Performance (Off-Design) Cycle Analysis for a Turbofan Engine With Interstage Turbine Burner

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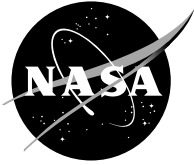
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Abstract

This report presents the performance of a steady-state, dual-spool, separate-exhaust turbofan engine, with an inter-stage turbine burner (*ITB*) serving as a secondary combustor. The *ITB*, which is located in the transition duct between the high- and the low-pressure turbines, is a relatively new concept for increasing specific thrust and lowering pollutant emissions in modern jet-engine propulsion. A detailed off-design performance analysis of *ITB* engines is written in Microsoft® Excel macrocode with Visual Basic Application to calculate engine performances over the entire operating envelope. Several design-point engine cases are pre-selected using a parametric cycle- analysis code developed previously in Microsoft® Excel, for off-design analysis. The off-design code calculates engine performances (i.e., thrust and thrust-specific-fuel-consumption) at various flight conditions and throttle settings.

Nomenclature

A	Flow area
a	Sound speed
C	Constants
c_p	Specific heat at constant pressure
F	Uninstalled thrust
f	Fuel/air ratio
g_c	Newton's constant
g_0	Acceleration of gravity
h	Altitude
h_{PR}	Low heating value of fuel
h_t	Total enthalpy
K	Constants
M	Mach number
$\dot{m}$	Mass flow rate
N	Engine speed

P_t	Total pressure
R	Gas constant or radius
S	Uninstalled specific fuel consumption
T	Temperature
T_t	Total temperature
V	Absolute velocity
W	Molecular weight
z	Geo-potential altitude
α	Bypass ratio
θ_0	Ratio of free-stream total temperature to SLS temperature of standard day
η	Efficiency
η_m	Mechanical Efficiency
γ	Ratio of specific heats
π	Total pressure ratio
τ	Total temperature ratio
τ_λ	Total enthalpy ratio

Subscripts

b	Main burner
c	Properties between upstream and main burner, engine core or compressor
cH	High-pressure compressor
cL	Low-pressure compressor
d	Diffuser
f	Fan
HP	High-pressure spool
i	Layer of atmosphere
itb	Properties between exit *ITB* and downstream, or *ITB*
LP	Low-pressure spool
max	Maximum
mix	Mixture of combustion product with air
n	Nozzle
nf	Fan nozzle
O	Overall
P	Propulsive
$prod$	Combustion product
R	Reference values
$Rspec$	Military specification
r	Ram
std	standard properties
TH	Thermal
t	Properties between main burner exit and *ITB*, or total properties
tH	High-pressure turbine
tL	Low-pressure turbine

Abbreviations

CSH	Constant Specific Heat
FTP	Full Throttle Performance

HPC	High-pressure Compressor
HPT	High-pressure Turbine
ITB	Interstage Turbine Burner
LP	Lapse Rate
LPC	Low-pressure Compressor
LPT	Low-pressure Turbine
MFP	Mass Flow Parameter
MSH	Modified Specific Heat
PTP	Partial Throttle Performance
SLS	Sea Level Static
TR	Throttle Ratio

1. Introduction

This engine performance analysis relies heavily on and builds upon the engine performance analysis in references 1, 2, and 3. As eloquently stated in reference 2, the parametric cycle analysis of the previous study (refs. 4 and 5) showed "how reference point (*design point* or *on-design*) engine performance was determined by design choices." This made it possible to examine trends in engine specific performance (*specific thrust* and *thrust specific fuel consumption*) with changes in design variables and to begin to narrow the desirable range for each design parameter. Now that the first step has been taken toward finding an optimum engine for a particular application, it is time to determine an engine's steady state operating performance.

The objective of performance cycle analysis is to determine the engine's performance over its operating envelope. The performance of several different engines can then be compared to reveal trends in engine performance with design choices. It is thus possible to focus on the most promising design and to ultimately find the engine reference point that has the best-balanced performance over the entire mission spectrum.

In general, performance analysis differs significantly from parametric analysis. In parametric (*reference* or *on-design point*) cycle analysis, all of the design choices (including the flight conditions) are free to be selected by the designer, and the specific engine performance characteristics are determined for each selected set of choices. In contrast, in performance cycle analysis or off-design analysis, the design choices have been set, and the performance of this specific reference point engine is then analyzed at all possible operating conditions. The independent variables in performance analysis are different flight conditions, throttle settings, and nozzle settings. Once the engine is sized and the mass flow rate is specified, performance analysis is employed to determine how a selected engine performs at all operating conditions within its flight envelope.

At the stage of performance analysis, it is considered that an engine of fixed configuration has been manufactured for testing in a facility that can simulate the desired flight conditions. Although the absolute size of the engine may be arbitrary during performance analysis, and we may lack most of the details of the geometry of each individual component, it is still best to think of it as pieces of hardware that have been built in order to explore the behavior of this reference point engine.

In off-design performance analysis, there are two classes of predicting individual component performance. First, each individual component performance can be modeled as a function of operating conditions. Secondly, actual component characteristics can be obtained from component hardware performance data for better estimate of the expected engine performance. These two approaches correspond to varying degrees of accuracy. In the absence of actual component hardware in a preliminary engine design, simple models of component performance in terms of operating conditions are used to give preliminary estimate of the engine performance.

In the following section, relationships are developed for determining the engine performance of the engine cycle whose parametric (*reference point*) performance was covered in the previous study (refs.4 and 5). The analysis is an extension of that presented in references 1, 2, and 3. The task now is to develop design tools in the form of equations and their solution procedures that will permit determination of the component, and hence engine, performance at any condition.

2. Aircraft Engine Performance Parameters

2.1 Performance Parameters

2.1.1 Thrust.—Thrust is the force used to sustain the flight (thrust = drag), accelerated flight (thrust > drag), deceleration (thrust < drag). Using figure 3.1 for the control volume, we can apply momentum balance to the control volume. Uninstalled thrust F of a jet engine (single inlet and single exhaust) is given by

$$F = \frac{(\dot{m}_0 + m_{fuel})V_e - \dot{m}_0 V_0}{g_c} + (P_e - P_o)A_e \qquad (1a)$$

where $\dot{m}_0, \dot{m}_{fuel}$ are mass flow rates of inlet air and fuel respectively

V_o, V_e are velocities at inlet and exit respectively

P_o, P_e are pressure at inlet and exit respectively

2.1.2 Specific Fuel Consumption.—It is the rate of fuel use by the propulsion system per unit of thrust produced. The uninstalled specific fuel consumptions, S, are given by

$$S = \frac{\dot{m}_{fuel}}{F}$$

2.2 Notations

Some useful quantity notation for compressible flow will be used in this report namely, stagnation temperature, stagnation pressure, and Mach number.

Stagnation temperature or total temperature T_t is defined as the temperature obtained when steadily flowing fluid is brought to rest adiabatically without involvement of works. Applying first law of thermodynamic to calorically perfect gas gives:

$$h_e = h_i = h_t = h + \frac{V^2}{2}$$

where,

$$h_t = \text{enthalpy at stagnation condition}$$

With the assumption of constant specific heat coefficient, the above equation can be written as:

$$T_t = T + \frac{V^2}{2 C_p}; \quad T_t = T\left(1 + \frac{\gamma-1}{2} M^2\right)$$

where,

T_t =   stagnation temperature

T =   static temperature

M =   Mach number

γ =   ratio of specific heats

Stagnation pressure or total pressure P_T is defined as pressure reached when a steady flowing fluid is brought to rest adiabatically and reversibly. Using the isentropic relation, total pressure is given by

$$P_t = P\left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$$

Ratio of total temperature (τ) and ratio of total pressure (π) across a component is denoted by subscript: d for diffuser, cL for low pressure compressor, cH for high pressure compressor, b for main burner, itb for inter-stage turbine burner, tL for low pressure turbine, tH for high pressure turbine, n for nozzle, and f for fan.

For example:

$$\pi_{cH} = \frac{\text{total pressure leaving high} - \text{pressure compressor}}{\text{total pressure entering high} - \text{pressure compressor}}$$

$$\tau_{cH} = \frac{\text{total temperature leaving high} - \text{pressure compressor}}{\text{total temperature entering high} - \text{pressure compressor}}$$

For the free stream, ram, we define τ_r as a ratio of total temperature/static temperature and π_r as a ratio of total pressure/static pressure.

$$\tau_r = \frac{T_{t0}}{T_0} = 1 + \frac{\gamma - 1}{2}M_0^2$$

$$\pi_r = \frac{P_{t0}}{P_0} = \left(1 + \frac{\gamma - 1}{2}M_0^2\right)^{\gamma/(\gamma-1)}$$

A ratio between total enthalpy of the burner exit and ambient enthalpy, denoted by τ_λ, is defined such that it will be one of the input parameters.

$$\tau_{\lambda-b} = \frac{\left(c_p T_t\right)_{burner\ exit}}{\left(c_p T_t\right)_{ambient}}$$

$$\tau_{\lambda-itb} = \frac{\left(c_p T_t\right)_{ITB\ exit}}{\left(c_p T_t\right)_{ambient}}$$

2.3 Component Performance

In this analysis it is acceptable to assume that the working fluid in the engine can be idealized as a perfect gas. Properties of an ideal gas strongly depend on the temperature. This cycle allows fluid properties variation across the engine which assumes constant fluid properties from the main burner entrance upstream (c_{pc}, γ_c), from *ITB* entrance to main burner exit (c_{pt}, γ_t), and from *ITB* exit downstream (c_{pt}, γ_t).

2.3.1 Inlet and Diffuser.—Pressure losses occur due to the friction with the inlet wall. The pressure total ratio, π_d, is always less than 1.

In supersonic flight, the pressure losses cause shock waves which produce greater pressure losses. The inlet total pressure is defined as the product of the ram pressure ratio and the diffuser pressure ratio. Therefore the portion of the pressure losses due to the shock waves and wall friction is defined by:

$$\pi_d = \pi_{d\,max}\ \eta_r$$

From the Military Specification 5008B (ref. 6), the following relation is obtained:

$$\eta_R = \begin{cases} 1 & \text{for} & M_0 \leq 1 \\ 1 - 0.075(M_0 - 1)^{1.35} & \text{for} & 1 < M_0 < 5 \end{cases}$$

2.4 Parametric Cycle Analysis

The performance of a selected reference point of a turbofan engine is desired at any flight conditions, throttle settings, and nozzle settings. It is assumed that a parametric (*design point*) cycle analysis has been performed for the reference point engine using the method of the previous study (ref. 4) to give the so-called *reference conditions* (subscript R) for the engine (S_R, $[F/\dot{m}_0]_R$, *etc.*), for each engine component (π_{fR}, τ_{fR}, etc.), and for the flight conditions (M_{0R}, P_{0R}, and T_{0R}).

To better understand the methodology of studying the performance cycle analysis problem, it is instructive to review how the parametric analysis proceeds. For example, in the parametric cycle analysis problem as provided in (ref. 4), there are 11 independent equations for solving the values of the 11 dependent component parametric variables (e.g., τ_f, τ_{cL}, τ_{cH}, f_b, τ_{tL}, π_{tH}, τ_{tL}, π_{tL}, f_{itb}, M_9, *and* M_{19}). These 11 equations are solved for the given values of the independent quantities consisting of flight conditions (i.e., M_0 and T_0) and design choices (i.e., π_f, π_{cL}, π_{cH}, and α). With values of these 11 component variables in hand, the engine reference point performance in terms of $F/\dot{m}_0$, S, η_P, and η_{TH} are readily found. A more complete description of the on-design code is given in reference 4 which is published with this manual.

Similarly, to find the off-design engine performance, the operational performance values of the 18 dependent variables listed in table 1 must be determined and then, in turn, related by 18 independent equations to obtain a solution for each component performance variable. Since performance analysis is an indirect problem as opposed to the direct problem of parametric analysis, the solution of the 18 performance equations is not as straightforward as in the parametric case. Regardless of the difficulty in solving the equations, once the values of the 18 dependent variables in table 1 are known, the engine performance in terms of $F/\dot{m}_0$, S, η_P, and η_{TH} follows immediately. In the "**Constant or Known**" column of table 1, π_b, τ_{itb}, π_{nf}, and π_n are assumed to remain constant, and τ_d, τ_n, and τ_{nf} are assumed equal to one.

Table 2.1.—Engine Performance Variables

Component	Independent	Constant or Known	Dependent
Engine	M_0, T_0, P_0		$\dot{m}_0$, α
Diffuser		$\pi_{d\,max} = f(M_0)$	
Fan		η_f	π_f; τ_f,
Low-pressure compressor		η_{cL}	π_{cL}, τ_{cL}
High-pressure compressor		η_{cH}	π_{cH}, τ_{cH}
Main burner	T_{t4}	π_b	f_b
High-pressure turbine		η_{tH}	π_{tH}, τ_{tH}
Inter-stage turbine burner	$T_{t4.5}$	π_{itb}	f_{itb}
Low-pressure turbine		η_{tL}, $\dfrac{A_{4.5}}{A_{4.5R}} = \left(\dfrac{\tau_{itb}}{\tau_{itbR}}\right)^n$	π_{tL}, τ_{tL}
Core nozzle		π_n, $\dfrac{A_8}{A_{8R}} = \left(\dfrac{\tau_{itb}}{\tau_{itbR}}\right)^m$	M_8, P_0/P_9
Fan nozzle		π_{nf}	M_{18}, P_0/P_{19}
Total number	5		18

3. Separate Exhaust Turbofan with ITB Cycle Analysis

3.1 Assumptions

The type of engine considered in this study is a separate-exhaust, two-spool turbofan engine with an Interstage Turbine Burner (*ITB*) serving as a secondary combustor as shown in figure 3.1. *ITB*, is located at the transition duct between the high- and the low-pressure turbines, is a relatively new concept for improving engine performance.

The station numbers of each location as indicated in figure 3.1 are in accordance with Aerospace Recommended Practice (ARP) 755A (ref. 6).
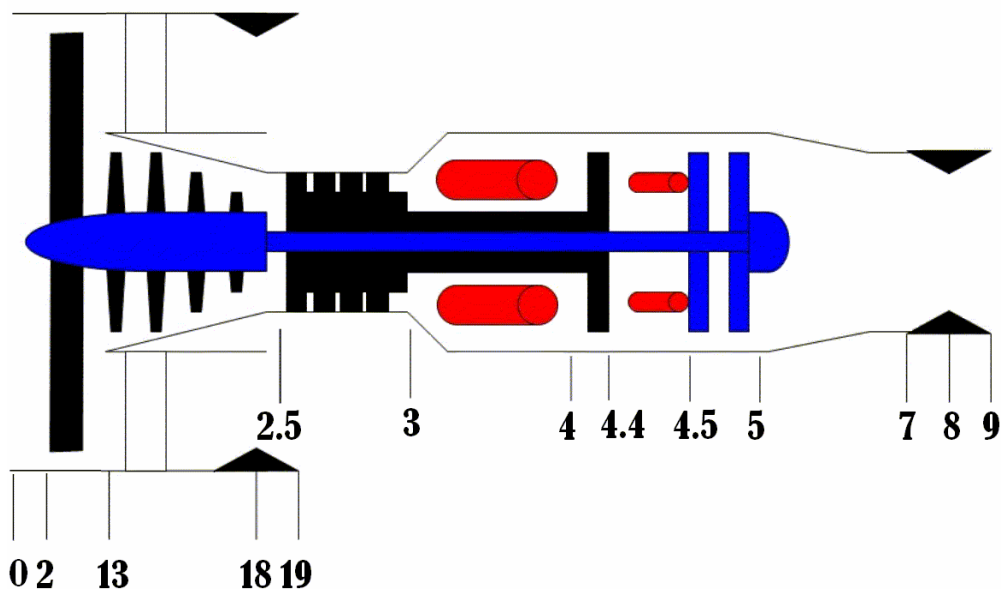

Figure 3.1.—Station Numbering for Separate-Exhaust Turbofan Engine
with Inter-stage Turbine Burning (ITB) (ref. 6)

In addition to the assumptions summarized in the parametric analysis (ref. 4), the following assumptions are employed in performance analysis:

- The flow area is constant at station 4, i.e., main burner exit or high-pressure turbine entrance;
- The flow is choked at the high-pressure turbine entrance nozzles (station 4), at the low-pressure turbine entrance nozzles (station 4.5), and at the throat of the exhaust nozzles (stations 8 and 18). Since the exhaust nozzles may unchoke at low throttle settings and influence the fan operating line, the cases of the unchoked exhaust nozzles (stations 8 and 18) are also included in this analysis;
- The component efficiencies (η_f, η_{cL}, η_{cH}, η_{tH}, η_{tL}, η_b, η_{itb}, η_{mH}, and η_{mL}) are constant;
- The total pressure ratios ($\pi_{d\,max}$, π_b, π_{itb}, π_n, and π_{nf}) remains the same as the values in (ref. 4).
- The simplifying gas model of a calorically perfect gas is used in the analysis. It assumes that the gases are calorically perfect upstream and downstream of the burner and inter-stage turbine burner (*ITB*) and values of γ_t, c_t, γ_{itb}, and c_{itb} do not vary with throttle setting. For this model, variations in fuel-air ratios (f_b and f_{itb}) are ignored when compared to unity.
- There is no turbine cooling.
- The exit areas (A_9 and A_{19}) of the exhaust nozzles are adjustable so that the corresponding pressure ratios (P_9/P_0 and P_{19}/P_0) can be set to predetermined values.
- The area at each engine station is constant. However the areas of stations 4.5 and 8 change with the *ITB* setting to follow a specified function of τ_{ITB}.
- The diffuser total pressure ratio, π_d, is given by

$$\pi_d = \pi_{d\,max}\,\eta_R$$

where $\pi_{d\,max}$ is the total pressure ratio due to friction and η_R is the total pressure recovery for the shocking system given by MIL-E-5008B (ref. 7).

3.2 Referencing

The functional relations for engine cycle analysis are based on the application of mass, energy, momentum, and entropy considerations to the one-dimensional steady flow of a perfect gas at an engine reference or off-design steady state operating point. Thus, if, at any off-design point,

$$f(\tau, \pi) = \text{constant} \tag{3.1a}$$

represents a relationship between the two performance variables τ and π at a steady state operating point, then the constant can be evaluated at the reference point, so that

$$f(\tau, \pi) = f(\tau_R, \pi_R) = \text{constant} \tag{3.1b}$$

since $f(\tau,\pi)$ applies to both on-design and off-design points. This technique for replacing constants with reference point values is used frequently in the performance analysis to follow. For example, see eq. (3.9).

3.3 Mass Flow Parameter

The mass flow parameter (*MFP*) is derived by combining mass flow per unit area with the perfect gas law, *Mach* number, sound speed, and equations for total temperature and pressure. The resulting expression is

$$MFP(M) = \frac{\dot{m}\sqrt{T_t}}{P_t A} = M\sqrt{\frac{\gamma}{R}\left(1 + \frac{\gamma - 1}{2}M^2\right)} \tag{3.2}$$

The total pressure mass flow parameter may be used to find any single flow quantity when the other four quantities and the calorically perfect gas constants (γ and R) are known at that station. The *MFP* is often used, for example, to determine the flow area required to choke a given flow (i.e., at $M = 1$). The *MFP* can also be used to develop valuable relationships between the flow properties at two different stations, especially when the mass flow is conserved between them.

3.4 High-Pressure Turbine

For our engine model, the mass flow entering the high-pressure turbine equals that entering the main burner plus the fuel added in the main burner. Thus

$$\dot{m}_4 = \dot{m}_3 + \dot{m}_b = \dot{m}_3\left(1 + f_b\right) \tag{3.3}$$

Likewise, the mass flow entering the low-pressure turbine equals that entering the high-pressure turbine plus the fuel added in the *ITB*. Thus

$$\dot{m}_{4.5} = \dot{m}_4 + \dot{m}_{itb} = \dot{m}_3\left(1 + f_b + f_{itb}\right) \tag{3.4}$$

Writing these using the mass flow parameter and the flow properties at stations 4 and 4.5 gives

$$\dot{m}_4 = \frac{P_{t4}}{T_{t4}}A_4 MFP(M_4) = \dot{m}_3\left(1 + f_b\right) \tag{3.5}$$

and

$$\dot{m}_{4.5} = \frac{P_{t4.5}}{T_{t4.5}}A_{4.5} MFP(M_{4.5}) = \dot{m}_3\left(1 + f_b + f_{itb}\right) \tag{3.6}$$

Solving each of these two expressions for $\dot{m}_3$ and equating yields

$$\frac{P_{t4}}{T_{t4}}\frac{A_4 MFP(M_4)}{\left(1 + f_b\right)} = \frac{P_{t4.5}}{T_{t4.5}}\frac{A_{4.5} MFP(M_{4.5})}{\left(1 + f_b + f_{itb}\right)} \tag{3.7}$$

which can be rewritten in terms of ratios as follows

$$\frac{P_{t4.4}}{P_{t4}}\frac{\sqrt{T_{t4}}}{\sqrt{T_{t4.5}}}A_{4.5} = A_4\frac{MFP(M_4)}{MFP(M_{4.5})}\frac{\left(1 + f_b + f_{itb}\right)}{\left(1 + f_b\right)}\frac{P_{t4.4}}{P_{t4.5}} \tag{3.8}$$

The right-hand side of the above equation is considered a constant since we make the following assumption: the flow is choked at stations 4 and 4.5, the flow areas at station 4 is constant, variations in

fuel-air ratios (f_b and f_{itb}) are ignored when compared to unity, and that the total pressure ratio of the *ITB* does not change. Using the referencing, we have

$$\frac{P_{t4.4}}{P_{t4}} \frac{\sqrt{T_{t4}}}{\sqrt{T_{t4.5}}} A_{4.5} = \left(\frac{P_{t4.4}}{P_{t4}} \frac{\sqrt{T_{t4}}}{\sqrt{T_{t4.5}}} A_{4.5} \right)_R \tag{3.9}$$

which can be rewritten in terms of component total pressure and total temperature ratios as

$$\pi_{tH} = \frac{\sqrt{\tau_{tH}\tau_{itb}}}{\left(\sqrt{\tau_{tH}\tau_{itb}}\right)_R} \frac{A_{4.5R}}{A_{4.5}} \pi_{tHR} \tag{3.10}$$

This gives one relationship between the high-pressure turbine's total pressure and total temperature ratios to meet mass conservation as a function of the *ITB*'s total temperature ratio. A second equation between π_{tH} and τ_{tH} comes from the high-pressure turbine efficiency equation or

$$\tau_{tH} = 1 - \eta_{tH} \left\{ 1 - \pi_{tH}^{(\gamma_t - 1/\gamma_t)} \right\} \tag{3.11}$$

For reference values of π_{tH}, τ_{tH}, $A_{4.5}$, and τ_{itb}, and the corresponding value of η_{TH}, equations (3.10) and (3.11) yield the high-pressure turbine's operating point (τ_{tH}, π_{tH}) for a given value of τ_{itb} and $A_{4.5}$ as sketched figure 3.2. The operating point is the intersection of equations (3.10) and (3.11).

One relationship for $A_{4.5}/A_{4.5R}$ is to assume that it is related to the total temperature ratio of the *ITB* (τ_{itb}) raised to the power *n* or

$$A_{4.5}/A_{4.5\ R} = \left(\tau_{itb}/\tau_{itb\ R}\right)^n \tag{3.12}$$

For the case where $A_{4.5}/A_{4.5\ R} = \sqrt{\left(\tau_{itb}/\tau_{itb\ R}\right)}$, then the solution to equations (3.10) and (3.11) is $\pi_{tH} = \pi_{tHR}$ and $\tau_{tH} = \tau_{tHR}$.
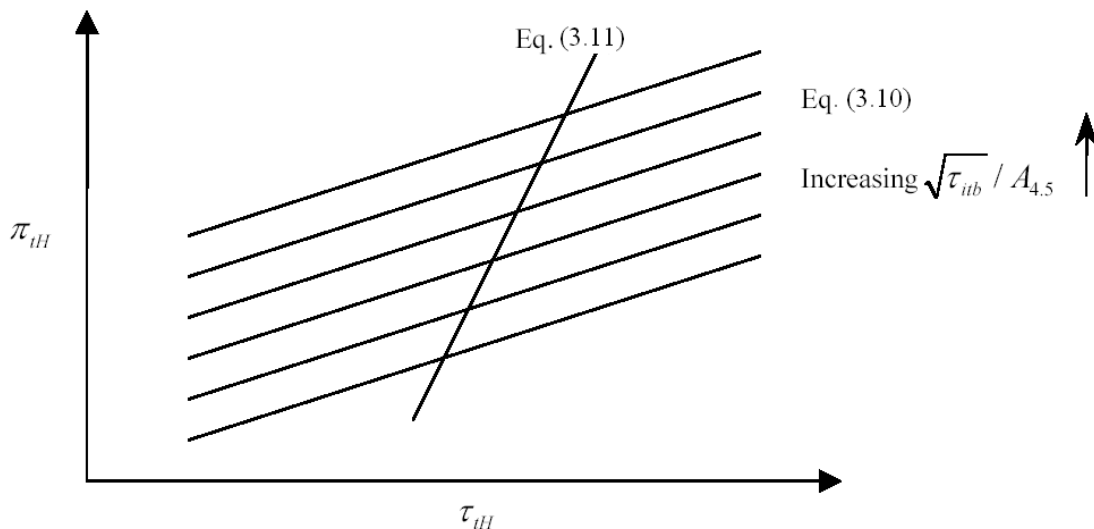

Figure 3.2.—Variation in High-Pressure Turbine Operating Conditions with Increasing $\sqrt{\tau_{itb}} / A_{4.5}$

3.5 Low-pressure Turbine

The mass flow rates at station 4.5 equals that at station 8. Writing the mass conservation at stations 4.5 and 8 using the mass flow parameter and gives

$$\frac{P_{t4.5}}{\sqrt{T_{t4.5}}} A_{4.5} MFP(M_{4.5}) = \frac{P_{t8}}{\sqrt{T_{t8}}} A_8 MFP(M_8) \tag{3.13}$$

This can be rewritten as

$$\frac{\pi_{tL}}{\sqrt{\tau_{tL}}} = \frac{A_{4.5}}{\pi_n A_8} \frac{MFP(M_{4.5})}{MFP(M_8)} \tag{3.14}$$

Noting that π_n, and $M_{4.5}$ (=1) are assumed constant, then the above equation can be rewritten using referencing as

$$\pi_{tL} = \pi_{tLR} \sqrt{\frac{\tau_{tL}}{\tau_{tLR}}} \frac{A_{8R}}{A_8} \frac{A_{4.5}}{A_{4.5R}} \frac{MFP(M_{8R})}{MFP(M_8)} \tag{3.15}$$

This gives one relationship between the low-pressure turbine's total pressure and total temperature ratios to meet mass conservation as a function of the core exhaust nozzle throat area (A_8/A_{8R}). A second equation between π_{tL} and τ_{tL} comes from the low-pressure turbine efficiency equation or

$$\tau_{tL} = 1 - \eta_{tL} \left\{ 1 - \pi_{tL}^{(\gamma_{itb} - 1)/\gamma_{itb}} \right\} \tag{3.16}$$

The operating condition (τ_{tL}, π_{tL}) of the low-pressure turbine can be determined using equations (3.15) and (3.16) along with the exhaust Mach number M_8 and specified functional relationships for the low pressure turbine nozzle throat area ($A_{4.5}/A_{4.5R}$) and the core exhaust nozzle throat area (A_8/A_{8R}). One relationship for A_8/A_{8R} is to assume that it is related to the total temperature ratio of the *ITB* (τ_{itb}) raised to the power *m* or

$$A_8/A_{8R} = (\tau_{itb}/\tau_{itbR})^m \tag{3.17}$$

With this functional relationship, the engine's low-pressure turbine performance will vary the same as the turbofan without the *ITB*, when the *ITB* is turned off. For reference values of π_{tL}, τ_{tL}, and τ_{itb} and the corresponding value of η_{tL}, equations (3.15) and (3.16) yield the low-pressure turbine's operating point (τ_{tL}, π_{tL}) for a given value of $\dfrac{A_{8R}}{A_8} \dfrac{A_{4.5}}{A_{4.5R}} \dfrac{MFP(M_{8R})}{MFP(M_8)}$ as sketched in figure 3.3. The operating point is the intersection of equations (3.15) and (3.16). For the case where $A_8/A_{8R} = A_{4.5}/A_{4.5R}$ and M_8/M_{8R}, then the solution to equations (3.15) and (3.16) is $\pi_{tL} = \pi_{tLR}$ and $\tau_{tL} = \tau_{tLR}$.

$$\frac{A_8}{A_{8R}} \frac{A_{4.5R}}{A_{4.5}} \frac{MFP(M_8)}{MFP(M_{8R})}$$
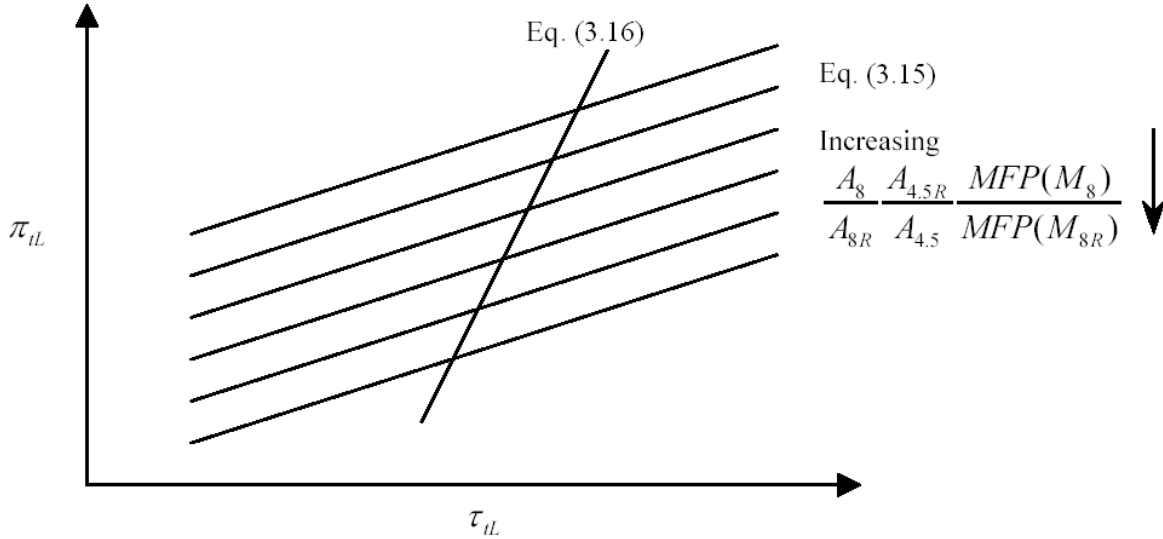
Figure 3.3.—Variation in Low-Pressure Turbine Operating Conditions with Increasing

3.6 Engine Bypass Ratio

An expression for the engine bypass ratio (α) follows directly from its definition written in terms of properties at stations 4 and 18.

$$\alpha = \frac{\dot{m}_f}{\dot{m}_c} \tag{3.18}$$

where

$$\dot{m}_f = \frac{P_{t18} A_{18}}{\sqrt{T_{t18}}} MFP_{18} = \frac{P_0 \pi_r \pi_d \pi_f \pi_{nf} A_{18}}{\sqrt{T_{t13}}} MFP_{18} \tag{3.19}$$

and

$$\dot{m}_4 = \dot{m}_c (1 + f_b) = \frac{P_{t4} A_4}{\sqrt{T_{t4}}} MFP_4 = \frac{P_0 \pi_r \pi_d \pi_{cL} \pi_{cH} \pi_b A_4}{\sqrt{T_{t4}}} MFP_4 \tag{3.20}$$

Thus the engine bypass ratio can be written as

$$\alpha = \frac{(1 + f_b)}{\pi_{cL} \pi_{cH} \pi_b / (\pi_f \pi_{nf})} \sqrt{\frac{T_{t4}}{T_{t13}}} \frac{MFP(M_{18})}{MFP(M_4)} \frac{A_{18}}{A_4} \tag{3.21}$$

Since $(1+f_b)$, π_b, π_{nf}, A_{18}, A_4, and M_4 (=1) are assumed constant and equal to their reference values, the bypass ratio can be rewritten using referencing as

$$\alpha = \alpha_R \frac{\pi_{cLR} \, \pi_{cLR} / \pi_{fR}}{\pi_{cL} \, \pi_{cH} / \pi_f} \sqrt{\frac{T_{t4}/T_{t4R}}{\tau_r \, \tau_f / (\tau_{rR} \tau_{fR})}} \frac{MFP(M_{18})}{MFP(M_{18R})} \tag{3.22}$$

3.7 Fan and Low-pressure Compressor

The performance equation for the total temperature ratio of the fan follows directly from the power balance of the low-pressure spool. Equating the power received by the fan and low-pressure (*LP*) compressor to that delivered by the low-pressure turbine gives

$$\dot{m}_f c_{pc}(T_{t13} - T_{t2}) + \dot{m}_c c_{pc}(T_{t2.5} - T_{t2}) = \eta_{mL}\dot{m}_{4.5}c_{pitb}(T_{t4.5} - T_{t5}) \tag{3.23}$$

or

$$\dot{m}_f c_{pc}T_{t2}\{\alpha(\tau_f - 1) + (\tau_{cL} - 1)\} = \eta_{mL}\dot{m}_c(1 + f_b + f_{itb})c_{pitb}T_{t4.5}(1 - \tau_{tL}) \tag{3.24}$$

Then dividing by $\dot{m}_c c_{pc}T_{t2}$, gives

$$\alpha(\tau_f - 1) + (\tau_{cL} - 1) = \eta_{mL}(1 + f_b + f_{itb})\frac{\tau_{\lambda-itb}}{\tau_r}(1 - \tau_{tL}) \tag{3.25}$$

The temperature rise of the low-pressure compressor can be approximated as proportional to that of the fan (refs. 2 and 3) and thus

$$\frac{\tau_{cL} - 1}{\tau_f - 1} = \left(\frac{\tau_{cL} - 1}{\tau_f - 1}\right)_R \tag{3.26}$$

Solving equation (3.26) for ($\tau_{cL} - 1$), substituting into equation (3.25), and then solving for the fan total temperature ratio (τ_f), gives

$$\tau_f = 1 + (\tau_{fr} - 1)\eta_{mL}\frac{\tau_{\lambda-itb}}{\tau_r}\left\{\frac{(1 - \tau_{tL})(1 + f_b + f_{itb})}{\tau_{cLR} - 1 + \alpha(\tau_{fr} - 1)}\right\} \tag{3.27}$$

Using the fan efficiency, the fan total pressure ratio is given by

$$\pi_f = \{1 + \eta_f(\tau_f - 1)\}^{\gamma_c/(\gamma_c - 1)} \tag{3.28}$$

The low-pressure compressor total temperature ratio follows from equation (3.26) and thus

$$\tau_{cL} = 1 + (\tau_f - 1)\frac{(\tau_{cL} - 1)_R}{(\tau_f - 1)_R} \tag{3.29}$$

Using the *LP* compressor efficiency, the *LP* compressor total pressure ratio is expressed as

$$\pi_{cL} = \{1 + \eta_{cL}(\tau_{cL} - 1)\}^{\gamma_c/(\gamma_c - 1)} \tag{3.30}$$

3.8 High-Pressure Compressor

The performance equation for the total temperature ratio of the high-pressure (*HP*) compressor follows directly from the power balance of the high-pressure spool. Solving equation (37) of the parametric analysis (ref. 4) for the total temperature ratio of the high-pressure compressor (τ_{cH}) gives

$$\tau_{cH} = 1 + \eta_{mH}\left(1 + f_b\right)\frac{\tau_{\lambda-b}\left(1 - \tau_{tH}\right)}{\tau_r \tau_{cL}} \tag{3.31}$$

Using the *HP* compressor efficiency, the *HP* compressor total pressure ratio is given by

$$\pi_{cH} = \left\{1 + \eta_{cH}\left(\tau_{cH} - 1\right)\right\}^{\gamma_c/\left(\gamma_c - 1\right)} \tag{3.32}$$

3.9 Exhaust Nozzles

The total to static pressure ratio of the core exhaust nozzle is given by equation (3.31) of reference 4 as

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9}\pi_r\pi_d\,\pi_{cL}\pi_{cH}\pi_b\pi_{tH}\,\pi_{itb}\pi_{tL}\pi_n \tag{3.33}$$

The exit Mach number follows directly using

$$M_9 = \sqrt{\frac{2}{\gamma_{itb} - 1}\left\{\left(\frac{P_{t9}}{P_9}\right)^{\frac{\gamma_{itb} - 1}{\gamma_{itb}}} - 1\right\}} \quad \text{If } M_9 > 1, \text{ then } M_8 = 1, \text{ else } M_8 = M_9. \tag{3.34}$$

The equation for the area ratio of the core exhaust nozzle follows from mass conservation between stations 8 and 9 and use of the mass flow parameter, giving

$$\frac{A_9}{A_8} = \frac{1}{\pi_n}\frac{MFP\left(M_8\right)}{MFP\left(M_9\right)} \tag{3.35}$$

The total to static pressure ratio of the fan exhaust nozzle is given by equation (3.36) of reference 4 as

$$\frac{P_{t19}}{P_{19}} = \frac{P_0}{P_{19}}\pi_r\,\pi_d\pi_f\pi_{nf} \tag{3.36}$$

The exit Mach number follows directly using

$$M_{19} = \sqrt{\frac{2}{\gamma_c - 1}\left\{\left(\frac{P_{t19}}{P_{19}}\right)^{\frac{\gamma_c - 1}{\gamma_c}} - 1\right\}} \quad \text{If } M_{19} > 1, \text{ then } M_{18} = 1, \text{ else } M_{18} = M_{19}. \tag{3.37}$$

The equation for the area ratio of the core exhaust nozzle follows from mass conservation between stations 18 and 19 and use of the mass flow parameter, giving

$$\frac{A_{19}}{A_{18}} = \frac{1}{\pi_{nf}} \frac{MFP(M_{18})}{MFP(M_{19})} \tag{3.38}$$

3.10 Engine Mass Flow Rate

An expression for the overall engine mass flow rate ($\dot{m}_0$) follows by the use of the mass flow parameter at station 4, giving

$$\dot{m}_0 = \dot{m}_0 \frac{1+\alpha}{1+\alpha_R} \frac{P_0 \, \pi_r \pi_d \pi_{cL} \pi_{cH}}{(P_0 \, \pi_r \pi_d \pi_{cL} \pi_{cH})_R} \sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{3.39}$$

3.11 Combustor Fuel-to-Air Ratios

The fuel-to-air ratios for the main burner (f_b) and the *ITB* (f_{itb}) are determined by the user's choice. There are two models of the gas properties available in this analysis, i.e., *Constant Specific Heat* (*CSH*) model and the *Modified Specific Heat* (*MSH*) model. Both models include the products of combustion in the heat capacity. For the details, see appendix B.

4. Summary of Equations

This section presents the complete performance cycle analysis equations for non-afterburning, separate exhaust flow, two-spool turbofan engine with inter-stage turbine burning.

Inputs

Performance choices

> *Flight parameters*: M_0, h [$T_0 = T_{amb}(h)$ (K, °R), $P_0 = P_{amb}(h)$ (Pa, psia)]
> *Throttle settings*: T_{t4}, $T_{t4.5}$ (K, °R)
> *Engine control*: *TR*

Design Constants

> *Perfect gas constants*: γ_c, γ_t, γ_{itb}, c_{pc}, c_{pt}, c_{pitb} [kJ/(kg·K), Btu/(lbm·°R)]
> π's: $\pi_{d\,max}$, π_b, π_{itb}, π_n, π_{nf}
> η's: η_f, η_{cL}, η_{cH}, η_{tH}, η_{tL}, η_b, η_{itb}, η_{mH}, η_{mL}
> *Fuel*: h_{PR} (kJ/kg, Btu/lbm)

Reference conditions

> *Flight parameters*: M_{0R}, h_R *(km, kft)*[$T_{0R} = T_{amb}(h_R)$ (K,°R), $P_{0R} = P_{amb}(h_R)$ (Pa, psia)], τ_R, π_R
> *Component behavior*: π_{fR}, π_{cLR}, π_{cHR}, π_{tHR}, π_{tLR}, τ_{fR}, τ_{cLR}, τ_{cHR}, τ_{tHR}, τ_{tLR}
> *Others*: T_{t4R}, $T_{t4.5R}$ (K, °R), f_{bR}, f_{itbR}, M_{8R}, M_{18R}, α_R, F_R (N, lbf), $\dot{m}_{0R}$ (kg/sec, lbm/sec), S_R
> $$\left(\frac{mg/s}{N}, \frac{lbm/hr}{lbf}\right),$$

Engine Control Limits:

$$T_{t4\,max}, T_{t4.5\,max}, \pi_{f\,max}, \pi_{cH\,max}, T_{t3\,max} \text{ (K, °R)}, P_{t3\,max} \text{ (Pa, psia)}, \%N_{LP}, \%N_{HP}$$

LP Turbine Nozzle Throat Area Functional Relationship:

$$A_{4.5} / A_{4.5R} = (\tau_{itb} / \tau_{itbR})^n$$

Core Exhaust Throat Area Functional Relationship:

$$A_8 / A_{8R} = (\tau_{itb} / \tau_{itbR})^m$$

Outputs

Overall Performance

$$F \text{ (N, lbf)}, \dot{m}_0 \text{ (kg/sec, lbm/sec)}, S\left(\frac{mg/\sec}{N}, \frac{lbm/hr}{lbf}\right), f_o, \eta_P, \eta_{TH}, \eta_O, V_9/a_0, V_{19}/a_0, \alpha, P_{t9}/P_9,$$

$$T_9/T_0,$$

$$P_{t19}/P_{19}, T_{19}/T_0$$

Component Behavior

$$\pi_f, \pi_{cL}, \pi_{cH}, \pi_{tH}, \pi_{tL}, \tau_f, \tau_{cL}, \tau_{cH}, \tau_{tH}, \tau_{tL}, \tau_{\lambda\text{-}b}, \tau_{\lambda\text{-}itb}, f_b, f_{itb}, M_8, M_{18}, M_9, M_{19}, A_9/A_8, A_{19}/A_{18}$$

Equations

Reference Conditions

If (*SI unit*) then

$$T_{ref} = 288.6 \text{ K}$$

Else

$$T_{ref} = 516.67 \text{ °R}$$

End If

$$\tau_{rR} = 1 + \frac{\gamma_c - 1}{2} M_{0R}^2 \tag{4.1R}$$

$$\pi_{rR} = \tau_{rR}^{\gamma_c/(\gamma_c - 1)} \tag{4.2R}$$

$$\eta_{RspecR} = 1 \qquad \text{for} \qquad M_{0R} \leq 1$$
$$\eta_{RspecR} = 1 - 0.075(M_{0R} - 1)^{1.35} \qquad \text{for} \qquad 1 < M_{0R} < 5 \tag{4.3R}$$

$$\pi_{dR} = \pi_{d\,max}\eta_{RspecR} \qquad (4.4R)$$

$$\tau_{itbR} = \frac{T_{t4.5R}}{T_{t4R}\tau_{tHR}} \qquad (4.5R)$$

$$MFP(M_{8R}) = M_{8R}\sqrt{\frac{\gamma_{itb}g_c}{R_{itb}}}\left(1 + \frac{\gamma_{itb}-1}{2}M_{8R}^2\right)^{\frac{\gamma_{itb}+1}{2(1-\gamma_{itb})}} \qquad (4.6R)$$

$$MFP(M_{18R}) = M_{18R}\sqrt{\frac{\gamma_c g_c}{R_c}}\left(1 + \frac{\gamma_c-1}{2}M_{18R}^2\right)^{\frac{\gamma_c+1}{2(1-\gamma_c)}} \qquad (4.7R)$$

$$\theta_{0R} = \tau_{rR}\frac{T_{0R}}{T_{ref}} \qquad (4.8R)$$

If $\theta_{0R} \geq TR$ then

$$\begin{aligned} T_{t4\,max} &= T_{t4R} \\ T_{t4.5\,max} &= T_{t4.5R} \end{aligned} \qquad (4.9R)$$

Else

$$\begin{aligned} T_{t4\,max} &= T_{t4R}\left(\frac{\theta_{0R}}{TR}\right) \\ T_{t4.5\,max} &= T_{t4.5R}\left(\frac{\theta_{0R}}{TR}\right) \end{aligned} \qquad (4.10R)$$

Endif

Preliminary computations 1

$$\tau_r = 1 + \frac{\gamma_c - 1}{2}M_0^2 \qquad (4.1)$$

$$\pi_r = \tau_r^{\gamma_c/(\gamma_c-1)} \qquad (4.2)$$

$$\begin{aligned} \eta_R &= 1 & for \quad M_0 \leq 1 \\ \eta_R &= 1 - 0.075(M_0-1)^{1.35} & for \quad 1 < M_0 < 5 \end{aligned} \qquad (4.3)$$

$$\pi_d = \pi_{d\,max}\eta_R \qquad (4.4)$$

$$R_c = \frac{\gamma_c - 1}{\gamma_c}C_{pc} \qquad (4.5)$$

$$R_t = \frac{\gamma_t - 1}{\gamma_t} C_{pt} \tag{4.6}$$

$$R_{itb} = \frac{\gamma_{itb} - 1}{\gamma_{itb}} C_{pt} \tag{4.7}$$

$$a_0 = \sqrt{\gamma_c R_c g_c T_0} \tag{4.8}$$

$$P_{t0} = P_0 \tau_r^{\gamma_c/(\gamma_c - 1)} \tag{4.9}$$

$$T_{t0} = T_0 \tau_r \tag{4.10}$$

$$MFP(M_0) = M_0 \sqrt{\frac{\gamma_c g_c}{R_c}} \left(1 + \frac{\gamma_c - 1}{2} M_0^2\right)^{\frac{\gamma_c + 1}{2(1 - \gamma_c)}} \tag{4.11}$$

Control limits

$$\theta_0 = \tau_r \frac{T_0}{T_{ref}} \tag{4.12}$$

If $\theta_0 \geq TR$ then

$$T_{t4} = T_{t4\max} \tag{4.13a}$$

Else

$$T_{t4} = T_{t4\max}\left(\frac{\theta_0}{TR}\right) \tag{4.13b}$$

Endif

If (4.*ITBon*) then
   If $\theta_0 \geq TR$ then

$$T_{t4.5} = T_{t4.5\max} \tag{4.14a}$$

   Else

$$T_{t4.5} = T_{t4.5\max}\left(\frac{\theta_0}{TR}\right) \tag{4.14b}$$

   Endif
Endif

Set initial values:
$\dot{m}_0 = \dot{m}_{0R}$, $f_b = f_{br}$, $f_{itb} = f_{itbR}$,

$\pi_{tH} = \pi_{tHR}$, $\tau_{tH} = \tau_{tHR}$, $\pi_{tL} = \pi_{tLR}$, $\tau_{tL} = \tau_{tLR}$

$\pi_f = \pi_{fr}$, $\pi_{cL} = \pi_{cLR}$, $\pi_{cH} = \pi_{cHR}$

$\tau_f = \tau_{fR}$, $\tau_{cL} = \tau_{cLR}$, $\tau_{cH} = \tau_{cHR}$

$$\tau_{\lambda-b} = \frac{C_{pt}T_{t4}}{C_{pc}T_0} \tag{4.15}$$

Inter-stage Turbine Burner

1

If (4.*ITBon*) then

$$\tau_{itb} = \frac{T_{t4.5}}{T_{t4}\,\tau_{tH}} \tag{4.16a}$$

$$\tau_{\lambda-itb} = \frac{C_{pitb}T_{t4.5}}{C_{pc}T_0} \tag{4.16b}$$

Else

$$\tau_{itb} = 1 \tag{4.16c}$$

$$\tau_{\lambda-itb} = \frac{C_{pt}T_{t4}}{C_{pc}T_0}\,\tau_{tH} \tag{4.16d}$$

$$T_{t4.5} = T_{t4}\tau_{tH} \tag{4.16e}$$

Endif

$$A_{4.5}\,/\,A_{4.5R} = \left(\tau_{itb}\,/\,\tau_{itbR}\right)^n \tag{4.17}$$

$$A_8\,/\,A_{8R} = \left(\tau_{itb}\,/\,\tau_{itbR}\right)^m \tag{4.18}$$

High-pressure turbine

$$\tau_{tH} = 1 - \eta_{tH}\left\{1 - \pi_{tH}^{(\gamma_t-1)/\gamma_t}\right\} \tag{4.19}$$

$$\pi_{tHN} = \frac{\sqrt{\tau_{tH}\,\tau_{itb}}}{\left(\sqrt{\tau_{tH}\,\tau_{itb}}\right)_R}\,\frac{A_{4.5R}}{A_{4.5}}\,\pi_{tHR} \tag{4.20}$$

Is $\left|\pi_{tHN} - \pi_{tH}\right| \leq 0.00001$? If so, then continue. If not, set $\pi_{tH} = \pi_{tHN}$ and return to 1

High-pressure compressor
2

$$\tau_{cH} = 1 + \eta_{mH} \, (1 + f_b) \, \frac{\tau_{\lambda - b} \, (1 - \tau_{tH})}{\tau_r \tau_{cL}} \tag{4.21}$$

$$\pi_{cH} = \left\{ 1 + \eta_{cH} \, (\tau_{cH} - 1) \right\}^{\gamma_c/(\gamma_c - 1)} \tag{4.22}$$

Main burner
   If *CSH* model then

$$f_b = \frac{\tau_{\lambda - b} - \tau_r \tau_{cL} \tau_{cH}}{\eta_b h_{PR} / h_0 - \tau_{\lambda - b}} \tag{4.23a}$$

   Else

$$T_{t3} = T_0 \tau_r \tau_{cL} \tau_{cH} \tag{4.23b}$$

   ENTHALPY $\left(T_{t3}, \, 0, \, h_{t3} \right)$

   Set initial value of fuel/air ratio at station 4, $f_{4i} = \dfrac{\tau_{\lambda - b} - \tau_r \tau_{cL} \tau_{cH}}{\eta_b h_{PR} / h_0 - \tau_{\lambda - b}}$ $\tag{4.23c}$

   3 ENTHALPY $\left(T_{t4}, f_{4i}, h_{t4} \right)$

$$f_b = \frac{h_{t4} - h_{t3}}{\eta_b h_{PR} - h_{t4}} \tag{4.23d}$$

   If $\left| f_b - f_{4i} \right| > 0.0001$, then $f_{4i} = f_b$ and go to 3; else continue.
   End if

Inter-stage Turbine Burner
   If *CSH* model then

$$f_{itb} = (1 + f_b) \frac{\tau_{\lambda - itb} - \tau_{\lambda - b} \tau_{tH}}{\eta_{itb} h_{PR} / h_0 - \tau_{\lambda - itb}} \tag{4.24a}$$

   Else

$$T_{t4.4} = T_{t4} \tau_{tH} \tag{4.24b}$$

   ENTHALPY $\left(T_{t4.5}, \, f_{4.5i}, \, h_{t4.4} \right)$

Set initial value of fuel/air ratio at station 4.4, $f_{4.5i}(1+f_b)\dfrac{\tau_{\lambda-itb}-\tau_{\lambda-b}\tau_{tH}}{\eta_{itb}\,h_{PR}/h_0-\tau_{\lambda-itb}}$ (4.24c)

4   ENTHALPY $\left(T_{t4.5},\,f_{4.5i},\,h_{t4.5}\right)$

$$f_{itb}=(1+f_b)\frac{h_{t4.5}-h_{t4.4}}{\eta_{itb}h_{PR}-h_{t4.5}}$$ (4.24d)

If $\left|f_b+f_{itb}-f_{4.5i}\right|>0.0001$, then $f_{4.5i}=f_b+f_{itb}$ and go to 4; else continue.

End if

If $f_{itb}=0$ then $\gamma_{itb}=\gamma_t,C_{pitb}=C_{pt}$, and $R_{itb}=R_t$ (4.34)

Fan and low-pressure compressor pressure ratios

$$\pi_f=\left\{1+\eta_f\left(\tau_f-1\right)\right\}^{\gamma_c/(\gamma_c-1)}$$ (4.25)

$$\pi_{cL}=\left\{1+\eta_{cL}\left(\tau_{cL}-1\right)\right\}^{\gamma_c/(\gamma_c-1)}$$ (4.26)

Exhaust nozzles

$$\frac{P_{t19}}{P_{19}}=\frac{P_0}{P_{19}}\pi_r\pi_d\pi_f\pi_{nf}$$ (4.27)

$$\frac{P_{t9}}{P_9}=\frac{P_0}{P_9}\pi_r\pi_d\pi_{cL}\pi_{cH}\pi_b\pi_{tH}\pi_{itb}\pi_{tL}\pi_n$$ (4.28)

$$M_9=\sqrt{\frac{2}{\gamma_{itb}-1}\left\{\left(\frac{P_{t9}}{P_9}\right)^{\frac{\gamma_{itb}-1}{\gamma_{itb}}}-1\right\}}$$ (4.29)

If $M_9>1$, then $M_8=1$, else $M_8=M_9$

$$MFP(M_8)=M_8\sqrt{\frac{\gamma_{itb}g_c}{R_{itb}}}\left(1+\frac{\gamma_{itb}-1}{2}M_8^2\right)^{\frac{\gamma_{itb}+1}{2(1-\gamma_{itb})}}$$ (4.30)

$$MFP(M_9)=M_9\sqrt{\frac{\gamma_{itb}g_c}{R_{itb}}}\left(1+\frac{\gamma_{itb}-1}{2}M_9^2\right)^{\frac{\gamma_{itb}+1}{2(1-\gamma_{itb})}}$$ (4.31)

$$\frac{A_9}{A_8}=\frac{1}{\pi_n}\frac{MFP(M_8)}{MFP(M_9)}$$ (4.32)

$$M_{19} = \sqrt{\frac{2}{\gamma_c - 1}\left\{\left(\frac{P_{t19}}{P_{19}}\right)^{\frac{\gamma_c - 1}{\gamma_c}} - 1\right\}} \qquad (4.33)$$

If $M_{19} > 1$, then $M_{18} = 1$, else $M_{18} = M_{19}$

$$MFP(M_{18}) = M_{18}\sqrt{\frac{\gamma_c g_c}{R_c}}\left(1 + \frac{\gamma_c - 1}{2}M_{18}^2\right)^{\frac{\gamma_c + 1}{2(1 - \gamma_c)}} \qquad (4.34)$$

$$MFP(M_{19}) = M_{19}\sqrt{\frac{\gamma_c g_c}{R_c}}\left(1 + \frac{\gamma_c - 1}{2}M_{19}^2\right)^{\frac{\gamma_c + 1}{2(1 - \gamma_c)}} \qquad (4.35)$$

$$\frac{A_{19}}{A_{18}} = \frac{1}{\pi_{nf}}\frac{MFP(M_{18})}{MFP(M_{19})} \qquad (4.36)$$

Engine bypass ratio

$$\alpha = \alpha_R \frac{\pi_{cLR}\pi_{cHR}/\pi_{fR}}{\pi_{cL}\pi_{cH}/\pi_f}\sqrt{\frac{T_{t4}/T_{t4R}}{\tau_r\tau_f/(\tau_{rR}\tau_{fR})}}\frac{MFP(M_{18})}{MFP(M_{18R})} \qquad (4.37)$$

Fan and low-pressure compressor temperature ratios

$$\tau_{fN} = 1 + (\tau_{fR} - 1)\eta_{mL}\frac{\tau_{\lambda-itb}}{\tau_r}\left\{\frac{(1 - \tau_{tL})(1 + f_b + f_{itb})}{\tau_{cLR} - 1 + \alpha(\tau_{fR} - 1)}\right\} \qquad (4.38)$$

$$\tau_{cL} = 1 + (\tau_{fN} - 1)\frac{(\tau_{cL} - 1)_R}{(\tau_f - 1)_R} \qquad (4.39)$$

Low-pressure turbine

$$\tau_{tL} = 1 - \eta_{tL}\left\{1 - \pi_{tL}^{(\gamma_{itb}-1)/\gamma_{itb}}\right\} \qquad (4.40)$$

$$\pi_{tL} = \pi_{tLR}\sqrt{\frac{\tau_{tL}}{\tau_{tLR}}}\frac{A_{8R}}{A_8}\frac{A_{4.5}}{A_{4.5R}}\frac{MFP(M_{8R})}{MFP(M_8)} \qquad (4.41)$$

Is $\left|\tau_{fN} - \tau_f\right| \le 0.0001$? If so, then continue. If not, set $\tau_f = \tau_{fN}$ an return to 2

Remainder of Calculations

Engine mass flow $\left(\dot{m}_0\right)$

$$\dot{m}_0 = \dot{m}_{0R} \frac{1+\alpha}{1+\alpha_R} \frac{P_0 \pi_r \pi_d \pi_{cL} \pi_{cH}}{(P_0 \pi_r \pi_d \pi_{cL} \pi_{cH})_R} \sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{4.42}$$

Overall engine performance

$$f_0 = \left(f_b + f_{itb}\right)/(1+\alpha) \tag{4.43}$$

$$\frac{T_9}{T_0} = \frac{T_{t4.5} \tau_{tL}/T_0}{\left(P_{t9}/P_9\right)^{(\gamma_{itb}-1)/\gamma_{itb}}} \tag{4.44}$$

$$\frac{T_{19}}{T_0} = \frac{\tau_r \tau_f}{\left(P_{t19}/P_{19}\right)^{(\gamma_c-1)/\gamma_c}} \tag{4.45}$$

$$a_9 = \sqrt{\gamma_{itb} R_{itb} g_c T_9} \tag{4.46}$$

$$a_{19} = \sqrt{\gamma_c R_c g_c T_{19}} \tag{4.47}$$

$$V_9 = M_9 a_9 \tag{4.48}$$

$$V_{19} = M_{19} a_{19} \tag{4.49}$$

$$\frac{F}{\dot{m}_0} = \frac{a_0}{g_c(1+\alpha)} \left\{ \begin{array}{l} \left[1 + f_0(1+\alpha)\right]\dfrac{V_9}{a_0} + \alpha\,\dfrac{V_{19}}{a_0} - (1+\alpha)M_0 \\[2mm] + \left[1 + f_0(1+\alpha)\right]\dfrac{R_{itb}}{R_c}\dfrac{T_9/T_0}{V_9/a_0}\dfrac{(1-P_0/P_9)}{\gamma_c} \\[2mm] + \alpha\,\dfrac{T_{19}/T_0}{V_{19}/a_0}\dfrac{(1-P_0/P_{19})}{\gamma_c} \end{array} \right\} \tag{4.50}$$

$$F = \dot{m}_0\left(\frac{F}{\dot{m}_0}\right) \tag{4.51}$$

$$S = \frac{f_0}{F/\dot{m}_0} \tag{4.52}$$

$$\eta_P = \frac{2 g_c M_0 (1+\alpha)\dfrac{F}{\dot{m}_0}}{a_0\left\{\left[1 + f_0(1+\alpha)\right]\left(\dfrac{V_9}{a_0}\right)^2 + \alpha\left(\dfrac{V_{19}}{a_0}\right)^2 - (1+\alpha)M_0^2\right\}} \tag{4.53}$$

$$\eta_{TH} = \frac{a_0^2}{2 g_c} \frac{\left[1 + f_0(1+\alpha)\right]\left(\dfrac{V_9}{a_0}\right)^2 + \alpha\left(\dfrac{V_{19}}{a_0}\right)^2 - (1+\alpha)M_0^2}{(1+\alpha) f_0 h_{PR}} \qquad (4.54)$$

$$\eta_O = \eta_{TH} \eta_P \qquad (4.55)$$

$$A_0 = \frac{\dot{m}_0 \sqrt{T_{t0}}}{P_{t0} MFP(M_0)} \qquad (4.56)$$

$$\%RPM_{LPspool} = 100 \sqrt{\frac{T_0 \tau_r \left(\tau_f - 1\right)}{\left[T_0 \tau_r \left(\tau_f - 1\right)\right]_R}} \qquad (4.57)$$

$$\%RPM_{HPspool} = 100 \sqrt{\frac{T_0 \tau_r \tau_{cL} \left(\tau_{cH} - 1\right)}{\left[T_0 \tau_r \tau_{cL} \left(\tau_{cH} - 1\right)\right]_R}} \qquad (4.58)$$

If any of the control limits (π_c, T_{t3}, P_{t3}, etc.) are exceeded, first reduce $T_{t4.5}$ until $\tau_{itb} = 1$ or reduce T_{t4} and then go to 1. A Newtonian iteration scheme is used to rapidly converge on the values of $T_{t4.5}$ and T_{t4} that meets the most constraining control limit.

5. User's Manual

The Microsoft® Excel program is written in combination between spreadsheet neuron cells, visual basic, and macro code. These three combinations provide user-friendly software that compilation and preprocessing are no longer necessary. The input is to well-labeled Microsoft® Excel cells. Most of the values are prespecified and an example case is displayed. It is good practice to save this case by another name before modifying the spread sheet in order to retrieve the starting case. User obtains result plots right away just by clicking some buttons.

The program mainly comprises of three sections, namely, *Input*, *Full Throttle Performance* and *Partial Throttle Performance*. The Microsoft® Excel spreadsheets associated with each section are listed in the table 2 as shown below:

Table 5.1.—List of Microsoft® Excel Spreadsheets

No	Section	Function	Microsoft® Excel Spreadsheet
1	*Input*	Introduction	Coverpage, Instruction
		Data Input	Input
2	*Full Throttle Performance*	Control	Control_FTP, SinglePt_FTP
		Data Output	D_FTP, D_FvsM0, D_SvsM0, D_picHvsM0, D_Tt4vsM0, D_mdot0, D_pif, D_alp, D_Tt4p5
3	*Partial Throttle Performance*	Control	Control_PTP
		Data Output	D_PTP

5.1 Input

5.1.1 CoverPage.—The 'CoverPage' sheet contains the information about the authors of this program. Any questions regarding the program can be addressed to the authors through the email or phone.

5.1.2 Instructions.—First-time users are strongly recommended to read this sheet before running the program. In this sheet, you will find details of how to run the program. This sheet also explains several assumptions made in the equations so that the users are aware of some cases in the equations that have been idealized to simplify the problems.

5.1.3 Input.—This sheet is where most of the inputs are specified. All cells in 'dark green' (█) color need to be filled in with input values, except those cells in 'hotpink' (█) color, where the built-in values are set. The program will check all the inputs value in this sheet to make sure that the inputs are specified correctly. Window dialog boxes will pop up and notice the user if there are input values that are not specified or zero input values are found.

Once the design choice of a specific reference-point engine has been made from a parametric cycle analysis (previously developed in ref. 4), the input values for the so-called '*Reference Conditions*' (i.e., the flight condition, throttle settings, engine component, etc.) and the '*Design Constants*' are readily obtained. By clicking the '*Import data from ITB on-design code*' button, the code will automatically load the reference engine data file at a specified path and filename. All reference engine data will be displayed at all 'burlywood' cells (█). Please refer to Appendix F for further explanation and instruction on this. Alternatively, users can choose to manually input each input value, which is a time-consuming process.

There are four combo boxes in the 'Input' sheet (*Combo Box* is a list box that displays a list of values and lets the users select one of the value in the list) namely Unit System, *ITB Switch*, *Fuel Burn Model*, and *Perform Computation in* as shown in figure 5.1. You need first to specify the value in combo box *Unit System*, combo box *ITB Switch*, and combo box *Fuel Burn Model* before moving on to combo box *Perform Computation in*.

Combo box *Unit System* lets you specify the input and output unit system. Currently, the program can handle two units systems, namely, *English* and *SI* units. Combo box *ITB Switch* lets user to turn ON or OFF the Inter-stage Turbine Burner (*ITB*). This feature provides a flexibility to choose two types of engine and they are the engine with ITB-ON and with ITB-OFF. With this feature, you will be able to see how much engine performance you can get with ITB-ON and with ITB-OFF. Combo Box *Fuel Burn Model* lets user to choose two models for the computation of the fuel-air-ratio inside the burners, i.e., Constant Specific Heat (*CSH*) model and the Modified Specific Heat (*MSH*) model. For the details, see appendix B.

Once all the inputs values are specified and all combo box values on *Unit System*, *ITB Switch*, and *Fuel Burn Model* are specified, user can specify a value in combo box Combo box *Perform Computation in*. This combo box provides three option lists to choose from, and they are:

1. Full Throttle Performance (FTP): Multiple-point
2. Full Throttle Performance (FTP): Single-point
3. Partial Throttle Performance (PTP)

Once you select either one of them, you will be directed to some new sheets. The macro code associated with this combo box will open a relevant control sheet, and some hidden data sheets depending on the selection you make.

The following sections describe the instruction of how to run the program for each different option.

Table 5.2.—List of Combo Box in '*Input*' sheet

No	Combo Box	List value
1	*Unit System*	SI
		English
2	*ITB Switch*	ON
		OFF
3	*Fuel Burn Model*	*CSH* (Constant Specific Heat)
		MSH (Modified Specific Heat)
4	*Perform Computation in*	Full Throttle Performance (FTP): Multiple-point
		Full Throttle Performance (FTP): Single-point
		Partial Throttle Performance (PTP)

Figure 5.1.—Microsoft® Excel *Input* Sheet Screenshot

5.2 Full Throttle Performance over M_0 and Altitude (FTP)

5.2.1 Multiple-point Calculations (sheet 'Control_FTP').—When this option is chosen, the program opens several sheets, namely, one control sheet (i.e., either 'Control_FTP'), and nine data sheets (i.e., 'D_FTP', 'D_FvsM0', 'D_SvsM0', 'D_picHvsM0', 'D_Tt4vsM0', 'D_mdot0', 'D_pif', 'D_alp', and 'D_Tt4p5').

In 'Control_FTP' sheet, user expects to see a sheet as shown in figure 5.2. Before the computation, user needs to complete two input parameters, i.e., Flight parameters and Altitude, as indicated in 'dark green' cells.

NOTE: In this code, there is only one design variable, namely, *Flight Mach number*. Other iteration variables, such as ambient conditions (T_0 and P_0), altitude (h), total exit temperature of main burner or *ITB,* etc., may be added to the code with minor code modification in the future.

To specify a range for flight Mach number (M_0), user needs to input the values for the lower limit, upper limit, and an increment of M_0. If input lower limit of M_0 is zero, the code will internally change it from zero to 0.01.

Click the ***Altitude (h)*** combo box to select number of plots for different altitudes. Selected number of cells will be created adjacent to the ***Altitude (h)*** combo box. Once all the 'dark green' cells are filled, it is ready to run the program and generate the plots, simply by clicking ***Calculate*** button. Data computed will be stored in nine data sheets.

Clicking the ***Clear data & plot*** button will clear up all the computed solutions in data sheets as well as the eight plots in 'Control_FTP' sheet.
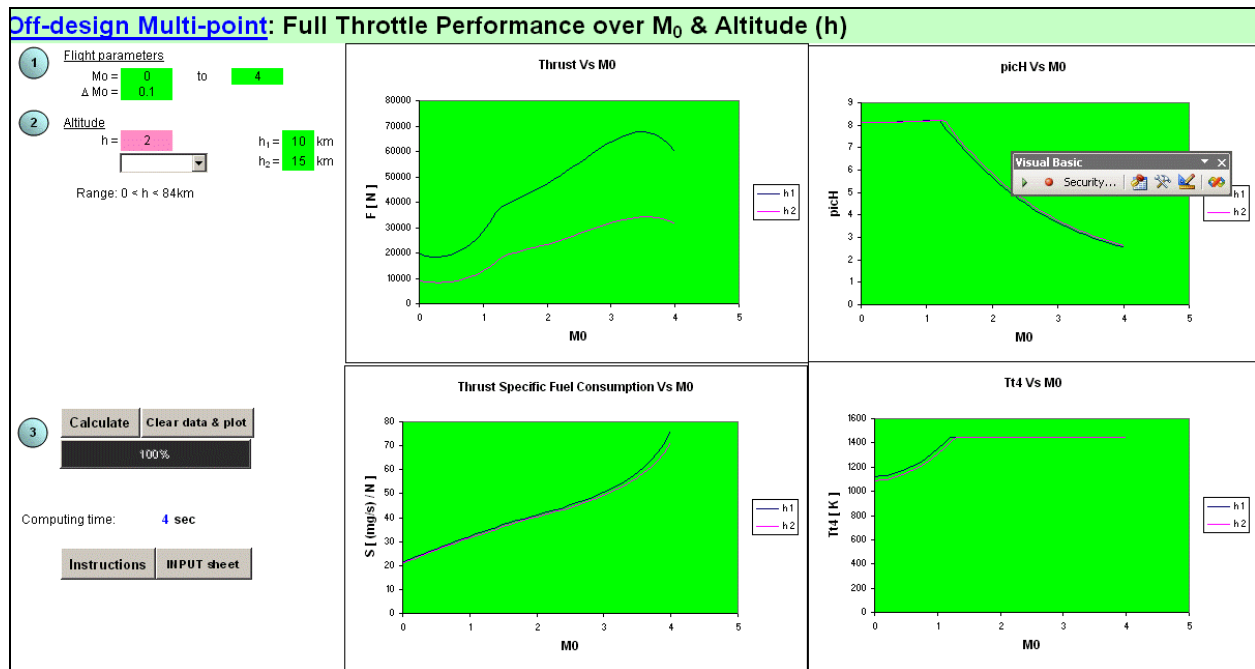
Figure 5.2.—Screenshot of Full-Throttle Performance: Multi-point calculations

5.2.2 Single-point Calculations (sheet 'SinglePt_FTP').—When this option is chosen, the program opens one control sheet (i.e., 'SinglePt_FTP'). This option allows the user to predict the off-design performance of the selected engine at a fixed M_0 and a specified altitude. As indicated in figure 5.3, user just needs to specify a single value for each M_0 and altitude. Then, simply clicking the **Calculate** button will run the code. All the input values from 'Input' sheet and the solutions will be displayed in the same sheet.

Off-Design Single Point: Full Throttle Performance over M0 & Altitude (h)

M_0 =	0		ITB ON
Altitude, h =	0 km		
- Ambient:	T_0 =	288.15 K	Calculate / Clear Data
	P_0 =	101325.00 Pa	

Instructions / INPUT sheet

Input Constants: *Note*: Always check values for T_0 and P_0

π_{dmax} =	0.99	π_b =	0.95	π_{itb} =	1	π_t =	0.99
π_{itf} =	0.98	η_b =	0.99	η_{itb} =	0.99	η_f =	0.92076
η_{tH} =	0.911284	η_{tL} =	0.92474	η_{cH} =	0.87849	η_{cL} =	0.85719
η_{mH} =	0.92	η_{mL} =	0.93	P_9/P_0 =	1	P_{19}/P_0 =	1
h_{PR-b} =	43124 kJ/kg						

Control Limits:

T_{t4max} =	1450	$T_{t4.5max}$ =	1350	T_{t3max} =	0	P_{t3max} =	0
π_{cHmax} =	8.230453	%N_H =	100	%N_L =	100		

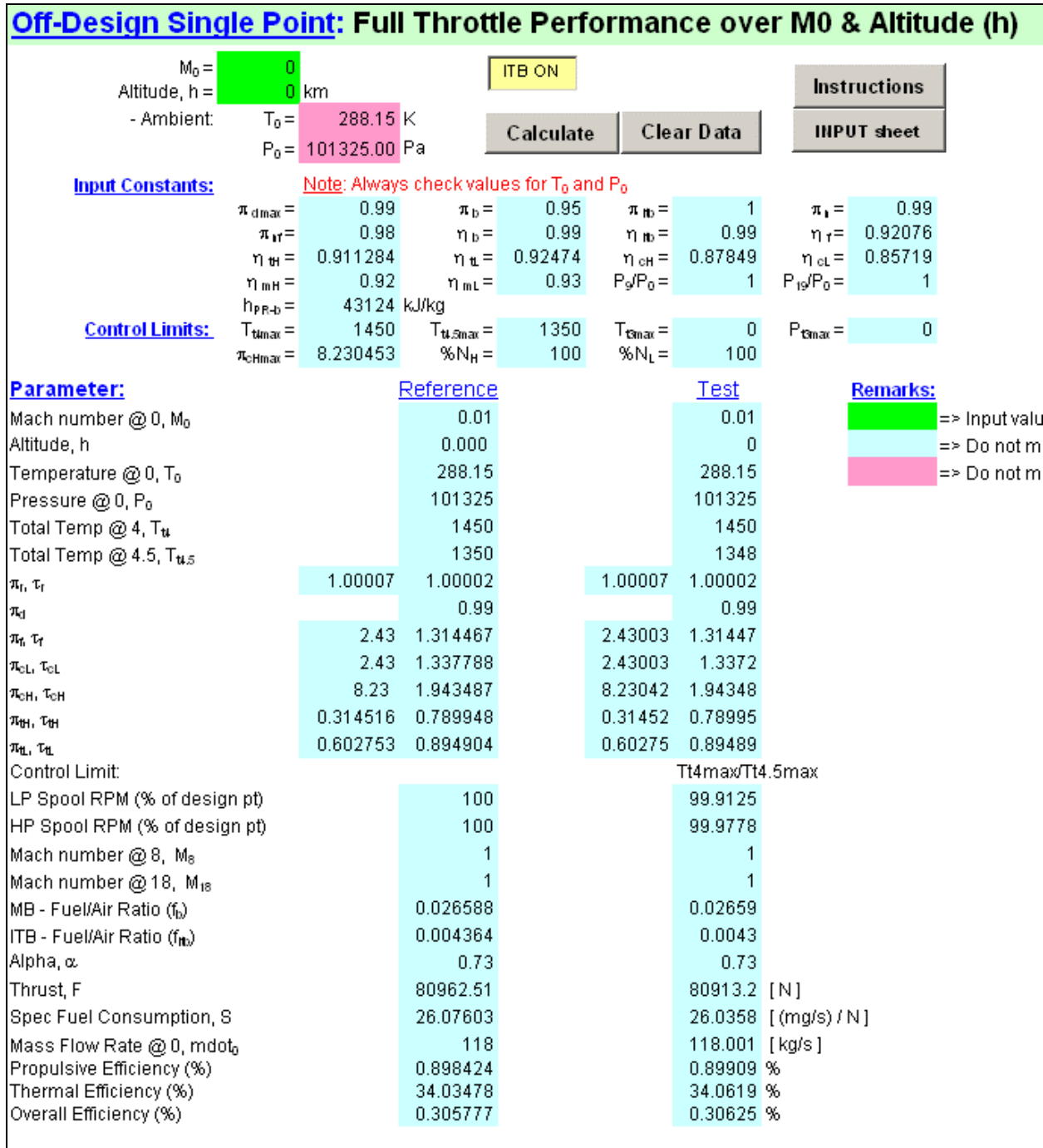| **Parameter:**	**Reference**		**Test**		**Remarks:**
Mach number @ 0, M_0		0.01		0.01	=> Input valu
Altitude, h		0.000		0	=> Do not m
Temperature @ 0, T_0		288.15		288.15	=> Do not m
Pressure @ 0, P_0		101325		101325	
Total Temp @ 4, T_{t4}		1450		1450	
Total Temp @ 4.5, $T_{t4.5}$		1350		1348	
π_r, τ_r	1.00007	1.00002	1.00007	1.00002	
π_d		0.99		0.99	
π_f, τ_f	2.43	1.314467	2.43003	1.31447	
π_{cL}, τ_{cL}	2.43	1.337788	2.43003	1.3372	
π_{cH}, τ_{cH}	8.23	1.943487	8.23042	1.94348	
π_{tH}, τ_{tH}	0.314516	0.789948	0.31452	0.78995	
π_{tL}, τ_{tL}	0.602753	0.894904	0.60275	0.89489	
Control Limit:				Tt4max/Tt4.5max	
LP Spool RPM (% of design pt)		100		99.9125	
HP Spool RPM (% of design pt)		100		99.9778	
Mach number @ 8, M_8		1		1	
Mach number @ 18, M_{18}		1		1	
MB - Fuel/Air Ratio (f_b)		0.026588		0.02659	
ITB - Fuel/Air Ratio (f_{itb})		0.004364		0.0043	
Alpha, α		0.73		0.73	
Thrust, F		80962.51		80913.2	[N]
Spec Fuel Consumption, S		26.07603		26.0358	[(mg/s) / N]
Mass Flow Rate @ 0, $mdot_0$		118		118.001	[kg/s]
Propulsive Efficiency (%)		0.898424		0.89909	%
Thermal Efficiency (%)		34.03478		34.0619	%
Overall Efficiency (%)		0.305777		0.30625	%

Figure 5.3.—Screenshot of Full-Throttle Performance: Single-point calculation sheet

5.3 Partial Throttle Performance (PTP)

When this option is chosen, the program will open two sheets, namely, a control sheet (i.e., 'Control_PTP'), and a data sheet (i.e., 'D_PTP').

5.3.1 Control_PTP.—In 'Control_PTP' sheet as shown in figure 5.4, user needs to specify four input parameters, i.e., Altitude, Min % thrust, Flight parameters, and increment of temperature reduction, as indicated in 'dark green' cells.

Click the **Flight parameters** (M_0) combo box to select number of plots for different M_0. Specified number of cells will be created and displayed adjacent to the **Flight parameters** (M_0) combo box. Once all the 'dark green' cells are filled, it is ready to run the program and generate the plot.

Simply pressing the **Calculate** button, the code will calculate the engine performance from 100% down to minimum thrust entered in *Min % thrust* cell. The minimum value of *min % thrust* is internally preset to be 10%. If any value lower than 10% is found, user will be noticed and asked to re-enter the value.

Other than setting a *Min % thrust*, user may select to enter a lowest allowable exit temperature of the main burner, T_{t4}. The default value of T_{t4} is 1620 °R (or 900 °K).
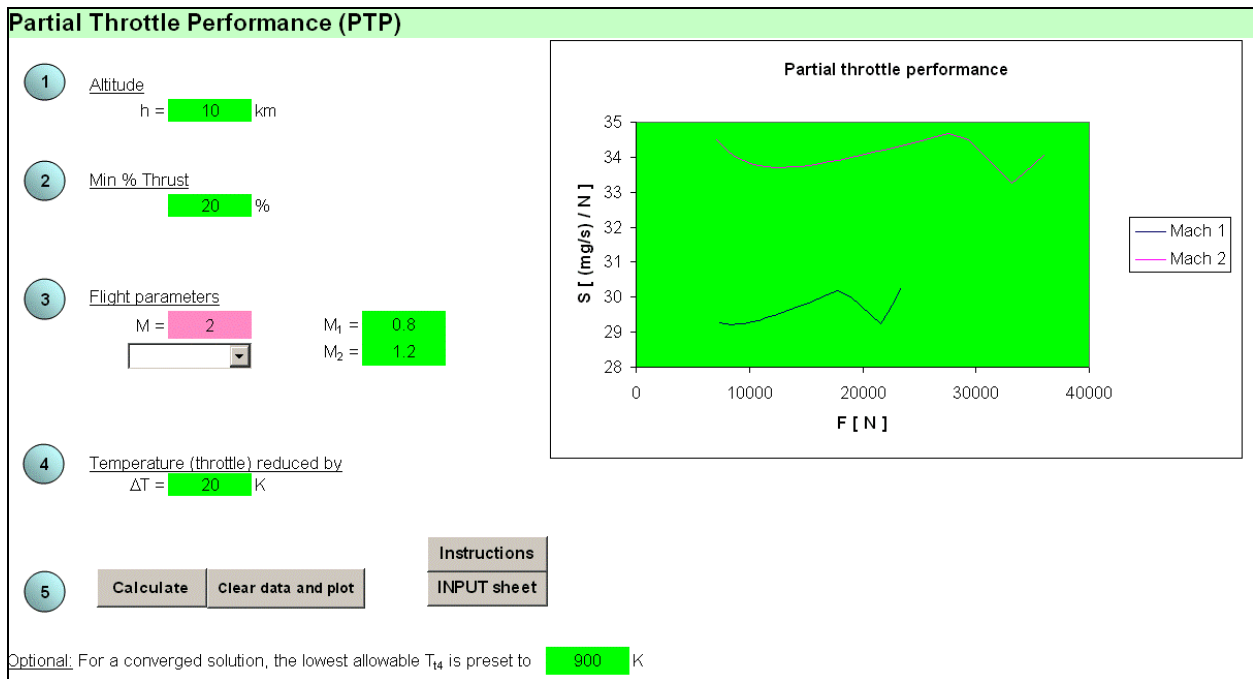

Figure 5.4.—Screenshot of Partial-Throttle Performance

5.4 Discussions

5.4.1 Storing Data.—Whenever storing the output data, user can do the followings:

1. Always keep an original file of the Microsoft® Excel code.
2. Save the file with a new filename.
3. From the menu bar, Select **File→Save As** and name the file, e.g., *offdesign-031704.xls*.
4. Click ***Save*** button to confirm saving.

    5.4.2 Troubleshooting.—This program has been debugged several times. Therefore whenever you encounter computation errors due to either zero division or square root of a negative quantity, you will be notified through a pop up window indicating where the computation problem is. If you have any comments or bug problems you encounter in the program, you can report them to us for further improvement. Detail about the contact number can be found in the 'CoverPage' sheet of the program.
    In multiple-point calculation, users may always see some of these pop-up windows as shown in figure 5.5(a) to (d).
    There are three possibilities of having this error or warning messages:

    (i) During the multiple-point computation, the design variable may be out of the range. It will happen, especially when the design variable such as flight Mach number is out of the range. For instance, the thrust of an engine is approaching zero as the flight Mach number is 5.1. If the upper limit of flight Mach number is set to be greater than 5.1 (e.g., 6), the code will not be able to proceed at Mach number higher than 5.1 and thus give the warning messages shown in figure 5.5(c).
    (ii) The iteration is not converging for some engine configurations at the specified flight Mach number and altitude.
    (iii) The input values are incorrect. For instance, the main burner exit temperature (T_{t4}) may be too small for an engine to produce enough thrust. Please double check all input values and try again.
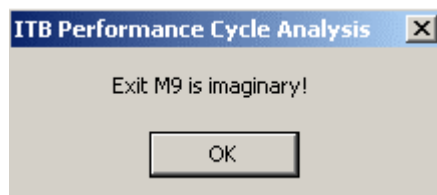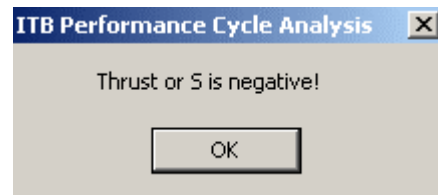
(a) Example of warning messages	(c) Example of warning messages
(c) Equation where the error occurs	(a) Current values of flight condition

Figure 5.5.—Pop-up windows showing warning messages

5.5 Summary

In summary, user needs to do the followings to run the program:
.

    Go to the "Input" sheet.
    Specify the **Unit System** (SI or English?)
    Specify the **ITB Switch**. (Is the computation for engine with ITB-ON or for engine with
         ITB-OFF?)
    Specify the **Fuel Burn Model**. (*CSH* or *MSH* model?)
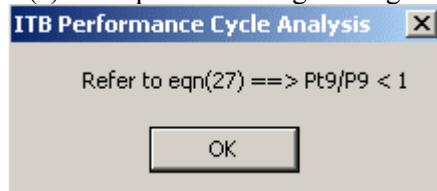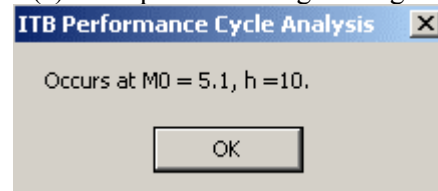   Import *Reference Conditions* data from on-design code.
    Enter all the input parameters indicated in 'dark green' cells (do not modify or change the value
         indicated in 'hotpink' cells).
    Specify **Perform Computation in**. User will be directed to a new control sheet depending on the
         selection.

    If you select "**Full Throttle Performance (FTP): Multiple-point**," do the followings in 'Control_FTP' sheet:

    Specify a range for flight Mach number (M_0).
    Specify number of altitude (new 'dark green' cells will be created).
    Specify all input parameters indicated in 'dark green'.
    Click **Calculate** button to compute and plot the results.
    Repeat the above steps for different input parameters.
    Go to the 'Input' sheet if you want to change some engine input parameters.

    If you select "**Full Throttle Performance (FTP): Single-point**," do the followings in 'SinglePt_FTP' sheet:
    Enter a value in *flight Mach number, (M_0)* cell.
    Enter a value in *Altitude (h)* cell.
    Click **Calculate** button to compute and display the results.

    If you select "**Partial Throttle Performance (PTP)**," do the followings:

    a. Specify a value for *altitude* (*h*).
    b. Enter a value in *Min % thrust* cell.
    c. Specify number of *flight Mach number* (new 'dark green' cells will be created).
    d. Specify all input parameters indicated in 'dark green'.
    e. Click **Calculate** button to compute and plot the result.
    f. Repeat the above steps for different input parameters.
    g. Go to the 'Input' sheet if you want to change some engine input parameters.

Appendix A—U.S. Standard Atmosphere 1976

A.1 Introduction

The US Standard Atmosphere 1976 is an average, piece-wise continuous function for density, pressure and temperature profiles of the earth's standard atmosphere. The standard atmosphere is mathematically defined in seven layers from sea level to 84.852km. It is a function of geo-potential altitude, which is represented by:

$$z = \frac{h \cdot R_{earth}}{h + R_{earth}} \tag{A.1}$$

where h – geometric altitude [km]
R_{earth} – radius of the earth [= 6356.66km]

The table below summarizes the values of several parameters at each of the defined levels.

Table A1.—Values of several parameters at each defined levels of the earth's atmosphere.

Layer (i)	Name	Lower Geopotential Altitude (z) [km]	Upper Geopotential Altitude [km]	Lapse rate ($LR_{std,i}$) [K/km]	Standard Pressure, ($P_{std,i}$) [Pa]	Standard Temperature, ($T_{std,i}$) [K]
1	Troposphere	0	11	−6.5	101325	288.15
2	Stratosphere	11	20	0.0	22632.06	216.65
3	-	20	32	+1.0	5474.89	216.65
4	-	32	47	+2.8	868.02	228.65
5	-	47	51	0.0	110.91	270.65
6	Mesosphere	51	71	−2.8	66.94	270.65
7	-	71	84.852	−2.0	3.96	214.65

A.2 Temperature

The variation of temperature, $T(z)$, with geo-potential altitude is expressed by a continuous and piecewise linear relation. If Lapse rate is non-zero ($LR_{std,i} \neq 0$),

$$T(z) = T_{std,i} + LR_{std,i} \cdot \left(z - Z_{std,i}\right) \tag{A.2}$$

If Lapse rate is zero ($LR_{std,i} = 0$),

$$T(z) = T_{std,i} \tag{A.3}$$

A.3 Pressure

The pressure, $P(z)$, is also a continuous piecewise function of geo-potential altitude. If Lapse rate is non-zero ($LR_{std,i} \neq 0$),

$$P(z) = P_{std,i} \left(\frac{T_{std}(z)}{T_{std,i}} \right)^{\left(\frac{-1000 \cdot g_0 \cdot W_0}{R^* \cdot LR_{std,i}} \right)} \qquad (A.4)$$

If Lapse rate is zero ($LR_{std,i} = 0$),

$$P(z) = P_{std,i} \cdot \exp\left(\frac{-1000 \cdot g_0 \cdot W_0 \cdot (z - Z_{std,i})}{R^* \cdot T_{std,i}} \right) \qquad (A.5)$$

where

g_0 – acceleration of gravity (= 9.80665 m/s^2)
R^* – gas constant for air (= 8314.32 J/kmol-K)
W_0 – Molecular weight of air (= 28.9644 kg/kmol)

Appendix B—Combustion Fuel Burn Models

B.1 Types of Fuel Burn Models

Two models (p. 116 of ref. 2) are being used to calculate the fuel burned in the main burner and the *ITB*:

(1) **Constant Specific Heat** (*CSH*) model—The air and combustion gases at inlet and exit of each component are modeled as "calorically perfect gases" with constant specific heats. The values of the specific heats are different at inlet and exit of two combustors (main burner and *ITB*)

(2) **Modified Specific Heat** (*MSH*) model—All engines properties are calculated using *CSH* model except the fuel used. In this model, the inlet total temperature (T_{t3} and $T_{t4.4}$) of combustors are calculated using *CSH* model while the exit total temperatures (T_{t4} and $T_{t4.5}$) are directly obtained from the user inputs. Nevertheless, the total enthalpies (h_{t3}, h_{t4}, $h_{t4.4}$, and $h_{t4.5}$) will be calculated directly from the *Variable Specific Heat* (VSH) model (p. 116 of ref. 2). Provided h_{PR} is given, the amount of fuel burned is then calculated using eq. (B4) and (B7), respectively. Therefore, the improvement on enthalpy calculation gives better estimates of fuel used.

The following *ENTHALPY* subroutine is based on pages 105 and 106 of reference 3 and used in the *MSH* model to calculate the enthalpy (h) at engine stations $t3$, $t4$, $t4.4$, and $t4.5$.

Subroutine Enthalpy

ENTHALPY (T, f, h)

Inputs: T and f

Output: h

$$h_{air} = \left[h_{ref} + A_0 T + \frac{A_1}{2} T^2 + \frac{A_2}{3} T^3 + \frac{A_3}{4} T^4 + \frac{A_4}{5} T^5 + \frac{A_5}{6} T^6 + \frac{A_6}{7} T^7 + \frac{A_7}{8} T^8 \right]_{air} \tag{B.1}$$

$$h_{prod} = \left[h_{ref} + A_0 T + \frac{A_1}{2} T^2 + \frac{A_2}{3} T^3 + \frac{A_3}{4} T^4 + \frac{A_4}{5} T^5 + \frac{A_5}{6} T^6 + \frac{A_6}{7} T^7 + \frac{A_7}{8} T^8 \right]_{prod} \tag{B.2}$$

$$h_{mix} = \frac{h_{air} + f h_{prod}}{1 + f} \tag{B.3}$$

where the constants A_0, etc. are given in table B1 (p.106 of ref. 3).

Table B1.—Constants for air and combustion products used in subroutine ENTHALPY(0, T, f, h, c_p, γ).

Constant	Air alone	Combustion products of air and $(CH_2)_n$ fuels
A_0	2.5020051×10^{-1}	7.3816638×10^{-1}
A_1	$-5.1536879 \times 10^{-5}$	1.2258630×10^{-5}
A_2	6.5519486×10^{-8}	$-1.3771902 \times 10^{-8}$
A_3	$-6.7178376 \times 10^{-12}$	$9.9686793 \times 10^{-12}$
A_4	$-1.5128259 \times 10^{-14}$	$-4.2051104 \times 10^{-14}$
A_5	$7.6215767 \times 10^{-18}$	$1.0212913 \times 10^{-18}$
A_6	$-1.4526770 \times 10^{-21}$	$-1.3335668 \times 10^{-21}$
A_7	$1.0115540 \times 10^{-25}$	$7.2678710 \times 10^{-25}$
h_{ref}	-1.7558886 Btu/lbm	30.58153 Btu/lbm

The equations needed to calculate the fuel-air ratios of the main burner (f_b) and the *ITB* (f_{itb}) are listed below for both the *CSH* and *MSH* models.

Main burner

If *CSH* model then (eq. (27) of ref. 5)

$$f_b = \frac{\tau_{\lambda-b} - \tau_r \tau_{cL} \tau_{cH}}{\eta_b h_{PR}/h_0 - \tau_{\lambda-b}}$$ (B.4)

Else

$$T_{t3} = T_0 \tau_r \tau_{cL} \tau_{cH}$$ (B.5)

ENTHALPY $(0, T_{t3}, 0, h_{t3}, 0, 0)$

Set initial value of fuel/air ratio at station 4 $= f_{4i}$

3   ENTHALPY $(0, T_{t4}, f_{4i}, h_{t4}, 0, 0)$

$$f_b = \frac{h_{t4} - h_{t3}}{\eta_b h_{PR} - h_{t4}}$$ (B.6)

If $|f_b - f_{4i}| > 0.0001$, then $f_{4i} = f_b$ and go to 3; else continue.

End if

Interstage Turbine Burner

If *CSH* model then (eq. (30) of ref. 5)

$$f_{itb} = (1 + f_b) \frac{\tau_{\lambda-itb} - \tau_{\lambda-b} \tau_{tH}}{\eta_{itb} h_{PR}/h_0 - \tau_{\lambda-itb}}$$ (B.7)

Else

$$T_{t4.4} = T_{t4}\,\tau_{tH}$$

ENTHALPY $\left(0, T_{t4.4}, f_b, h_{t4.4}, 0, 0\right)$

Set initial value of fuel/air ratio at station 4.4 $= f_{4.5i}$

4   **ENTHALPY** $\left(0, T_{t4.5}, f_{4.5i}, h_{t4.5}, 0, 0\right)$

$$f_{itb} = \left(1 + f_b\right)\frac{h_{t4.5} - h_{t4.4}}{\eta_{itb}\,h_{PR} - h_{t4.5}} \tag{B.8}$$

If $\left|f_b + f_{itb} - f_{4.5i}\right| > 0.0001$, then $f_{4.5i} = f_b + f_{itb}$ and go to 4; else continue.

End if

If $f_{itb} = 0$ then $\gamma_{itb} = \gamma_t$, $C_{pitb} = C_{pt}$, and $R_{itb} = R_t$

Appendix C—Control Limits

C.1 Introduction

Normally, the designers of an aircraft gas turbine engine need to consider the engine control system so that the newly-designed or improved gas turbine engine operates well within its flight envelope. For example, the throttle setting needs to be controlled so that the burner exit temperature will never exceed the turbine blade maximum temperature. Another example is that fan or compressor surges or stalls should not occur during the operation.

Therefore, engine's control system plays a significant role in this engine performance cycle analysis because of the several advantages: (1) maintain stable airflow, internal pressures and temperatures, and rotor speeds within safe operating limits; (2) avoid significant speed, pressure or temperature variation, and stalls or surges (ref. 2). In addition, it also demonstrates how an aircraft engine behaves away from their reference or on-design conditions.

The engine's control system in this study is specifically designed for a separate-flow, dual-spool, uncooled turbofan engine with *ITB*. Moreover, the development of the equations is based on the foundational materials found in references 1 and 2.

C.2 Variables of Engine Control System

C.2.1 Dimensionless Freestream Total Temperature.—Due to its significant influence on overall behavior of the turbofan engine, a dimensionless form of the freestream total temperature is defined, i.e., the ratio of the freestream total temperature to the sea-level static (*SLS*) temperature of the standard atmosphere, as shown below (ref. 2):

$$\theta_0 = \frac{T_{t0}}{T_{ref}} = \frac{T_0\left(1 + \frac{\gamma_c - 1}{2}M_0^2\right)}{T_{ref}} = \tau_r \frac{T_0}{T_{ref}} \tag{C.1}$$

(Note: $T_{ref} = T_{std}$)

This property is called *theta* 0 (θ_0) and is used to combine two parameters, i.e., altitude (T_0) and flight condition (M_0) into a single parameter. Therefore, every point in the flight envelope has a specific value of θ_0.

In addition, θ_0 has the following properties:

(1) $\theta_0 = 1$ at sea level static conditions;
(2) θ_0 can be greater or less than 1.0;
(3) θ_0 depends only on Mach number above between 36.5 and 65 kft (11.12 and 19.81 km).

C.2.2 Throttle Ratio.—The other useful property of an engine control system is the throttle ratio (*TR*). It is defined as the ratio of the maximum T_{t4} value to the T_{t4} value at *SLS* conditions. It can be expressed as:

$$TR = \frac{T_{t4\max}}{T_{t4SLS}} \tag{C.2}$$

When the reference altitude is at *SLS* (*h=0*), then $T_{t4SLS} = T_{t4R}$. Otherwise, *TR* has to be manually input by user.
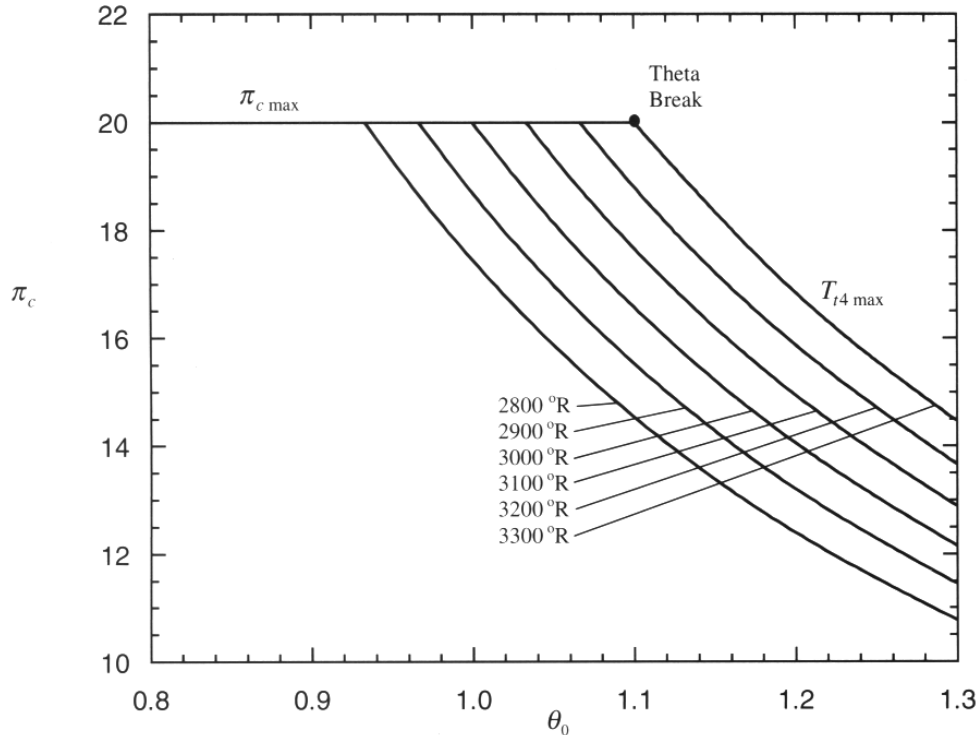
Figure C1.—Compressor Ratio as a Function of θ_0 and T_{t4} of a Single-Spool Turbojet
Engine for a Compressor with a Reference Point of $\pi_c = \pi_{c\,max} = 20$,
$T_{t4} = T_{t4\,max} = 3300\,°R$, and $\theta_0 = 1.1$ (ref. 2).

C.2.3 Theta Break.—Normally, the change in shape of the performance curve occurs at simultaneous maximum of π_c and T_{t4}. In figure 3.1, it is clear that the control logic must switch from limiting π_c to limiting T_{t4} at this point of maximum π_c and T_{t4}. This unique point is known as the *theta break* (θ_{0break}).

C.3 Types of Control Limits

In this study, only three types of engine control limits will be considered, i.e., turbine blade maximum temperature, compressor limitations and engine speed.

C.3.1 Material Limitation.—It is known that an increase in permissible turbine inlet temperature will lead to a reduced *TSFC* and increased specific thrust (ref. 4, p. 357). Although the current advanced technology in material allows higher inlet turbine temperature, it is still essential to limit T_{t4} or $T_{t4.5}$ for safety purpose.

C.3.2 Compressor Pressure Ratio.—Three limits, namely, $\pi_{c\,max}$, $P_{t3\,max}$ and $T_{t3\,max}$, are selected to stabilize the internal behavior of the compressors and avoid compressor surges or stalls.

C.3.3 Engine Speed.—As shown in section 5.2.7 of reference 2, the change in total enthalpy across a compressor or a fan is proportional to the rotational speed N squared for a calorically perfect gas.

C.3.3.1 Low-pressure Spool: For a low-pressure compressor (*LPC*), it can be expressed as follow:

$$h_{t2.5} - h_{t2} = K_1 N_{LP}^2 \qquad\text{(C.3)}$$

Using referencing, it can be rewritten as:

$$\frac{N_{LP}}{N_{LPR}} = \sqrt{\frac{h_{t2.5} - h_{t2}}{h_{t2.5R} - h_{t2R}}} = \sqrt{\frac{h_{t2}(\tau_{cL} - 1)}{h_{t2R}(\tau_{cLR} - 1)}}$$

$$= \sqrt{\frac{h_{t0}(\tau_{cL} - 1)}{h_{t0R}(\tau_{cLR} - 1)}} \cong \sqrt{\frac{T_{t0}(\tau_{cL} - 1)}{T_{t0R}(\tau_{cLR} - 1)}}$$

$$= \sqrt{\frac{\theta_0(\tau_{cL} - 1)}{\theta_{0R}(\tau_{cLR} - 1)}} \tag{C.4}$$

Since the *LPC* and the fan are on the same shaft, the enthalpy rise across the *LPC* will be proportional to the enthalpy rise across the fan during normal operation. (ref. 2)

$$\frac{\tau_{cL} - 1}{\tau_f - 1} = \left(\frac{\tau_{cL} - 1}{\tau_f - 1}\right)_R \tag{C.5}$$

Then, equation (C.4) can be rewritten as

$$\frac{N_{LP}}{N_{LPR}} = \sqrt{\frac{\theta_0(\tau_f - 1)}{\theta_{0R}(\tau_{fR} - 1)}} \tag{C.6}$$

C.3.3.2 High-pressure Spool: For high-pressure compressor (*HPC*), it is in the similar form:

$$h_{t3} - h_{t2.5} = K_1 N_{HP}^2 \tag{C.7}$$

Using referencing, it can be rewritten as:

$$\frac{N_{HP}}{N_{HPR}} = \sqrt{\frac{h_{t3} - h_{t2.5}}{h_{t3R} - h_{t2.5R}}} = \sqrt{\frac{h_{t2.5}(\tau_{cH} - 1)}{h_{t2.5R}(\tau_{cHR} - 1)}}$$

$$= \sqrt{\frac{h_{t0}\tau_{cL}(\tau_{cH} - 1)}{h_{t0R}\tau_{cLR}(\tau_{cHR} - 1)}} \cong \sqrt{\frac{T_{t0}\tau_{cL}(\tau_{cH} - 1)}{T_{t0R}\tau_{cLR}(\tau_{cHR} - 1)}} \tag{C.8}$$

$$\frac{N_{HP}}{N_{HPR}} = \sqrt{\frac{\theta_0\tau_{cL}(\tau_{cH} - 1)}{\theta_{0R}\tau_{cLR}(\tau_{cHR} - 1)}} \tag{C.9}$$

C.4 Relationship between T_{t4}, $T_{t4.5}$ and π_c

C.4.1 High-pressure compressor.—From a power balance between *HPT* and *HPC*, one will obtain the performance equation for the total temperature ratio of the *HPT*:

$$\tau_{cH} = 1 + \eta_{mH}(1 + f_b)\frac{\tau_{\lambda-b}(1 - \tau_{tH})}{\tau_r\tau_{cL}} \tag{C.10}$$

Using *HP* compressor efficiency, the *HP* compressor total pressure is given by:

$$\pi_{cH} = \left\{ 1 + \eta_{cH}\left(\tau_{cH} - 1\right)\right\}^{\gamma_c/\gamma_c - 1} \qquad (C.11)$$

From equation (5) in reference 4,

$$\tau_{\lambda-b} = \frac{C_{pt}\,T_{t4}}{C_{pc}\,T_0} \qquad (C.12)$$

Rearranging equation (C.1), one obtains

$$\tau_r = \theta_0 \frac{T_{ref}}{T_0} \qquad (C.13)$$

Substitute these two equations into (C.10) yields

$$\tau_{cH} = 1 + \eta_{mH}\,(1+f_b)(1-\tau_{tH})\frac{C_{pt}T_{t4}}{C_{pc}T_0}\left(\frac{T_0}{T_{ref}\theta_0}\right)\frac{1}{\tau_{cL}} \qquad (C.14)$$

It is assumed that choking occurs at station 4. Consequently, for an uncooled turbine with constant specific heat in this study, the value of τ_{tH} will remain constant. (ref. 2, pp. 143–144). In addition, f_b is negligible when compared to unity. Then, let

$$C_0 = \eta_{mH}\,(1+f_b)(1-\tau_{tH})\left(\frac{C_{pt}}{C_{pc}}\right)\left(\frac{1}{T_{ref}}\right) \qquad (i)$$

and eq. (C.14) becomes

$$\tau_{cH} = 1 + C_0\frac{T_{t4}}{\theta_0}\frac{1}{\tau_{cL}} \qquad (C.15)$$

Substituting (C.15) into (C.11) gives

$$\pi_{cH} = \left\{ 1 + C_1\frac{T_{t4}}{\theta_0}\frac{1}{\tau_{cL}}\right\}^{\gamma_c/\left(\gamma_c - 1\right)} \qquad (C.16)$$

where

$$C_1 = C_0\,\eta_{cH} \qquad (ii)$$

Compared to equation (D.3) of reference2 (pp525), π_{cH} is not only a function of throttle setting (T_{t4}) and flight conditions (θ_0), but also a function of τ_{cL}:

$$\pi_{cH} = f\left(\frac{T_{t4}}{\theta_0}, \tau_{cL}\right) \tag{C.17}$$

C.4.2 Low-pressure compressor

From a power balance between *LPC*, fan and *LPT*, one will obtain the performance equation for the total temperature ratio of the *LPT*:

$$\alpha(\tau_f - 1) + (\tau_{cL} - 1) = \eta_{mL}(1 + f_b + f_{itb})\frac{\tau_{\lambda-itb}}{\tau_r}(1 - \tau_{tL}) \tag{C.18}$$

Substituting equation (C.5) into (C.18), it yields:

$$\tau_f = 1 + (\tau_{fR} - 1)\eta_{mL}\frac{\tau_{\lambda-itb}}{\tau_r}\left\{\frac{(1 - \tau_{tL})(1 + f_b + f_{itb})}{\tau_{cLR} - 1 + \alpha(\tau_{fR} - 1)}\right\} \tag{C.19}$$

The *LPC* total temperature ratio follows from equation (C.5):

$$\tau_{cL} = 1 + (\tau_f - 1)\frac{(\tau_{cL} - 1)_R}{(\tau_f - 1)_R} \tag{C.20}$$

Using *LPC* efficiency, the *LPC* total pressure ratio is given by:

$$\pi_{cL} = \{1 + \eta_{cL}(\tau_{cL} - 1)\}^{\gamma_c/(\gamma_c - 1)} \tag{C.21}$$

From equation (6) of reference 4,

$$\tau_{\lambda-itb} = \frac{C_{pitb}T_{t4.5}}{C_{pc}T_0} \tag{C.22}$$

Substituting equation (22) into equation (19) yields

$$\tau_f = 1 + (\tau_{fR} - 1)\eta_{mL}\frac{C_{pitb}T_{t4.5}}{C_{pc}T_0}\left(\frac{T_0}{T_{ref}\theta_0}\right)\left\{\frac{(1 - \tau_{tL})(1 + f_b + f_{itb})}{\tau_{cLR} - 1 + \alpha(\tau_{fR} - 1)}\right\} \tag{C.23}$$

Similar to the assumption in *HPT*, the flow is choked at station 4.5. Therefore, the value of τ_{tL} will remain constant. In addition, f_b and f_{itb} are negligible when compared to unity.

Unfortunately, α is not a constant but a function of $(\pi_{cL}, \pi_{cH}, \pi_f, M_{18})$. Let

$$C_2 = (\tau_{fR} - 1)\eta_{mL}\frac{C_{pitb}}{C_{pc}}\left(\frac{1}{T_{ref}}\right)(1 - \tau_{tL})(1 + f_b + f_{itb}) \tag{iii}$$

Then,

$$\tau_f = 1 + C_2 \frac{T_{t4.5}}{\theta_0} \left\{ \frac{1}{\tau_{cLR} - 1 + \alpha(\tau_{fR} - 1)} \right\} \tag{C.24}$$

Substituting (C.24) into (C.20) yields

$$\tau_{cL} = 1 + C_3 \frac{T_{t4.5}}{\theta_0} \left\{ \frac{1}{\tau_{cLR} - 1 + \alpha(\tau_{fR} - 1)} \right\} \tag{C.25}$$

where

$$C_3 = \eta_{mL} \frac{C_{pitb}}{C_{pc}} \left(\frac{1}{T_{ref}} \right) (1 - \tau_{tL})(1 + f_b + f_{itb})(\tau_{cLR} - 1) \tag{iv}$$

Substituting (C.25) into (C.21) yields

$$\pi_{cL} = \left[1 + C_4 \frac{T_{t4.5}}{\theta_0} \left(\frac{1}{\tau_{cLR} - 1 + \alpha(\tau_{fR} - 1)} \right) \right]^{\gamma_c/(\gamma_c - 1)} \tag{C.26}$$

where

$$C_4 = \eta_{cL} C_3 \tag{v}$$

Again, when compared to equation (D.3) of reference 2 (pp525), π_{cL} is not only a function of throttle setting (T_{t4}) and flight conditions (θ_0), but also a function of α :

$$\pi_{cL} = f\left(\frac{T_{t4.5}}{\theta_0}, \alpha \right) \tag{C.27}$$

C.5 Limiting procedures

C.5.1 Reference conditions.—The values of $T_{t4\,max}$ and $T_{t4.5\,max}$ (control limit) can only be smaller or equal to the values of T_{t4R} and $T_{t4.5R}$ at the reference (*on-design*) condition. Furthermore, when the reference condition is at a reduced T_{0R} due to the altitude, the reference T_{t4R} and $T_{t4.5R}$ have to be reduced too. This may be followed by the reduction in the values of $T_{t4\,max}$ and $T_{t4.5\,max}$ too.

$$\theta_{0R} = \tau_{rR} \frac{T_{0R}}{T_{ref}} \tag{C.28}$$

If $\theta_{0R} \geq TR$ then

$$T_{t4\,max} = T_{t4R}$$

$$T_{t4.5\,\text{max}} = T_{t4.5R} \tag{C.29a}$$

else

$$T_{t4\,\text{max}} = T_{t4R}\left(\frac{\theta_{0R}}{TR}\right)$$

$$T_{t4.5\,\text{max}} = T_{t4.5R}\left(\frac{\theta_{0R}}{TR}\right) \tag{C.29b}$$

C.5.2 Limiting T_{t4} **and** $T_{t4.5}$.—As shown in appendix D of reference 2 (p. 525), as θ_0 is decreased to the left of the *theta break*, or θ_{0break}, T_{t4} must be reduced in order to protect the compressor from poor operation. By doing so, the term $\dfrac{T_{t4}}{\theta_0}$ must remains constant.

A simple limiting procedure as shown below is included into the code in order to limit T_{t4} values equal to or less than $T_{t4\,\text{max}}$.

$$\theta_0 = \tau_r \frac{T_0}{T_{ref}} \tag{C.31}$$

If $\theta_0 \geq TR$ then

$$T_{t4} = T_{t4\,\text{max}} \tag{C.32}$$

else

$$T_{t4} = T_{t4\,\text{max}}\left(\frac{\theta_0}{TR}\right) \tag{C.33}$$

When *ITB* is considered, a separate limiting procedure is required to limit $T_{t4.5}$ values equal to or less than $T_{t4.5\,\text{max}}$.

If $\theta_0 \geq TR$ then

$$T_{t4.5} = T_{t4.5\,\text{max}} \tag{C.34}$$

else

$$T_{t4.5} = T_{t4.5\,\text{max}}\left(\frac{\theta_0}{TR}\right) \tag{C.35}$$

C.5.3 Limiting π_f **and** π_{cH}.—For a two-spool turbofan engine with *ITB*, there are two engine fuel flows. The fuel flow to the main burner directly influences the speed of the *HP* spool and the *HP* compressor pressure ratio. The fuel flow to the *ITB* directly influences the speed of the *LP* spool and the fan and *LP* compressor pressure ratios.

In Full Throttle Performance, when *HP* compressor pressure ratio is greater than its given maximum, the actual engine control will reduce the required fuel flow to the main burner in real time until the given limit is reached. In this study, the code does not calculate the reduction in fuel flow directly. Instead, the code will reduce T_{t4} (main burner) or $T_{t4.5}$ (*ITB*) to a lower value.

Therefore, two control limits have been selected in Full Throttle Performance, i.e., fan pressure ratio, $\pi_{f\,max}$ (*LP* spool) and *HP* compressor pressure ratio $\pi_{cH\,max}$ (*HP* spool). Based on this, two throttle control loops have been added to the code as shown in table C1.

When the fan pressure ratio (*FPR*) is greater than its limit, $\pi_{f\,max}$ (case 1)**,** the control loop will reduce $T_{t4.5}$ (reduce fuel flow to the *ITB*). When the *HP* compressor pressure ratio (*HPC PR*) is greater than its limit, $\pi_{cH\,max}$ (case 2), the control loop will reduce T_{t4} (reduce fuel flow to the main burner).

If *ITB* is on, the control loops for *FPR* and *HP*C *PR* are sequential, i.e., *FPR* limit control goes first, followed by *HPC PR* limit control.

If the *ITB* is off and the *FPR* is still greater than its limit, $\pi_{f\,max}$ (case 3)**,** the control loop will reduce T_{t4} (reduce fuel flow to the main burner). This reduction in T_{t4} will reduce both *FPR* and *HPC PR*. Similarly, this reduction in T_{t4} applies to a case where the *FPR* is still too high even when $T_{t4.5}$ is reduced until *ITB* is off (case 1 shifted to 3).

Table C1.—The logic sequence of reducing temperature (reducing fuel flow to the burners)

Parameter 'reduce'	Reduce $T_{t4.5}$ when		Reduce T_{t4} when	
Case	$\pi_f \geq \pi_{f\,max}$	$\pi_{cH} \geq \pi_{cH\,max}$	$\pi_f \geq \pi_{f\,max}$	$\pi_{cH} \geq \pi_{cH\,max}$
ITB on	1	-	-	2
ITB off	-	-	3	4

Figure C2.—Flow-Chart of Control Limits for Fan Pressure Ratio
and High-Pressure Compressor Pressure Ratio.

48

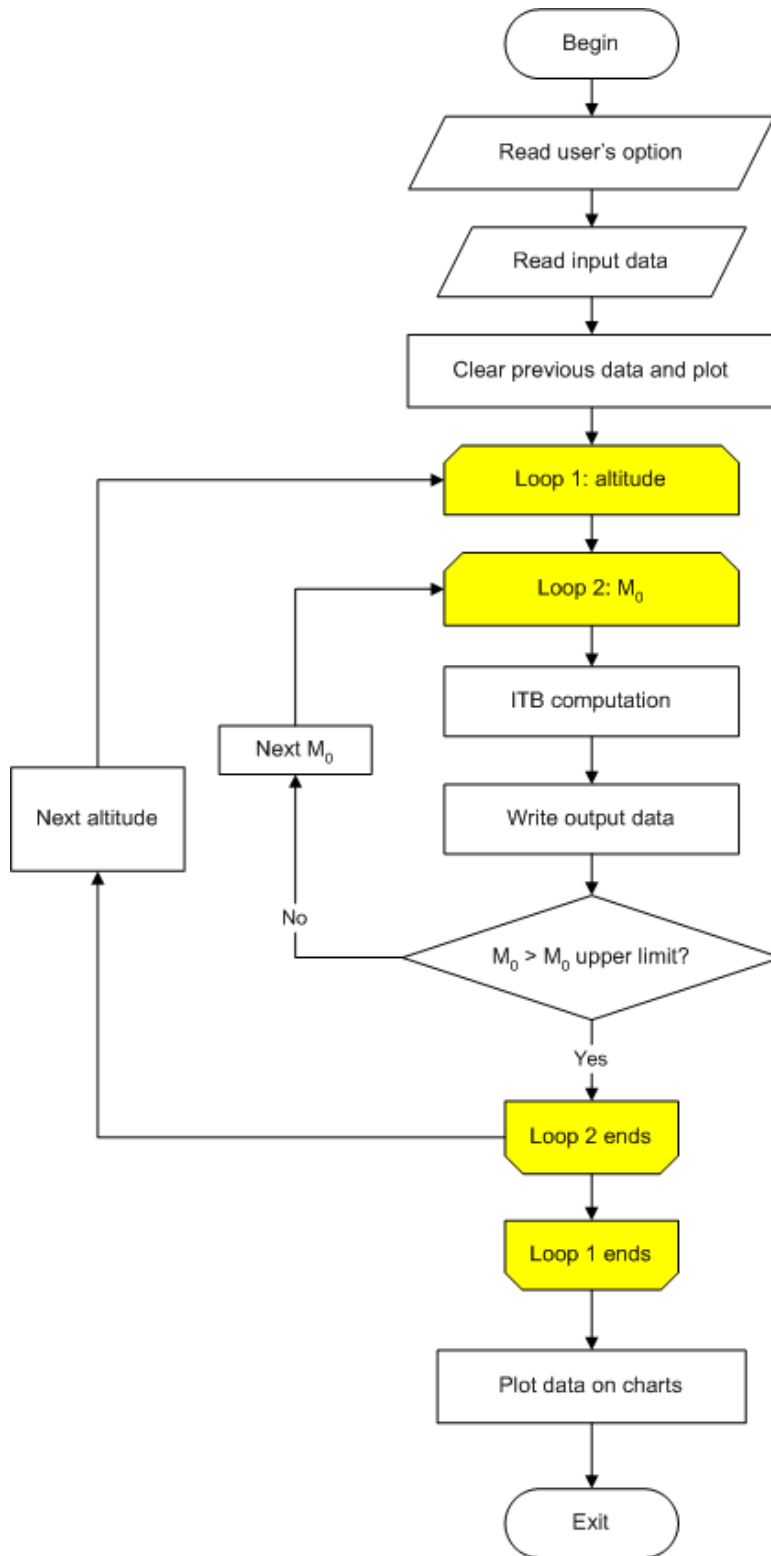# Appendix D—Flowchart of Iterative Solution Scheme

Figure D1.—Flow-Chart of the Iterative Solution Scheme
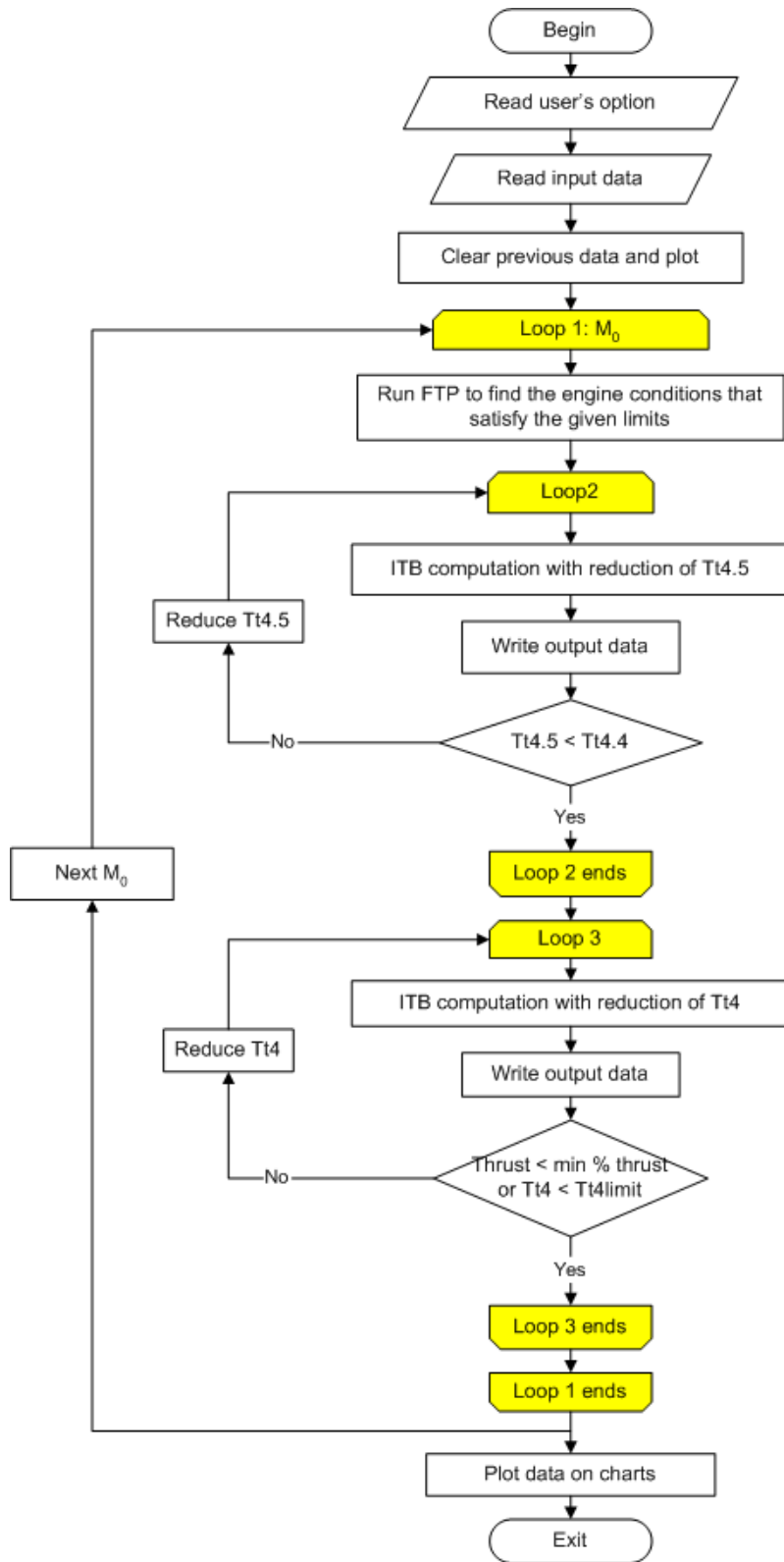for Full-Throttle Performance.

Figure D2.—Flow-Chart of the Iterative Solution Scheme
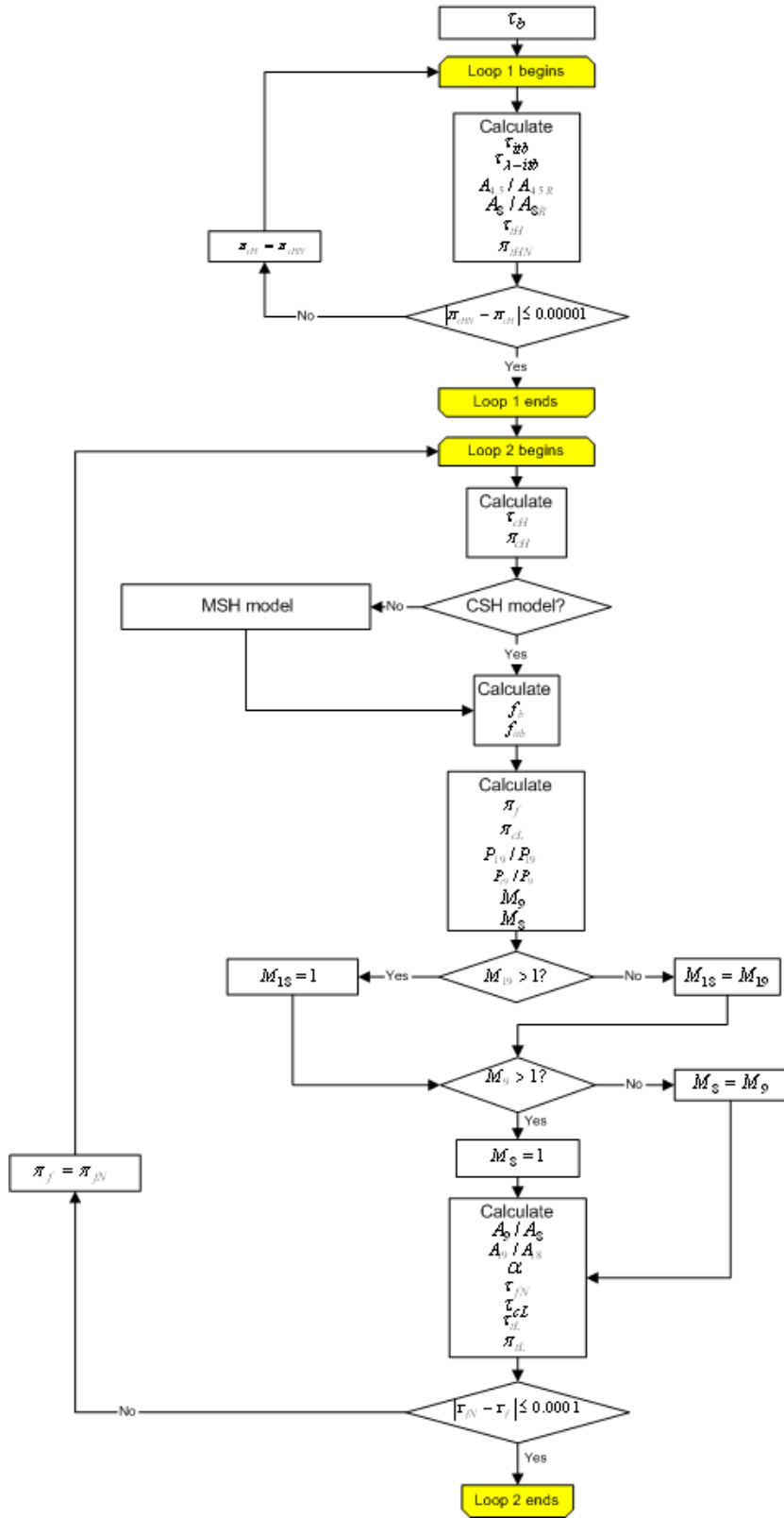for Partial-Throttle Performance.

Figure D3.—Flow-Chart of the Iterative Solution Scheme for
the Subroutine *itbperf2* (Central *ITB* Computation)

Appendix E—Reference Engine Data From On-Design Code

E.1 Introduction

In general, on-design (parametric) cycle analysis differs significantly from off-design (performance) cycle analysis. Thus, two separate codes using Microsoft® Excel and VBA are written for each cycle analysis.

Once the most promising engine is selected by user (i.e., all engine components are fixed, e.g., CPR, BPR, altitude, FPR etc.), it is then run using on-design code to get the engine performance at the selected operating conditions.

These output data represents the selected engine performance at a design-point. In order to predict the engine performance at off-design conditions, these outputs are then used as the reference conditions in off-design code. This step is important because different reference conditions (i.e., different engines) will give different performance over the whole flight spectrum.

For convenience, authors have modified the on-design code to include 'Input/Output files' features in each code. By doing so, the output from on-design code can be saved as the reference engine data file, which is required in off-design calculation. As a result, users do not need to manually input the reference conditions, which is a time-consuming process.

E.2 Instructions to get reference engine data

Follow the step 1 to 12 to generate a reference engine data file.

1. Generate the sample reference engine data as shown in table E1 using on-design code (ref. 4).
2. Open off-design code.
3. Select 'Input' sheet as shown in figure 5.1.
4. Click on '*Import data from ITB ondesign code*' button.
5. A dialog box similar to figure E1 will appear and request for specifying the path and filename.
6. The default path and filename is "c:*\reference.txt*". Users may need to enter a new path if necessary, for example, "*c:\ITB\reference.txt*".
7. An input dialog box as shown in figure E2 appears when one of these situations happens:
   (7a) The specified directory (default drive is '*h:*') does not exist.
   (7b) The file (e.g., "*refe1rence.txt*" in figure E2) does not exist.
   Check for existing drive path or/and specify another filename.
8. A dialog box as shown in figure E3 appears when the unit system in reference data file is different from the current unit system in off-design code. Change the unit system in 'Unit System' pull-down list. Then, start from step 2.
9. An input dialog box as shown in figure E4 appears and request for entering the value of mass flow rate. The default value is 100 kg/s.
10. Enter value and click '*Ok*' button to proceed.
11. All reference conditions data and engine characteristics of a specific design-point engine are successfully imported. The specified pathname will be shown in 'Reference-point engine path:"
12. It is ready to perform off-design calculation.
13. Repeat step 1 to 10 for different engines.

Table E1.—Input of Reference Data for on-design code

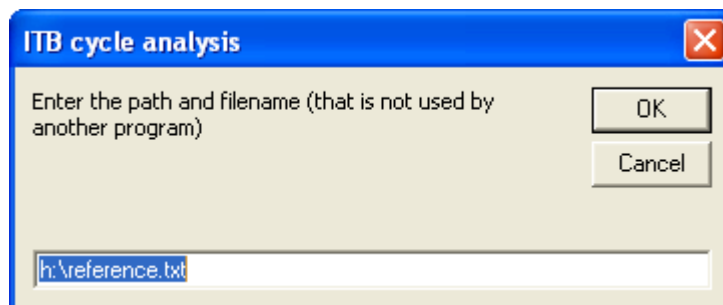| Reference Conditions	Input
Mach number (M_{0R})	0
Altitude (h, kft)	0
Main burner exit total temperature (T_{t4R}, °R)	3600
ITB exit temperature ($T_{t4.5R}$, °R)	3275
LPC pressure ratio (π_{cLR})	2.31
Compressor pressure ratio (π_{cR})	26.8
Fan pressure ratio (π_{fR})	2.31
Fan bypass ratio (α_R)	1.91
Mass flow rate ($\dot{m}_{0R}$, lbm/s)	1097

Figure E1.—Dialog Box for Specifying the Path and Filename
where the Reference Conditions Data Will Be Stored.
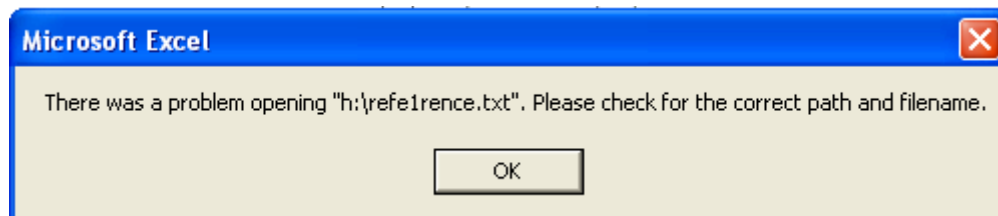
Figure E2.—Dialog Box when there is An Error Reading Reference
Conditions Data from the Specified Path and Filename.
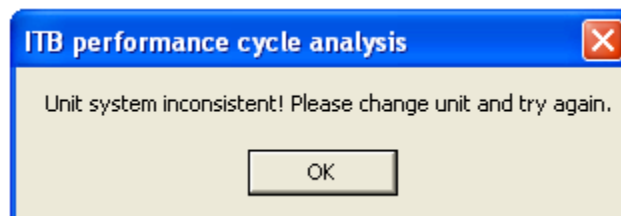
Figure E3.—Dialog Box when there is Unit System Inconsistency.
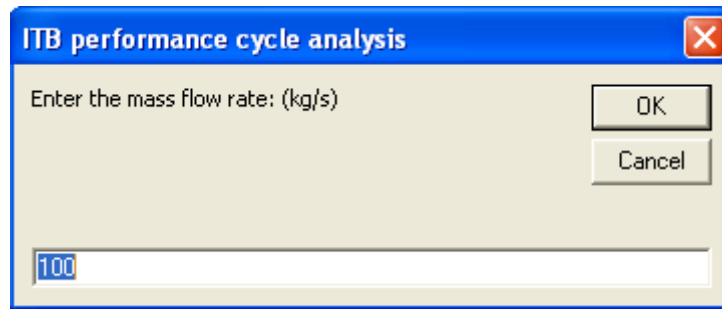
Figure E4.—Dialog Box for Inputting
Value of Mass Flow Rate.

Appendix F—Visual Basic Codes

The Visual Basic codes can be easily seen on the spreadsheet under "Tools\Macro\Visual Basic Editor".

```
Sub itbperf1()
  Dim eta_r
  Style = vbYes
  Title = "ITB Performance Cycle Analysis"
  Style1 = vbYesNo
'
  If plot_type = 1 Then
      msg0 = "Occurs at " & "M0 = " & M0 & ", h =" & alt & "."
  Else
      msg0 = "Occurs at " & "M0 = " & M0 & ", Tt4 =" & Tt4
  End If
  bigit = vbNo
'
'Set initial values for some engine components
  mdot0 = mdot0R
  f_b = f_bR
  f_itb = f_itbR
  pi_tH = pi_tHR
  pi_tL = pi_tLR
  pi_cH = pi_cHR
  pi_cL = pi_cLR
  pi_f = pi_fR
  tau_tH = tau_tHR
  tau_tL = tau_tLR
  tau_cH = tau_cHR
  tau_cL = tau_cLR
  tau_f = tau_fR
'
'Restore values for some parameters if ITBswitch is internally ON again in next M0 iteration
  If ITBswitch = "ON" Then
      tau_itbR = Tt4p5R / (Tt4R * tau_tHR)        'eqn(5r)
      'cp_itb = Sheets("Input").Cells(7, 14) * conversion 'removed 3/8/05
      'gama_itb = Sheets("Input").Cells(10, 14) '
  Else
      tau_itbR = 1#
  End If
'
'Preliminary computations
  tau_r = 1# + cnst5 * M0 * M0 'eqn(1)
  pi_r = tau_r ^ cnst8    'eqn(2)
  If M0 <= 1# Then
    eta_r = 1#                    'eqn(3)
  End If
  If M0 > 1# And M0 <= 5 Then
    eta_r = 1# - 0.075 * (M0 - 1#) ^ 1.35
  End If
  pi_d = pi_dmax * eta_r 'eqn(4)
  'R_itb = (gama_itb - 1#) * cp_itb / gama_itb 'eqn(7)
  a0 = (gama_c * R_c * gc * T0) ^ (0.5)      'eqn(8)
```

```
    MFP_M0 = M0 * cnst4 * tau_r ^ cnst6        'eqn(11)
    Tt0 = T0 * tau_r                    'eqn(10)
    Pt0 = P0 * tau_r ^ cnst8            'eqn(9)
    itblink = 1
10
End Sub
Sub itbperf2()
  Dim pi_tHN, tau_fN
  Dim MFP_M8, MFP_M9, MFP_M18, MFP_M19
  Dim ebr1, ebr2, flpc
  Dim h_t3, f_4i, Tt4p4, h_t4p4, f_4p5i
  Title = "ITB Performance Cycle Analysis"
  Style = vbYes
  Style1 = vbYesNo
'
  h_0 = cp_c * T0
  If h_0 = 0# Then
    Msg11 = "      Division by zero in 'h_0 equation'. "
    response = MsgBox(Msg11, Style, Title)
    Msg11 = "        Check if ' T0 = 0 '"
    response = MsgBox(Msg11, Style, Title)
    Msg11 = " Current values => cpc =" & cp_c & ", T0=" & T0
    response = MsgBox(Msg11, Style, Title)
    response = MsgBox(msg0, Style, Title)
    If singlept = False Then jump = MsgBox(Msg_jump, Style1, Title)
    itblink = 0
    GoTo 10
  End If
'
  limit = 0
  loopif = 0
  loopich = 0
  loop1 = 0
  reduce = 1
  pi_cHo = pi_cHR
  tau_cHo = tau_cHR
  GoTo 19
'liew = 1
'Check intermediate values for debugging
  Sheets("test").Cells(1, 1) = "M0"
  Sheets("test").Cells(1, 4) = "loop2"
  Sheets("test").Cells(1, 2) = "loopif"
  Sheets("test").Cells(1, 3) = "loopicH"
  Sheets("test").Cells(1, 5) = "Tt4"
  Sheets("test").Cells(1, 6) = "Tt4p5"
  Sheets("test").Cells(1, 7) = "tau_fN"
  Sheets("test").Cells(1, 8) = "tau_cH"
  Sheets("test").Cells(1, 9) = "pi_f"
  Sheets("test").Cells(1, 10) = "pi_cH"
  Sheets("test").Cells(1, 11) = "overpi_f"
  Sheets("test").Cells(1, 12) = "overpi_cH"
  Sheets("test").Cells(1, 13) = "ITB"
  Sheets("test").Cells(1, 14) = "reduce"
19
  If ITBswitch = "OFF" Then reduce = 3
12
```

```
    GoTo 29
'Check intermediate values for debugging
If loopich > 0 Then
  Sheets("test").Cells(liew + 1, 1) = M0
  Sheets("test").Cells(liew + 1, 4) = loop2
  Sheets("test").Cells(liew + 1, 2) = loopif
  Sheets("test").Cells(liew + 1, 3) = loopich
  Sheets("test").Cells(liew + 1, 5) = Tt4
  Sheets("test").Cells(liew + 1, 6) = Tt4p5
  Sheets("test").Cells(liew + 1, 7) = tau_f
  Sheets("test").Cells(liew + 1, 8) = tau_cH
  Sheets("test").Cells(liew + 1, 9) = pi_f
  Sheets("test").Cells(liew + 1, 10) = pi_cH
  Sheets("test").Cells(liew + 1, 11) = overpi_f
  Sheets("test").Cells(liew + 1, 12) = overpi_cH
  Sheets("test").Cells(liew + 1, 13) = ITBswitch
  Sheets("test").Cells(liew + 1, 14) = reduce
  liew = liew + 1
  End If
29
'Recalculate cp_t, gama_t and R_t after either (i) controlimit or (ii) Tt4 cut
  Call enthalpy(1, Tt4, f_b, 0, cp_t, gama_t) '3/8/05
  cnst9 = (gama_t - 1#) / gama_t '3/8/05
  R_t = cnst9 * cp_t '3/8/05

  tau_lb = cp_t * Tt4 / h_0               'eqn(15)
'
'Component: INTERSTAGE-TURBINE BURNER
1
  If loop1 > 20 Then
    loop1 = 21
    itblink = 0
    GoTo 10
  End If
11
'Recalculate cp_itb, gama_itb and R_itb after either (i) controlimit or (ii) Tt4 cut
  Call enthalpy(1, Tt4p5, f_itb, 0, cp_itb, gama_itb) '3/8/05
  R_itb = (gama_itb - 1#) / gama_itb * cp_itb '3/8/05

  loop2 = 0
  If ITBswitch = "ON" Then itbinit = 1     'added at 6/11/04
  tau_litb = cp_itb * Tt4p5 / h_0        'eqn(16b) 'added at 6/11/04
'Check if ITB is still operating
  If tau_litb <= (tau_lb * tau_tH) Then itbinit = 0 'added at 6/11/04
  If ITBswitch = "ON" And itbinit = 0 And reduce = 1 Then 'added at 6/11/04
  'If ITBswitch = "ON" And Tt4p5 / (Tt4 * tau_tH) <= 1 And reduce = 1 Then
    ITBswitch = "OFF"
    loopif = 0
    reduce = 3
  End If
'Make changes to some parameters while ITB is turned off
  If ITBswitch = "ON" And reduce = 1 Or reduce = 2 Then
    tau_itb = Tt4p5 / (Tt4 * tau_tH)       'eqn(16a)
  ' tau_litb = cp_itb * Tt4p5 / h_0       'eqn(16b)'removed at 6/11/04
  Else
    tau_itb = 1#                    'eqn(16c)
```

```
        tau_itbR = 1#
        tau_litb = cp_t * Tt4 * tau_tH / h_0     'eqn(16d), ignore tau_itb=1
        Tt4p5 = Tt4 * tau_tH                'eqn(16e)
        Call enthalpy(1, Tt4p5, 0, 0, cp_itb, gama_itb) '3/8/05
        R_itb = (gama_itb - 1#) / gama_itb * cp_itb '3/8/05
      End If
  A4p5oR = (tau_itb / tau_itbR) ^ n_a          'eqn(17)
  A8oR = (tau_itb / tau_itbR) ^ m_a           'eqn(18)
'
'Component: HIGH-PRESSURE TURBINE
  tau_tH = 1# - eta_tH * (1# - pi_tH ^ cnst9)  'eqn(19)
  pi_tHN = (tau_tH * tau_itb / (tau_tHR * tau_itbR)) ^ (0.5) / A4p5oR * pi_tHR 'eqn(20)
  If Abs(pi_tHN - pi_tH) > 0.00001 Then
      pi_tH = pi_tHN
      loop1 = loop1 + 1
      GoTo 1
  End If
2
    If loop2 > 50 Then
      Msg17 = "Eqn(38) => 'tau_fN' does not converge!"
      response = MsgBox(Msg17, Style, Title)
      Msg17 = "Reduce or increase 'pi_f_max' value and try again."
      response = MsgBox(Msg17, Style, Title)
      response = MsgBox(msg0, Style, Title)
      If singlept = False Then jump = MsgBox(Msg_jump, Style1, Title)
      itblink = 0
      GoTo 10
    End If
'
'Component: HIGH-PRESSURE COMPRESSOR
  tau_cH = 1# + eta_mH * (1# + f_b) * tau_lb * (1# - tau_tH) / (tau_r * tau_cL) 'eqn(21)
  pi_cH = (1# + eta_cH * (tau_cH - 1#)) ^ cnst8 'eqn(22)
  err_picH = Abs(pi_cH - pi_cHmax)
   If limit = 4 And err_picH > 0.001 Then
      Dpi_cH = pi_cHo - pi_cH
      If Dpi_cH = 0 Then Dpi_cH = 0.00001 'avoid division by zero
      DTt4 = Tt4o - Tt4
      pi_cHo = pi_cH
      Tt4o = Tt4
      DifTt4 = (pi_cH - pi_cHmax) * DTt4 / Dpi_cH
      If Abs(DifTt4) > 100# Then DifTt4 = DifTt4 / Abs(DifTt4) * 100#
      Tt4 = Tt4 - DifTt4
      limit = 4
      GoTo 12
    End If
   'reduce = 2 Or reduce = 4 Then
   'skip control limit when PTP
'
'Component: MAIN BURNER
  temp_fb = eta_b * h_PR / h_0 - tau_lb
  If temp_fb = 0# Then
    Msg12 = "     Division by zero => Refer to eqn(21a)."
    response = MsgBox(Msg12, Style, Title)
    Msg12 = "Eqn(21a) =>  Check if ' eta_b * h_PR / h_0 = tau_lb '"
    response = MsgBox(Msg12, Style, Title)
```

```
        Msg12 = "Current values => eta_b=" & eta_b & ", h_PR=" & h_PR & ", h_0=" & h_0 & ", tau_lb=" &
tau_lb
        response = MsgBox(Msg12, Style, Title)
        response = MsgBox(msg0, Style, Title)
        If singlept = False Then jump = MsgBox(Msg_jump, Style1, Title)
        itblink = 0
        GoTo 10
      End If
    'Fuel model 1: CSH
      If fuel_model_name = "CSH" Then
        f_b = (tau_lb - tau_r * tau_cL * tau_cH) / temp_fb 'eqn(23a)
      Else
    'Fuel model 2: MSH
        loop3 = 0
        Tt3 = T0 * tau_r * tau_cL * tau_cH                'eqn(23b)
        'Call enthalpy(Tt3, 0, h_t3)'3/8/05
        Call enthalpy(0, Tt3, 0, h_t3, 0, 0)
        f_4i = (tau_lb - tau_r * tau_cL * tau_cH) / temp_fb 'eqn(23c)'initial value for f_4i
3
        If loop3 > 20 Then
          itblink = 0
          GoTo 10
        End If
        'Call enthalpy(Tt4, f_4i, h_t4)'3/8/05
        Call enthalpy(0, Tt4, f_4i, h_t4, 0, 0)
        temp_fb = eta_b * h_PR - h_t4
        If temp_fb = 0# Then
          Msg12 = "Division by zero => Refer to eqn(21d)."
          response = MsgBox(Msg12, Style, Title)
          Msg12 = "  Check if ' eta_b * h_PR = h_T4 '"
          response = MsgBox(Msg12, Style, Title)
          Msg12 = "Current values =>  eta_b=" & eta_b & ", h_PR=" & h_PR & ", h_T4=" & h_t4
          response = MsgBox(Msg12, Style, Title)
          response = MsgBox(msg0, Style, Title)
          If singlept = False Then jump = MsgBox(Msg_jump, Style1, Title)
          itblink = 0
          GoTo 10
        End If
        f_b = (h_t4 - h_t3) / temp_fb                'eqn(23d)
        If Abs(f_b - f_4i) > 0.0001 Then
          f_4i = f_b
          loop3 = loop3 + 1
          GoTo 3
        End If
      End If
      If ITBswitch = "OFF" Then 'skip fitb computation
        Call enthalpy(1, Tt4, f_b, 0, cp_t, gama_t) '3/8/05
        cnst9 = (gama_t - 1#) / gama_t '3/8/05
        R_t = cnst9 * cp_t '3/8/05
        f_itb = 0#
        GoTo 5
      End If
      '
      'Component: INTERSTAGE-TURBINE BURNER
      temp_fitb = eta_itb * h_PR / h_0 - tau_litb
      If temp_fitb = 0# Then
```

```
        Msg13 = "Division by zero => Refer to eqn(22a). Check if 'eta_itb * h_PR / h_0 = tau_litb'"
        response = MsgBox(Msg13, Style, Title)
        response = MsgBox(msg0, Style, Title)
        If singlept = False Then jump = MsgBox(Msg_jump, Style1, Title)
        itblink = 0
        GoTo 10
      End If
    'Fuel model 1: CSH
      If fuel_model_name = "CSH" Then
        f_itb = (1# + f_b) * (tau_litb - tau_lb * tau_tH) / (temp_fitb) 'eqn(24a)
      Else
    'Fuel model 2: MSH
        loop4 = 0
        Tt4p4 = Tt4 * tau_tH                          'eqn(24b)
        'Call enthalpy(Tt4p4, f_b, h_t4p4)'3/8/05
        Call enthalpy(0, Tt4p4, f_b, h_t4p4, 0, 0)
        f_4p5i = (1# + f_b) * (tau_litb - tau_lb * tau_tH) / (temp_fitb) 'eqn(24c)initial value for f_4p5i
    4
        If loop4 > 20 Then
          loop4 = 21
          itblink = 0
          GoTo 10
        End If
        'Call enthalpy(Tt4p5, f_4p5i, h_t4p5)'3/8/05
        Call enthalpy(0, Tt4p5, f_4p5i, h_t4p5, 0, 0)
        temp_fitb = eta_itb * h_PR - h_t4p5
        If temp_fitb = 0# Then
          response = MsgBox(Msg13, Style, Title)
          response = MsgBox(msg0, Style, Title)
          If singlept = False Then jump = MsgBox(Msg_jump, Style1, Title)
          itblink = 0
          GoTo 10
        End If
        f_itb = (1# + f_b) * (h_t4p5 - h_t4p4) / (temp_fitb)          'eqn(24d)
        If Abs(f_b + f_itb - f_4p5i) > 0.0001 Then
          f_4p5i = f_b + f_itb
          loop4 = loop4 + 1
          GoTo 4
        End If
      End If
    5
      If f_itb = 0# Then
        gama_itb = gama_t
        cp_itb = cp_t
        R_itb = R_t
      End If
    'Constants
      cnst1 = (gama_itb / R_itb * gc) ^ (0.5)
      cnst2 = (gama_itb - 1#) / 2#
      cnst3 = (gama_itb + 1#) / (2# * (1# - gama_itb))
      MFP_M8R = M8R * cnst1 * (1# + cnst2 * M8R * M8R) ^ cnst3          'eqn(6r)
    '
    'Component: FAN & LOW-PRESSURE COMPRESSOR PRESSURE RATIOS
      pi_f = (1# + eta_f * (tau_f - 1#)) ^ cnst8    'eqn(25)
      pi_cL = (1# + eta_cL * (tau_cL - 1#)) ^ cnst8 'eqn(26)
    '
```

```
  'Component: EXHAUST NOZZLES
  'added at 5/12/04 -1 (Exit Pressure condition)
   'Pt9oP0 = pi_r * pi_d * pi_cL * pi_cH * pi_b * pi_tH * pi_itb * pi_tL * pi_n
   'choked = (0.5 * (gama_itb + 1#)) ^ (gama_itb / (gama_itb - 1#))
   'If Pt9oP0 >= choked Then
   '  M9 = 1#
   '  Pt9oP9 = choked
   '  P0oP9 = Pt9oP9 / Pt9oP0
   'Else
   '  P0oP9 = 1#
   '  Pt9oP9 = Pt9oP0
   '  temp_M9 = Pt9oP9 ^ ((gama_itb - 1#) / gama_itb)
   '  If temp_M9 < 1# Then
   '    response = MsgBox(Msg14, Style, Title)
   '    response = MsgBox(msg0, Style, Title)
   '    jump = MsgBox(Msg_jump, Style1, Title)
   '    itblink = 0
   '    GoTo 10
   '  End If
   '  M9 = (2# * (temp_M9 - 1#) / (gama_itb - 1#)) ^ 0.5
   'End If
  'added at 5/12/04 -2
   Pt19oP19 = P0oP19 * pi_r * pi_d * pi_f * pi_nf 'eqn(27)
   Pt9oP9 = P0oP9 * pi_r * pi_d * pi_cL * pi_cH * pi_b * pi_tH * pi_itb * pi_tL * pi_n  'eqn(28)
   temp_M9 = Pt9oP9 ^ ((gama_itb - 1#) / gama_itb) 'eqn(29a)
   If temp_M9 < 1# Then
     Msg14 = "          Exit M9 is imaginary! "
     response = MsgBox(Msg14, Style, Title)
     Msg14 = "       Refer to eqn(27) ==> Pt9/P9 < 1"
     response = MsgBox(Msg14, Style, Title)
     Msg14 = "Current values => P0/P9=" & P0oP9 & ", pi_r=" & pi_r & ", pi_d=" & pi_d & ", pi_cL=" & pi_cL
 & ", pi_b=" & pi_b & ", pi_tH=" & pi_tH & ", pi_itb=" & pi_itb & ", pi_tL=" & pi_tH & ", pi_n=" & pi_n
     response = MsgBox(Msg14, Style, Title)
     response = MsgBox(msg0, Style, Title)
     If singlept = False Then jump = MsgBox(Msg_jump, Style1, Title)
     itblink = 0
     GoTo 10
   End If
   M9 = (2# * (temp_M9 - 1#) / (gama_itb - 1#)) ^ (0.5)   'eqn(29)
   If M9 > 1# Then
     M8 = 1#
   Else
     M8 = M9
   End If
   MFP_M8 = M8 * cnst1 * (1# + cnst2 * M8 * M8) ^ cnst3   'eqn(30)
   MFP_M9 = M9 * cnst1 * (1# + cnst2 * M9 * M9) ^ cnst3   'eqn(31)
   A9oA8 = MFP_M8 / (MFP_M9 * pi_n)                 'eqn(32)
  'added at 5/12/04 -1
  ' Pt19oP0 = pi_r * pi_d * pi_f * pi_nf
  ' choked = (0.5 * (gama_c + 1#)) ^ (gama_c / (gama_c - 1#))
  ' If Pt19oP0 >= choked Then
  '   M19 = 1#
  '   Pt19oP19 = choked
  '   P0oP19 = Pt19oP19 / Pt19oP0
  ' Else
  '   P0oP19 = 1#
```

```
'    Pt19oP19 = Pt19oP0
'    temp_M19 = Pt19oP19 ^ cnst7
'    If gama_c < 1# Or temp_M19 < 1# Then
'      response = MsgBox(Msg15, Style, Title)
'      response = MsgBox(msg0, Style, Title)
'      jump = MsgBox(Msg_jump, Style1, Title)
'      itblink = 0
'      GoTo 10
'    End If
'    M19 = ((temp_M19 - 1#) / cnst5) ^ 0.5
' End If
'added at 5/12/04 -2
temp_M19 = Pt19oP19 ^ cnst7                'eqn(33a)
If temp_M19 < 1# Then
  Msg14 = "          Exit M19 is imaginary! "
  response = MsgBox(Msg14, Style, Title)
  Msg14 = "     Refer to eqn(31) ==> Pt19/P19 < 1"
  response = MsgBox(Msg14, Style, Title)
  Msg14 = "Current values => P0/P19=" & P0oP9 & ", pi_r=" & pi_r & ", pi_d=" & pi_d & ", pi_f=" & pi_f
& ", pi_nf=" & pi_nf
  response = MsgBox(Msg14, Style, Title)
  response = MsgBox(msg0, Style, Title)
  If singlept = False Then jump = MsgBox(Msg_jump, Style1, Title)
  itblink = 0
  GoTo 10
End If
'
M19 = ((temp_M19 - 1#) / cnst5) ^ (0.5)      'eqn(33)
If M19 > 1# Then
  M18 = 1#
Else
  M18 = M19
End If
MFP_M18 = M18 * cnst4 * (1# + cnst5 * M18 * M18) ^ cnst6        'eqn(34)
MFP_M19 = M19 * cnst4 * (1# + cnst5 * M19 * M19) ^ cnst6        'eqn(35)
A19oA18 = MFP_M18 / (MFP_M19 * pi_nf)                'eqn(36)
'
'ENGINE BYPASS RATIO
ebr1 = (pi_cLR * pi_cHR * pi_f) / (pi_cL * pi_cH * pi_fR)      'eqn(37a)
ebr2 = (Tt4 * tau_rR * tau_fR / (Tt4R * tau_r * tau_f)) ^ (0.5)  'eqn(37b)
alp = alpR * ebr1 * ebr2 * MFP_M18 / MFP_M18R                'eqn(37)
'
'Component: FAN & LOW-PRESSURE COMPRESSOR TEMPERATURE RATIOS
temp_flpc = tau_cLR - 1# + alp * (tau_fR - 1#)              'eqn(38a)
If temp_flpc = 0# Then
  Msg16 = "          Division by zero in eqn(38) !!"
  response = MsgBox(Msg16, Style, Title)
  Msg16 = "Eqn(38) => Check if 'tau_cLR - 1 + alpha*(tau_fR - 1) = 0'"
  response = MsgBox(Msg16, Style, Title)
  Msg16 = "Current values =>  tau_cLR= " & tau_cLR & ", alpha=" & alp & ", tau_fR=" & tau_fR
  response = MsgBox(Msg16, Style, Title)
  response = MsgBox(msg0, Style, Title)
  If singlept = False Then jump = MsgBox(Msg_jump, Style1, Title)
  itblink = 0
  GoTo 10
End If
```

```
flpc = (1# - tau_tL) * (1# + f_b + f_itb) / (temp_flpc)        'eqn(38b)
tau_fN = 1# + (tau_fR - 1#) * eta_mL * tau_litb / tau_r * flpc   'eqn(38)orig
tau_cL = 1# + (tau_fN - 1#) * (tau_cLR - 1#) / (tau_fR - 1#)     'eqn(39)orig
'
'Component: LOW-PRESSURE TURBINE
tau_tL = 1# - eta_tL * (1# - pi_tL ^ ((gama_itb - 1#) / gama_itb)) 'eqn(40)orig
pi_tL = pi_tLR * (tau_tL / tau_tLR) ^ (0.5) * A4p5oR / A8oR * MFP_M8R / MFP_M8 'eqn(41)orig
'
'Checking: Writing intermediate values of Tt4, Tt4.5, pi_f and pi_cH
 GoTo 39
  If pi_f - pi_fmax > 0.001 Then
   overpi_f = "OVER"
   overpif = overpif + 1
  Else
   overpi_f = "no"
  End If
  If pi_cH - pi_cHmax > 0.001 Then
   overpi_cH = "OVER"
   overpicH = overpicH + 1
  Else
    overpi_cH = "no"
  End If
 'Check intermediate values for debugging
  Sheets("test").Cells(liew + 1, 1) = M0
  Sheets("test").Cells(liew + 1, 4) = loop2
  Sheets("test").Cells(liew + 1, 2) = loopif
  Sheets("test").Cells(liew + 1, 3) = loopich
  Sheets("test").Cells(liew + 1, 5) = Tt4
  Sheets("test").Cells(liew + 1, 6) = Tt4p5
  Sheets("test").Cells(liew + 1, 7) = tau_f
  Sheets("test").Cells(liew + 1, 8) = tau_cH
  Sheets("test").Cells(liew + 1, 9) = pi_f
  Sheets("test").Cells(liew + 1, 10) = pi_cH
  Sheets("test").Cells(liew + 1, 11) = overpi_f
  Sheets("test").Cells(liew + 1, 12) = overpi_cH
  Sheets("test").Cells(liew + 1, 13) = ITBswitch
  Sheets("test").Cells(liew + 1, 14) = reduce
  liew = liew + 1
39
  If reduce = 2 Or reduce = 4 Then GoTo 20
  temp_f = Abs(tau_fN - tau_f)
  If temp_f > 0.0001 Then
    tau_f = tau_fN
    loop2 = loop2 + 1
    GoTo 2
  End If
'
 'Skip control limits for throttle hook calculation (PTP)
  'If ctype = 2 Then GoTo 30 '3/12/04
  'If reduce = 2 Or reduce = 4 Then GoTo 20
  If pi_fmax > 0# Then
   If pi_f > pi_fmax And limit <> 3 And reduce = 1 Then
     pi_fo = pi_f
     Tt4p5o = Tt4p5
     Tt4p5 = Tt4p5 - 2#
     loopif = loopif + 1
```

```
        limit = 3
        GoTo 11
      End If
    err_pif = Abs(pi_f - pi_fmax)
    If limit = 3 And reduce = 1 And err_pif > 0.001 Then
        Dpi_f = pi_fo - pi_f
        If Dpi_f = 0 Then Dpi_f = 0.001 'avoid division by zero'3/10/05
        DTt4p5 = Tt4p5o - Tt4p5
        pi_fo = pi_f
        Tt4p5o = Tt4p5
        DifTt4p5 = (pi_f - pi_fmax) * DTt4p5 / Dpi_f
        If Abs(DifTt4p5) > 100# Then DifTt4p5 = DifTt4p5 / Abs(DifTt4p5) * 100#
        Tt4p5 = Tt4p5 - DifTt4p5
        limit = 3
        GoTo 11
      ElseIf reduce = 1 Then
        reduce = 2
        GoTo 20
      End If
    If pi_f > pi_fmax And limit <> 3 And reduce = 3 Then
        pi_fo = pi_f
        Tt4o = Tt4
        Tt4 = Tt4 - 2#
        loopif = loopif + 1
        limit = 3
        GoTo 12
      End If
    err_pif = Abs(pi_f - pi_fmax)
    If limit = 3 And reduce = 3 And err_pif > 0.001 Then
        Dpi_f = pi_fo - pi_f
        If Dpi_f = 0 Then Dpi_f = 0.00001 'avoid division by zero
        DTt4 = Tt4o - Tt4
        pi_fo = pi_f
        Tt4o = Tt4
        DifTt4 = (pi_f - pi_fmax) * DTt4 / Dpi_f
        If Abs(DifTt4) > 100# Then DifTt4 = DifTt4 / Abs(DifTt4) * 100#
        Tt4 = Tt4 - DifTt4
        limit = 3
        GoTo 12
      ElseIf reduce = 3 Then
        reduce = 4
      End If
    End If
20
  If reduce = 2 Or reduce = 4 Then
    If pi_cHmax > 0# Then
      If pi_cH > pi_cHmax And limit <> 4 Then
        'err_picH = pi_cH - pi_cHmax
        If loopich = 0 Then
          pi_cHo = pi_cH
          Tt4o = Tt4
          Tt4 = Tt4 - 2#
          If loopich = 0 Then loopich = 1
          limit = 4
          GoTo 12
        End If
```

```
        End If
      End If
    End If
30
  Pt3 = P0 * pi_r * pi_d * pi_cL * pi_cH
  Tt3 = T0 * tau_r * tau_cL * tau_cH
  'Sheets("SinglePt_FTP").Cells(23, 9) = PRPMhp
  If reduce = 2 Or reduce = 4 Then
    If Pt3max > 0# Then
      If Pt3 > Pt3max And limit <> 5 Then
        Pt3o = Pt3
        Tt4o = Tt4
        Tt4 = Tt4 - 2#
        If loopt3 = 0 Then loopt3 = 1
        limit = 5
        GoTo 12
      End If
      err_Pt3 = Abs(Pt3 - Pt3max)
      If limit = 5 And err_Pt3 > 0.04 Then
        DPt3 = Pt3o - Pt3
        DTt4 = Tt4o - Tt4
        Pt3o = Pt3
        Tt4o = Tt4
        DifTt4 = (Pt3 - Pt3max) * DTt4 / DPt3
        If Abs(DifTt4) > 100# Then DifTt4 = DifTt4 / Abs(DifTt4) * 100#
        Tt4 = Tt4 - DifTt4
        limit = 5
        GoTo 12
      End If
    End If
  End If
  If reduce = 2 Or reduce = 4 Then
    If Tt3max > 0# Then
      If Tt3 > Tt3max And limit <> 6 Then
        Tt3o = Tt3
        Tt4o = Tt4
        Tt4 = Tt4 - 2#
        If loopTt3 = 0 Then loopTt3 = 1
        limit = 6
        GoTo 12
      End If
      err_Tt3 = Abs(Tt3 - Tt3max)
      If limit = 6 And err_Tt3 > 0.1 Then
        DTt3 = Tt3o - Tt3
        If DTt3 = 0 Then DTt3 = 0.00001 'avoid division by zero
        DTt4 = Tt4o - Tt4
        Tt3o = Tt3
        Tt4o = Tt4
        DifTt4 = (Tt3 - Tt3max) * DTt4 / DTt3
        If Abs(DifTt4) > 100# Then DifTt4 = DifTt4 / Abs(DifTt4) * 100#
        Tt4 = Tt4 - DifTt4
        limit = 6
        GoTo 12
      End If
    End If
  End If
```

```
Temp = 100# * (T0 * tau_r / (T0R * tau_rR)) ^ (0.5)
'PRPMlp = Temp * ((tau_f - 1#) / (tau_fR - 1#)) ^ (0.5)   'eqn(57)
PRPMlp = Temp * ((tau_fN - 1#) / (tau_fR - 1#)) ^ (0.5)   'eqn(57), 3/11/05
PRPMhp = Temp * (tau_cL * (tau_cH - 1#) / (tau_cLR * (tau_cHR - 1#))) ^ (0.5) 'eqn(58)
If itbinit = 1 Then 'added (see khliew79's note 3/11/05)
  If PRPMlpmax > 0# Then
    If PRPMlp > PRPMlpmax And limit <> 7 Then
      PRPMlpo = PRPMlp
      Tt4p5o = Tt4p5
      Tt4p5 = Tt4p5 - 2#
      If loopPRPMlp = 0 Then loopPRPMlp = 1
      limit = 7
      tau_f = tau_fN '3/11/05
      GoTo 11
    End If
    err_PRPMlp = Abs(PRPMlp - PRPMlpmax)
    If limit = 7 And err_PRPMlp > 0.1 Then
      DPRPMlp = PRPMlpo - PRPMlp
      If DPRPMlp = 0 Then DPRPMlp = 0.001 'avoid division by zero'3/10/05
      DTt4p5 = Tt4p5o - Tt4p5
      PRPMlpo = PRPMlp
      Tt4p5o = Tt4p5
      DifTt4p5 = (PRPMlp - PRPMlpmax) * DTt4p5 / DPRPMlp
      If Abs(DifTt4p5) > 100# Then DifTt4p5 = DifTt4p5 / Abs(DifTt4p5) * 100#
      Tt4p5 = Tt4p5 - DifTt4p5
      limit = 7
      tau_f = tau_fN
      GoTo 11
    End If
  End If
End If
If reduce = 2 Or reduce = 3 Or reduce = 4 Then 'modified 3/11/05
'If reduce = 2 Or reduce = 4 Then
  If PRPMlpmax > 0# Then
    If PRPMlp > PRPMlpmax And limit <> 7 Then
      PRPMlpo = PRPMlp
      Tt4o = Tt4
      Tt4 = Tt4 - 2#
      If loopPRPMlp = 0 Then loopPRPMlp = 1
      limit = 7
      tau_f = tau_fN
      GoTo 12
    End If
    err_PRPMlp = Abs(PRPMlp - PRPMlpmax)
    If limit = 7 And err_PRPMlp > 0.1 Then
      DPRPMlp = PRPMlpo - PRPMlp
      If DPRPMlp = 0 Then DPRPMlp = 0.00001 'avoid division by zero
      DTt4 = Tt4o - Tt4
      PRPMlpo = PRPMlp
      Tt4o = Tt4
      DifTt4 = (PRPMlp - PRPMlpmax) * DTt4 / DPRPMlp
      If Abs(DifTt4) > 100# Then DifTt4 = DifTt4 / Abs(DifTt4) * 100#
      Tt4 = Tt4 - DifTt4
      limit = 7
      tau_f = tau_fN
      GoTo 12
```

```
      End If
    End If
  End If
  If reduce = 2 Or reduce = 4 Then
    If PRPMhpmax > 0# Then
      If PRPMhp > PRPMhpmax And limit <> 8 Then
        PRPMhpo = PRPMhp
        Tt4o = Tt4
        Tt4 = Tt4 - 2#
        If loopPRPMhp = 0 Then loopPRPMhp = 1
        limit = 8
        GoTo 12
      End If
      err_PRPMhp = Abs(PRPMhp - PRPMhpmax)
      'If limit = 7 And err_PRPMlp > 0.1 Then
      If limit = 8 And err_PRPMhp > 0.1 Then '3/11/05
        DPRPMhp = PRPMhpo - PRPMhp
        If DPRPMhp = 0 Then DPRPMhp = 0.00001 'avoid division by zero
        DTt4 = Tt4o - Tt4
        PRPMhpo = PRPMhp
        Tt4o = Tt4
        DifTt4 = (PRPMhp - PRPMhpmax) * DTt4 / DPRPMhp
        If Abs(DifTt4) > 100# Then DifTt4 = DifTt4 / Abs(DifTt4) * 100#
        Tt4 = Tt4 - DifTt4
        limit = 8
        GoTo 12
      End If
    End If
  End If
 End If
 itblink = 3
10
End Sub
Sub itbperf3()
 Dim a9, a19, area0, emf
 Dim fm01, fm02, fm03
 Title = "ITB Performance Cycle Analysis"
 Style = vbYes
 Style1 = vbYesNo
'
'Engine mass flow
 emf = P0 * pi_r * pi_d * pi_cL * pi_cH / (P0R * pi_rR * pi_dR * pi_cLR * pi_cHR) 'eqn(42a)
 mdot0 = mdot0R * (1# + alp) / (1# + alpR) * emf * (Tt4R / Tt4) ^ (0.5) 'eqn(42)
'
'Overall engine performance
 f_o = (f_b + f_itb) / (1# + alp)                        'eqn(43)
 T9oT0 = Tt4p5 / T0 * tau_tL / (Pt9oP9) ^ ((gama_itb - 1#) / gama_itb) 'eqn(44)
 T19oT0 = tau_r * tau_f / (Pt19oP19) ^ cnst7    'eqn(45)
 a9 = (gama_itb * R_itb * gc * T9oT0 * T0) ^ (0.5)           'eqn(46)
 a19 = (gama_c * R_c * gc * T19oT0 * T0) ^ (0.5)            'eqn(47)
 v9 = M9 * a9                              'eqn(48)
 v19 = M19 * a19                           'eqn(49)
 v9oa0 = v9 / a0                           'eqn(48a)
 v19oa0 = v19 / a0                          'eqn(49a)
'
 fm01 = (1# + f_o * (1# + alp)) * v9oa0 + alp * v19oa0 - (1# + alp) * M0 'eqn(50a)
 fm02 = (1# + f_o * (1# + alp)) * R_itb / R_c * T9oT0 / v9oa0 * (1# - P0oP9) / gama_c 'eqn(50b)
```

```vba
    fm03 = alp * T19oT0 / v19oa0 * (1# - P0oP19) / gama_c        'eqn(50c)
    Fomd0 = a0 / gc * (fm01 + fm02 + fm03) / (1# + alp)          'eqn(50)
    F = mdot0 * Fomd0                                'eqn(51)
    S = f_o / Fomd0 * unitS                          'eqn(52)
    If Sheets("Input").ComboBox1.Value = "SI" Then unitchange = 1000000#
    If F < 0# Or S < 0# Then
      wdata = "No"
      Msg14 = "        Thrust or S is negative! " 'modified 3/29/05
      response = MsgBox(Msg14, Style, Title)
      response = MsgBox(msg0, Style, Title)
      If singlept = False Then jump = MsgBox(Msg_jump, Style1, Title)
      GoTo 9
    End If


   ' added on 7/21/04 -1
    v0 = M0 * a0
    'f_itb = f_itb / (1# + f_b) 'then, 'f_itb' will be equivalent to 'fitb' in on-design code. This is needed for eta
computation. (removed 3/8/05)
    fo = f_b + f_itb
    corenume = (1# + fo) * v9 * v9 - v0 * v0
    fanume = alp * (v19 * v19 - v0 * v0)
    nume = 0.5 / gc * (corenume + fanume)
    denu = f_b * h_PR * eta_b + f_itb * h_PR * eta_itb

    eta_thermal = nume / denu * 100#             'eqn(54)
    eta_P = (1# + alp) * Fomd0 * v0 / nume * 100# 'eqn(53)
    eta_O = (1# + alp) * Fomd0 * v0 / denu * 100# 'eqn(54)
    'eta_O = eta_thermal * eta_P / 100#
   ' added on 7/21/04 -2
    area0 = mdot0 * Tt0 ^ (0.5) / (Pt0 * MFP_M0)  'eqn(56)
   9
    bigit = vbYes
   10
    End Sub
```

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13. ABSTRACT (Maximum 200 words)

This report presents the performance of a steady-state, dual-spool, separate-exhaust turbofan engine, with an interstage turbine burner (ITB) serving as a secondary combustor. The ITB, which is located in the transition duct between the high- and the low-pressure turbines, is a relatively new concept for increasing specific thrust and lowering pollutant emissions in modern jet-engine propulsion. A detailed off-design performance analysis of ITB engines is written in Microsoft® Excel (Redmond, Washington) macrocode with Visual Basic Application to calculate engine performances over the entire operating envelope. Several design-point engine cases are pre-selected using a parametric cycle-analysis code developed previously in Microsoft® Excel, for off-design analysis. The off-design code calculates engine performances (i.e. thrust and thrust-specific-fuel-consumption) at various flight conditions and throttle settings.

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