

Comparison of Numerical Schemes for a Realistic Computational Aeroacoustics Benchmark Problem

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Abstract

In this work, a nonlinear structured-multiblock CAA solver, the NASA GRC BASS code, will be tested on a realistic CAA benchmark problem. The purpose of this test is to ascertain what effect the high-accuracy solution methods used in CAA have on a realistic test problem, where both the mean flow and the unsteady waves are simultaneously computed on a fully curvilinear grid from a commercial grid generator. The proposed test will compare the solutions obtained using several finite-difference methods on identical grids to determine whether high-accuracy schemes have advantages for this benchmark problem.

Introduction

The field of Computational Aeroacoustics (CAA) is concerned with the time-accurate calculation of unsteady flow fields. In order to accurately propagate the unsteady acoustic, vortical, and entropy waves, high-accuracy numerical differencing schemes have been developed which require very few grid points per disturbance wavelength to calculate an accurate value of the spatial derivative (see Refs. 1 and 2 for an overview of CAA developments). These schemes have been extended for use in nonlinear flow calculations, and have produced very good results (e.g., Refs. 3-5).

However, for realistic flow calculations using curvilinear grids, it is not clear if these high-accuracy schemes retain the advantages that they show for model problems.

Previous work has indicated that the grid generator has an effect on the attainable accuracy of a numerical scheme⁶, even with a very smooth grid from a commercial grid generator.⁷

In the proposed work, the NASA BASS code will be applied to the CAA Benchmark problem of a vortical gust impinging on a loaded 2D cascade.⁸ The BASS code has four spatial differencing options: explicit 2nd order, explicit 6th order, optimized DRP⁹, and prefactored compact 6th order.¹⁰ While it is expected that the three high-accuracy schemes will perform adequately, the question is whether they will perform better than the low-order scheme on a realistic problem.

It must be noted at this point that this test problem may well be weighted in favor of the 2nd order explicit scheme because the wavelength of the vortical gust is very long and the computational boundaries are very close. Thus, if the high-accuracy schemes provide a measurably better answer, this will be a strong indication that high-accuracy schemes are useful for traditional CFD problems as well as CAA.

Governing Equations and Numerical Method

In this work, the Euler equations are solved. The 2D nonlinear Euler equations may be written in Cartesian form as:

$$Q_t + E_x + F_y = 0 \quad (1)$$

The NASA Glenn Research Center BASS code was used to solve this equation.^{4-6,11-12} The BASS code uses optimized explicit time marching combined with high-accuracy finite-differences to accurately compute the unsteady flow. The code is parallel, and uses a block-structured curvilinear grid to represent the physical flow domain. A constant-coefficient 10th order artificial dissipation model¹³ is used to remove unresolved high-frequency modes from the computed solution.

The BASS code solves the Euler equations using the nonconservative chain-rule formulation; previous experience has indicated that the formal lack of conservation is offset by the increased accuracy of the transformed equations.³⁻⁵ The chain-rule form of the Euler equations are:

$$\begin{aligned} Q_t + \xi_\xi Q_\xi + \eta_\eta Q_\eta \\ + \xi_x E_\xi + \eta_x E_\eta \\ + \xi_y F_\xi + \eta_y F_\eta = 0 \end{aligned} \quad (2)$$

For this work, the optimized low-storage RK56 scheme of Stanescu and Habashi¹⁴ was combined with the prefactored sixth-order compact differencing scheme of Hixon¹⁰.

Proposed Work

In this work, the CAA benchmark cascade problem given in Ref. 8 will be computed using the NASA GRC BASS code. The BASS code will be run using various spatial differencing schemes of different accuracies, and the results will be compared to determine the effectiveness of the high-accuracy finite-difference schemes currently used in CAA codes on a realistic test problem. The grid density and stretching will also be varied to investigate the grid density required for an accurate solution.

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