A Bayesian Treatment of Risk for Radiation Hardness Assurance

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ABSTRACT

We construct a Bayesian risk metric with a method that allows for efficient and systematic use of all relevant information and provides a rational basis for RHA decisions in terms of costs and mission requirements.

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Introduction

Because cost and schedule often preclude radiation testing with samples sufficient for statistical inference, Radiation Hardness Assurance (RHA) decisions often require combining test data with simulation results, expert opinion and other data. Reference 1 discussed use of archival radiation data to supplement test data and radiation lot acceptance testing (RLAT) and provide a sufficient statistical base for meaningful inference.[1] Here we examine whether use of such data in RHA may be amenable to a Bayesian statistical approach—often the most efficient treatment of diverse information when decisions must be made with limited data. (See figure 1.)

In Bayesian statistics, if the test data $\boldsymbol{B}$ are favor hypothesis A as reflected in the Prior distribution $P(\boldsymbol{A})$, the information in $P(\boldsymbol{A})$ supplements $\boldsymbol{B}$, yielding increased confidence.

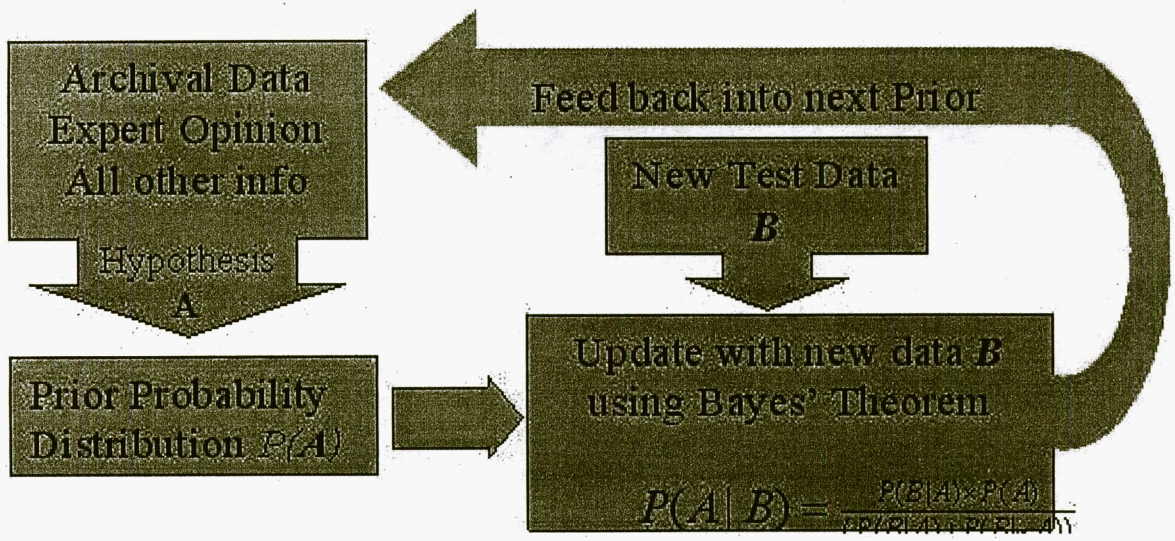

Fig. 1 Bayesian statistics constructs a Prior distribution, $P(A)$, for a hypothesis A, from information available prior to testing. We then update $P(A)$ with new test data, B, to form a posterior distribution $P(A|B)$ using Bayes' Theorem. On the right hand side of the equation, $P(A|B)$ is the likelihood of observing data B given hypothesis A, and the denominator is a normalizing factor interpreted as the likelihood of observing B whether A is true or not ($\sim A$—read "not A"). The process can be iterative.

Bayesian analysis has been suggested previously.[2] Indeed, a Bayesian treatment is natural, since many aspects of hardness assurance resemble deFinetti's classic problem on Bayesian probability.[3] (See figure 2.)

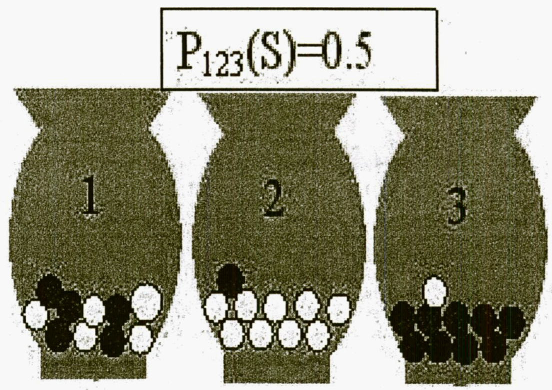

Fig. 2 The probability of drawing a black or white stone is not defined until one first selects an urn (or flight lot or launch date). The probability distribution itself is a random variable.

Bayesian Risk Formalism

A typical Bayesian RHA problem involves a part subjected to a stress (e.g. TID). One must decide whether to use a part as is or require additional remediation to increase the probability that the part survives the stress and meets its requirements. We may also require testing to better define the success probability. We make decisions using a risk metric. (See figure 3.) If we carry out a test, the test cost C_t is the minimum risk, since we incur this cost regardless of test results.[4,5] The risk metric is useful for planning test efforts and allocating scarce resources to achieve the greatest risk reduction. It can even be used (albeit with caution) to compare risks across disciplines (e.g. TID vs. SEE or reliability.) From a hardness assurance perspective, risk defines when we may fly a part "as is" and when we need more mitigation or testing to ensure adequate probability of mission success. Since C_t and C_r are known, and P_s=1- P_f, the problem reduces to defining C_f and p_f.

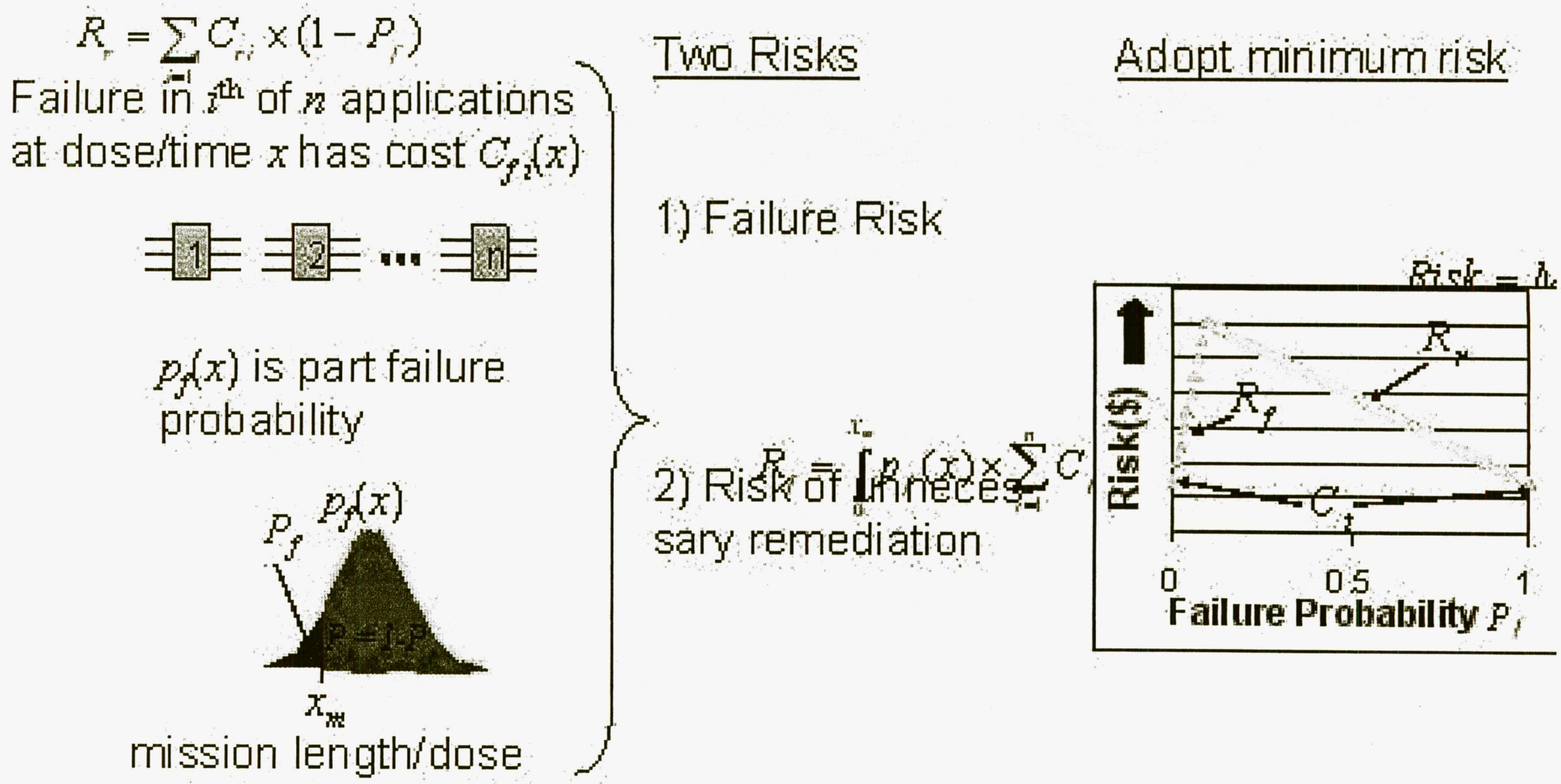

Fig 3 Bayesian RHA minimizes risk by testing or implementing remediation until the reduction in failure risk R_f is less than the risk R_r that future efforts will incur unnecessary additional cost.

Defining Costs

C_f, C_t and C_r scale the level of effort directed toward reducing risks. Binning risks by system impact with a cost assigned to each bin simplifies risk determination. This technique is similar to some of those used for SEE and reliability analyses.[6]

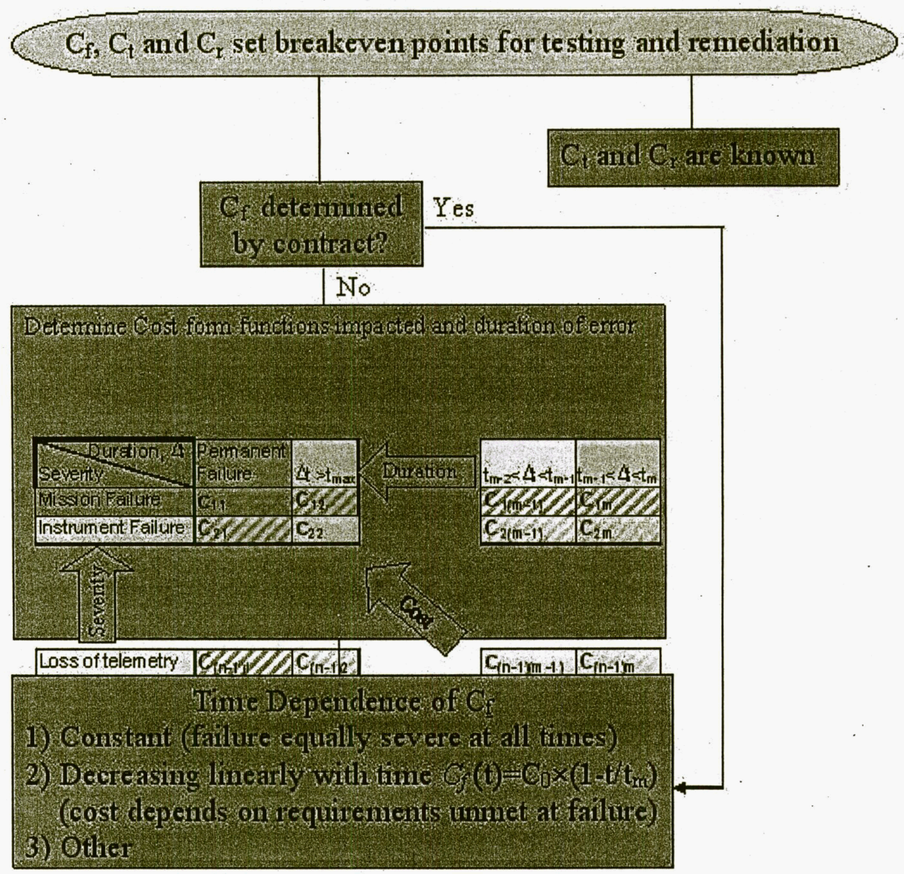

Fig. 4 Contractual details or a failure's system impact may determine the magnitude and time dependence of failure cost.

While C_f depends on time, a failure pdf depends on a radiation related stress (e.g. TID, LET). Thus, evaluating risk requires a treatment of time variation of the radiation environment.

Stress Time Dependence

While one can not predict detailed time behavior of the space radiation environment, one can seek to bound future variability for a given confidence level based on historical data. For Poisson processes such as single-event effects (SEE), one bounds the rate for a given CL by assuming an upward rate fluctuation consistent with that CL. (See figure 5.) This still gives rise to a constant error/failure probability in time, so R_f has the same time dependence as $C_f(t)$. Moreover, usually part-to-part variability is less severe for SEE than for TID, and risk estimation is a simpler task.

The cumulative nature of degradation mechanisms such as TID requires a different strategy—developing mission environments that are worst case in their time profile as well as magnitude.

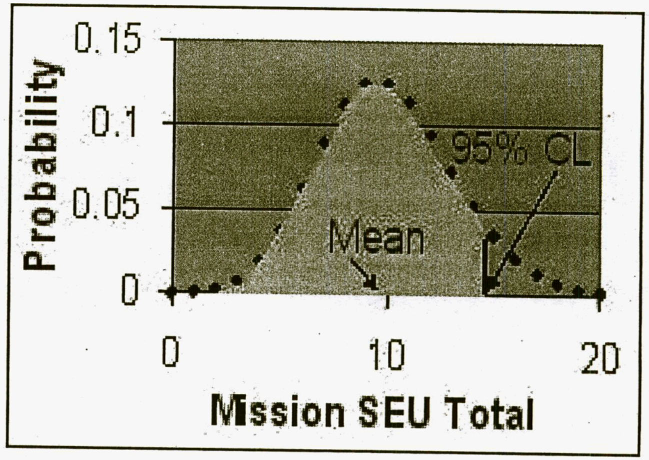

Fig. 4 For single-event effects, models such as CREME96 yield average rates, we can bound the rate for a given confidence level assuming upward Poisson fluctuations from this average rate.

We used the ESP solar proton model[7] to bound at some CL the time variation of the radiation environment for the James Webb Space Telescope (JWST is slated to operate for 5.5 years at the Earth-Sun second Lagrange Point L2). ESP gives proton fluences at any confidence level for solar proton events (SPE) and for missions from 1 to 7 solar-active years. Using ESP to generate the 95% CL mission fluences for missions from 1 to 7 years, we calculate the 1 year incremental fluences for each of the 7 (Solar Max) years. We then assume the worst year occurs during the first year of the mission, the second worst in the second year, and so on. (See figure 5.)

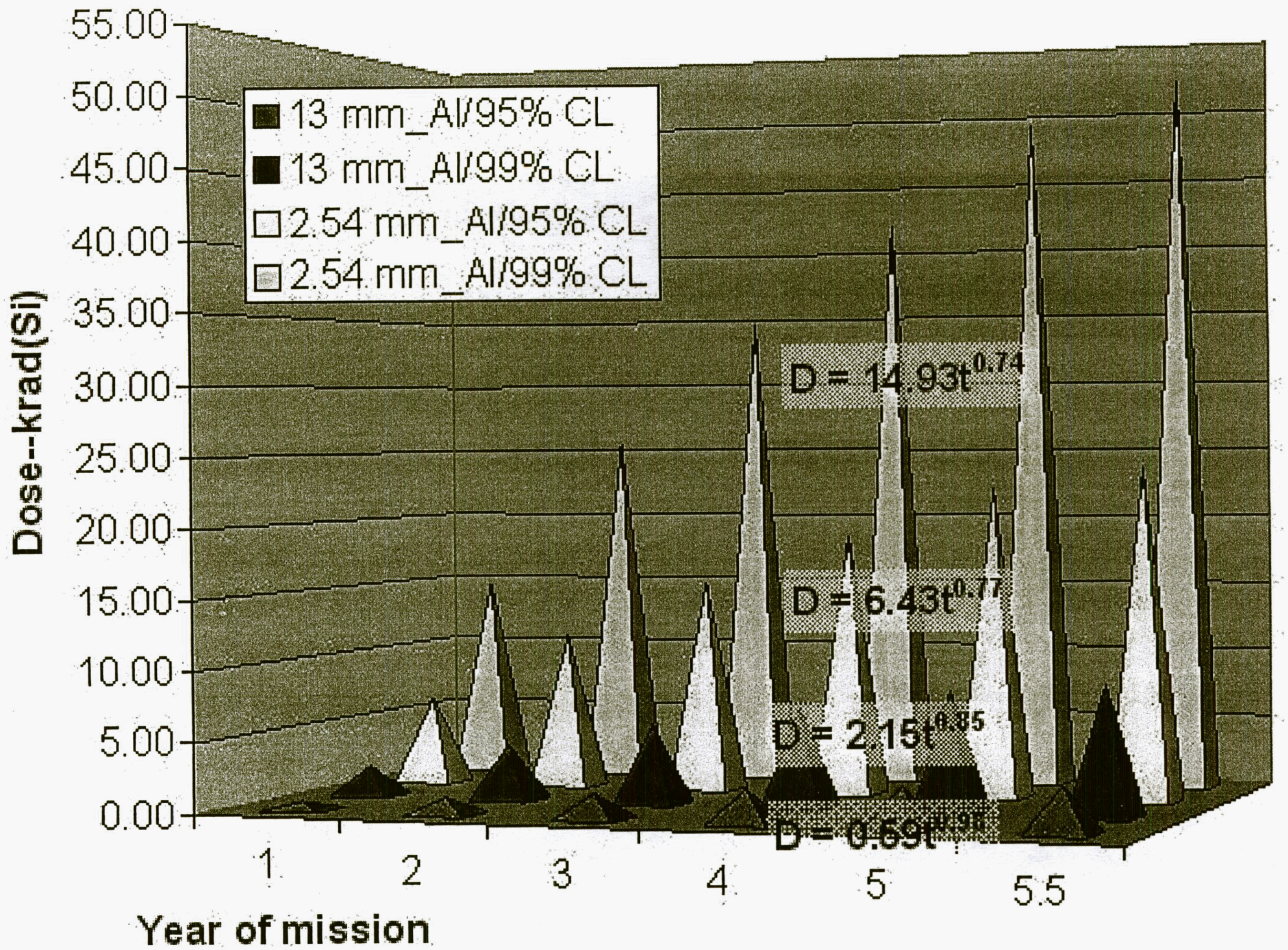

Figure 5 TID levels at the 95% and 99% CL the ESP model for 2.54 and 13 mm (~100 and 500 mils) Al equivalent spherical shielding show that year-to-year variability is greater for higher CL and thinner equivalent shielding.

Because large SPE may dominate TID for short missions (≤ 2 yrs.), we generated 10000 mission histories with SPE numbers fluctuating Poisson-wise about the mean, and randomly assigned a severity (based on CL) to each event. The 95% CL history is that with only 5% of all histories receiving more TID during the first 6 and 12 months, and subject to the constraint that the 2 year TID is consistent with the 95% CL ESP 2 year mission TID. We find the dose rises roughly as $t^{1/2}$, with only slight dependence on the shielding or confidence level. The good power law fit to the dose vs. time simplifies our task: If the failure distribution is lognormal in stress, it will also be lognormal in time. If it is Weibull, it will be proportional to a Weibull in time multiplied by a power of time t. For complex time dependence, failure risk can be calculated numerically.

Priors and Testing Decisions

While we can base a Prior on any information, we prefer to generate the Prior from a likelihood ratio for historical data (e.g., for a normal distribution, we take the ratio of likelihood for any mean and standard deviation (μ,σ) to the likelihood for the parameters that best fit the data). The log of the likelihood ratio is distributed as the $\chi 2$ distribution with degrees of freedom equal to the number of distribution parameters.

To illustrate this method, we use the same data set used in reference 1 (38 flight lots, 158 parts of Linear Technologies RH1014 quad op amps). Assuming a normal distribution, Figure 7 gives the Prior for μ and σ for excess bias current (Ibias) after 100 krad(Si). In figure 7, in addition to the best-fit μ and σ, we can also identify the worst-case distributions consistent with the 95% CL. The near agreement between the best-fit and 95% WC parameters shows that a large dataset gives significant immunity to statistical fluctuations.

μ \ σ	3.70	3.85	4.00	4.15	4.30	4.45	4.60	4.75	4.90	5.05
26.35	0.00	0.01	0.02	0.05	0.07	0.07	0.05	0.02	0.01	0.00
26.50	0.00	0.02	0.08	0.15	0.19	0.17	0.11	0.06	0.02	0.01
26.65	0.01	0.07	0.20	0.36	0.43	0.36	0.23	0.11	0.04	0.01
26.80	0.03	0.14	0.41	0.71	0.81	0.66	0.39	0.18	0.07	0.02
26.95	0.05	0.25	0.68	1.15	[illegible]	1.00	0.58	0.27	0.10	0.03
27.10	0.07	0.35	0.93	[illegible]	[illegible]	[illegible]	0.74	0.33	0.12	0.04
27.25	0.08	0.38	1.02	[illegible]	1.80	[illegible]	0.79	0.35	0.13	0.04
27.40	0.07	0.34	0.90	[illegible]	[illegible]	1.26	0.72	0.32	0.12	0.04
27.55	0.05	0.24	0.65	1.09	1.22	0.96	0.56	0.26	0.09	0.03
27.70	0.03	0.13	0.38	0.66	0.76	0.62	0.37	0.17	0.07	0.02
27.85	0.01	0.06	0.18	0.32	0.39	0.33	0.21	0.10	0.04	0.01
28.00	0.00	0.02	0.07	0.13	0.17	0.15	0.10	0.05	0.02	0.01
28.15	0.00	0.01	0.02	0.04	0.06	0.06	0.04	0.02	0.01	0.00

Fig. 7 The best-fit for a Normal Prior of excess Ibias in RH1014 op amps after 100 krads has μ=27.25, σ=4.3 (orange cells). The 95% WC fit is μ=28.15, σ=5.05 (turquoise cells).

The prior distribution has many uses—determining sample sizes, guiding resource allocation, etc. For parts like the RH1014, with good data process stability, we can use the Prior to develop margin requirements for waiving RLAT—based on the criterion that $R_f < C_t$. For constant C_f, the allowable failure probability, $P_f = C_t/C_f$. For C_f=\$1000000 and C_t=\$10000, then $P_f \leq 1\%$ integrated over the mission duration (see figure 8). The design-to leakage current, Id, is the value for which $P(\text{Ibias} > I_d) < 1\%$. For the RH1014 and many other parts, I_d and μ increase roughly linearly with dose from 0-200 krad(Si). Multiplying the ratio $I_d/\mu = 1.66$ by the usual 2x radiation design margin (RDM), the RDM for 95/99 assurance is roughly 3.3.

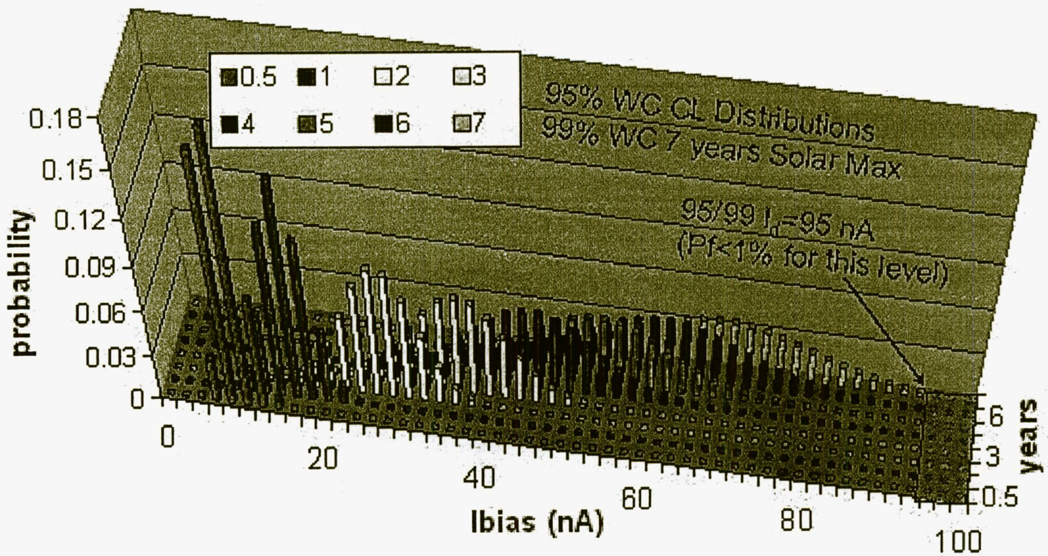

Fig. 8 The 95% CL worst-case Ibias pdf for RH 1014s exposed to a 99% CL ESP environment behind 1.3 mils Al shifts upward and broadens during 7 years of Solar Max. Setting the design-to Ibias (I_d) to 95 nA, P(Ibias<I_d) (pink rectangle) is less than1%. This defines the 95/99 CL I_d.

If Cf is time dependent—e.g. decreasing linearly with time—the calculation is more complicated, but the criterion (R_f<C_t) is the same. Again for $C_f(t=0)$=\$1000000 and C_t=\$10000, figure 9 illustrates that introducing the time dependence of C_f reduces Id to 84 nA and the required RDM to 2.95x. Regardless of the time dependence of C_f, the large dataset and reasonable radiation response of the RH1014 allow us to substitute moderate RDMs for RLAT. While no historical data can protect against adverse process changes, this approach can at least justify substituting periodic testing for RLAT in some cases. Smaller datasets require larger RDMs.

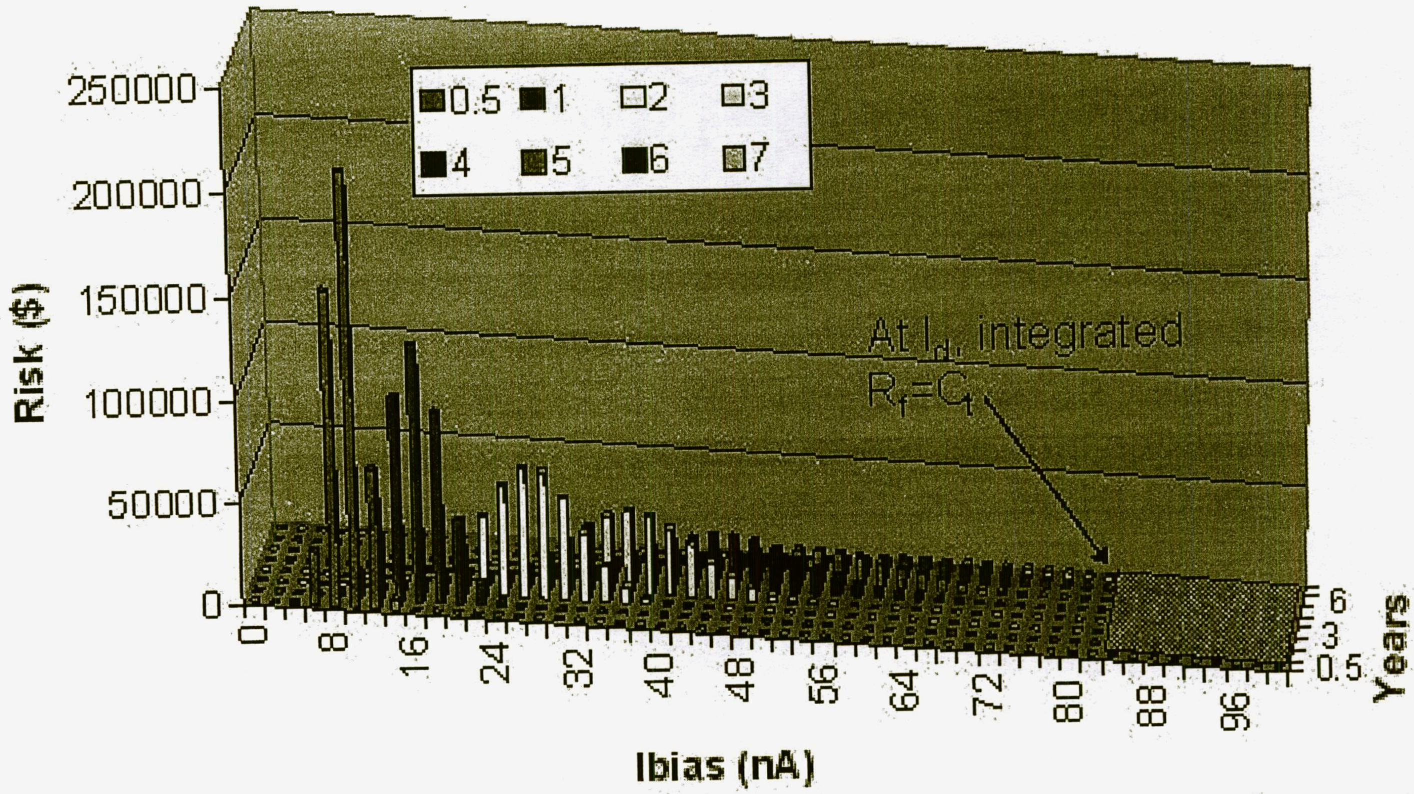

Fig. 9 At the design-to Ibias, I_d, integrated failure risk R_f equals the test cost C_t, so more testing does not reduce risk. A failure cost that decreases linearly with time lowers I_d by about 12%.

Priors with Limited Data

Bayesian methods allow us to develop Priors using expert opinion, modeling, etc. to supplement, or even replace, data. We validate our assumptions by updating the Prior with test data. The Prior should reflect our model's uncertainty, and tests should distinguish between our model and alternatives (A and $\sim A$ in Bayes' Theorem). If $P(B|A) > P(B|\sim A)$, the posterior $P(A|B)$ will more clearly favor hypothesis A. If B conflicts with A, the posterior becomes more diffuse. This suggests our model is incorrect and argues for more testing.

When data are sparse, the Prior is useful for test planning and resource allocation. To illustrate, we consider TID response of CD4000 family parts baselined for control units for JWST optics. These parts must operate at 25 Kelvin for 5.5 years in the L2 radiation environment (22 krad(Si) @ 95% CL by EOL). Although CD4000s perform well to >100 krad(Si) at ambient temperatures, similar performance at 25 K is questionable, given the suppression of interface state formation and annealing at cryogenic temperatures.

In testing to date, the CD4051 mux and CD4066 switch revealed considerable variability, with all 5 CD4051s failing parametrically between 10 and 20 krad(Si), one CD4066 failing at 30 krad(Si), another at 70 krad(Si) and the other 3 still functional with testing suspended at 75 krad(Si).

These results raised concerns for hardness of the CD40109 level shifter—still to be tested. Failure to qualify this part would result in redesign with a significant mass penalty. We define failure as leakage current exceeding specifications and assume C_f decreases linearly in time: Cf=$1E8×(1-t/5.5). Constructing a Prior for the CD4066 from the previous cryo TID data is straightforward, although the wide Prior makes a Normal approximation unphysical. Table I shows TID as a function of time (1st 2 columns--95% CL), P_f for the Prior (3rd column--90% CL) and P_f with 5 and 8 more parts failing above 75 krad(Si) (last 2 columns—90% CL). Other guidelines from studying the Prior and its response to new data sets are:

1) Limiting the failure distribution breadth (given the 30 krad(Si) failure) requires testing the next lot to failure.

2) 5 parts are adequate to attain 90/99 statistics if the distribution looks Weibull; 8 are needed if it looks lognormal.

Table I: CD4066 90% CL Cumulative Weibull P_f

Mission Year	TID krad(Si)	P(Fail)-Prior	P(Fail)-Prior + 5 parts >75 krad(Si)	P(Fail)-Prior + 8 parts >75 krad(Si)
1	6.14	0.16	0.00007	0
2	10.7	0.095	0.0005	0.00001
3	14.6	0.125	0.0017	0.00005
4	17.7	0.145	0.0035	0.00015
5	20.7	0.166	0.006	0.0004
5.5	22	0.175	0.008	0.0005

The different results for the CD4066 and CD4051 complicate generation of a Prior for the CD40109. Based on cryogenic performance data and expert opinion, we construct the Prior weighting results for the CD4066 by 90% and the CD4051 by 10%. We give CD4000 family parts half the weight for CD40109 data. The resulting Prior in figure 10 shows that broad failure distributions with high mean hardness are as probable as narrow distributions peaked 70-90 krad(Si). Again, testing to failure is important to narrow the distribution.

W \ S	0.20	0.95	1.70	2.45	3.20	3.95	4.70	5.45	6.20	6.95
25	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
37	0.03	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
49	0.03	0.10	0.06	0.02	0.00	0.00	0.00	0.00	0.00	0.00
61	0.03	0.15	0.15	0.10	0.06	0.03	0.01	0.00	0.00	0.00
73	0.03	0.18	[redacted]	0.19	0.13	0.09	0.06	0.03	0.02	0.01
85	0.04	[redacted]	0.26	[redacted]	0.16	0.11	0.07	0.04	0.02	0.01
97	0.04	[redacted]	0.27	[redacted]	0.15	0.09	0.05	0.02	0.01	0.01
109	0.04	[redacted]	0.26	[redacted]	0.12	0.06	0.03	0.01	0.01	0.00
121	0.04	[redacted]	[redacted]	0.17	0.09	0.04	0.02	0.01	0.00	0.00
133	0.04	[redacted]	[redacted]	0.14	0.07	0.03	0.01	0.00	0.00	0.00
145	0.04	[redacted]	[redacted]	0.12	0.05	0.02	0.01	0.00	0.00	0.00
157	0.04	[redacted]	[redacted]	0.10	0.04	0.01	0.00	0.00	0.00	0.00
169	0.04	[redacted]	0.18	0.08	0.03	0.01	0.00	0.00	0.00	0.00
181	0.04	[redacted]	0.16	0.07	0.02	0.01	0.00	0.00	0.00	0.00

Figure 10 Generic CD4XXXX Prior Weibull distribution, weighted 90% on theCD4066 and 10% on the CD4051.

Adding data (5 parts w/ failure >75 krad(Si)), narrows the distribution in both W and S (see Fig. 11). Table II gives the CD40109 P_f in a format similar Table I. Again, 5 parts with failure levels >75 krad(Si) are adequate to establish 90/99 statistics if the failure distribution resembles a Weibull, while 8 parts are needed if the distribution looks more lognormal.

W \ S	2.45	3.20	3.95	4.70	5.45	6.20	6.95	7.70	8.45	9.20	9.95	10.70
65	0.01	0.02	0.02	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
67.5	0.01	0.03	0.05	0.05	0.05	0.03	0.02	0.01	0.01	0.00	0.00	0.00
70	0.02	0.04	0.08	0.10	0.11	0.11	0.09	0.07	0.05	0.04	0.02	0.01
72.5	0.02	0.05	0.10	0.15	0.18	[redacted]	[redacted]	0.19	0.16	0.13	0.10	0.08
75	0.02	0.06	0.11	0.17	[redacted]	[redacted]	0.26	[redacted]	[redacted]	[redacted]	0.17	0.13
77.5	0.02	0.06	0.11	0.17	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	0.16	0.13	0.10
80	0.02	0.06	0.10	0.15	0.18	0.18	0.17	0.14	0.11	0.09	0.06	0.04
82.5	0.02	0.05	0.09	0.12	0.13	0.12	0.10	0.08	0.05	0.04	0.02	0.01
85	0.02	0.05	0.07	0.08	0.08	0.07	0.05	0.03	0.02	0.01	0.01	0.00
87.5	0.02	0.04	0.05	0.06	0.05	0.04	0.02	0.01	0.01	0.00	0.00	0.00
90	0.02	0.03	0.04	0.04	0.03	0.02	0.01	0.01	0.00	0.00	0.00	0.00
92.5	0.01	0.02	0.03	0.02	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00
95	0.01	0.02	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Fig 11 CD40109 Posterior Weibull distribution, assuming the Prior from figure 11 and 5 parts failing at levels >75 krad(Si).

Table I: CD40109 90% CL Cumulative Weibull P_f

Mission Year	TID krad(Si)	P(Fail)-Prior	P(Fail)-Prior + 5 parts >75 krad(Si)	P(Fail)-Prior + 8 parts >75 krad(Si)
1	6.14	0.115	0.0001	0
2	10.7	0.175	0.0007	0.000007
3	14.6	0.22	0.0022	0.00005
4	17.7	0.25	0.0045	0.00015
5	20.7	0.275	0.008	0.0004
5.5	22	0.28	0.01	0.0006

Discussion and Conclusions

Bayesian risk is a rigorous and efficient way of using diverse information and a metric for allocating resources, judging RHA efficacy and comparing risks. Still, Bayesian methods, while rigorous, do not banish subjectivity. Subjectivity arises in the Prior—e.g. the information to include, the weights for different data, the models assumed, and so on. Since all subsequent probabilities are conditional on the Prior, a conservative Prior definition is likely to lead to a bounding analysis. Moreover, by making subjectivity explicit in the analysis, we can validate it and correct it as needed. The iterative nature of the method reflects ongoing nature of RHA.

There is also subjectivity in defining failure costs, whether that subjectivity arises from assessment of mission objectives or is imposed by contract. Such subjectivity is inherent in any effort to treat radiation effects at the system level. Increasing the fidelity of our model of failure impacts on mission requirements leads to more realistic assessment of failure risks and optimizes resource allocation to maximize mission success.

In this work, we have used parametric degradation data for RH1014s to develop design guidelines (design-to values and equivalent RDMs). We have used Priors constructed with data on cryogenic performance of a logic family to plan testing efforts for parts in the family. In pursuing these goals, we have also outlined a method for bounding at a given CL the temporal variations in the radiation environment. These examples illustrate the utility of the methodology. We are confident that additional applications will be found. One promising area is analysis of relatively rare, but high impact failure/error modes such as severe ELDRS response and extremely long transients in analog devices.[8] It may prove feasible to use models to estimate Priors for microcircuits that may exhibit these effects and use the Priors to develop design guidelines that maximize success probability and to develop qualification regimes that validate the model's assumptions as efficiently as possible.

Although this work has focused on TID degradation, the methods are equally valid for displacement damage, SEE, reliability and any other threat that can be characterized by a pdf depending on some stress. This characteristic facilitates comparison of risks across disciplines—at least when the analyses have consistent assumptions and comparable degrees of conservatism. Also, since historical data, expert opinion and other information underlying the Prior are unlikely to anticipate unexpected threats, the methods outlined here are complementary to conventional RHA methods,[9] rather than a replacement for them.

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