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INCLUDING AEROELASTIC EFFECTS IN THE CALCULATION OF X-33 LOADS AND CONTROL CHARACTERISTICS

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Introduction

Up until now, loads analyses of the X-33 RLV have been done at Marshall Space Flight Center (MSFC) using aerodynamic loads derived from CFD and wind tunnel models of a rigid vehicle. Control forces and moments are determined using a rigid vehicle trajectory analysis and the detailed control load distributions for achieving the desired control forces and moments, again on the rigid vehicle, are determined by Lockheed Martin Skunk Works. However, static aeroelastic effects upon the load distributions are not known. The static aeroelastic effects will generally redistribute external loads thereby affecting both the internal structural loads as well as the forces and moments generated by aerodynamic control surfaces. Therefor, predicted structural sizes as well as maneuvering requirements can be altered by consideration of static aeroelastic effects.

Objective

The objective of the present work is the development of models and solutions for including static aeroelasticity in the calculation of X-33 loads and in the determination of stability and control derivatives. Since structural analysis of the X-33 vehicle is being done in NASTRAN, it was decided that the models and analyses would be done using NASTRAN Aeroelastic Supplement's Static Aeroelasticity solution sequence (SOL 144) (Ref. 1). Essentially, NASTRAN's trim solutions are considered incremental changes to those represented by the rigid loads and control force solutions. Load distributions determined by these rigid analyses are applied to the model as "external" loads during the aeroelastic solution. Alone, the aeroelastic load redistributions would bring the vehicle out of the desired flight condition. However, the aeroelastic trim solution can determine changes in the trim variables (e.g. angle of attack, sideslip, control deflections) that will maintain the desired trim condition. Values of stability and control derivatives for the rigid and flexible vehicle are also produced by the SOL 144. A second result of the static aeroelastic solution is internal loads/stress recovery. This latter part of SOL 144 is essentially identical to the regular static solution in NASTRAN (SOL 101).

Mathematical Foundation

Following is a mathematical description of the problem to be solved. While the form of the equations presented herein is not identical to that presented in the NATRAN Aeroelastic Supplement User's Guide, it is mathematically equivalent and more readily followed.

The case of an unrestrained vehicle is a good bit more complicated than that of the restrained structure for several reasons. One is the fact that the load-deflection relation involves the "free body flexibility" matrix. The deflection under load of an unrestrained structure can't be found simply by inverting the stiffness matrix because the stiffness matrix is singular. A second complication arises from the need to consider such details of the flight condition as trim conditions.

The unrestrained nature of the structure is handled by the so-called inertia relief formulation, which will be discussed briefly. The structure is assumed restrained at a support point against rigid body motion, and corresponding load-deflection relation of the restrained structure is expressed in the usual way but with certain differences in detail,

$$\mathbf{P}_{net} = \mathbf{K}\delta_{res} \tag{1}$$

$\mathbf{P}_{net}$ is the vector of *net* loads being the combination of externally applied loads (aerodynamic, propulsive, etc.) and mass-intensive loads, such as gravity and inertial loads. These loads are self-equilibrating for a free body. If we expand equation (1) ,

$$\begin{Bmatrix} \mathbf{P}_{0net} \\ \mathbf{P}_{support} \end{Bmatrix} = \begin{bmatrix} \mathbf{K}_{\ell\ell} & \mathbf{K}_{\ell r} \\ \mathbf{K}_{r\ell} & \mathbf{K}_{rr} \end{bmatrix} \begin{Bmatrix} \delta_{0\ell} \\ \mathbf{0} \end{Bmatrix} \tag{2}$$

$\mathbf{K}_{\ell\ell}$ is the restrained structure stiffness matrix. $\mathbf{P}_{support}$ is the load vector at the "support" which, for a free body *should* be zero, again because the net loads are self-equilibrating. The deflections of the unrestrained points are related to the net loads by

$$\delta_{0\ell} = \mathbf{K}_{\ell\ell}^{-1}\mathbf{P}_{0net} \tag{3}$$

The important steps are to relate the net loads to the externally applied loads, and the restrained structure deflections to the unrestrained structure deflections.

The global deflections δ of the unrestrained vehicle are the sum of the eff ects of the actual displacements of the support (including rotations), and the deflections $\delta_{0\ell}$ of the unrestrained points relative to the support. This is expressed by

$$\delta = \Phi_r\delta_r + \delta_{res} \tag{4}$$

where Φ_r is the rigid body modeshape matrix that distributes rigid body motions at the support to all the nodes and δ_r is the vector of rigid body motions at the support points. The crucial step is to stipulate that the body axes that the rigid body displacements describe are *mean axes*. The condition that has to be met in this case is that the vector of total nodal displacements and the rigid body modes are mass orthogonal. This "mean axis contraint" is enforced by

$$\Phi_r^T\mathbf{M}\delta = \Phi_r^T\mathbf{M}\Phi_r\delta_r + \Phi_r^T\mathbf{M}\delta_{res} = \mathbf{M}_r\delta_r + \Phi_r^T\mathbf{M}\delta_{res} = \mathbf{0} \tag{5}$$

where $\mathbf{M}_r$ is the rigid body mass matrix (generally a 6x6, positive definite matrix). From eqn. (5) we obtain,

$$\delta_r = -\mathbf{M}_r^{-1}\Phi_r^T\mathbf{M}\delta_{res}$$

When the above is substituted back into (4), we obtain,

$$\delta = \left[\mathbf{I} - \Phi_r\mathbf{M}_r^{-1}\Phi_r^T\mathbf{M}\right]\delta_{res} = \mathbf{R}\delta_{res} \tag{6}$$

The matrix $\mathbf{R}$ is the inertia relief matrix, although the reason for this name becomes clearer when the relation of the net loads to the externally applied loads is determined as follows.

The resultant overall external loads at the support are given by

$$\mathbf{F}_r = \Phi_r^T\mathbf{P}$$

The vector $\mathbf{F}_r$ is typically comprised of the three aerodynamic forces, the three aerodynamic moments, and propulsive forces and moments. The accelerations produced by these loads are found by premultiplying $\mathbf{F}_r$ by the inverse of the rigid body mass matrix,

$$\mathbf{a}_r = \mathbf{M}_r^{-1}\mathbf{F}_r = \mathbf{M}_r^{-1}\Phi_r^T\mathbf{P}$$

Note that $\mathbf{a}_r$ includes the effect of gravity. This is true by the equivalence principle that states that we cannot distinguish between the inertial load resulting from an acceleration and the load resulting from gravitation. Now the rigid accelerations at all of the node points can be found from

$$\ddot{\delta} = \Phi_r \mathbf{a}_r = \Phi_r \mathbf{M}_r^{-1} \Phi_r^T \mathbf{P}$$

The inertial loads, or more correctly the "mass-intensive" or "body" loads (including gravity), at each node point are then

$$\mathbf{P}_{inertial} = -\mathbf{M}\ddot{\delta} = -\mathbf{M}\Phi_r \mathbf{M}_r^{-1} \Phi_r^T \mathbf{P}$$

Finally the net loads are the combination of the externally applied loads and the body loads,

$$\mathbf{P}_{net} = \mathbf{P} + \mathbf{P}_{inertial} = \left[\mathbf{I} - \mathbf{M}\Phi_r \mathbf{M}_r^{-1} \Phi_r^T\right]\mathbf{P} = \mathbf{R}^T \mathbf{P} \qquad (7)$$

We may find the restrained structure deflections resulting from the net loads as

$$\begin{Bmatrix} \delta_{0\ell} \\ \mathbf{0} \end{Bmatrix} = \begin{bmatrix} \mathbf{K}_{\ell\ell}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{P}_{0net} \\ \mathbf{P}_{sup\,ports} = \mathbf{0} \end{Bmatrix} \qquad \text{or} \qquad \delta_{res} = \mathbf{a}_{res} \mathbf{P}_{net} \qquad (8)$$

Substituting for the net load vector and premultiplying by $\mathbf{R}$

$$\delta = \mathbf{R}\mathbf{a}_{res} \mathbf{R}^T \mathbf{P} = \mathbf{a}_f \mathbf{P} \qquad (9)$$

The matrix $\mathbf{a}_f$ is the so-called "free body flexibility" matrix which is singular.

Thus far, there is nothing especially aeroelastic in the development for the unrestrained vehicle. We will begin the aeroelastic formulation by first considering the various components of the externally applied load vector, $\mathbf{P}$. $\mathbf{P}$ is composed of initial aerodynamic and propulsive loads as determined for the rigid vehicle, with incremental loads resulting from deformation of the structure, incremental changes in control surfaces and vehicle attitude. The initial loads can also account for initial vehicle camber, attitude, angular velocity, estimated control deflections, etc. Preferably, these initial loads are determined through means more sophisticated and comprehensive than the lifting surface theories used in aeroelastic modeling. We may then write the load vector as

$$\mathbf{P} = \mathbf{P}_0 + \Delta\mathbf{P}^{prop} + \bar{q}\mathbf{Q}\delta + \bar{q}\mathbf{Q}_t \Delta\mathbf{v}_t + \bar{q}\mathbf{Q}_\omega \Delta\omega \qquad (10)$$

$\mathbf{P}_0$ is the initial aerodynamic and propulsive load vector, $\Delta\mathbf{P}^{prop}$ is the incremental propulsive load vector (if applicable), $\Delta\mathbf{v}_t$ is increments in the aerodynamic trim variables (angles of attack and sideslip, and control deflections), $\mathbf{Q}_t$ is a matrix of GAF's for these trim variables, $\Delta\omega$ is increments in the vehicle's angular rates, and $\mathbf{Q}_\omega$ is the corresponding matrix of GAF's. $\bar{q}\mathbf{Q}\delta$ is the vector of aerodynamic loads resulting from deformation.

Using equation (10) in equation (9), we may obtain the deformation vector,

$$\delta = \mathbf{A}^{-1}\mathbf{a}_f \mathbf{P}_0 + \mathbf{A}^{-1}\mathbf{a}_f \Delta\mathbf{P}^{prop} + \bar{q}\mathbf{A}^{-1}\mathbf{a}_f \mathbf{Q}_t \Delta\mathbf{v}_t + \bar{q}\mathbf{A}^{-1}\mathbf{a}_f \mathbf{Q}_\omega \Delta\omega \qquad (11)$$

where $\mathbf{A}^{-1} = \left[\mathbf{I} - \bar{q}\mathbf{a}_f \mathbf{Q}\right]^{-1}$ is the aeroelastic deflection amplification matrix.

Substituting equation (11) into equation (10),

$$\mathbf{P} = \mathbf{B}\mathbf{P}_0 + \mathbf{B}\Delta\mathbf{P}^{prop} + \bar{q}\mathbf{B}\mathbf{Q}_t\Delta\mathbf{v}_t + \bar{q}\mathbf{B}\mathbf{Q}_\omega\Delta\omega \qquad (12)$$

where $\mathbf{B} = \left[\mathbf{I} + \bar{q}\mathbf{Q}\mathbf{A}^{-1}\mathbf{a}_f\right] = \left[\mathbf{I} - \bar{q}\mathbf{Q}\mathbf{a}_f\right]^{-1}$ is the aeroelastic load amplification matrix. Equation (12) gives the external loads acting on the vehicle including aeroelastic effects. Now we consider trim.

Six Degree-of-Freedom Equations of Motion

The six DOF EOM of the vehicle may be written,

$$\mathbf{F} = \mathbf{M}_r g\Theta + \mathbf{M}_r\dot{\mathbf{v}} + \tilde{\Omega}\mathbf{M}_r\mathbf{v} \qquad (13)$$

F is comprised of the three external force componets and three external moment. As such, it can be expressed in terms of the external nodal forces of equation (12) as

$$\mathbf{F} = \Phi_r^T\mathbf{P} \qquad (14)$$

$\mathbf{M}_r$ is the rigid body mass matrix, seen earlier. g is the magnitude of the acceleration of gravity, and the vector Θ is defined, for the support point at (or at least near) the vehicle's c.g., by

$$\Theta = \lfloor sin\theta \quad -cos\theta\, sin\phi \quad -cos\theta\, cos\phi \quad \mathbf{0}_{1\mathrm{X}3} \rfloor^T$$

which arises from the vector transformation of the gravity vector from the local vertical frame to the body frame. Hence, θ and ϕ are two of the Euler angles for this transformation. If the support point were not at the c.g., then the zero vector in the lower partition would contain offset components between the c.g. and the support point. Note that a traditional flight mechanics body frame is assumed here: $+x$ - axis forward; $+y$ - axis right; $+z$ - axis out the underside. The matrix $\tilde{\Omega}$ is defined by

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\omega} & \mathbf{0} \\ \mathbf{0} & \tilde{\omega} \end{bmatrix} \qquad \text{where} \qquad \tilde{\omega} = \begin{bmatrix} 0 & -r & p \\ r & 0 & -q \\ -p & q & 0 \end{bmatrix}$$

and p, q, and r are the body axis components of the angular velocity vector. The effect of $\tilde{\omega}$ is to accomplish the vector product of the angular velocity vector with another vector. Finally, **v** is comprised of the body axis components of the translational and angular velocities of the vehicle,

$$\mathbf{v} = \lfloor u \quad v \quad w \quad p \quad q \quad r \rfloor^T = \lfloor V cos\alpha\, cos\beta \quad V sin\beta \quad V sin\alpha\, cos\beta \quad p \quad q \quad r \rfloor^T$$

For relatively mild maneuvers and small aerodynamic angles we may make the approximations,

$$\tilde{\Omega}\mathbf{M}_r\mathbf{v} \approx m\lfloor 0 \quad Vr \quad -Vq \quad \mathbf{0}_{1\mathrm{X}3} \rfloor^T = m\mathbf{a}_c \qquad \text{and} \qquad \mathbf{v} \approx \lfloor V \quad V\beta \quad V\alpha \quad p \quad q \quad r \rfloor^T$$

recognizing the first three entries in $\mathbf{a}_c$ as centripital acceleration components. Combining equations (12), (13), and (14),