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APPLICATION OF PARAMETRIZED POST-NEWTONIAN METHODS TO THE GRAVITATIONAL IS OF SATELLITE ENERGY EXCHANGE DATA

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Introduction

Project Satellite Energy Exchange (SEE) is a free-flying, high altitude satellite that utilizes space to construct a passive, low-temperature, nano-g environment in order to accurately measure the poorly known gravitational constant G plus other gravitational parameters that are difficult to measure in an earth-based laboratory Eventually data received from SEE must be analyzed using a model of the gravitational interaction including parameters that describe deviations from general relativity and experiment. One model that can be used to *fit* the data is the Parametrized post-Newtonian (PPN) approximation [4] of general relativity (GR) which introduces ten parameters which have specified values in (GR). It is the lowest-order, consistent approximation that contains non linear terms. General relativity predicts that the Robertson parameters, γ (light deflection), and β (advance of the perihelion), are both 1 in GR. Another eight parameters, α_k , j=1,2,3, and ζ_k, k=1,2,3,4 and ξ, are all zero in GR. Non zero values for α_k parameters predict preferred frame effects; for ζ_k , violations of globally conserved quantities such as mass, momentum and angular momentum; and for ξ, a contribution from the Whitehead theory of gravitation, once thought to be equivalent to GR. In addition, there is the possibility that there may be a preferred frame for the universe. If such a frame exists, then all observers must measure the velocity w of their motion with respect to this universal rest frame. Such a frame is somewhat reminiscent of the concept of the *ether* which was supposedly the frame in which the velocity of light took the value *c* predicted by special relativity. The SEE mission can also look for deviations from the r^{-2} law of Newtonian gravity, adding parameters α and λ for non Newtonian behavior that describe the magnitude and range of the r^{-2} deviations respectively. The foundations of the GR supposedly agree with Newtonian gravity to first order so that the parameters α and λ are zero in GR. More important, however, GR subsequently depends on this Newtonian approximation to build up the non linear higher-order terms which forms the basis of the PPN frame work.

The SEE Encounter and Post-Newtonian Approximation

Other than establishing the physical design of the SEE platform, it is also necessary to determined how the data taken can be analyzed. Ostensibly the major experiment that occurs is called Satellite Energy Exchange which occurs when a small mass satellite in a slightly lower lying orbit overtakes a very much larger mass satellite, called a shepherd, in a slightly higher orbit. As the small mass nears the shepherd, the mutual attraction increases the total energy of the smaller satellite which subsequently moves to a *higher* orbit and *slows* down. The effect on the much more massive shepherd is nearly negligible. In the frame of the shepherd, the smaller mass satellite approaches in a slightly lower orbit, moves to a slightly higher orbit and then appears to recede in the direction from which it came subsequently meeting the shepherd again 360° away where another SEE encounter occurs again. Such *orbits*, as seen in the rotating frame of the shepherd, are called *horseshoe* orbits. Experimentally, the closest point of approach of the small satellite to the shepherd is a direct measurement of the *local* gravitational constant G_L. In the somewhat similar Cavendish experiment, which measures the relative acceleration of two masses, the equivalent local gravitation constant depends on *seven* PPN parameters plus the relative motion of the of the laboratory with respect to the universal rest frame. This analysis must be completed for the SEE encounter itself. Several more experiments can be conducted in the SEE capsule such as a long term measurement of the constancy of the gravitational "constant" itself. This introduces another parameter which relates any deviation to the Hubble constant. The motion of the shepherd with respect to the sun and the moon (as well as the Earth which is the primary gravitational source for the shepherd) allows the possibility of polarization of orbits and the measurement of the difference between active and passive gravitational mass, or equivalently the direct measurement of the possible violation of the weak equivalence principle. These effects can be described in terms of the Nordtvedt effect which itself depends on nine PPN parameters.

Experiments and PPN Additional Parameters

A short list of (possible) additional experiments that can be accomplished in the SEE platform is given by:

- The possibility of non r^{-2} behavior (two additional parameters).
- New experiments such as the behavior of an initially symmetric, gravitationally bound Particle Cloud in a zero-g, drag-free orbit (at least two PPN parameters).
- Perihelion shift of the shepherd (at least six PPN parameters).
- Orbital resonances with the Earth's ordinary geopotential field for harmonic components of the rotation period of the Earth (resonance for α_2).
- Scalar—Vector—Tensor theories (etc.) (additional parameter(s)).
- The effect of the spin of specifically prepared test particles(polarized and shielded from the magnetic field) can be measured (five PPN parameters).

Status of PPN analysis for SEE

In most cases the actual contributions of the PPN parameters have not been completely analyzed for experiments proposed for the SEE mission. Will has complied a list of the applications of the PPN formalism for the classical test of relativity, such as the perihelion advance and light deflection plus more modern tests such as time delay of radar signals and lunar laser ranging. Thus without a basic foundation in the PPN formalism, it will be impossible to extract meaningful consequences of the SEE data.

Introduction to and Consequences of PPN Analysis

The design of the SEE experimental package beyond measurement of G is in the formative stages. It is estimated In order to understand the PPN formalism, a program was initiated this summer to familiarize members of the SEE team on the fundamentals of the PPN framework. This review consisted of a basic introduction to the essentials of gravitational theory plus an introduction to the post-Newtonian approximation and subsequently the extension of general relativity to a general framework called the *Parametrized* Post-Newtonian approximation from which likely deviation of GR can be tested. The importance of this familiarization will become evident below.

From the design characteristics of the SEE satellite that the absolute value of G can be measured to at least one part in 10^6 for each event; and the temporal behavior, to one part in $10^{12\text{-}13}$ depending on the duration of the tracking of the shepherd, i.e. the total path length of the shepherd. It turns out that the limits on the temporal behavior of G is close to the value necessary to test speculations concerning unified theories such as string or supergravity theories. However the actual dependence on parameters (PPN or otherwise) has not been calculated for most of the proposed experiments. This means, for example, that the actual SEE encounter (or any other experiment proposed for the SEE satellite) must be recalculated in terms of the PPN metric in order to investigate the actual dependence on the PPN parameters. These calculations were not attempted during the period of this research effort but are now well within the capability of the team.

An important aspect of the above calculations allows the establishment of the connection between the various proposed experiments that will be conducted during a SEE Mission. It is obvious from the discussions above, that a single experiment will not, for the most part, be able to set the value of the any single PPN parameter by itself. In general, each experiment will yield data that is a complicated function of PPN parameters that must be compared with every other experiment in the reduction of data to experimental limits on the PPN parameters. **In general, nothing should be assumed *a priori* concerning the value of these parameters.** If the set of experiments is not sufficient to determine **all** the individual parameters, then experimental limits can only be placed upon functional relationships among the parameters.

A second aspect, once these parametric functions for each experiment have been analyzed, is the crucial input in the necessary error analysis of the SEE mission itself. **This is an essential task necessary for the detailed design of the mission itself.**

Modified Malin Theory

A new experimental configuration for SEE was developed as a test case for extending the types of experiments that could give an independent relationships amongst the PPN parameters. Such an example is a configuration of particles that could be observed in the SEE capsule. The thrust of this work is in a slight modification of Malin's theory which is a non viable, alternate theory of gravity. As it stands, Malin's theory does not satisfy most of the requirements for a viable theory of gravity such as the correct Newtonian limit and globally conserved quantities such as total momentum and angular momentum. The field equation for Malin's theory is given by [3]

$$R_{k\ell} = \kappa T_{k\ell} \tag{1}$$

where $\kappa = 4\pi G$, G is the gravitational constant, $R_{k\ell}$ is the Ricci tensor, and $T_{k\ell}$ is the matter energy-momentum tensor. If the left hand side of the above equation consisted of the Einstein tensor $\frac{1}{2}G_{k\ell} = \frac{1}{2}\left(R_{k\ell} - \frac{1}{2}g_{k\ell}R\right)$, where $G_{k\ell}$ is the Einstein Tensor, $g_{k\ell}$ is the metric tensor and R is the Riemann scalar, then all the limits and conservation laws described above would be satisfied provided that $\nabla_i T_j^{\ i} = 0$. In general, $\nabla_i R_j^{\ i} \neq 0$ which implies that the divergence of the energy-momentum is also not zero. However it turns out that if the divergence of the energy-momentum satisfies the condition $\nabla_i T_j^{\ i} = \lambda R_j$ where λ is a function of O(2) in the gravitational potentials, then it is possible to slightly modify Eq.(1) so that the effective energy-momentum tensor T_{eff}^{jk}

$$G^{jk} = \kappa T^{jk}_{eff} = \kappa\{T^{jk}_{PF} + \frac{1}{2} g^{jk}[\zeta_3 \rho\Pi + 3\zeta_4 p]\} \quad (2)$$

which does not satisfy global conservation laws,. The ζ_3 and ζ_4 PPN parameters then represent fourth order corrections to Euler's equations of motion and represents a theoretical framework for testing the magnitude of these corrections. A possible experimental test bed is discussed below for their measurement.

Experimental Configuration

Consider a collection of *N* small, spherically symmetric, electrically neutral particles of uniform density. Let the particles be collected into a spherical shape of Radius *R*. In zero g, this could be a *bag* that that positively holds that particles in a spherical until the experiment begins. If the bag could be remove, and the spherical shape were initially maintained, the particles would have the effective low temperatures of about 80 K of the SEE environment. The gravitational force on individual particles of mass *m* at radius $r < R$ is approximately given by

$$\mathbf{F}(r) \approx -G\rho(r)V(r)\frac{\hat{\mathbf{r}}}{r} \approx -G\left(\frac{Nm^2}{R^3}\right)\hat{\mathbf{r}} \quad (6)$$

for a spherically symmetric system with uniform density. Then for $R=20\ r_0$, where r_0 is the radius of a *20-g* ball of density about $5\ g/cm^3$, $N \approx 8000$ and $r_0 \approx 1$ cm. The approximate binding force on a particle at the surface of the spherical aggregation is $F(R) \approx 5 \times 10^{-2}$ *dynes.* At a temperature of about *80 K,* the mean motion of the particles in the aggregation will be about *40 nm/s.* This velocity should be compared with the escape velocity of a single ball from the surface of the spherical aggregate $v_{exc} \approx 3.3 \times 10^{-3}\ cm/s$ which shows that the thermal velocity is negligible in comparison with the kinetic energy necessary for escape from the spherical aggregate. Thus the spherical aggregate appears to be a bound system with regard to thermal perturbations. Over a long period of time, these tiny perturbations will alter both the shape and density distributions $\mathbf{F}(r) \to \mathbf{F}(\mathbf{r}),\ \ \rho(r) \to \rho(\mathbf{r})$ which will bring in multipole moments of the mass distribution. In orbit about the Earth, the gravity gradient forces will also perturb the spherical shape. This perturbation will eventually lead to collisions between the balls. The difference in orbital velocity from a small ball at the top of spherical aggregate and one at the bottom (*bt*) along the radius of the satellite to the Earth is given by $\Delta v_{bt} \approx 1.9 \times 10^{-2}\ cm/s$ and the relative motion of *nearest neighbor(nn)* particles is given $\Delta v_{nn} \approx 9.5 \times 10^{-4}\ cm/s$ where R_E is the orbital radius. The nearest neighbor velocity is an indication of stability; however, it is the ratio:: $\Delta \upsilon_{esc} :: \Delta \upsilon_{nn} \approx 3$ which will determine whether a particle will acquire escape velocities. Thus the system appears to be marginally stable. There are several methods that could be used to obtain useful data for this system.

- Test for anomalous (heat) energy dissipation. This would be directly related to the internal energy and pressure of the system and therefore to ζ_3 and ζ_4.
- Use the evolution of the morphology of the aggregate over a long period of time to study the microstructure of the aggregate[Gokhale, 1996].
- Analyze the fractal dimension of the aggregate which is a direct measure of the force law between the small balls [Slobodrian, 1996].

Post-Newtonian Approximations Data Base

One of the first goals in this ongoing study was to collect a comprehensive data base to facilitate the study of the post-Newtonian approximation and its application to Project SEE. This has resulted in a collection of papers and references, collected by Kelly Smith (Accompanying Student) comprising over 400 entries dedicated to research in the area of the post-Newtonian approximation and the PPN theory. This data base, compiled using Microsoft Access, is now available on disc from Larry Smalley, Department of Physics, University of Alabama in Huntsville, Huntsville AL 35899 or by email: smalley@pluto.cs.uah.edu.

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