a growing crack is not the same as that at an initial crack tip [11]. The physical values of parameters such as CTOD are measured on test specimen exterior, and a through thickness average interior value is generally inferred. This inferred average value might not be accurate because of 3D issues such as constraint variation, and material inhomogeneity. In addition current 2D theories used for stable crack growth assume flat, straight, through thickness crack front. The flat to slant transition is vastly a 3D process; thus it is likely that the CTOD values in the interior are different than the measured surface values [12]. Thus using these experimental methods may not yield better results than analytical and computational methods.

Another method of measuring crack opening is the microtopography method like the one described by [13, 14]. This method measures and maps three dimensional fracture surfaces. The analysis of these data provides direct extraction of the parameters of interest at any location within the specimen interior. The fracture process can be recreated in a virtual sense by numerical manipulation of the special data. This method can be expensive and time consuming compared to other methods used to infer CTOD.

Other methods like weight function, Green's function, and collocation methods are also used to determine CTOD [15, 16, 17, 18, 19]. These methods may be complicated depending of the geometry and application, and have to be derived for a variety of geometries and loads.

Most of the methods described above can be complicated, costly, time consuming, and may only apply to certain geometries and loading conditions. In this paper a new numerical method based on the strip-yield analysis approach and the dislocation density method was developed for calculating the Crack Tip Opening Displacement (CTOD). This new approach can be easily applied by fracture mechanics analysts to solve different crack configurations having finite and infinite geometries, complex geometrical problems, and arbitrary applied loading conditions. This can be done using significantly low modeling and computing time. In addition the results from this new approach can be more accurate than weight functions because it deals with directs solutions that are based on dislocation density method which is a moderately singular form of BEM. The new approach can also be easily modified to account for strain hardening effects as well.

3. Theory:

This section describes a method developed in this paper to obtain crack-face opening displacements at any point on a crack face, and thus, the capability to use the strip-yield model for general 2D crack problems. For this purpose, the technique developed here adapts the boundary element / dislocation density method, and succeeds in obtaining the requisite functional values as a series of definite integrals where the functional parts of each (including those that involve singular integrands) are evaluated exactly in closed form.

The technique is essentially a Boundary Element Method (BEM) in which the crack line integrals are treated in a special way to eliminate the difficulties associated with the application of the standard boundary element method to fracture problems. The resulting integral equations contain integrals associated with the fracture in terms of a distribution of point dislocations along the crack line.

The method is based on a utilization of the dislocation density function, $A(\zeta)$, which is related to the difference in displacements (i.e. the crack-opening displacements) of the crack faces.

The complex-valued displacement function is defined as

$$D = u_x + i\, u_y \tag{1}$$

where u_x and u_y are the x- and y-components, respectively, of displacement and ΔD is the difference in displacements between the upper and lower crack surfaces. The dislocation density function is related to displacement by

$$A(\zeta) = \frac{i\mu}{\pi(\kappa+1)} \frac{\partial[\Delta D(\zeta)]}{\partial s} \tag{2}$$

where κ equals $3-4\nu$ for plane strain and $(3-\nu)/(1+\nu)$ for plane stress, μ and ν are the shear modulus and Poisson's ratio, respectively, of the material, and *s* is the distance along the crack from the tip to the point of interest.

A technique has been developed to integrate (2), which yields nominally

$$\Delta D(\zeta) = \frac{\pi(\kappa+1)}{i\mu} \int_0^s A[\zeta(s)]ds \tag{3}$$

Evaluation of this integral requires the summing up of the contribution from each element between the crack tip and the point of interest.

In [20], the geometric interpolation along the crack line is piecewise linear; the crack line geometry over the *(j+1)*th element is approximated by

$$\zeta(t) = \frac{1-t}{2}\zeta_j + \frac{1+t}{2}\zeta_{j+1}; \quad -1 \le t \le 1 \tag{4}$$

as shown in Figure 2.

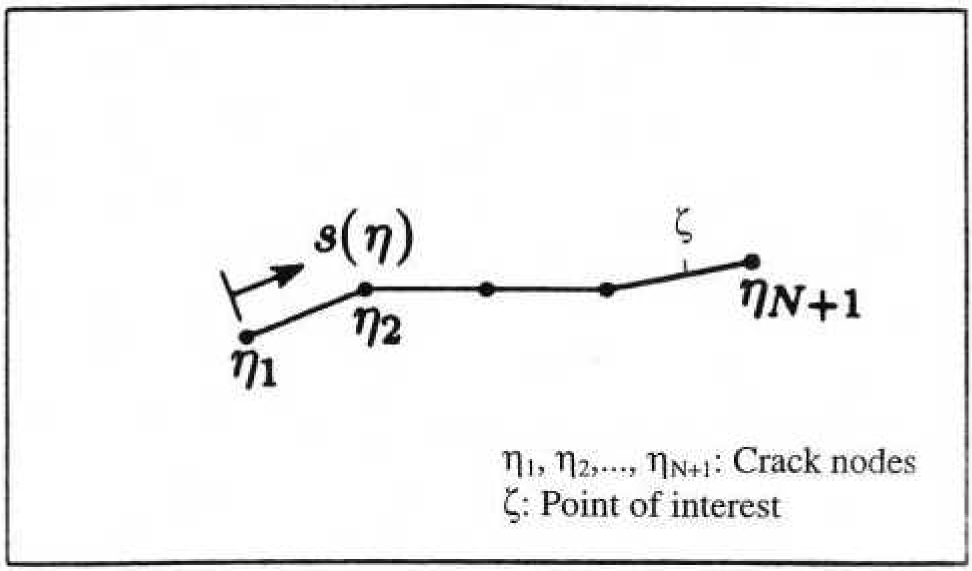


Figure 2. Piecewise Linear Interpolation for Crack Geometry

The dislocation density function on each element is approximated by functions that contain the requisite singularity multiplied by an interpolating part that depends on the natural element coordinate t,

$$A(t) = A[\zeta(t)] = \frac{1}{2\sqrt{\alpha_j}}\left(\frac{1-t}{\sqrt{\rho_j + t}}A_j + \frac{1+t}{\sqrt{\rho_j + t}}A_{j+1}\right); \quad 1 \le j \le N/2 \tag{5}$$

$$A(t) = A[\zeta(t)] = \frac{1}{2\sqrt{\alpha_j}}\left(\frac{1-t}{\sqrt{\rho_j - t}}A_j + \frac{1+t}{\sqrt{\rho_j - t}}A_{j+1}\right); \quad N/2 < j \le N \tag{6}$$

where it is assumed that the number of elements is even, and that the mid-numbered node bisects the crack. In (5) and (6), A_j are nodal quantities, which are obtained from the vector of unknowns, and numerical values of which become available when the system matrix is solved; α_j is the half-length of the *j*th element, and $\rho_j = \beta_j / \alpha_j$ in which β_j is the element-wise arc length from the center of the *j*th element to the nearest crack tip.

Substituting (4), (5) and (6) in (3) allows evaluation of • *D* as an element-wise sum of definite integrals. Most of the integrals thus obtained involve integrands that are non-singular and well behaved, and are generically of the form

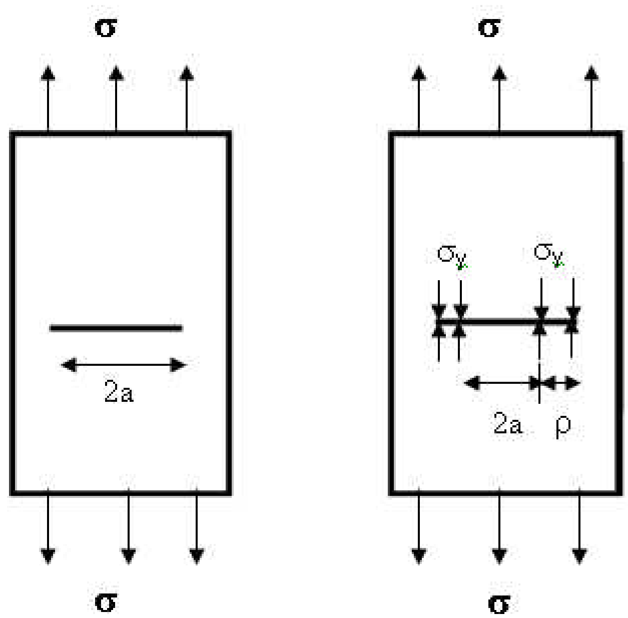
$$I_1(s)=\int_{-1}^{s}(1\mp t)/\sqrt{\rho+t}\;dt \tag{7}$$

and

$$I_2(s)=\int_{-1}^{s}(1\mp t)/\sqrt{\rho-t}\;dt \tag{8}$$

These are integrated exactly in closed form using elementary methods. The significant point is that even those integrals which have the crack tips at the end-points of integration (and consequently integrands that are singular) have been integrated exactly in closed form, yielding results that are in some sense "exact," and independent of the accuracy of the integration technique employed.

4. Methodology:



(a) (b)

Figure 3. The strip yield model representation

To derive the desired strip yield solution using the NASBEM software, two models for Boundary Element Method (BEM) analysis need to be constructed (see Figure 3). The first model, Fig 3 (a), includes the physical crack with loading applied to the boundary, and is used to calculate the value of K. The second model, Fig 3 (b) loaded as in Fig 3 (a) on the boundary, includes both the physical crack and the yield zone (loaded to the yield stress value) and is used for iteratively calculating the strip yield solution for the length of ρ and the CTOD. For the second model, the crack is modeled using segments consisting of the physical crack of length 2*a*, and the yield zone segments of length ρ, which are loaded by closure stresses equal to the assumed yield stress. For this second model, after a case is run, the resulting K value is checked, and if the K value is not sufficiently close to zero, the length of ρ is adjusted, and the process is repeated.

When constructing the BEM, special attention is required to the interface between the physical crack and the plastic zone. The number of elements on both sides of the interface needs to be equal in order to achieve highly accurate solutions. For this purpose the interface was divided into 2 segments of equal length with a relatively high number of elements. Currently this numerical approach is in the process of being automated, and will be able to compute CTOD, and CTOA in an easy and short manner.

5. Results/Discussion:

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a. Center Crack Case:

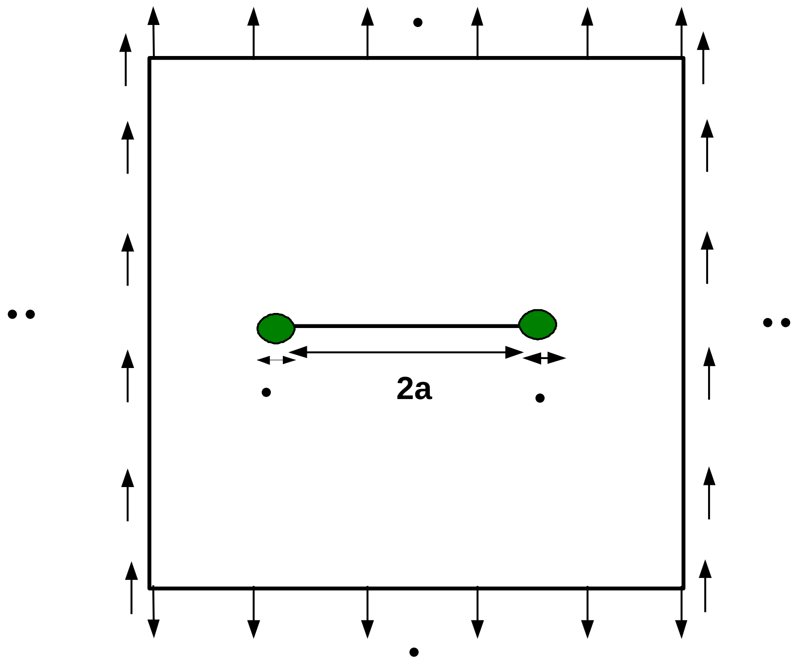

Figure 4 A center crack case configuration

The CTODs were calculated using the proposed BEM methodology, and compared to results obtained by the original Dugdale formula for an infinite plate to verify the accuracy of the BEM. Figure 5 shows the CTOD comparisons between the BEM and Dugdale solution for an infinite plate. The Maximum difference between the two solutions was about 1%, thus deeming this new method highly accurate and effective. In figure 6 the BEM solutions were also compared to Tada [21] for a finite width plate with a center crack (2a/w=0.2). The BEM and Tada solution had a maximum difference about 3.5%.

Figure 7 compared CTOD using BEM for an infinite plate under uni-axial and bi-axial loading conditions. CTOD values for uni-axial and bi-axial values were the same until (s / s_y)>0.7 when CTOD for the bi-axial started to decrease as the stress increased. This is because the horizontal load on the plate resulting when •=1 imposed a restraint on the lateral displacement in the y-direction thus generating lower values for CTOD.

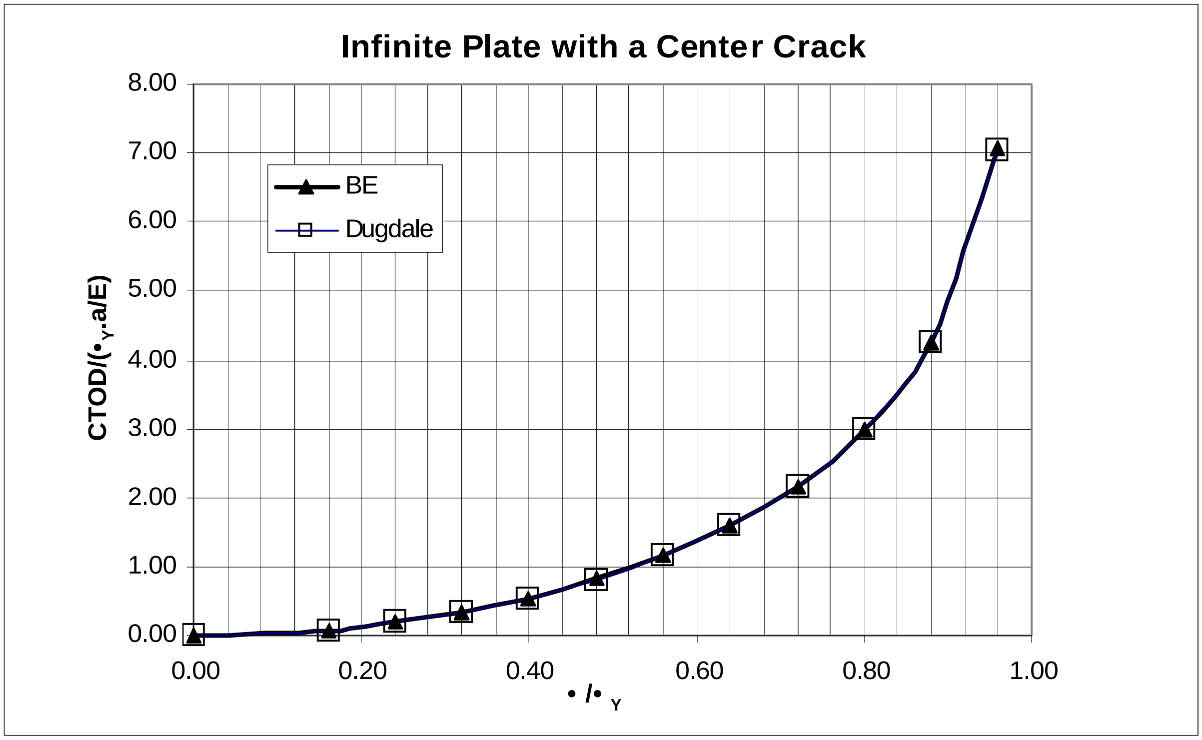

Figure 5 Boundary Element Method CTOD comparisons between the BE and Dugdale solution for an infinite plate

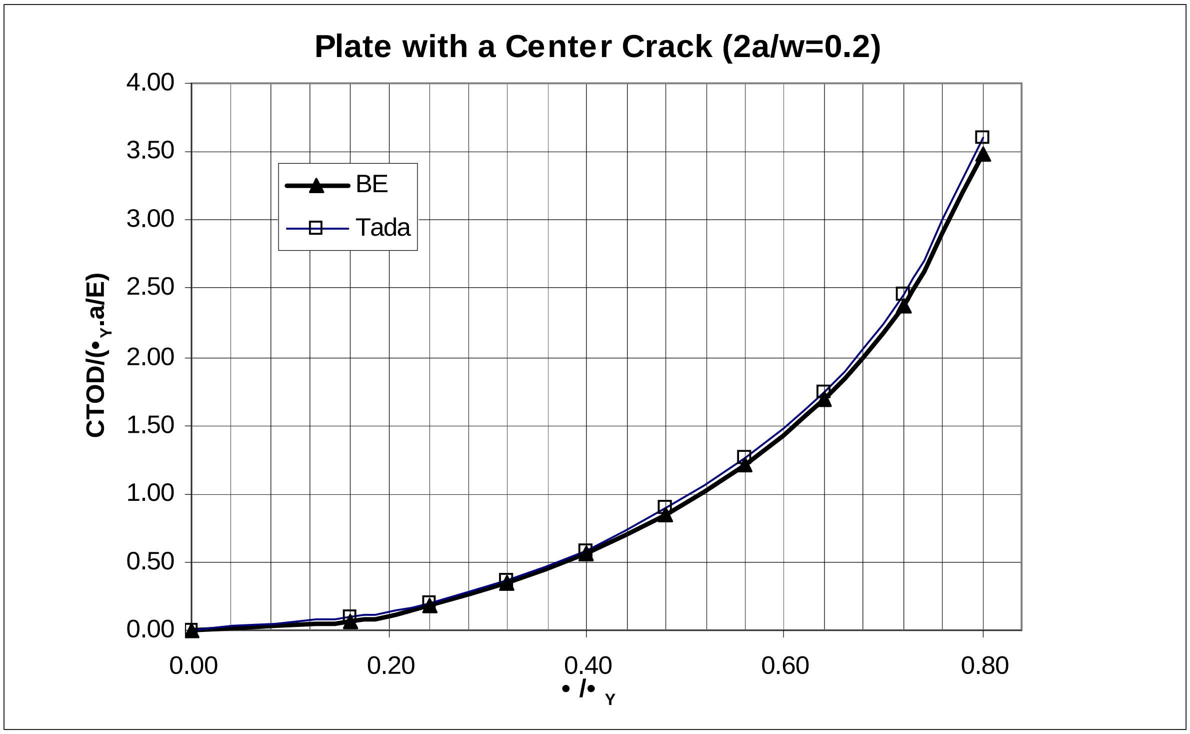

Figure 6 Boundary Element Method CTOD comparisons between the BE and Tada solution for 2a/w=0.2

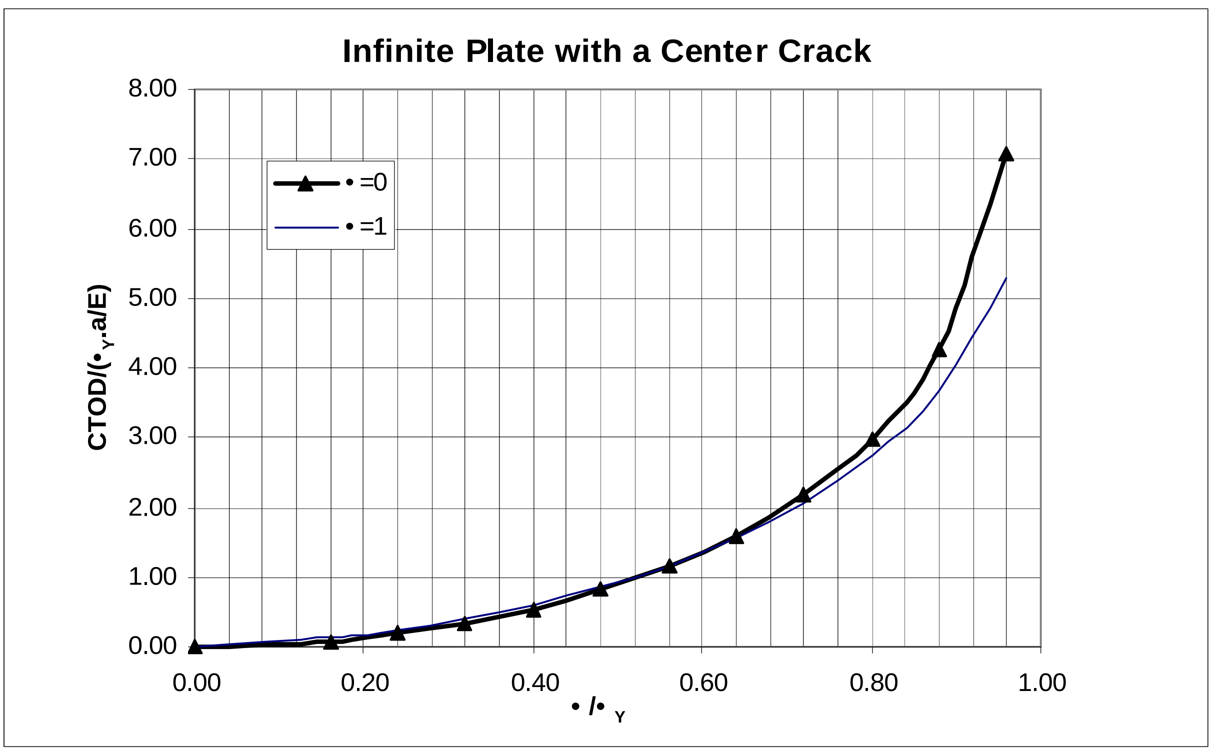

Figure 7 Boundary Element Method CTOD comparisons for an infinite plate under Uni-axial, and Bi-axial loading

b. Plate with a Center Hole Crack (2 Cracks):

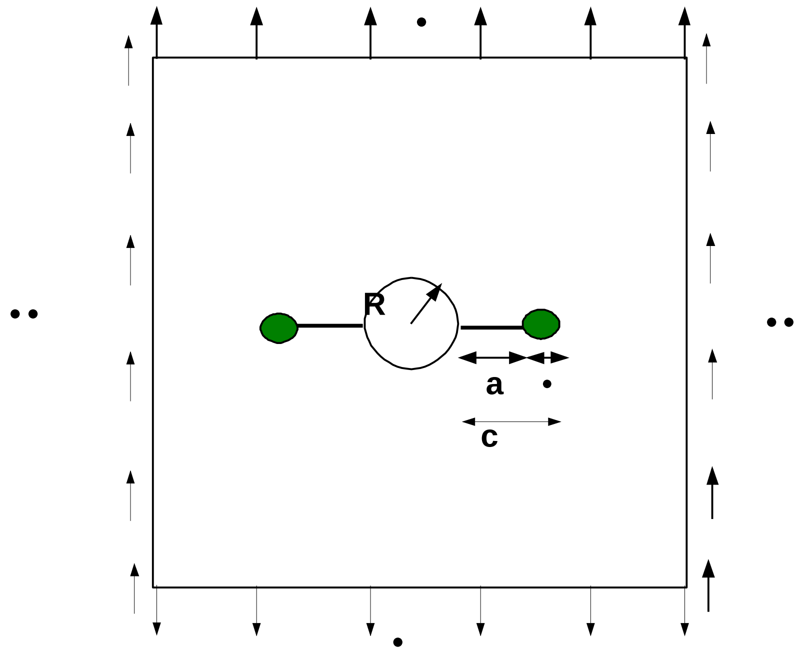

Figure 8 A plate with a center crack hole case configuration

Different crack length and plate widths were analyzed for the center crack hole plate case configuration. The CTODs and plastic zone sizes were calculated using the Boundary Element Method for axial and biaxial loading conditions. Before the BEM model was used, the accuracy of the BEM had to be verified by comparing the center crack hole plate case configuration results for one and two cracks to Rich [22] using an infinite plate with a center crack hole.

Uni-axial loading condition (• =0):

The plastic zone sizes were compared in figure 6 between the Boundary Element Method, and Rich's solution for a/R=0.5, 1, & 2 under uni-axial loading conditions. The BEM technique was very effective when compared to Rich's solutions, and the results had a maximum difference of less than 2.5%.

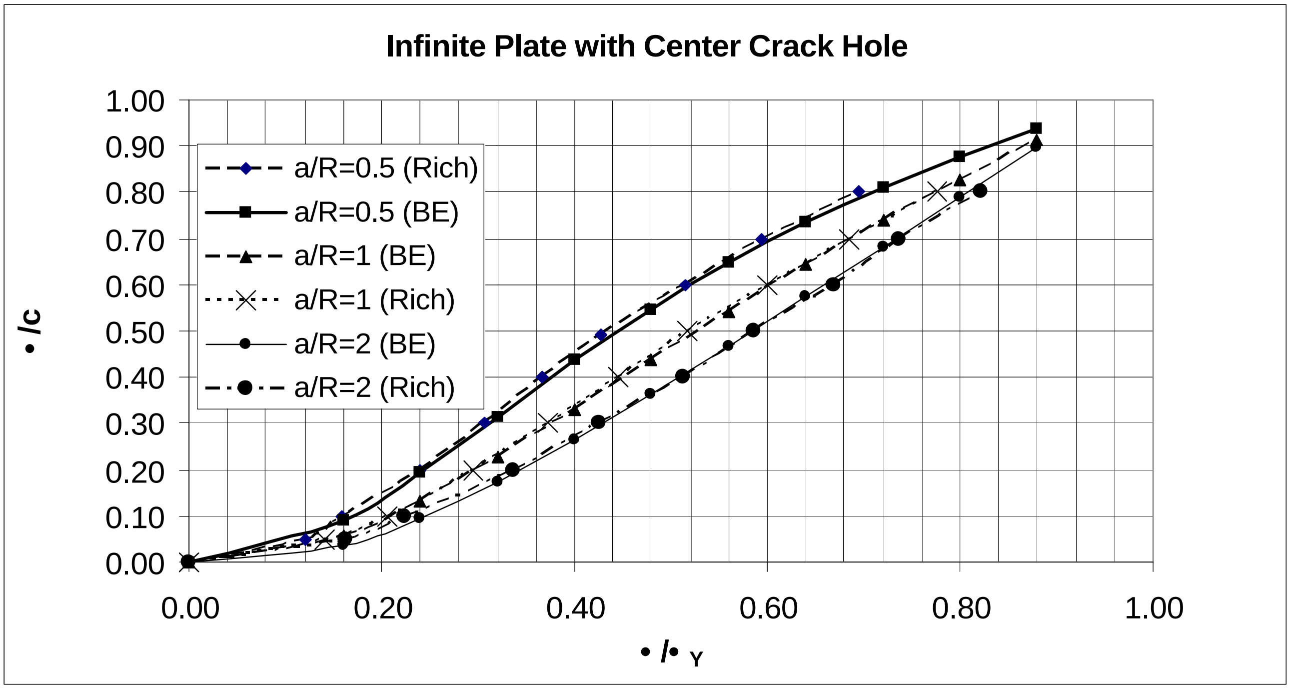

Figure 9 Plastic zone size comparisons between the Boundary Element Method, and Rich's solution for different crack sizes

The plastic zone sizes ratios for an infinite plate were compared in figure 9 for different crack lengths ranging from a/R=0.05 to 2. For smaller crack length like in the case of a/R=0.05 the uni-axial tension that is applied normal to the crack possesses higher stress concentration from the hole influencing the plastic zone size. As a/R ratio approaches a large value, the solution gets closer to Dugdale solution for internal crack in an infinite plate.

Bi-axial loading condition (• =1):

The effects of the bi-axial loading on the CTOD were investigated for different crack lengths as shown in figure 10-12. The CTOD for uni-axial tension was compared to the one with bi-axial tension. In the case of the bi-axial tension the CTOD was lower than the un-axial loading case. This is because the horizontal load on the plate resulting when • =1 imposed a restraint on the lateral displacement in the y-direction thus generating lower values for CTOD. The effect was more noticeable for small cracks as shown in figure 10 for a/R=0.05.

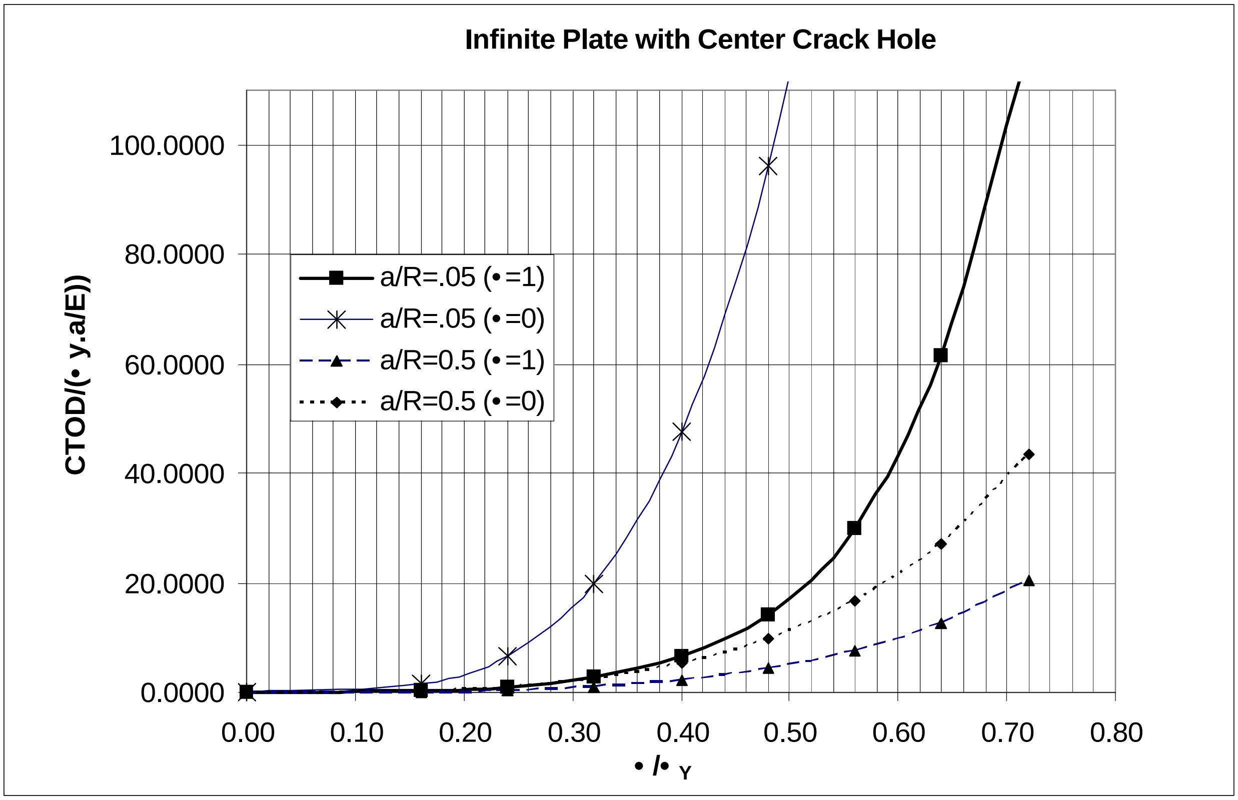

Figure 10 Boundary Element Method CTOD comparisons for different cracks using an infinite plate under Uni-axial, and Bi-axial loading

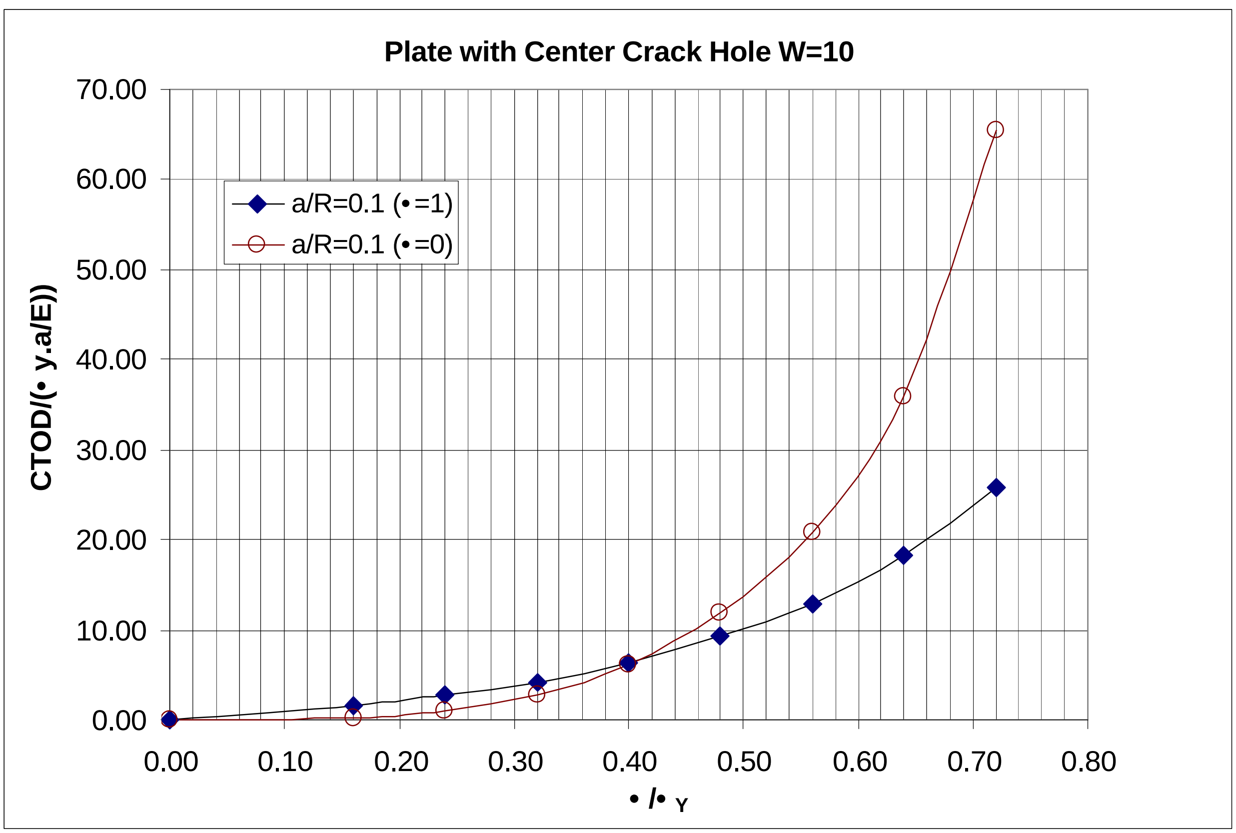

Figure 11 Boundary Element Method CTOD comparisons for different cracks under uni-axial, and Bi-axial loading

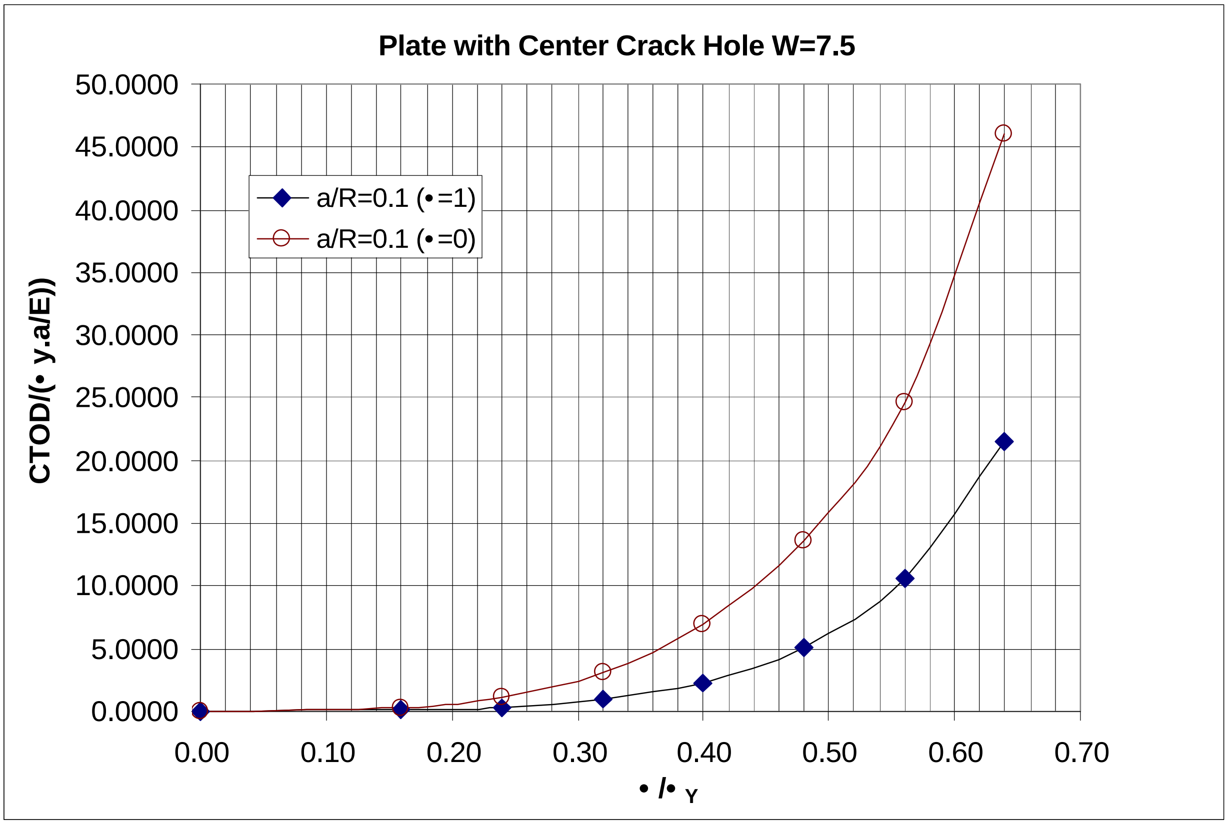

Figure 12 Boundary Element Method CTOD comparisons for different cracks under Uni-axial, and Bi-axial loading

c. Compact Specimen:

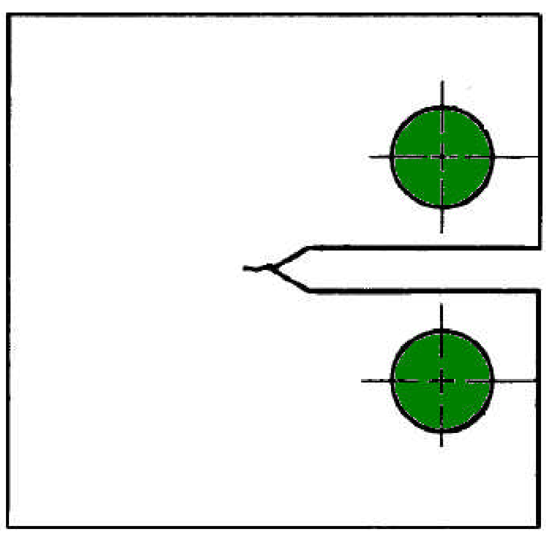

Figure 15 A Compact Specimen

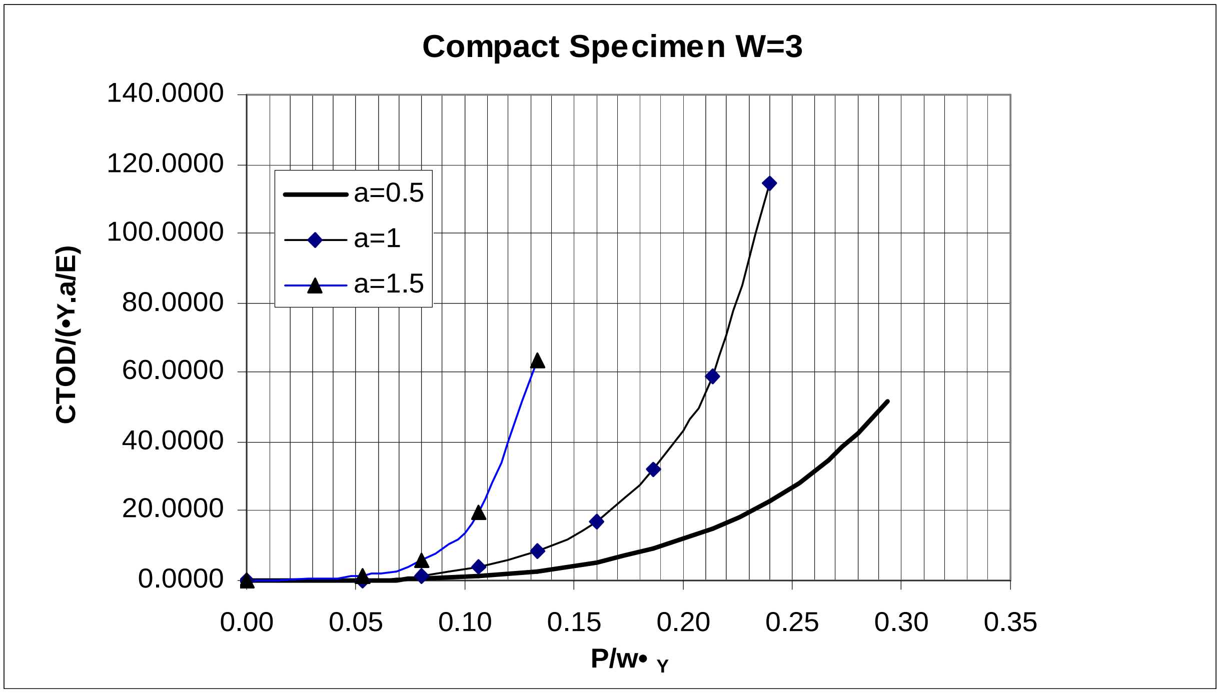

Figure 13 Boundary Element Method CTOD comparisons for different cracks using a w=3 plate

d. Three Hole Crack in Tension (THT):

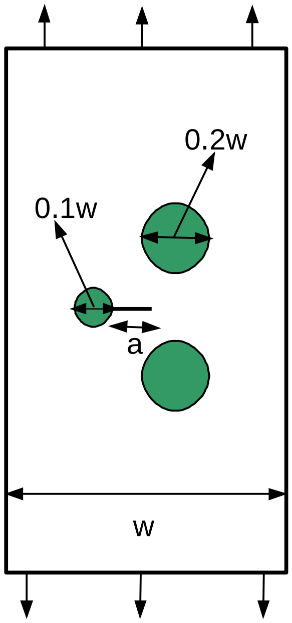

Figure 14 A plate with a three hole crack configuration

The CTOD was also calculated for complex geometries like the Three Hole Crack in tension

(THT) case as shown in figure 15. The CTOD were calculated for two plate widths. The first plate had a width of 10”, while the second plate had a width of 20”. Both geometries had a/R=1. The second plate with a width of 20” exhibited higher CTOD values compared to the narrower plate.

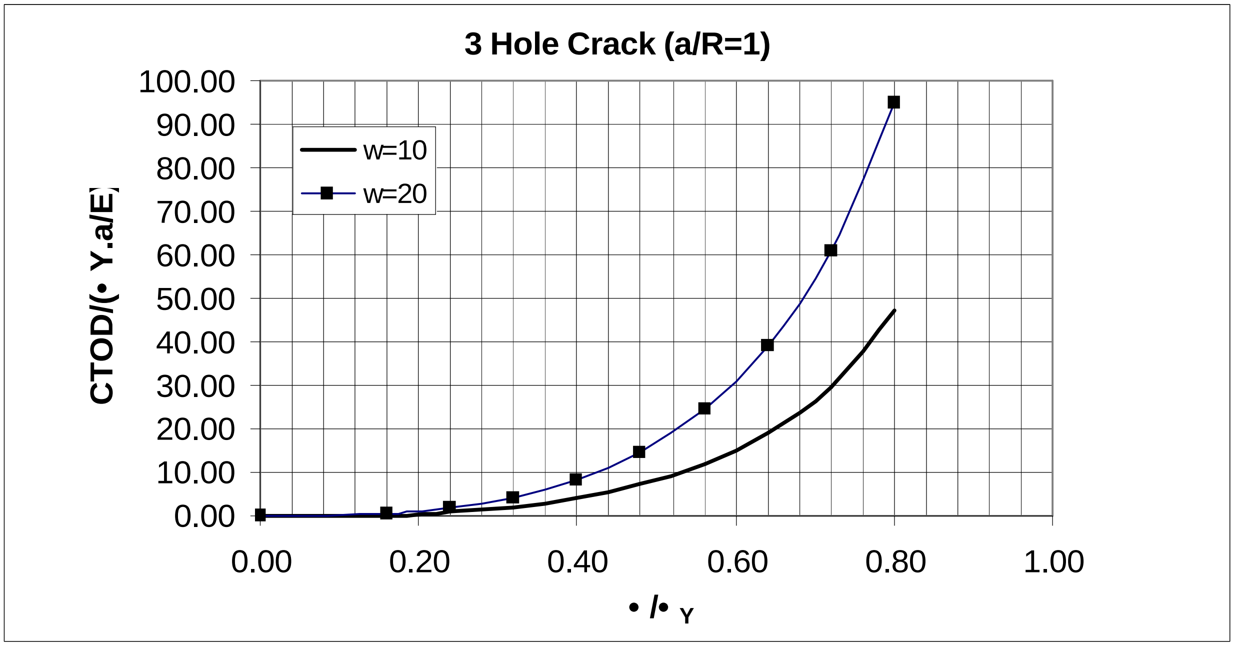

Figure 15 Boundary Element Method CTOD comparisons for different widths for a 3 hole crack configuration

e. Multi Site Damage:

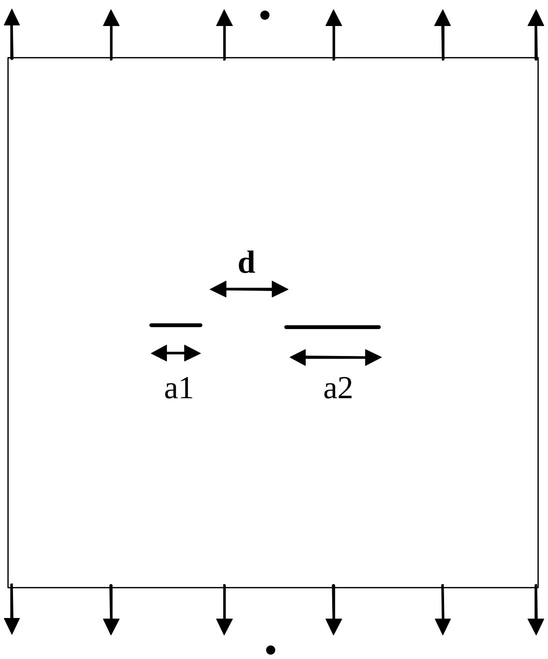

Figure 16 A plate with a three hole crack configuration

The CTOD and plastic zone sizes were investigated using the new method developed in this paper to study the effects of Multi Site Damage (MSD). Crack tips link up may be challenging when multiple cracks are involved like in the case of MSD. This is because as two tips approach, the energy-based methods may prove difficult to evaluate. Since CTOD is measured local to the crack tip, multiple crack interaction effects are expected to be negligible [23]. The geometry modeled here consisted of 2 cracks of different lengths. The first crack (a1) had a length of 0.5" while the second crack (a2) had a length of 1". The distance between the two cracks (d) was assumed as 2". The CTOD for the big crack was higher than the smaller crack as shown in figure 17. The maximum difference between the two cracks was 83% at the higher stress ratio. The plastic zone ratio was also investigated as shown in figure 18. The plastic ratios were approximately the same until the plastic zone for the big crack started to get higher at stress ratios larger than 0.67. The plastic zones connected with each other at a stress ratio of 0.78.

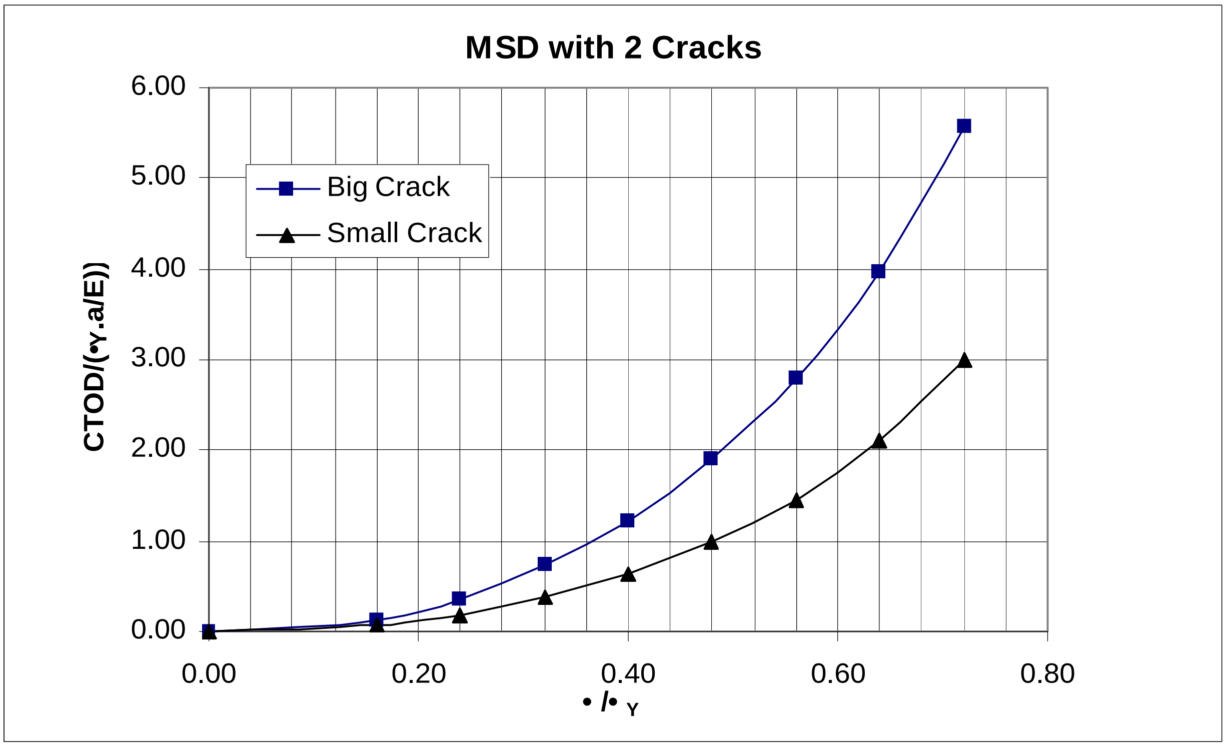

Figure 17 Boundary Element Method CTOD comparisons for MSD

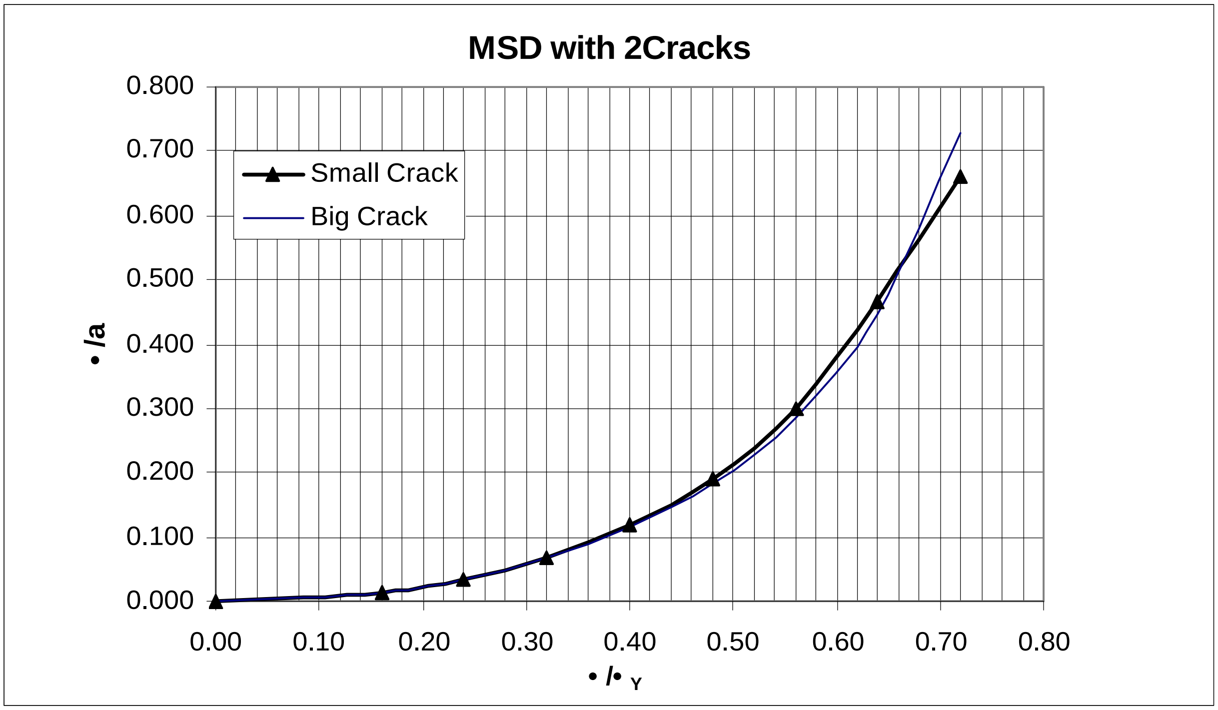

Figure 18 Boundary Element Method CTOD comparisons for the plastic zones

6. Summary/ Conclusion:

The applicability of a new numerical method for calculating the Crack Tip Opening Displacement (CTOD) was investigated. Some of the geometries included the complex Multi Site Damage (MSD), and the Three Hole Crack in Tension (THT) cases. The new technique developed in this paper adapted the Boundary Element / Dislocation Density Method to obtain crack-face opening displacements at any point on the crack. Most of the methods in literature used to calculate CTOD were complicated, costly, time consuming, and many of them only apply to certain geometries and loading conditions. The new approach developed in this paper was applied to different crack configurations having finite and infinite geometries, complex geometrical problems, and arbitrary applied loading conditions with great accuracy, and very minimum computation time and resources.

7. Acknowledgments:

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