# Stirling Analysis Comparison of Commercial VS. High-Order Methods

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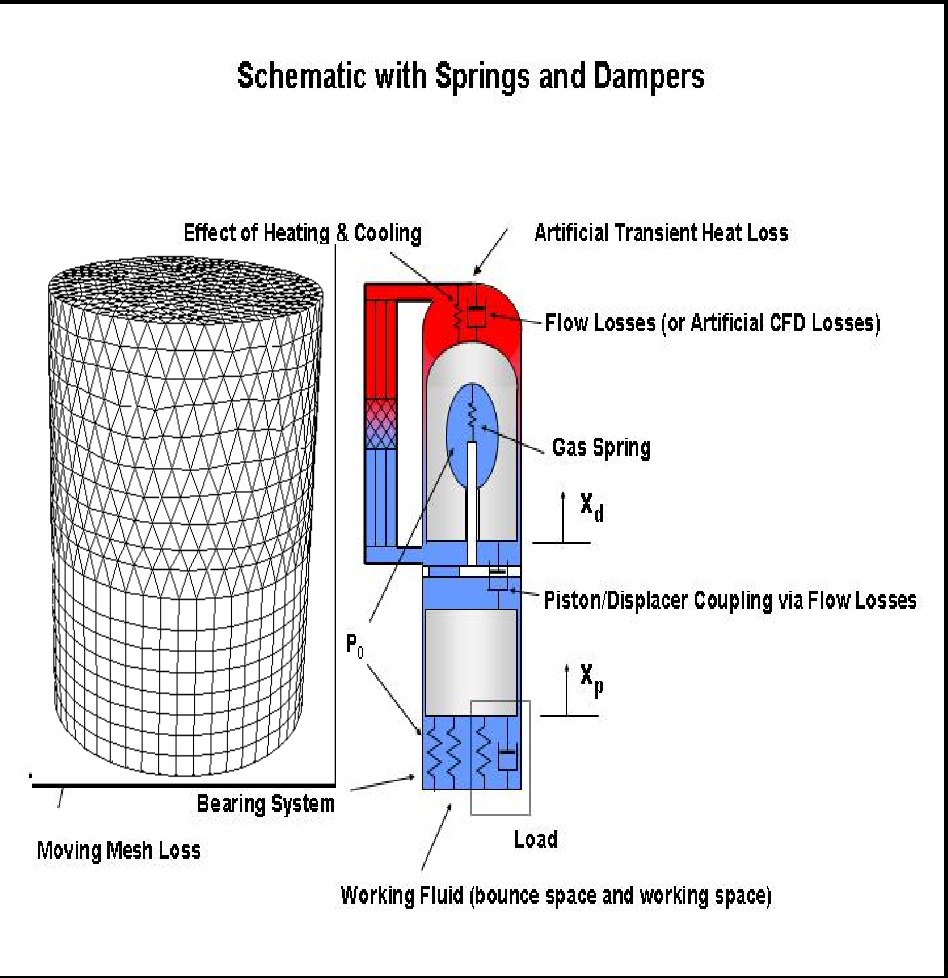
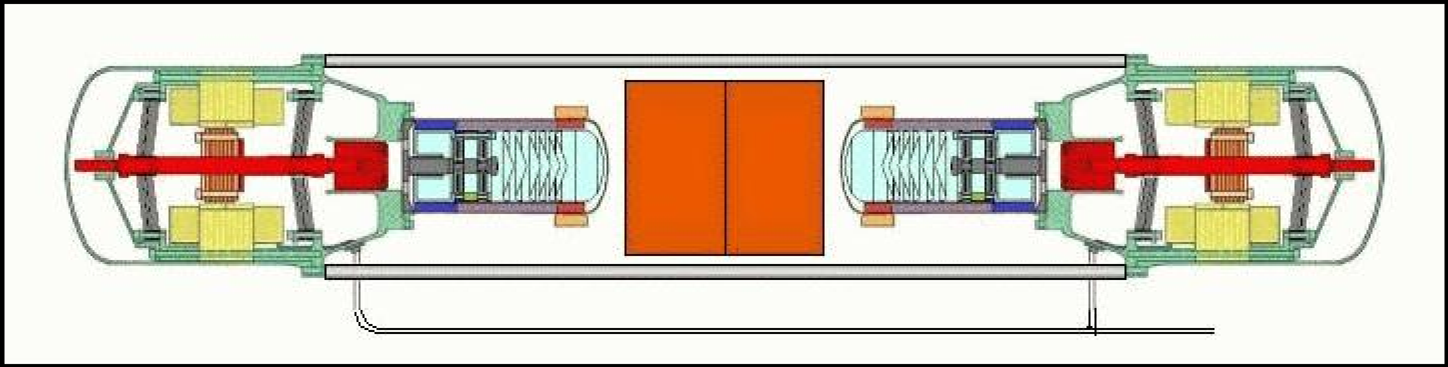
# Stirling Simulation Numerical Error

- •Remeshing interpolation
- •Layering interpolation
- •Diffusive time advance
- •Low grid quality/skewness
- •Sliding interface interpolation
- •Artificial entropy
- •Turbulence transition



# Dual Opposed Convertors

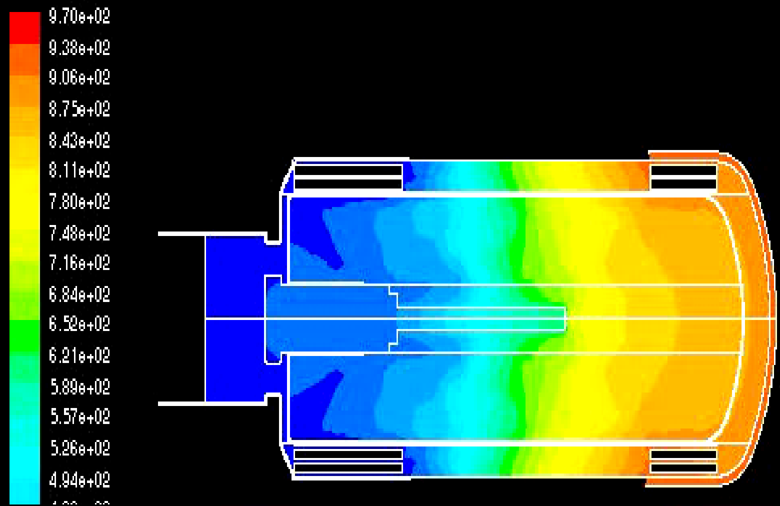
- High Efficiency – Low Mass Space Power

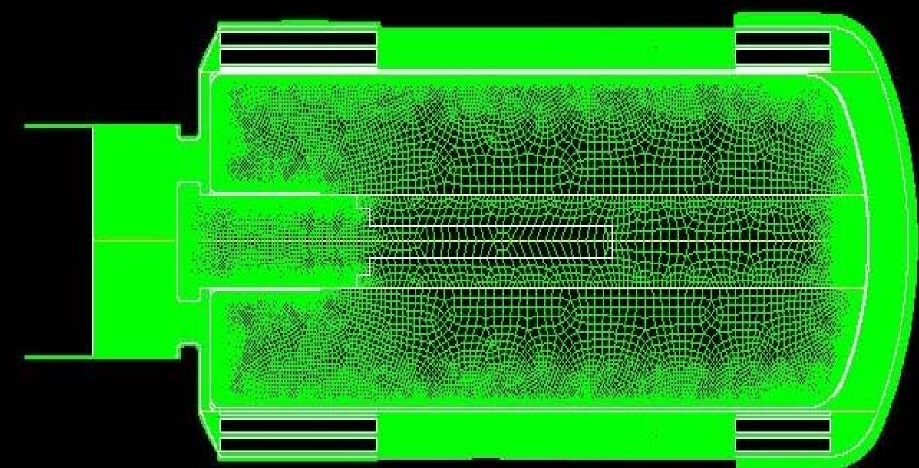


- Free Piston Geometry is Essentially Smooth



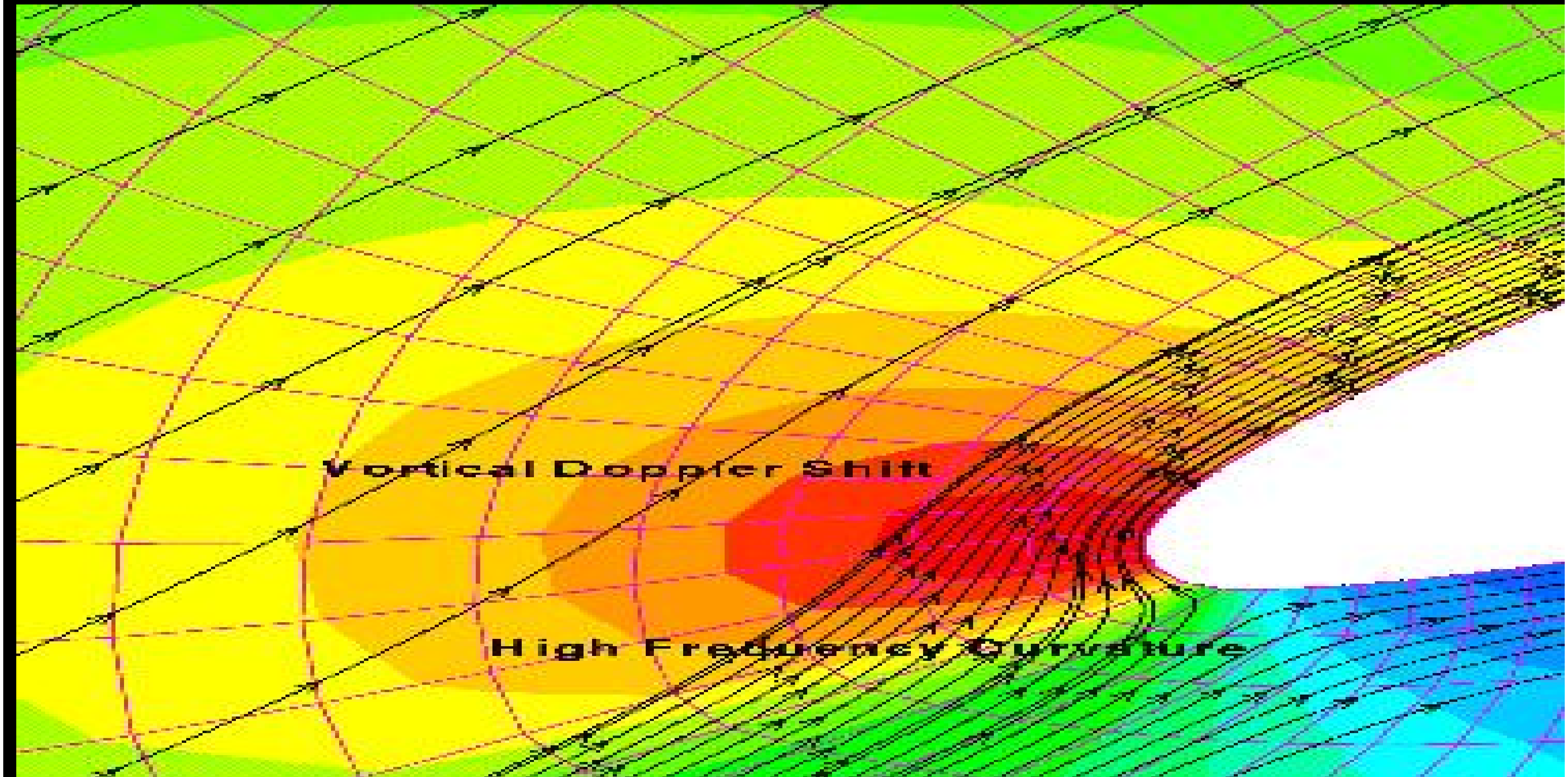
# Whole Engine Simulation



- Bounded by Walls – No need for nonreflecting B.C.
- Kolmogorov scales fairly large
- Steep thermal gradients
- No shocks/subsonic/transitioning
- High-order friendly

# Curvilinear Features



# Stability Analysis

- Courant (CFL) number,
  $r = c \, \Delta t / \Delta x$
- Von Neumann number,
  $v = \mu \, \Delta t / \Delta x^2$
- Linear Viscous Burger's Equation

$$\frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = \mu \frac{\partial^2 u}{\partial x^2}$$

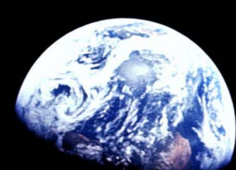
# Compact Scheme 6th Order in Space

$$\alpha\left(\frac{\partial u}{\partial x}\right)_{i-1}+\left(\frac{\partial u}{\partial x}\right)_{i}+\alpha\left(\frac{\partial u}{\partial x}\right)_{i+1}=a\,\frac{u_{i+1}-u_{i-1}}{2\Delta x}+b\,\frac{u_{i+2}-u_{i-2}}{4\Delta x}$$

$$\alpha=1/3,\ a=14/9,\ b=1/9$$

$$\alpha\left(\frac{\partial^2 u}{\partial x^2}\right)_{i-1}+\left(\frac{\partial^2 u}{\partial x^2}\right)_{i}+\alpha\left(\frac{\partial^2 u}{\partial x^2}\right)_{i+1}=a\frac{u_{i+1}-2u_i+u_{i-1}}{\Delta x^2}+b\frac{u_{i+2}-2u_i+u_{i-2}}{4\Delta x^2}$$

$$\alpha=2/11,\ a=12/11,\ b=3/11$$



# Runge-Kutta 4th Order

$$R(u) = -cu_x + \mu u_{xx}$$

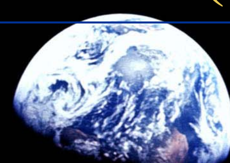
$$u^{(1)} = u^n + \frac{\Delta t}{2} R^n$$

$$u^{(2)} = u^n + \frac{\Delta t}{2} R^1$$

$$u^{(3)} = u^n + \Delta t R^2$$

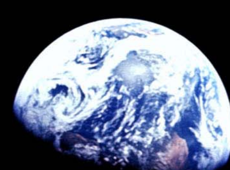
$$u^{(n+1)} = u^n + \frac{\Delta t}{6}\left(R^n + 2R^{(1)} + 2R^{(2)} + R^{(3)}\right)$$

$$R^{(1)} = R(u^{(1)}), R^{(2)} = R(u^{(2)}), R^{(3)} = R(u^{(3)})$$



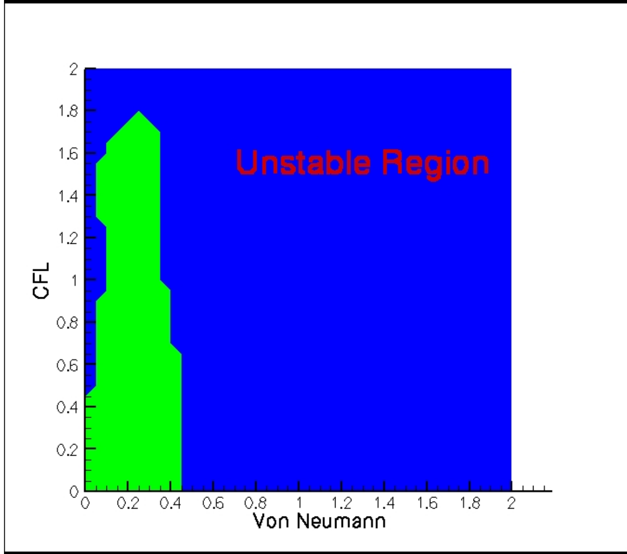
# Current Practice

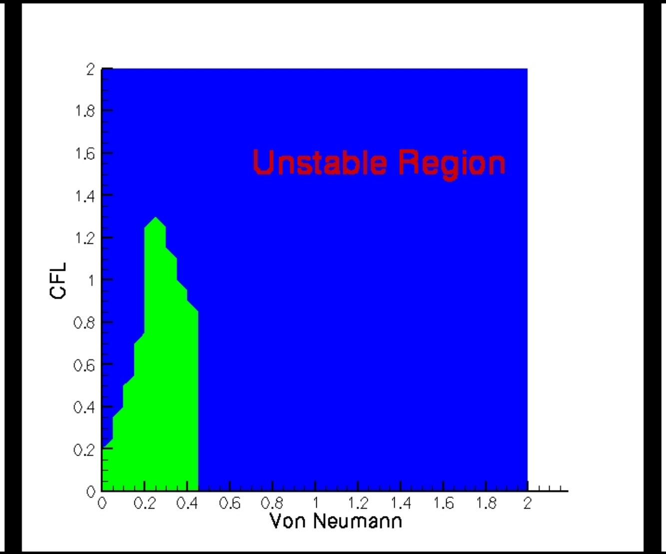
- Implicit 1$^{st}$ or 2$^{nd}$ order in time commercially
- 1$^{st}$ or 2$^{nd}$ order in space implicit
- Explicit/implicit 4$^{th}$ order in time academically
- Implicit 6$^{th}$ order compact scheme in space

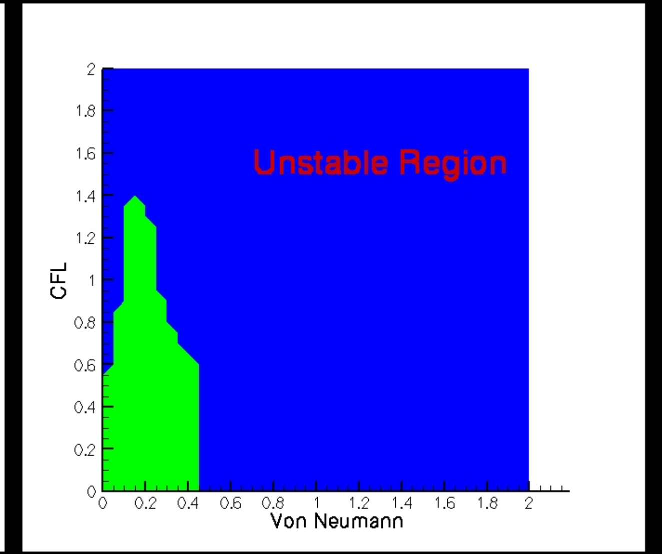


# Compact Scheme Stability Range

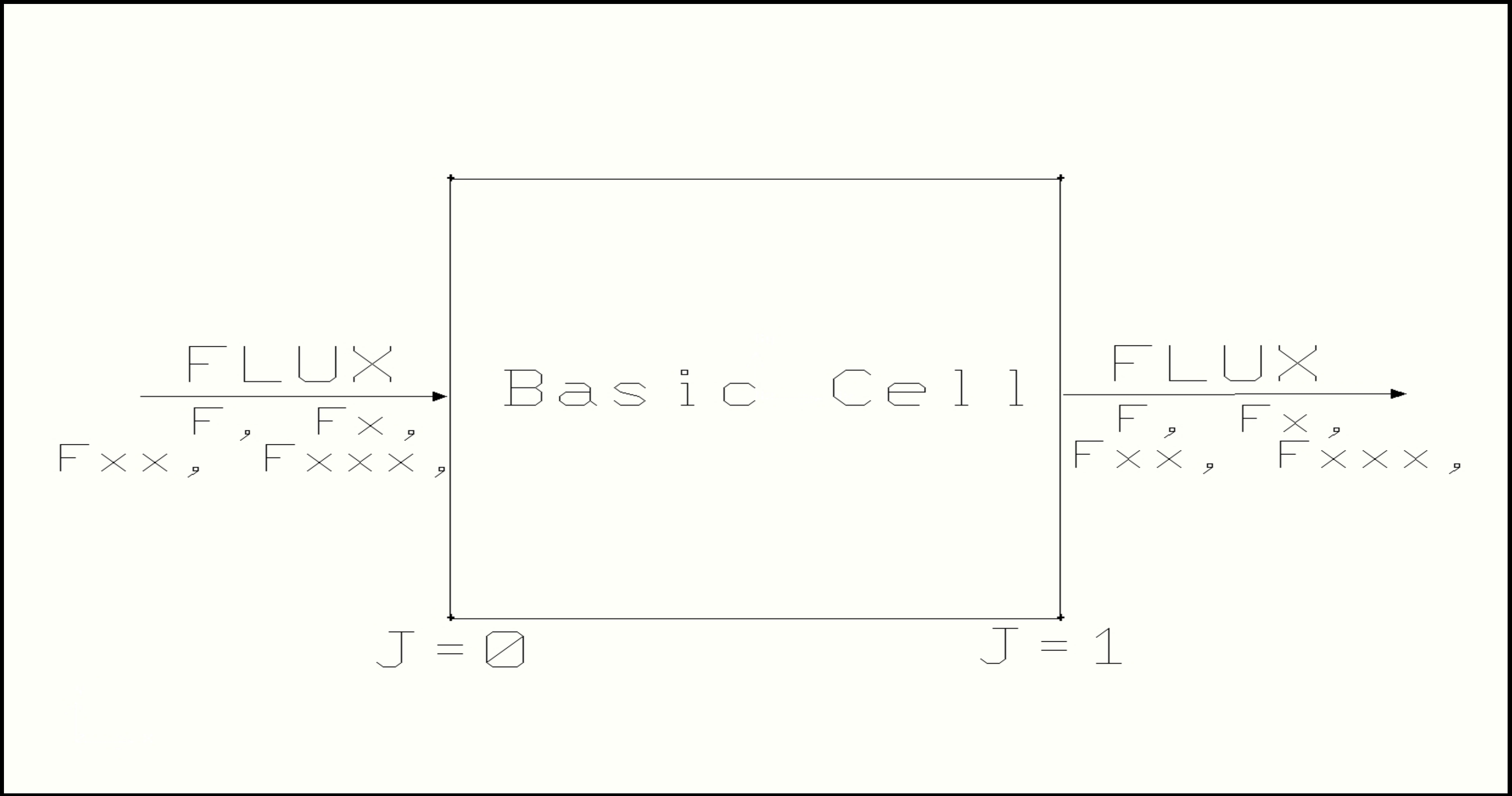
- Domain size affects stability since implicit



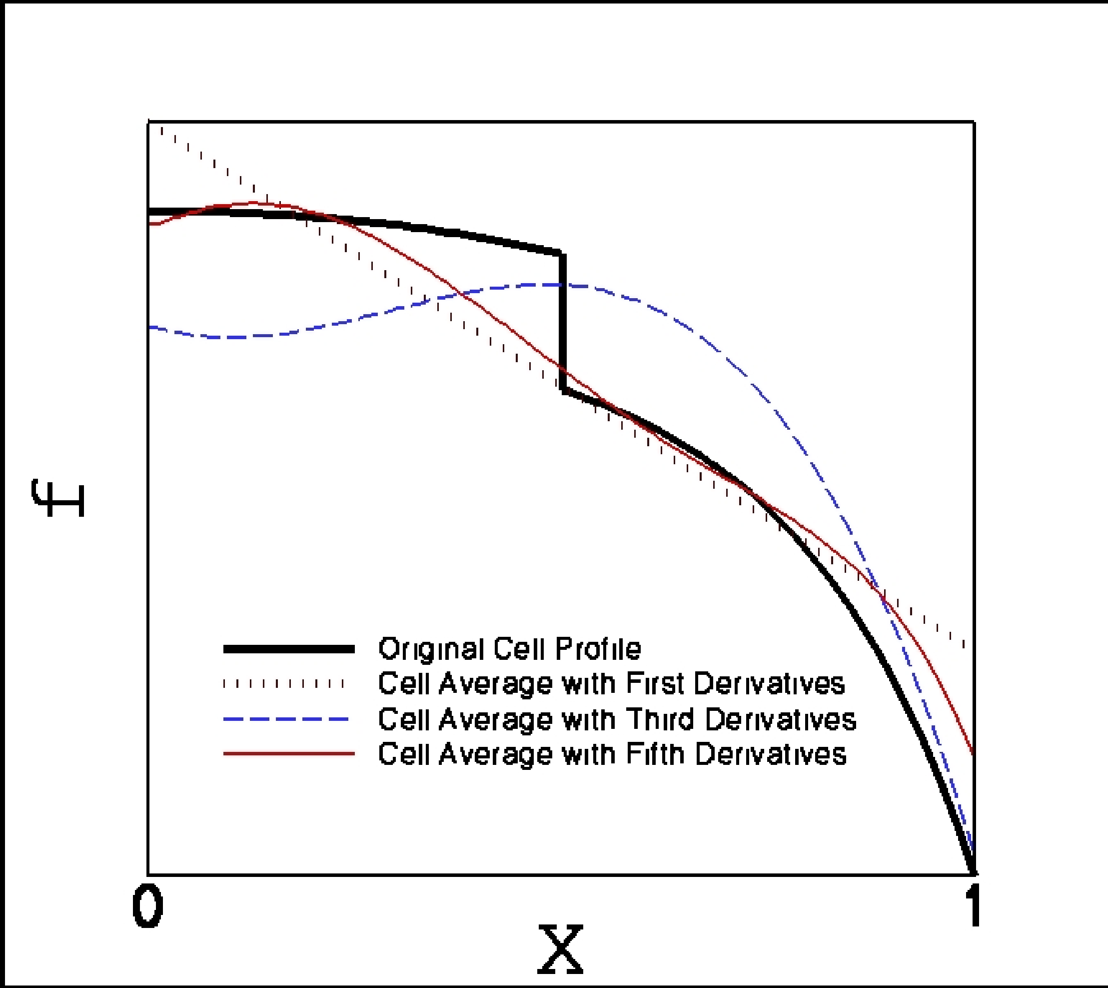



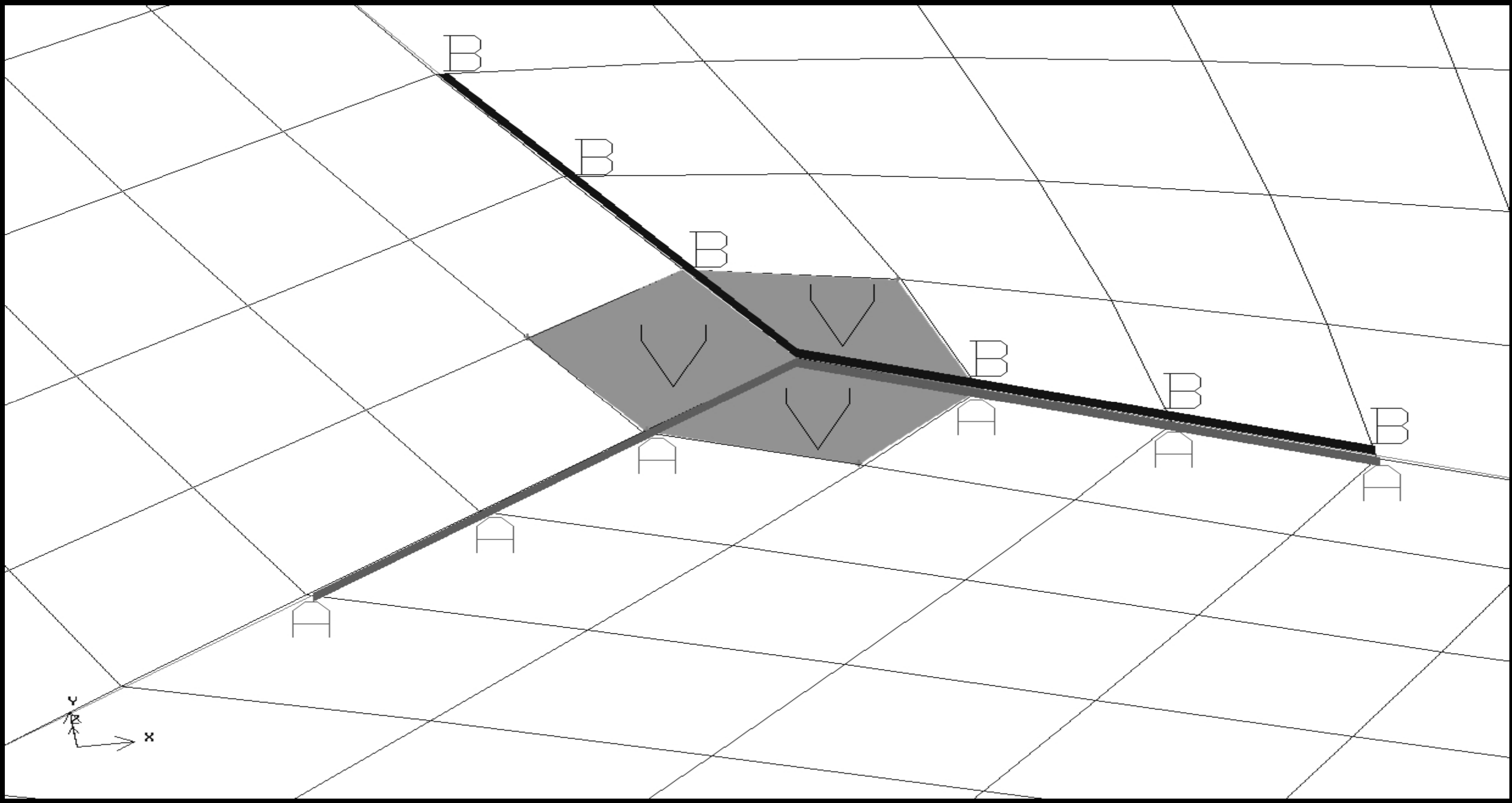
# Basic UHF Technique



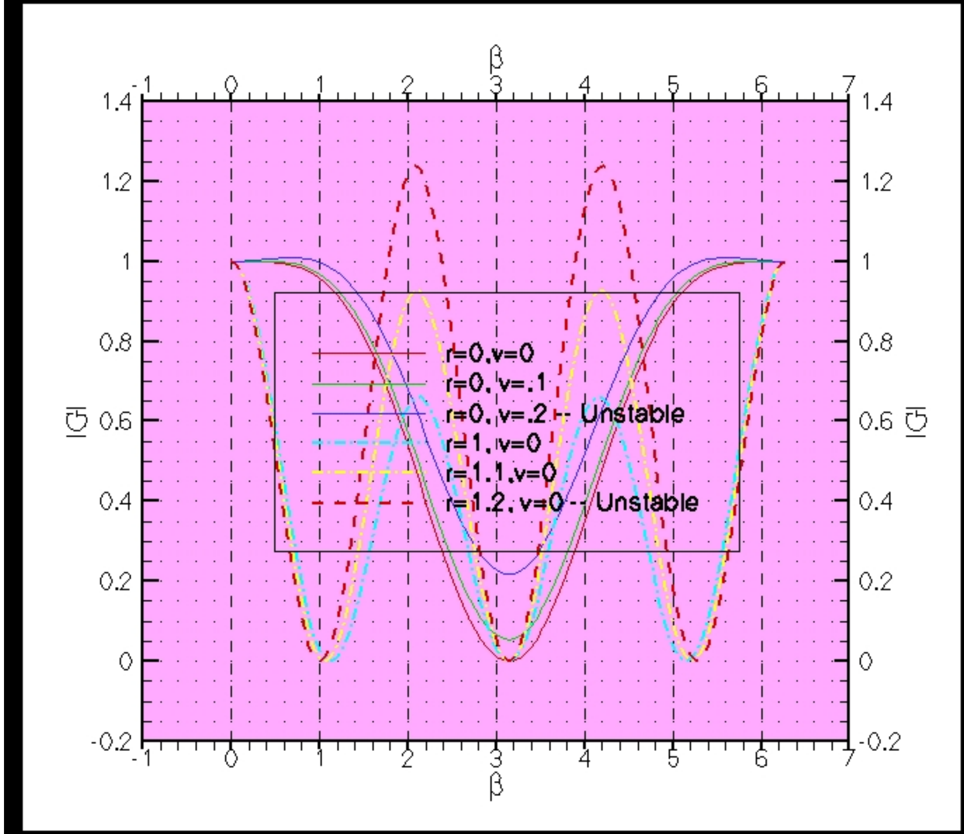
# Derivatives of Cell Averages

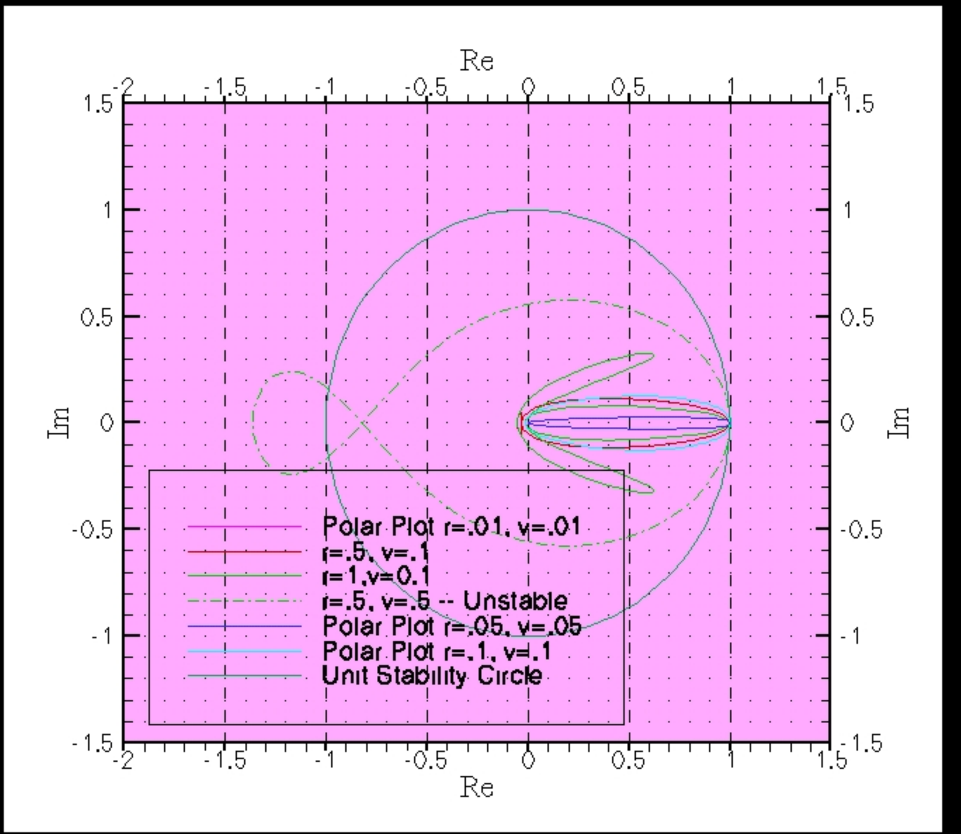


# Grid Singularity Resolution

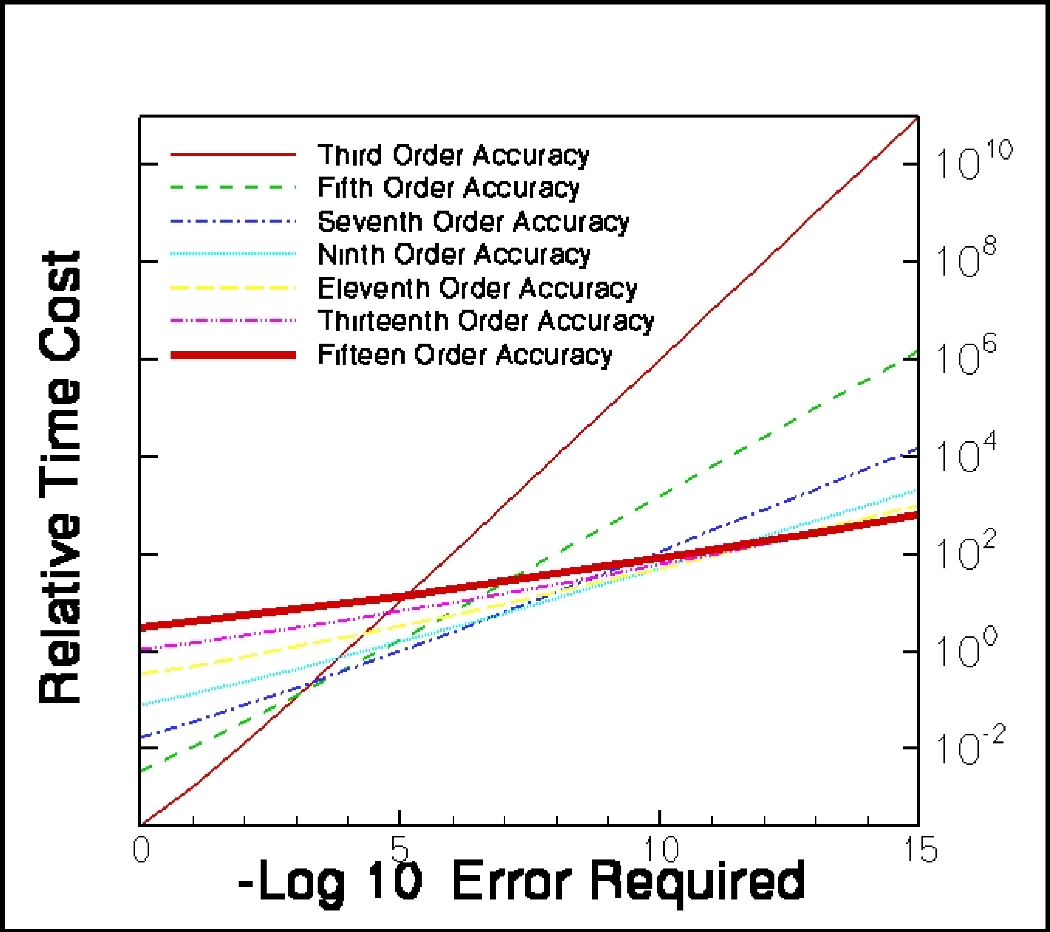


# C4o0 Linear Viscous Burger's Equation

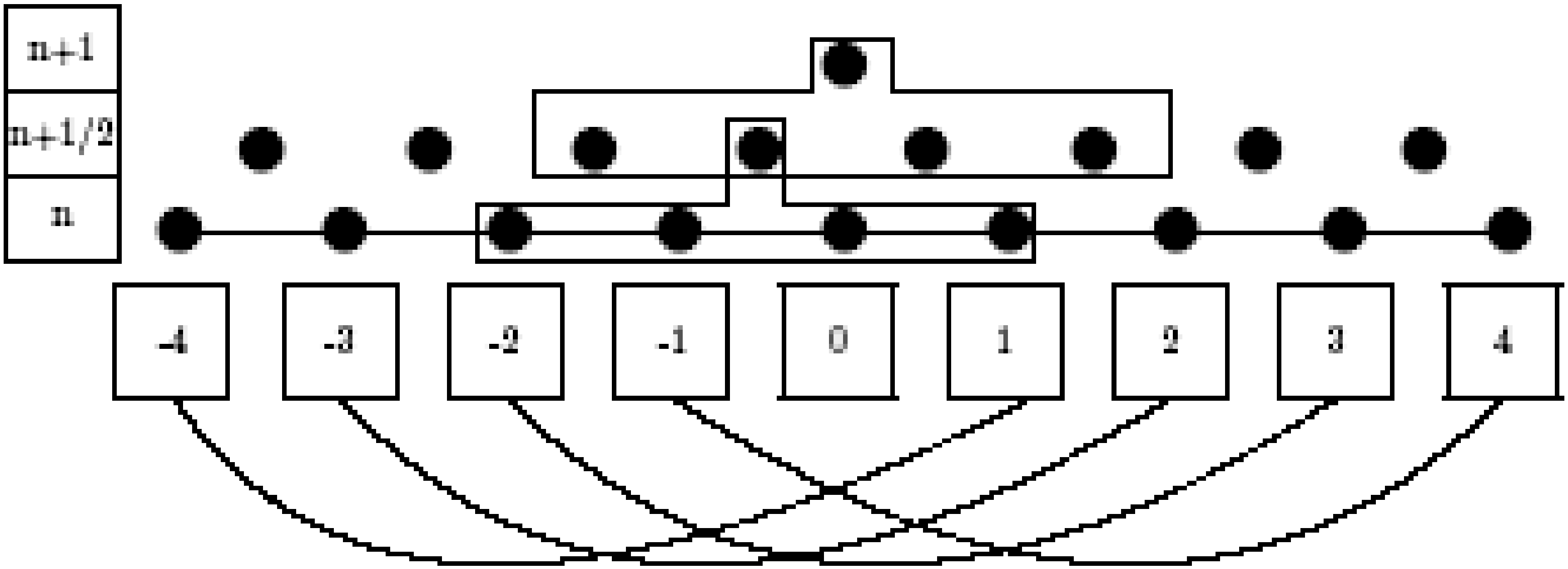


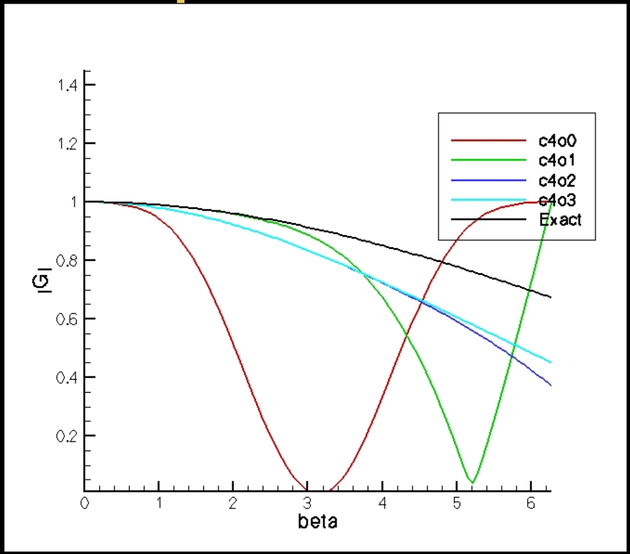
# Efficiency Improves with Accuracy

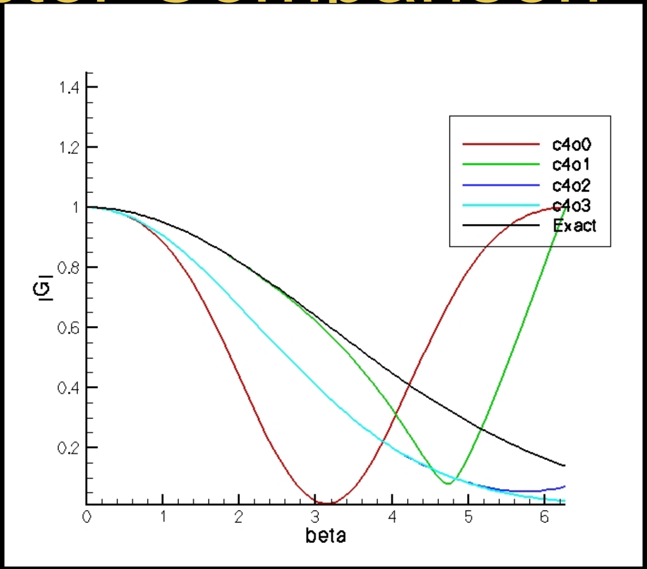


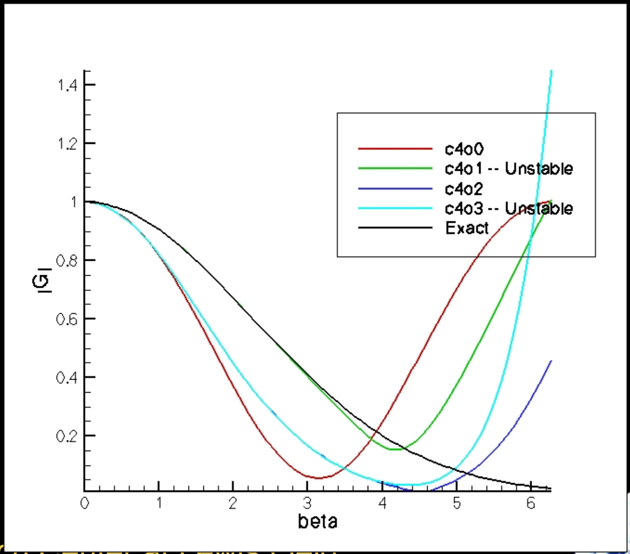
# Computational Domain Schematic

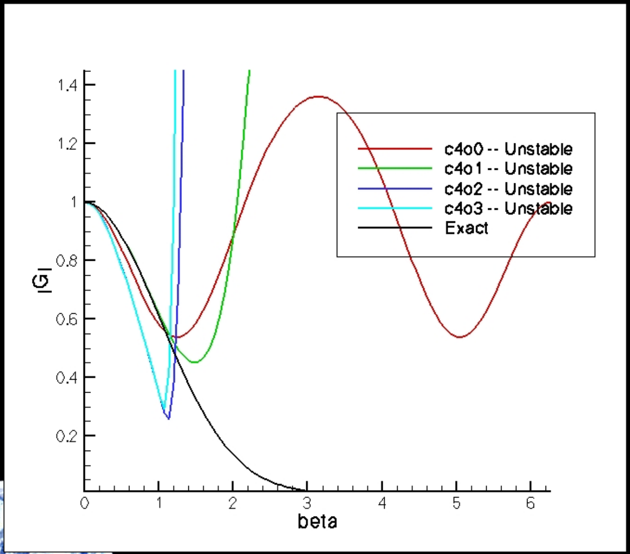


# Amplification Factor Comparison

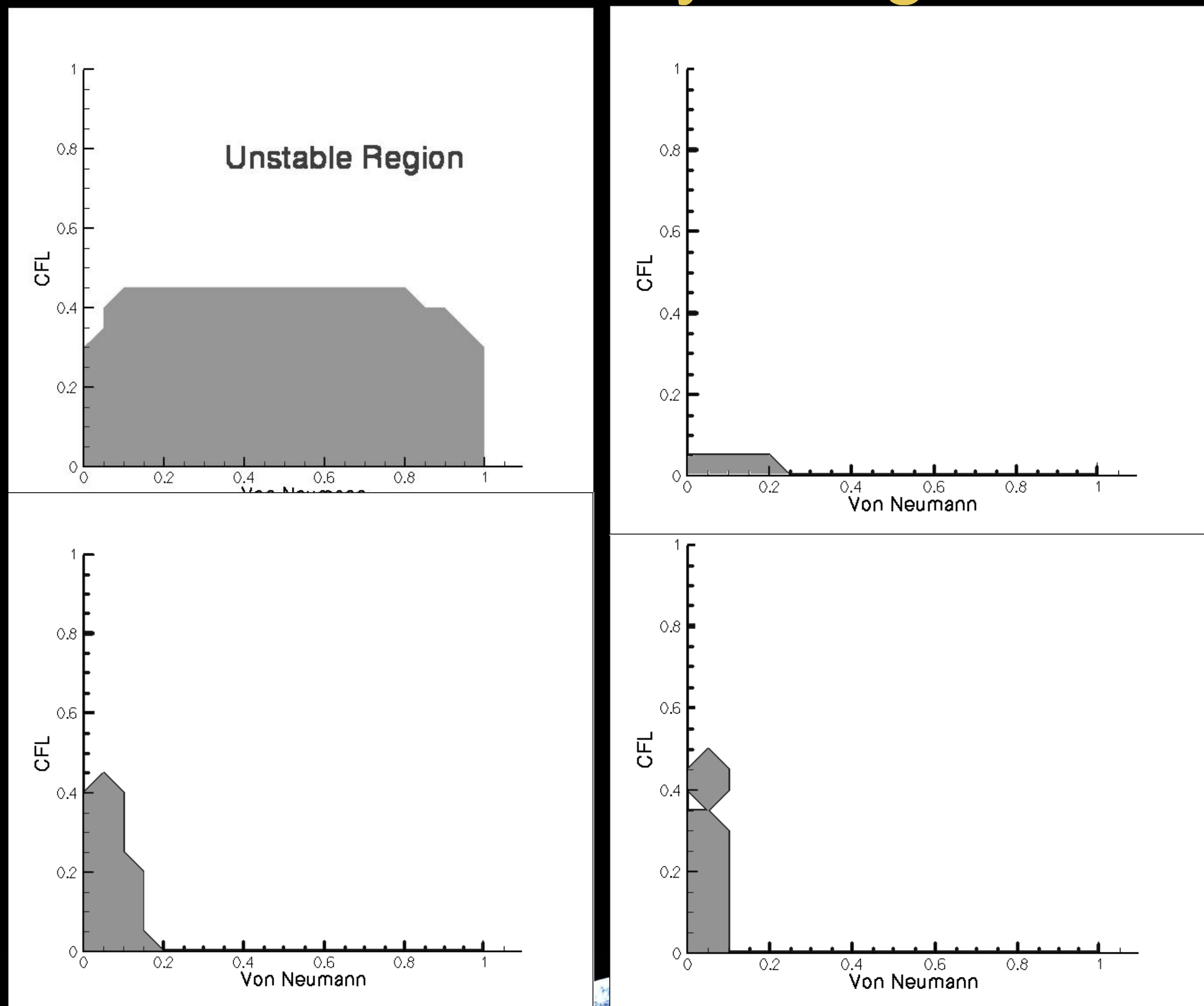


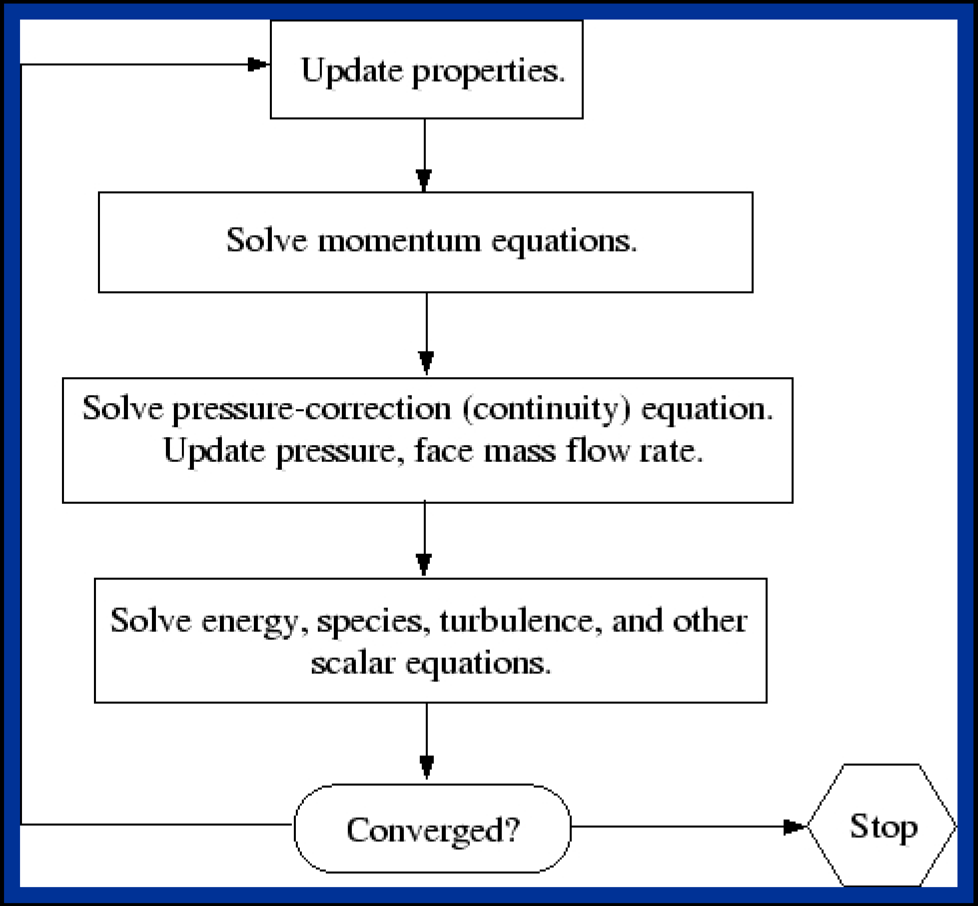



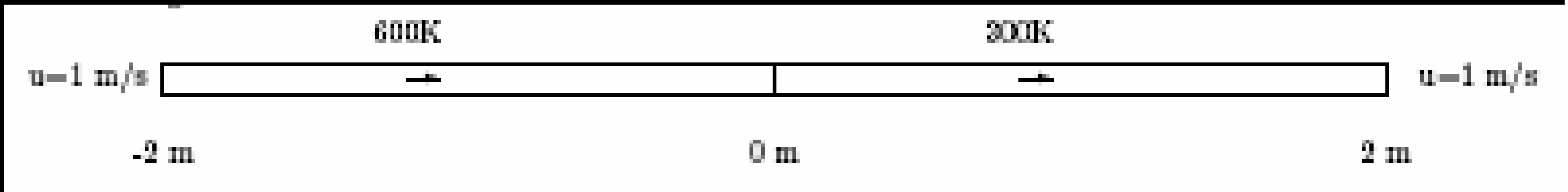
# UHF Stability Range



# Overview of Segregated Solution



# Heat Transfer Test



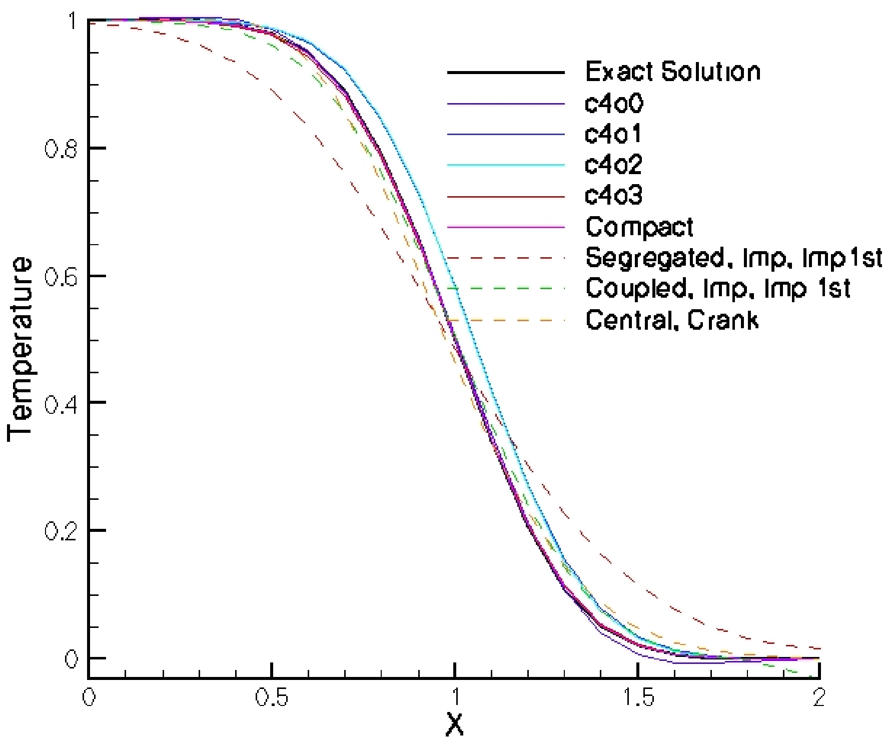

$$\frac{\partial E_t}{\partial t} + \frac{\partial}{\partial x}\left( (\rho C_v T + p)u - \frac{4}{3}\mu u u_x + q_x \right) = 0$$

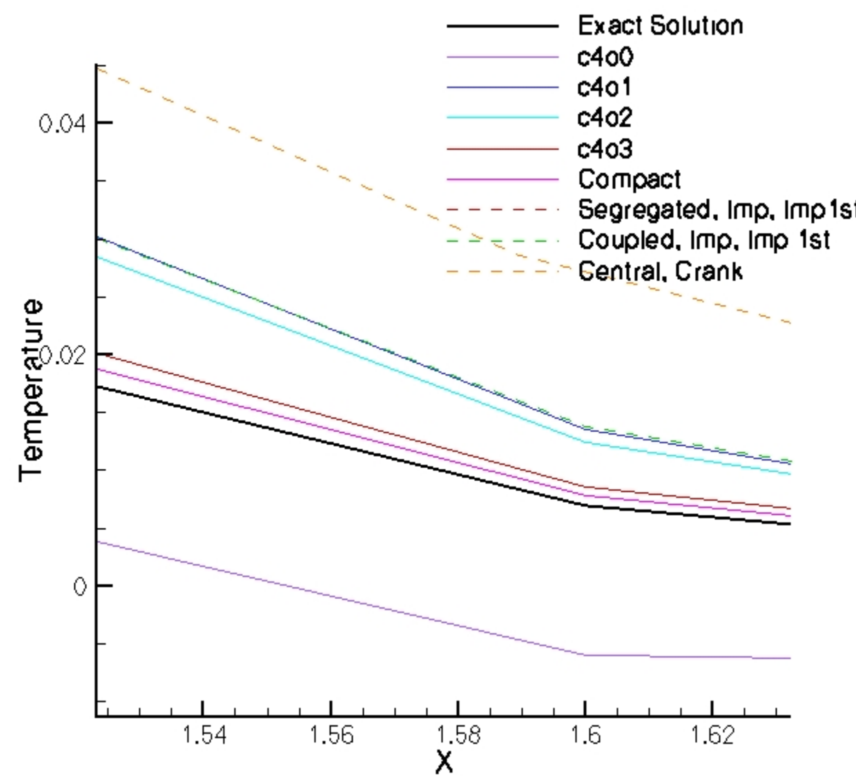
Reduces to:

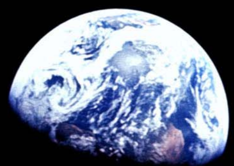
$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial x^2},$$

$$\alpha = \frac{k}{\rho C_p}$$

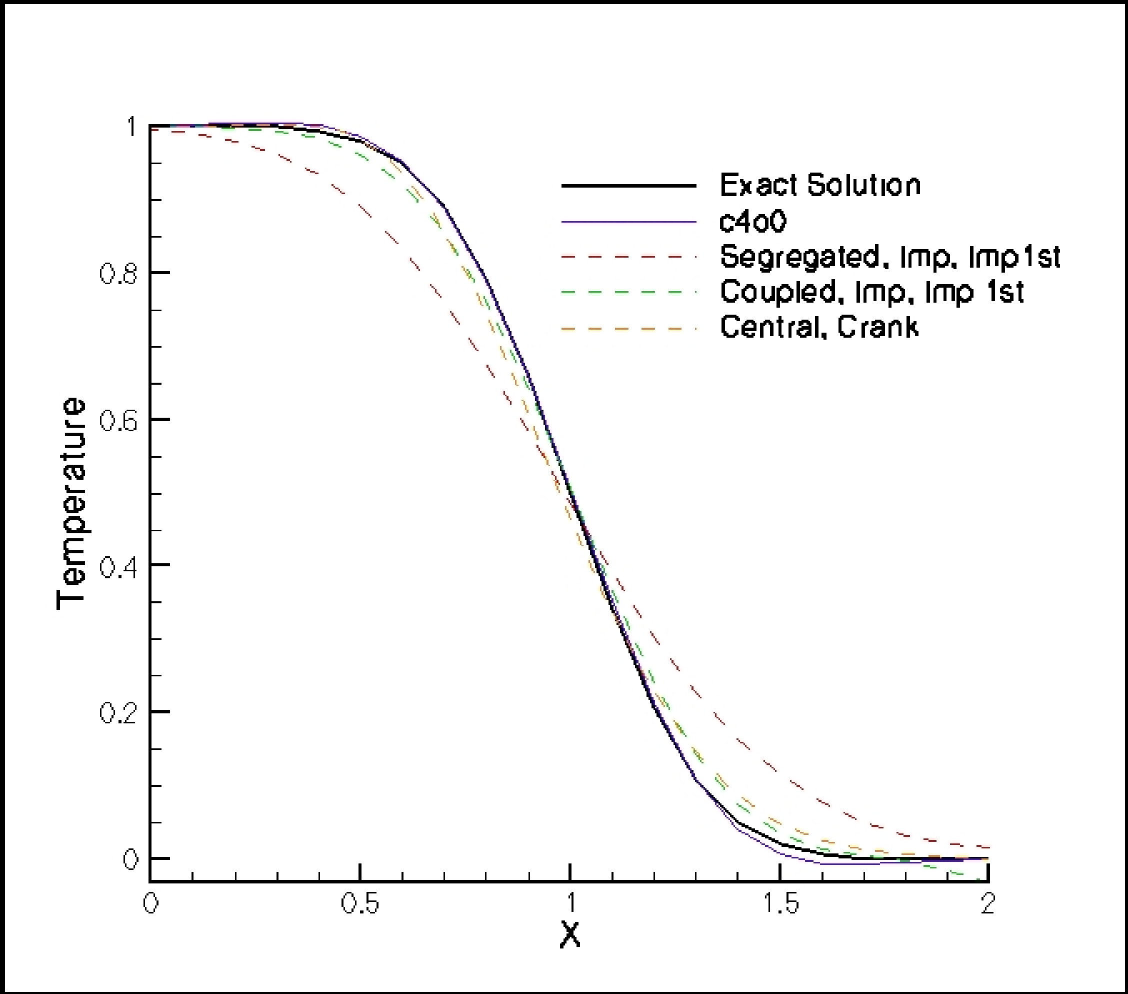
# Comparison of Commercial & Advanced



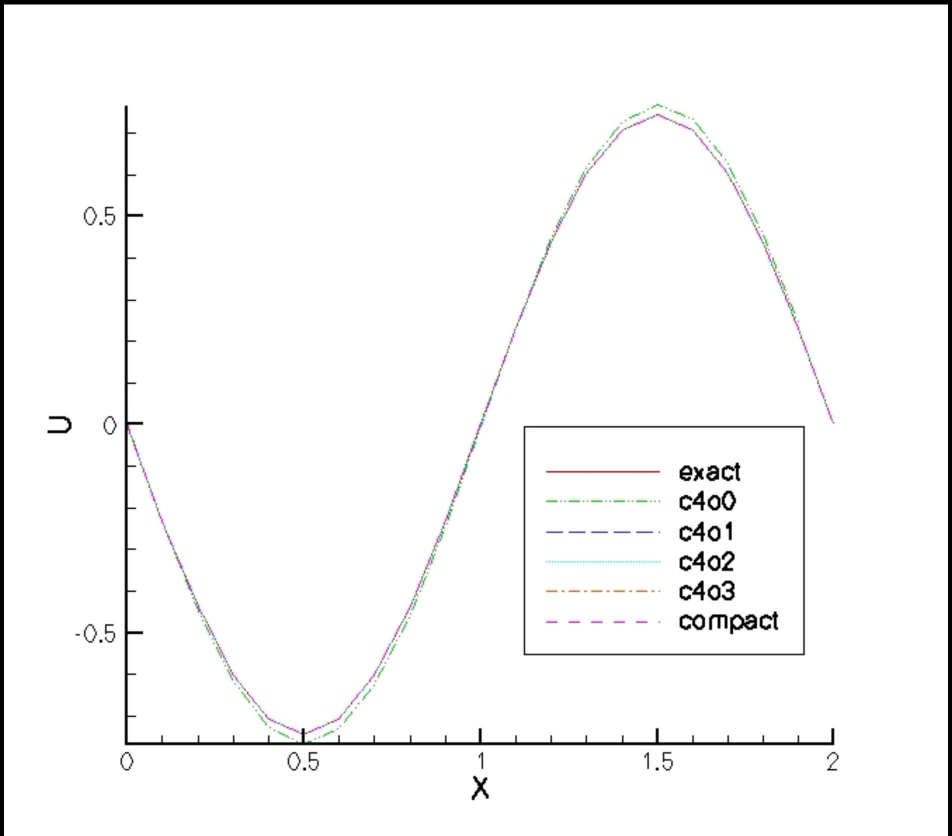



# Commercial Comparison

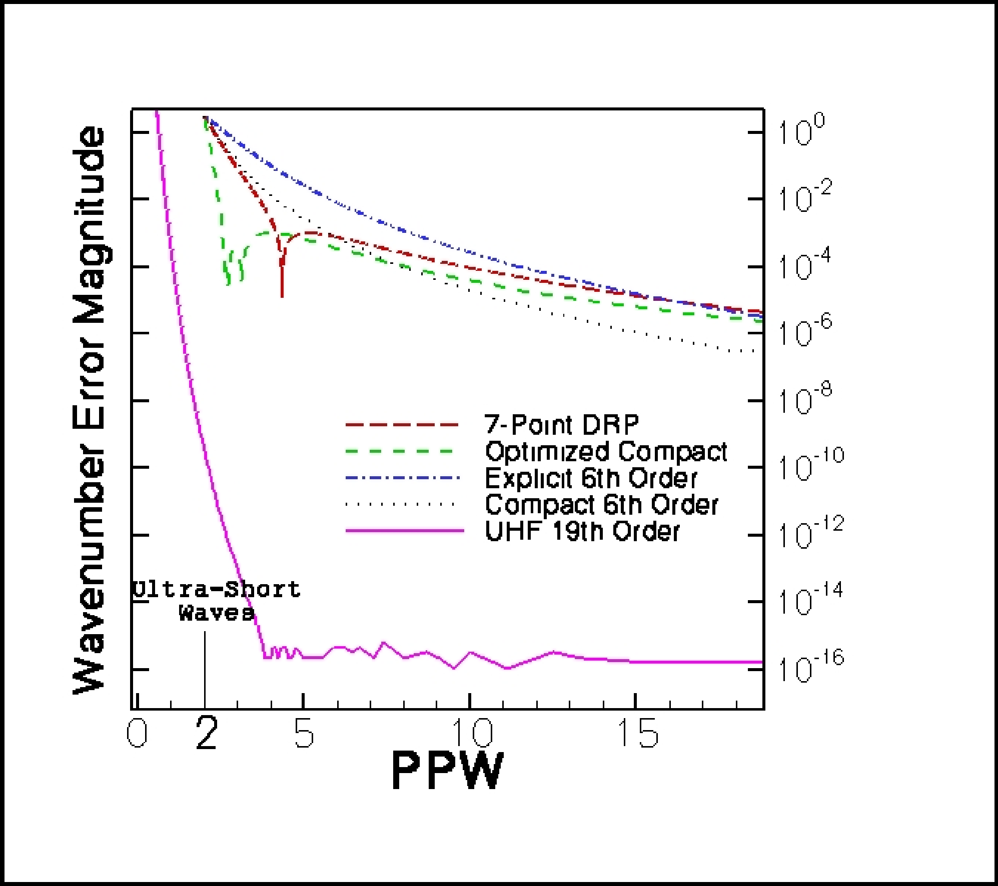


# Turbulence Transition Efficiency

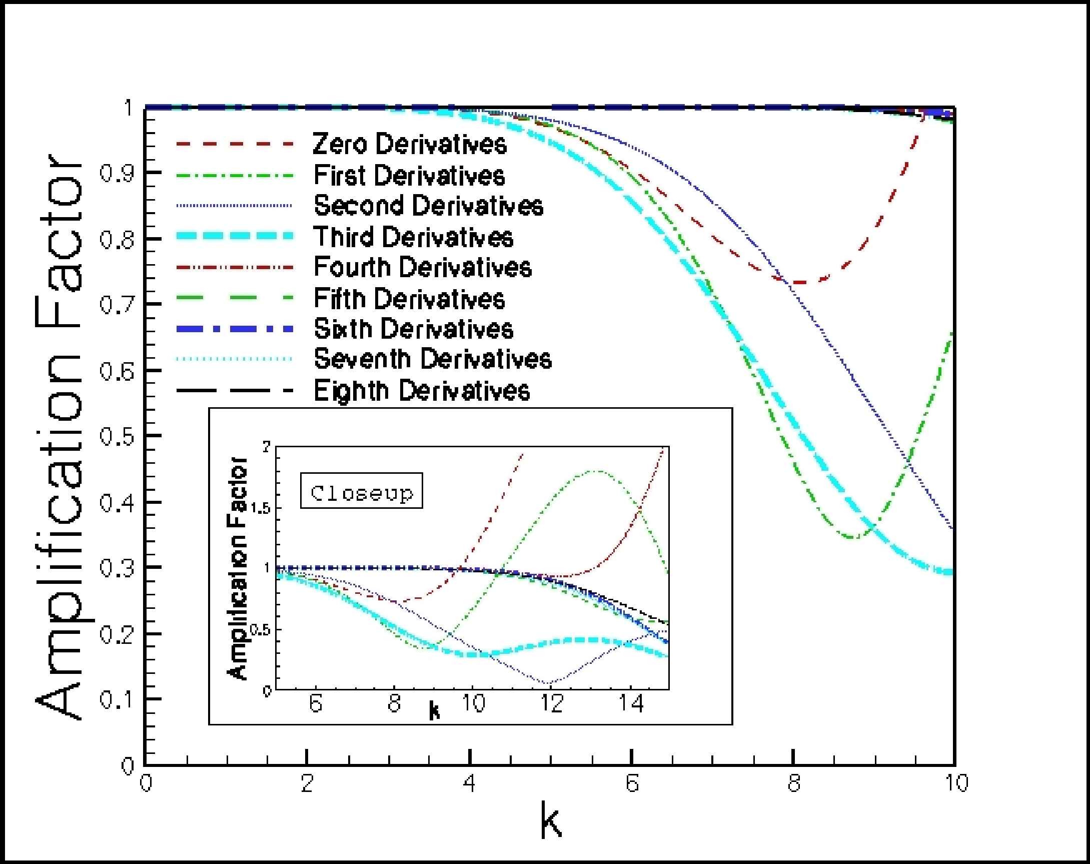



| Method | Spacing | Error |
|---|---|---|
| c4o0 | .1 | $2.54229 \, 10^{-2}$ |
| c4o0 | .2 | $4.27563 \, 10^{-2}$ |
| c4o0 | .4 | $4.68577 \, 10^{-2}$ |
| c4o1 | .1 | $3.11163 \, 10^{-6}$ |
| c4o1 | .2 | $2.96551 \, 10^{-5}$ |
| c4o1 | .4 | $8.4702 \, 10^{-4}$ |
| c4o2 | .1 | $1.0178 \, 10^{-10}$ |
| c4o2 | .2 | $3.1935 \, 10^{-9}$ |
| c4o2 | .4 | $6.41079 \, 10^{-8}$ |
| c4o3 | .1 | $3.44169 \, 10^{-15}$ |
| c4o3 | .2 | $2.27818 \, 10^{-13}$ |
| c4o3 | .4 | $3.12925 \, 10^{-11}$ |
| compact | .1 | $1.0993 \, 10^{-6}$ |
| compact | .2 | $7.10929 \, 10^{-5}$ |
| compact | .4 | $5.31245 \, 10^{-3}$ |

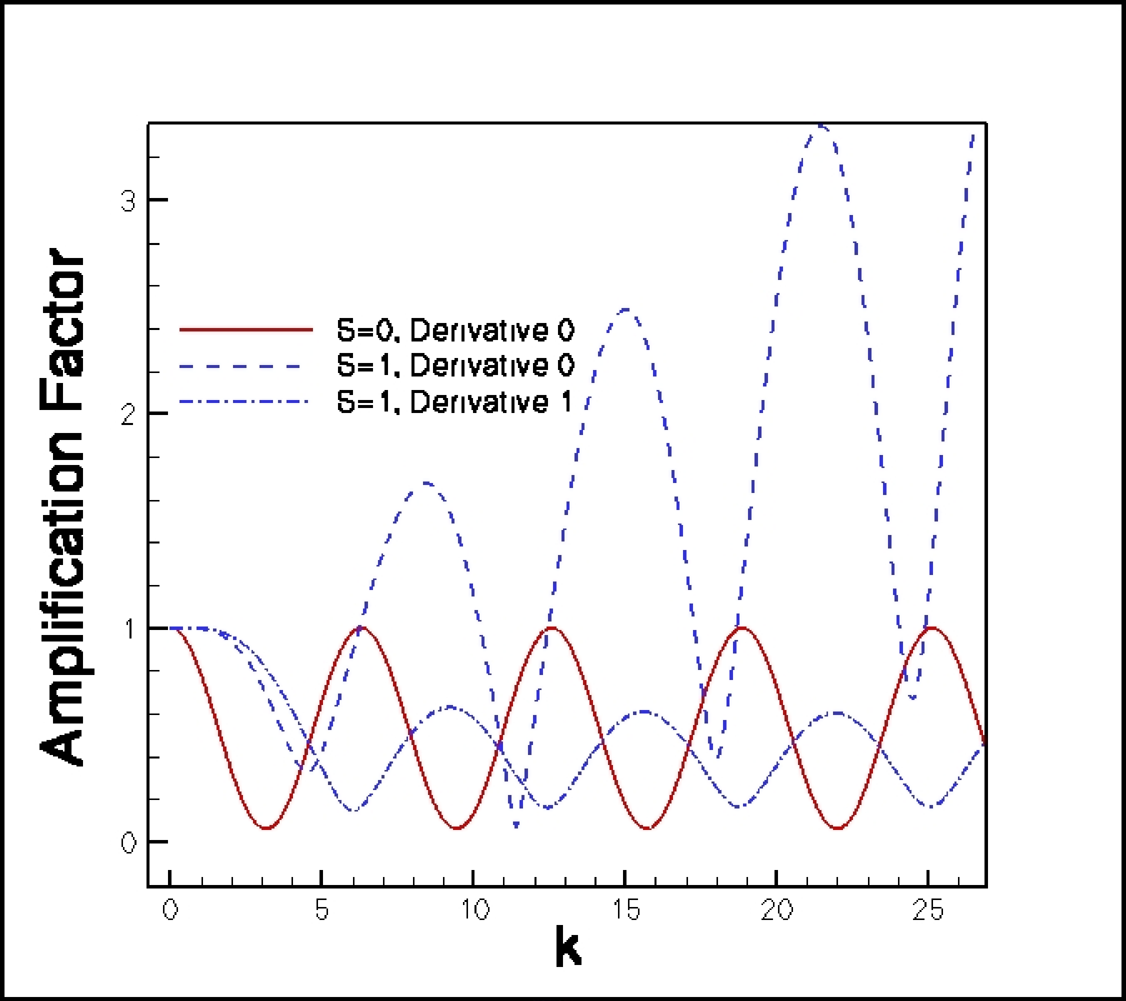
# Points per Kolmogorov Wavelength



# Wave Equation Amplification

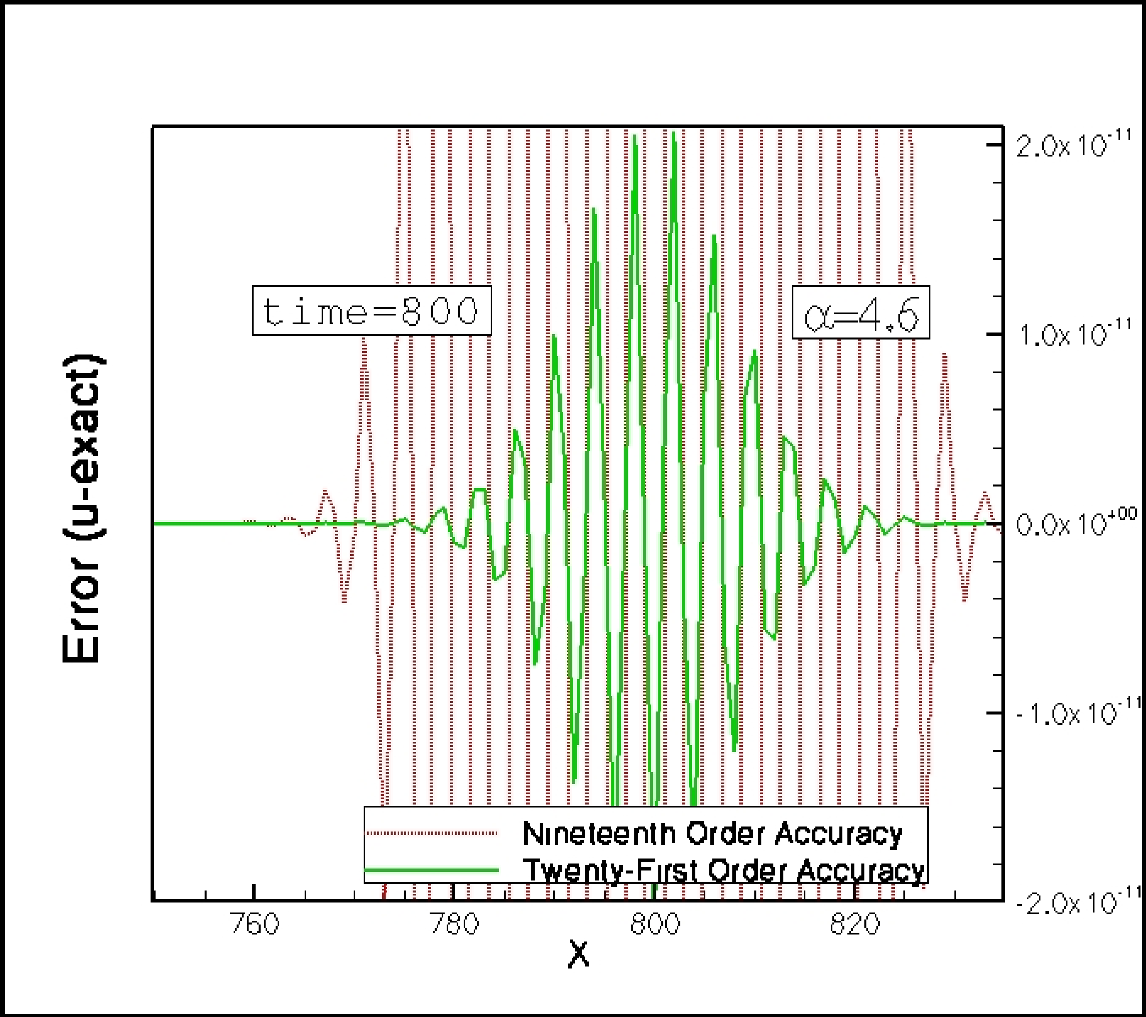


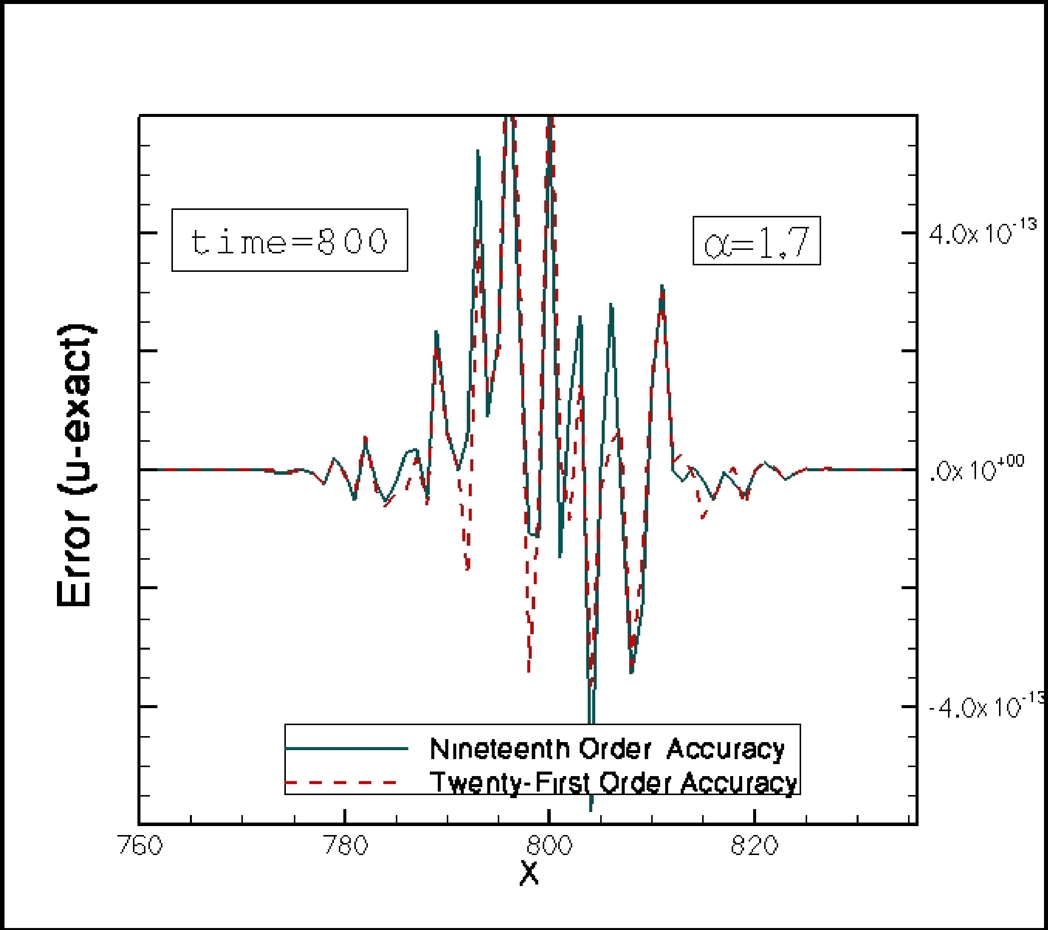
# Aliased Frequency Amplification



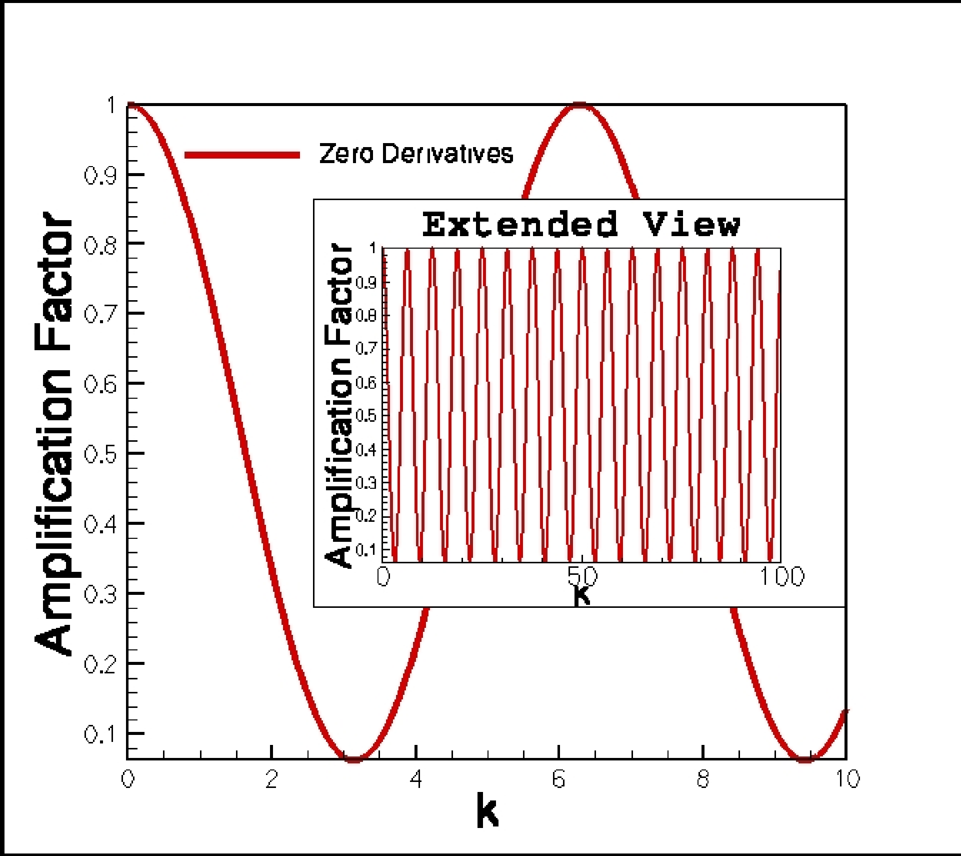
# Conclusions

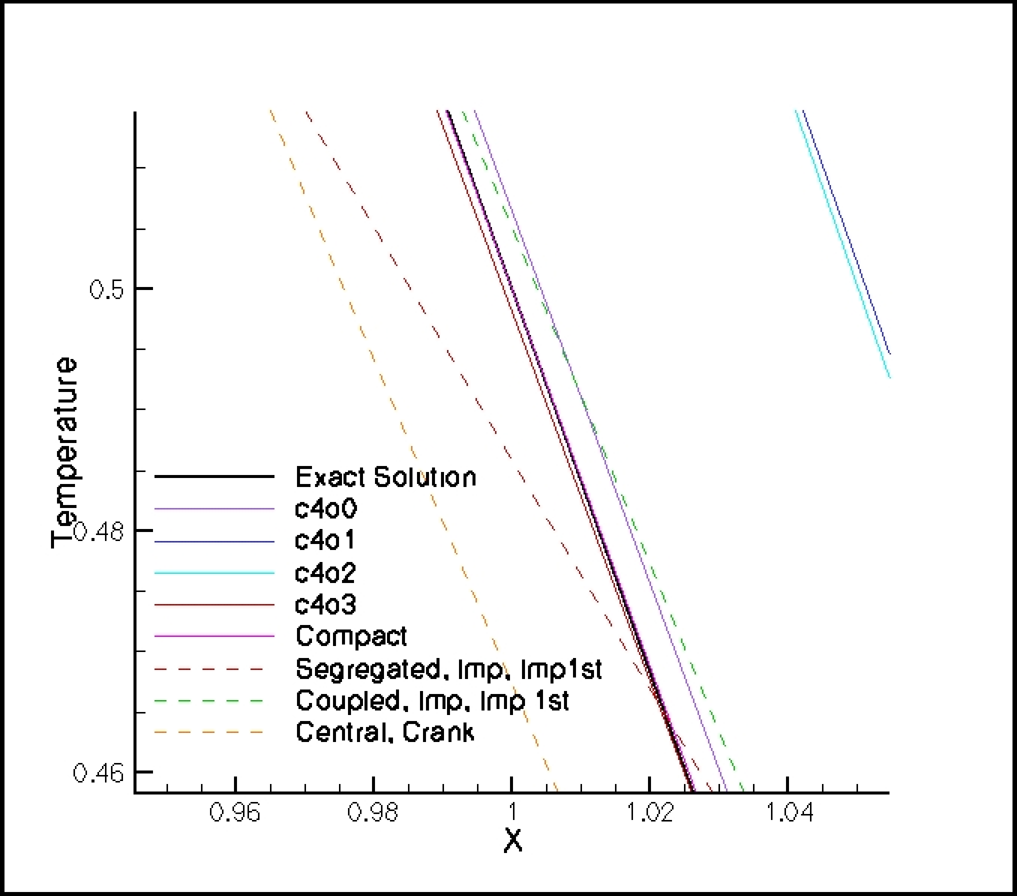
- Low Reynold's number, wall bounded flow allows economical use of large eddy simulation for turbulent transition modeling
- UHF and Compact comparable at conjugate heat transfer
- UHF much better for turbulence modeling
- Modern methods much more efficient than those currently available commercially


time=800
α=4.6
2.0x10⁻¹¹
1.0x10⁻¹¹
0.0x10⁺⁰⁰
-1.0x10⁻¹¹
-2.0x10⁻¹¹
Error (u-exact)
760
780
800
820
X
Nineteenth Order Accuracy
Twenty-First Order Accuracy

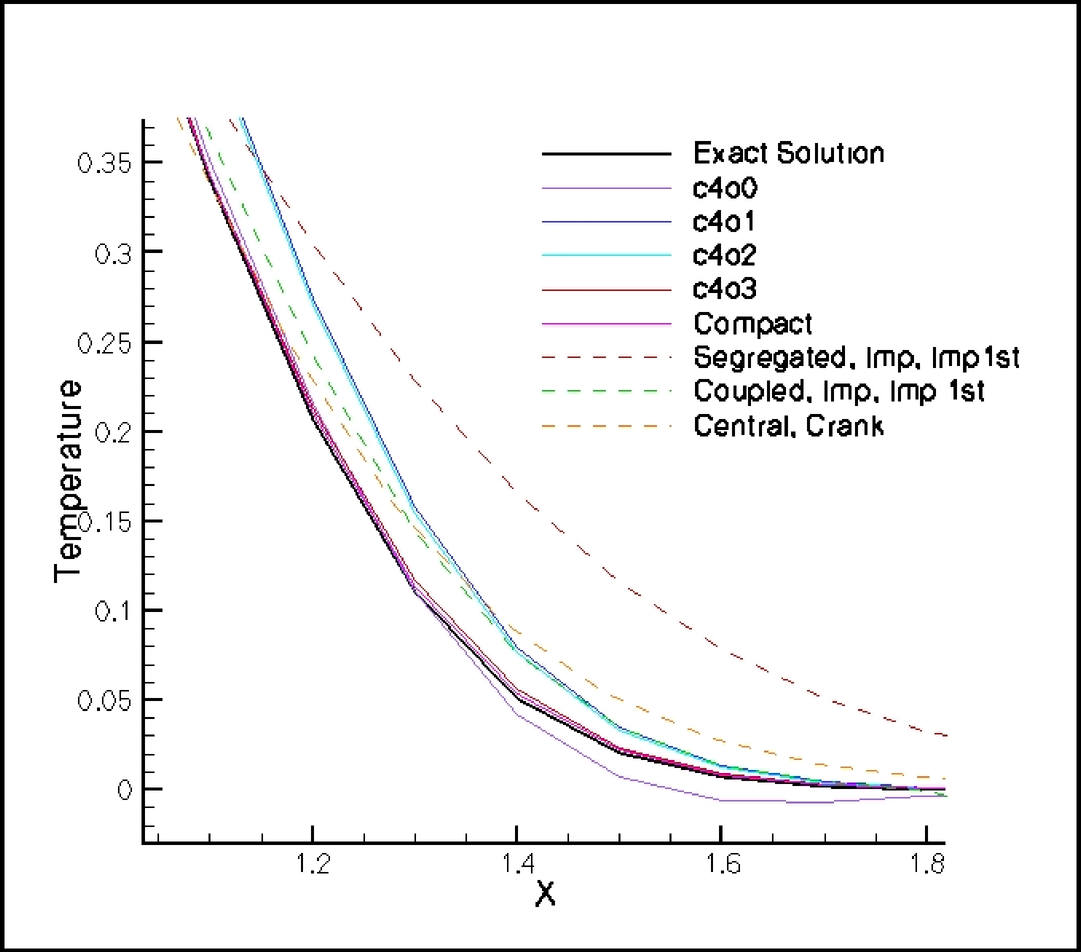


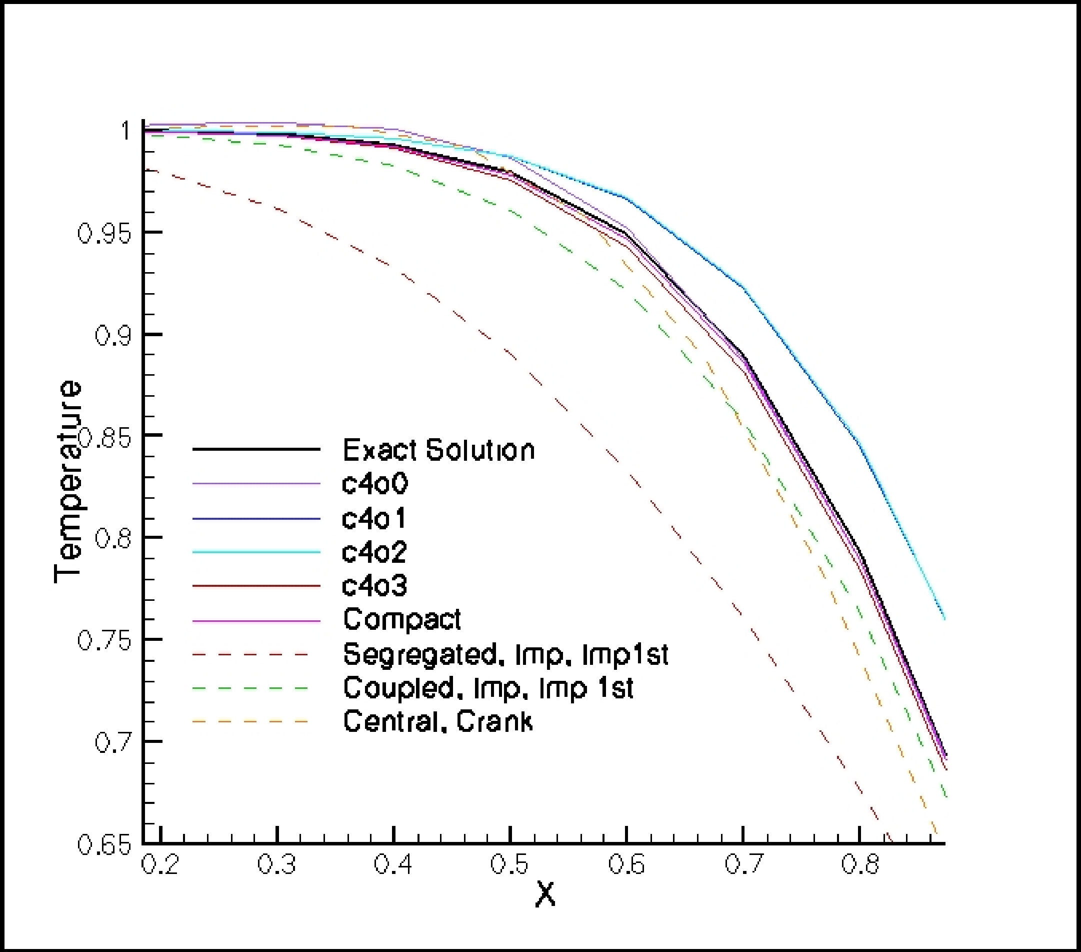
time=800
α=1.7
4.0x10^-13
.0x10^+00
-4.0x10^-13
Error (u-exact)
Nineteenth Order Accuracy
Twenty-First Order Accuracy
760
780
800
820
X


Zero Derivatives
Extended View
Amplification Factor
k
0
2
4
6
8
10
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
50
100



Temperature
0.5
0.48
0.46
0.96
0.98
1
1.02
1.04
X
Exact Solution
c4o0
c4o1
c4o2
c4o3
Compact
Segregated, Imp, Imp1st
Coupled, Imp, Imp 1st
Central, Crank


Exact Solution
c4o0
c4o1
c4o2
c4o3
Compact
Segregated, Imp, Imp1st
Coupled, Imp, Imp 1st
Central, Crank
Temperature
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1.2
1.4
1.6
1.8
X


1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
Temperature
0.2
0.3
0.4
0.5
0.6
0.7
0.8
X
Exact Solution
c4o0
c4o1
c4o2
c4o3
Compact
Segregated, Imp, Imp1st
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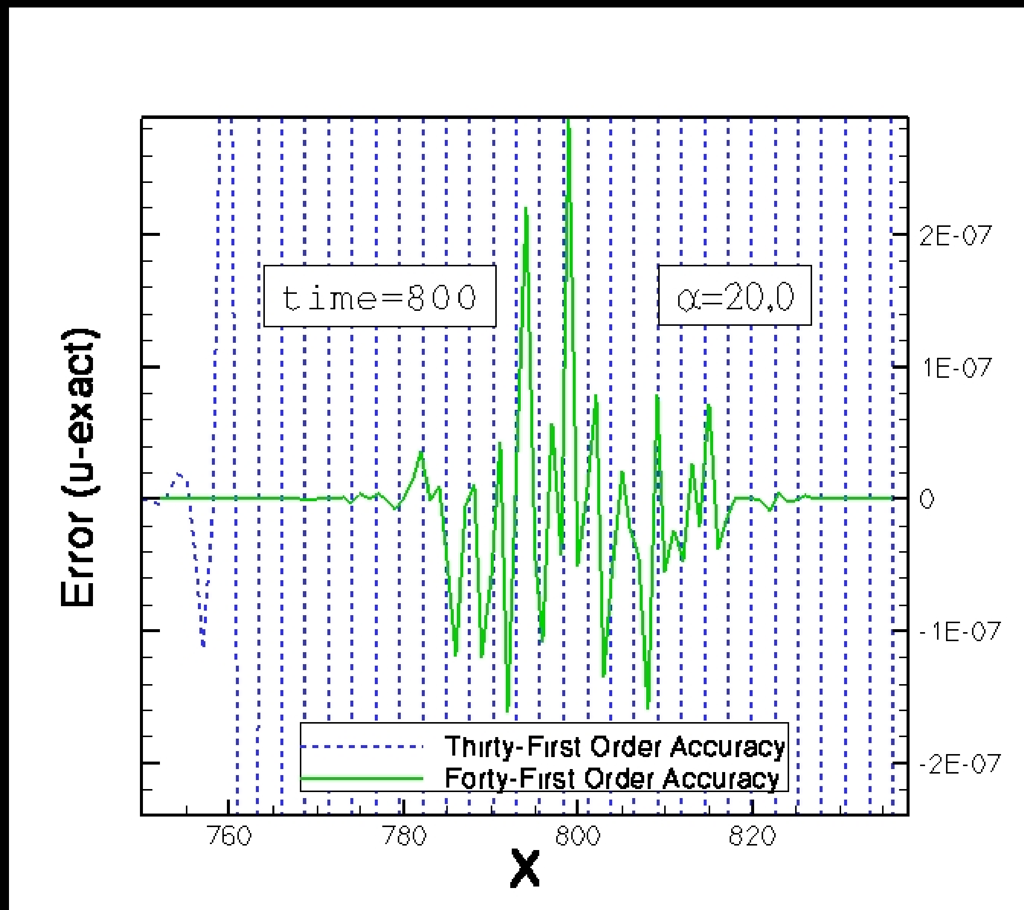
time=800
α=20.0
Error (u-exact)
2E-07
1E-07
0
-1E-07
-2E-07
760
780
800
820
X
Thirty-First Order Accuracy
Forty-First Order Accuracy