A SEARCH FOR r-MODES FROM 1825 TO THE PRESENT

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ABSTRACT

Global oscillations (r-modes) of the Sun's outer convective envelope with periods ~ 1 month and longer have been detected in several short data strings of several years duration. To test whether r-modes might persist beyond one 11 year cycle, the daily sunspot numbers from 1825 to the present were analyzed. Good evidence--but confidence level less than 3σ--was found for most of the 14 r-modes with spherical harmonic index $\ell \leq 5$ that can exist in the presence of solar differential rotation. The characteristic rotation rate of almost every such r-mode was detected, displaced systematically from its expected value by only 0.15%. If this probable detection is real, then most low harmonic r-modes have lifetimes exceeding one century and the rotation of the Sun's outer layers varies by < 0.05%, except possibly at solar minimum.

1. INTRODUCTION

Global oscillations of the convection zone with long oscillation periods (r-modes) that exceed one solar rotation can interact with and modulate the strength of convection and other flows. The modulation grows stronger as the characteristic time scale of the flow approaches that of the oscillation. The mainly horizontal motions of an r-mode provide vorticity and shear that assists or inhibits (at different locations) other flows in the Sun. For example, the effect of long wavelength r-modes on mixing length convection was derived by Wolff (1995). Sunspot activity should also be modulated by r-modes since the location and number of spots has to be associated with some kind of flow inside the Sun. Uncertainty about whether these effects are significant and whether r-modes live a long time are slowly being answered by analysis of observations.

The first small piece of evidence that there might be a few r-modes active in the Sun came from analyzing three years of irradiance data near solar maximum (Wolff and Hickey, 1987). The possibility that whole families of r-modes were being created or strengthened at certain instants in the past was studied by Wolff (1996, 1997) who found that, after an initial pulse of high solar activity, sunspot numbers varied for several years in a somewhat regular manner that could be explained by the properties of r-modes. Very roughly, 20% of the fluctuation in sunspot number seemed attributable to r-mode modulation during the first 500 days following a pulse.

None of these papers say that r-modes live indefinitely. Indeed, there is some indication of decay with a time constant of roughly two years. Strong damping due to turbulent viscosity in the outer 4 Mm of the solar radius (Wolff 1995) would destroy these modes even faster unless strong and frequent driving was present deeper inside the Sun. But possible driving mechanisms are not all known and none are worked out quantitatively. This leaves solar observations as the only immediate way to discover if r-modes can live for decades or longer.

In this paper, theoretical r-mode rotation rates are displayed in §2 and detected in the Fourier spectrum of daily sunspot numbers in §3.

2. COMPUTED ROTATION RATES OF r-MODES

2.1 Solar Rotation

Stellar rotation is necessary for an r-mode oscillation. The family of r-modes discussed herein are trapped in a spherical shell which occupies the outer few percent of the solar radius. In this shell, solar rotation varies with latitude and depth and varies by lesser amounts with time. The mean solar surface rotation as a function of latitude has been measured using sunspots as tracers (Newton and Nunn, 1951) or the Doppler effect which directly senses plasma speeds (Howard and Harvey, 1970). Now, analysis of the five minute oscillations (p-modes) can determine both the latitude and depth dependence of rotation (e. g., Thompson, et al., 1996) but only for recent years. At low and mid latitudes, the angular rotation rate of the outer solar layers increases with depth.

Rotation seems to vary during the 11 year cycle but the amount is not sure. Using sunspot positions from 1874 to 1976 Balthazar, et al. (1986) confirmed earlier reports that the fastest rotation occurs near solar minimum. This is not sinusoidal but a narrow peak lasting one or two years in which rotation of the outer layers is about 1% faster. Yoshimura and Kambry (1993) averaged 11 year intervals and found one secular change in rotation which accumulated to almost 2% after several decades. These results apply to the sunspot latitudes. When all latitudes are averaged using full-disk, Doppler techniques, Ulrich and Bertello (1996) found that their 6 years of data showed no variation in annual averages exceeding their detection threshold of 0.3%. While the literature gives no definitive result, the review by Schroter (1985) states that "a faster rotation of the equatorial belt around sunspot minimum seems therefore to be the first established fact regarding a cycle dependence of solar differential rotation".

2.2 Effect of Differential Rotation

When a nearly spherical star is rotating like a solid body at the rate ν_∞, a very simple law describes the rotation rate of an r-mode,

$$\nu_{rot} = \nu_{\infty}\left[1 - \frac{2}{\ell(\ell+1)}\right] \qquad (1)$$

where ℓ is the principal index of the spherical harmonic function $Y_{\ell}^{m}(\theta, \phi)$ that controls the surface distribution of the oscillation. But rotation that varies with latitude causes ν_{rot} to depend also on m since the various azimuthal states for a given ℓ sample different ranges of latitude. The squares on Figure 1 shows the rotation rates from equation (1) and how they are changed by the Sun's differential rotation into new rates (plus signs). The 14 lowest harmonic modes that can exist in the Sun are plotted and each is identified by the harmonic numbers (ℓ, m) of the mode. These rotation rates were computed and tabulated by Wolff (1998) using a generalized perturbation method. As their names imply, the fast mode rotates faster in an inertial frame than the slow mode with the same ℓ, m. Fast and slow modes have a clear physical distinction only in a uniformly rotating star where the fast modes are in geostrophic balance and remain fixed relative to the stellar fluid while slow modes drift backwards in longitude. Solar differential rotation causes all r-modes to drift in longitude relative to the fluid but the fast modes tend to drift less.

The presence of a global velocity field (e. g., an r-mode) can encourage or discourage local convection depending where the convective event is located. Such nonlinear interactions between r-modes and convection were discussed by Wolff (1995, 1997). Assuming that solar activity is also located where there is stronger convection, then it should also tend to occur at certain locations within the r-mode velocity field. Thus, solar activity should be modulated by the rotation rate of the r-mode. We will test for that now.

3. ANALYSIS OF SOLAR ACTIVITY SINCE 1825

3.1 Four day averages

The Royal Observatory of Belgium maintains the sunspot record at World Data Center-C1 where values of the international sunspot index, R_I , are listed for every day from 1849 to the present (web address, www.oma.be). From this data, four-day averages were computed which reduced the number of data points to < 14,000. An earlier period with incomplete data was also used. In the years 1825 through 1848, most four-day intervals contain at least one measured value of the daily sunspot index from which an average was computed. But, for those four-day intervals where no data was available, the one year running mean of R_I was used. The longest strings of consecutive days without sunspot data were: 36, 21, 17, 13, 13, 11, ..., days. The resulting time series of R_I studied in this paper contains 15,700 four-day intervals beginning at 1825.5 and ending at 1997.4. By including 24 years of early data and interpolating across its gaps, a bit of high frequency noise is introduced into the Fourier spectrum but 16% better spectral resolution is achieved.

3.2 Fourier Spectrum

Solar activity is closely linked to a number of nonlinear processes, such as deep convection, near-surface convection, vortex flow, and magnetic discharges during flares. Whatever regular time behavior may underly solar activity, its wave shapes are more likely to be pulses than simple sinusoids. Thus, Fourier analysis is not ideal and can produce several confusing harmonics for each true periodicity.

In spite of this limitation, the Fourier spectrum of fluctuations in sunspot number was used by Wolff (1983) to detect about 20 beat frequencies < 135 nHz which he attributed to g-modes in the Sun's interior. Since monthly mean data was used, the spectrum had a Nyquist frequency of 190 nHz, preventing study of the range 250 – 380 nHz where beats with the $\ell = 1$ mode would be located. Each of these many g-mode beats between 0 and 380 nHz should introduce several harmonics into the spectrum. These represent a significant noise source for the present study which will search for the synodic rotation periods of r-modes in the outer solar layers.

The lower curve on Figure 2 is a portion of the Fourier spectrum of four-day-averaged R_I described above. Smoothing with a 2 nHz boxcar filter (middle curve) shows more clearly the broad rise and fall of the spectrum beginning at 390 nHz and ending at either 470 or 500 nHz. The low-frequency part of this enhancement might be explained as the signature of long-lived sunspots whose synodic rotation rates are marked for several latitudes from 0° to 40°. These values come from the expression $13.38 - 2.68 \sin^2(\text{latitude})$ in units of degrees/day as measured by Newton and Nunn (1951) and averaged over six solar cycles from 1878 – 1944. The rate for the mean latitude of sunspots (about 18°) is not far from the frequency where the enhancement is a maximum. Since most sunspots occur between 5° and 30° latitude, they could be responsible for some of the tall lines in that portion of the lower curve. A wider range of frequencies is covered by the r-modes (see below). But neither sunspots, r-modes, or any known fluid layer inside the Sun rotates fast enough to explain the higher-frequency half of the enhancement.

The insert on Fig. 2 magnifies the frequency scale and shows the dominant 11 year signal near 3 nHz and its two sidelobes that are about half the height of the central peak. This apparent triplet, whose first harmonic is visible about 5.8 nHz, may also be aliasing to cause the other possible triplets centered at 1.1 and 3.8 nHz. Since the 3 nHz feature is roughly 40 times stronger than the lines of interest in this paper it can introduce many confusing triplets (aliases) at $\pm$ 3 nHz from any true frequency in the bottom curve on the figure.

For further general insight into the spectrum, be aware that the fast and slow branches of r-mode rotation rates on Figure 1 each converge toward the same series limit near 465.2 nHz as ℓ approaches infinity. An observer on earth will sense these rates as synodic frequencies

$$\nu_{syn} = \nu_{rot} - 31.69 \text{ nHz}$$

so the series limit becomes 433.5 nHz. This is about 7 nHz higher than the fastest sunspot rotation rates. The noticable little drop at this location in the smoothed curve could be coincidental or could mark the end of r-mode influence on the spectrum.

3.3 The r-mode Signals

We will test for r-modes by searching for matches between individual peaks in the R_I spectrum and the rotation rates of modes plotted on Figure 1. The synodic rates ν_{syn} apply to the sunspot record. The first test amounts to a Monte Carlo technique. A free parameter b was scanned near 1.0 in very fine steps of 5×10^{-5} to see when the quantities $b\nu_{syn}$ best match the observed spectrum. The success criterion is that the average height of the spectrum at the 14 r-mode rotation rates should be a maximum near b = 1. One can interpret b as correcting for some systematic error either in computed r-mode rates or in the assumed long term average solar rotation. In the latter case, the 172 year average rotation of the outer solar layers would be larger by the factor b than the recently measured curve of Thompson, et al. (1996) from which the r-mode rotation rates were computed.

Figure 3 shows the mean spectral height as a function of b. Each small maximum in the curve is due to shifting the 14 theoretical lines by about one resolution element in the observed spectrum. The curve has a prominent maximum at b = 1.0015 which is very close to the expected value of 1. This peak remained the highest when the plotted range of ± 3% was extended to ± 15%. Under a null hypothesis that r-mode rotation rates do not affect the sunspot record, each small maximum on Figure 3 would represent an independent trial of how well 14 random frequencies can match the observed spectrum. There are about 100 peaks on Figure 3 and 500 on the extended scan. Of these 500 trials in the range $0.85 < b < 1.15$, the highest peak lies very close to 1.0 as an r-mode model would predict. Thus the null hypothesis has only a small probability of being correct.

The second test is more precise and examines the 14 r-modes individually. Figure 4 magnifies the above shown R_I spectrum which is rich with peaks. While many peaks are undoubtedly due to random solar behavior or to noise in the early observations, I believe that many others are harmonics and aliases of true periodicities such as mentioned in §3.2. They should ultimately be understood and removed by a method designed for a nonlinear phenomenon. Vertical lines are drawn at the 14 r-mode frequencies, $1.0015\,\nu_{syn}$. In all but one or two cases, the theoretical line falls within the half width of a peak in the spectrum. Many of these peaks are relatively prominent. The line at 420.3 nHz is a clear failure. Possibly, it fails because the very strong peak at 421.1 has distorted the spectrum in its vicinity.

At even greater magnification, Figure 5 shows the spectral segment immediately surrounding each theoretical frequency. From the top down are the seven fast modes in order of descending frequency. Next are the slow modes in the same frequency order and finally the average of the 14 curves, magnified 3 times vertically for clarity. Seven of the 14 r-mode rotation rates are almost exactly centered on a spectral peak. To compute the probability that this agreement is accidental, make the simplifying assumption that the half widths of all spectral lines cover 50% of the frequency axis. Then the actual probability that k frequencies out of 14 randomly chosen ones will accidentally fall within the half width of a spectral line is given by the binomial expression

$$P(k) = \frac{14!}{k!(14-k)!}\left(\frac{1}{2}\right)^{14} \qquad (2)$$

For k = 13, 12, and 11 one has respectively, P = 0.0009, 0.007, and 0.029. Thus, the probability that the r-mode rates have accidentally agreed with the observed spectral lines is small, $\cong 10^{-2}$, but not negligible.

The central peak on the average curve at the bottom of Figure 5 has a half width of only 0.2 nHz. Under the r-mode interpretation, this implies that solar rotation varies by $\leq 0.05\%$ during times when the r-modes contributing to this peak are active. But this does not rule out sudden jumps to other rotation rates outside this range especially if they are short lived. For example, the sunspot number is so small close to solar minimum that it makes very little contribution to the spectrum at these frequencies. A small rotation change at solar minimum would be virtually undetectible by this analysis. Thus, the reports cited in §2.1 that rotation of the outer Sun might vary by ~1% might be consistent with the present study if the rotational variation begins and ends in the year or two when sunspot numbers are typically small.

4. Summary

A time series of the daily international sunspot number for the last 172 years was constructed consisting of one data point every four days. The Fourier spectrum of this series was used to test whether solar activity is weakly modulated by r-modes which show no variation in phase. Phase constancy would mean that the rotation of the outer solar envelope must be extremely repeatable from one 11 year maximum to any other.

Figures 3, 4 and 5 showed evidence that sunspot number was modulated by the theoretical rotation rates of the 14 lowest harmonic r-modes in the Sun. The evidence falls short of a three sigma confidence level and was estimated to have a chance, $\sim 10^{-2}$, of being accidental agreement. Shorter data sets analyzed in the past by this author and colleagues have shown stronger agreement which indicates that some of the r-modes may not live without interruption for 172 years.

If many long lived r-modes have been detected, as the probability indicates, then the narrowness of the identified spectral lines would put a very tight constraint on solar rotation except possibly near solar minimum where sunspots are too few to have much imapct on the spectrum. Over most of each 11 year cycle and cumulatively for all such intervals, rotation of the outer solar envelope could have varied only by $< 0.05\%$.

The broad rise and fall of the spectrum on Figure 2 between 390 and perhaps 500 nHz needs a fuller explanation. Both r-modes and recurrent sunspots can cause numerous signals in the lower-frequency half of that range. But if they alone were effective, one would expect a sharp drop down to the apparent noise level beyond 432 nHz (the synodic rate of the fastest rotating r-mode). Instead, the enhancement in the spectrum declines slowly for another 40 to 70 nHz. One possible way to model this is with prograde modes (modes that rotate faster than the fluid they are imbedded in). Such modes exist if one modifies the pure r-mode and allows it also to contain some g-mode characteristics. Lee and Saio (1987) derived such modes for the cores of massive stars. Inertial modes on earth also have both prograde and retrograde branches. Whether such mixed modes might have long term viability in the solar convection zone should be investigated in a nonlinear theory.

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Figure Captions

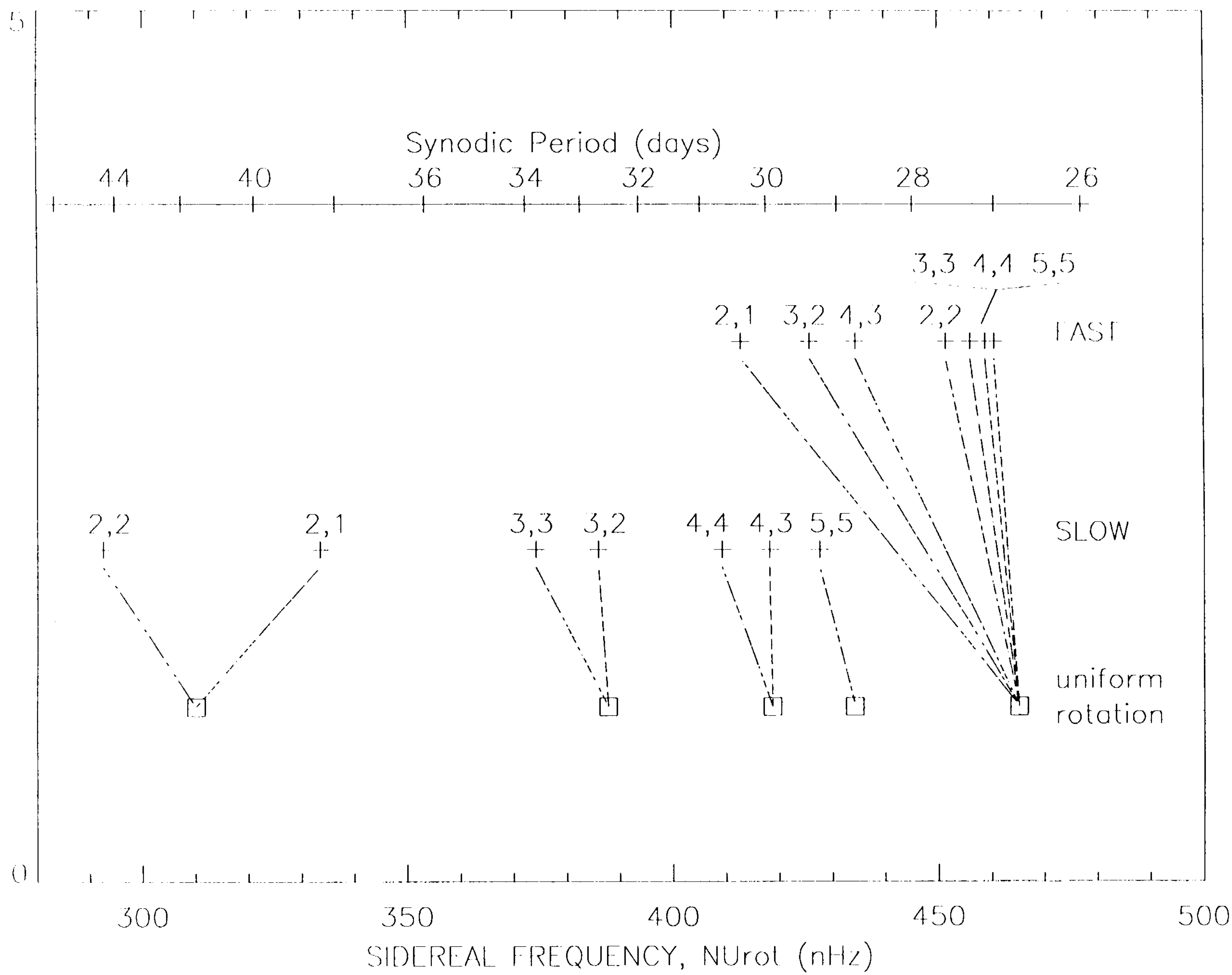
Figure 1. The rotation rates of solar r-modes (+) relative to an inertial frame. The spherical harmonic numbers (ℓ, m) are shown for each mode with $\ell \leq 5$ that is viable in the Sun. The squares apply to the same modes in a "Sun" without differential rotation.

Figure 2. The Fourier spectrum of 172 years of International Sunspot Numbers (bottom curve) and the same after 2 nHz smoothing (displaced upward 60 units). Marked above are the rotation rates of recurrent sunspots at four latitudes and the fastest-rotating r-mode. The insert graph shows that the 11 year line at 3 nHz is roughly 40 times stronger than lines in the bottom curve which will cause many aliases in the bottom curve.

Figure 3. The mean height of the sunspot spectrum at the frequencies, $b\nu_{syn}$, where the 14 values ν_{syn} are theoretically computed r-mode rotation rates. The distinct peak so close to b = 1 is a good indication that r-modes are detected by their modulation of solar activity.

Figure 4. A magnified view of the spectrum with the 14 r-mode frequencies marked for b = 1.0015. Most lines match a local maximum in the observed spectrum and many of the peaks tend to be high. The probability of matching so many lines by accident is very small but it does not reach a 3 sigma level of confidence.

Figure 5. The spectrum very near each of the 14 r-mode lines (each placed at zero on the abscissa). Curves for the seven fast modes are at the top in order of descending frequency, then the seven slow modes in the same order. The average of the 14 curves (bottom, magnified vertically by a factor of 3) shows that its central peak is not broadened by persistent changes in solar rotation exceeding 0.05%.



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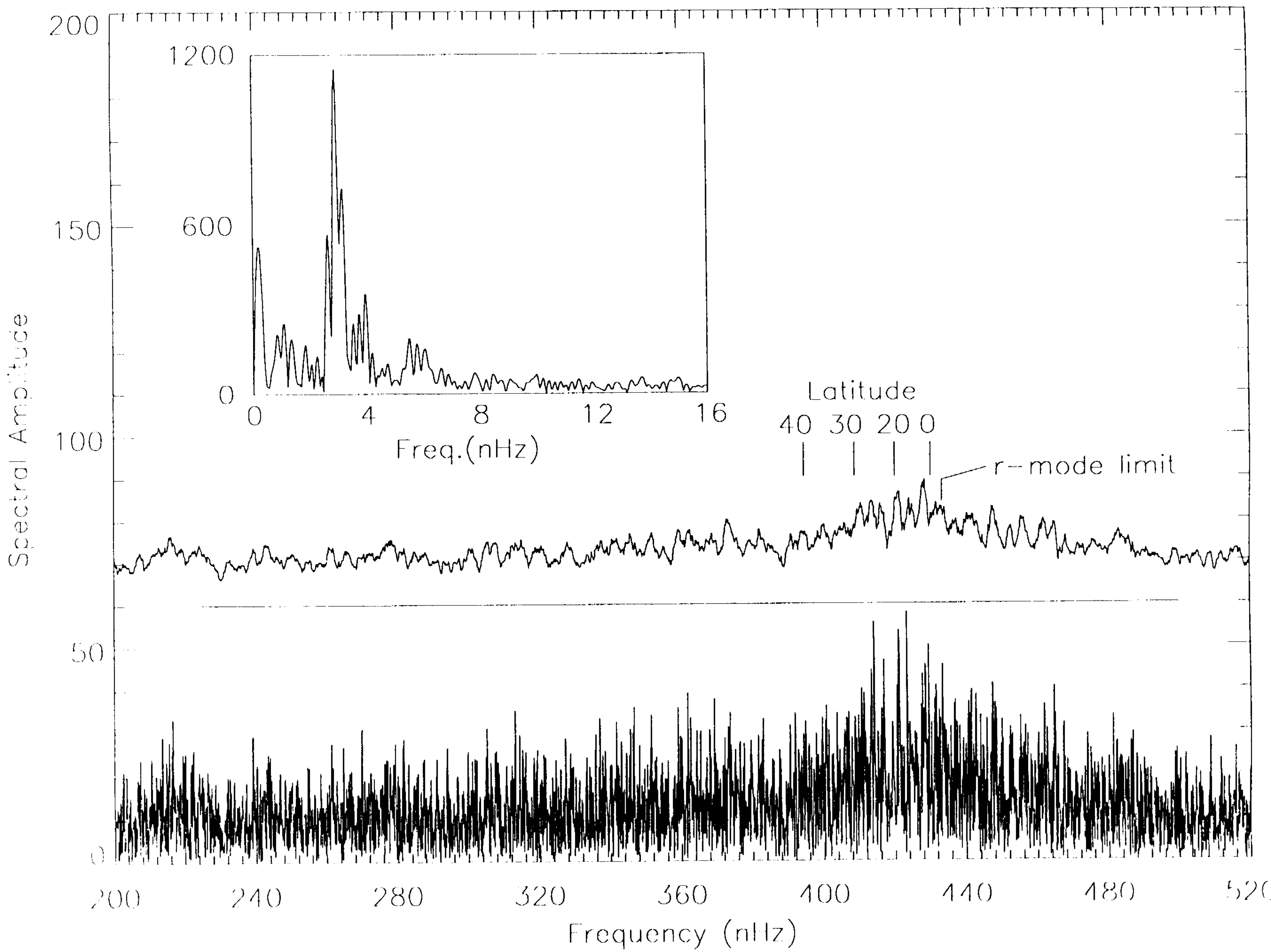

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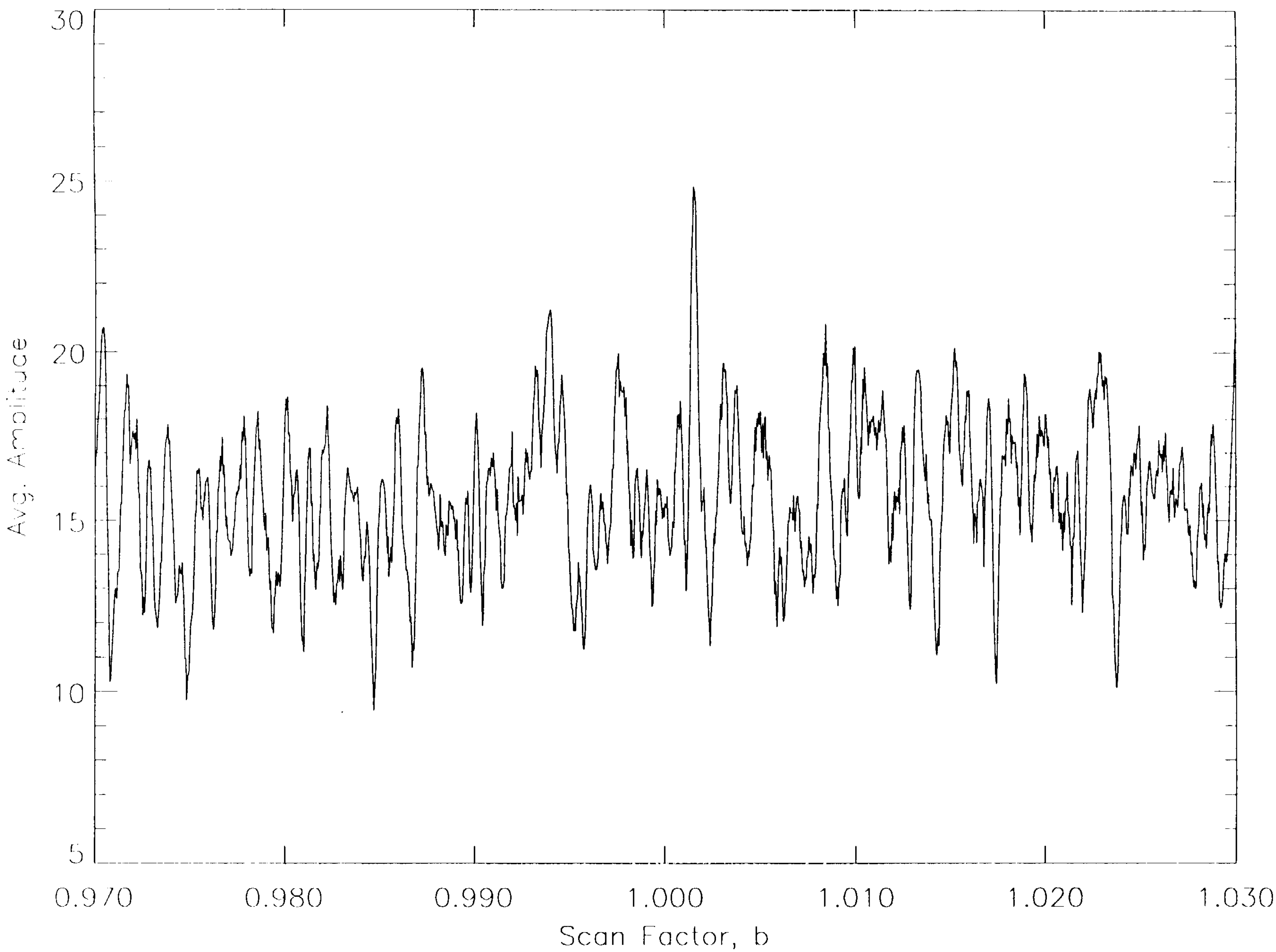

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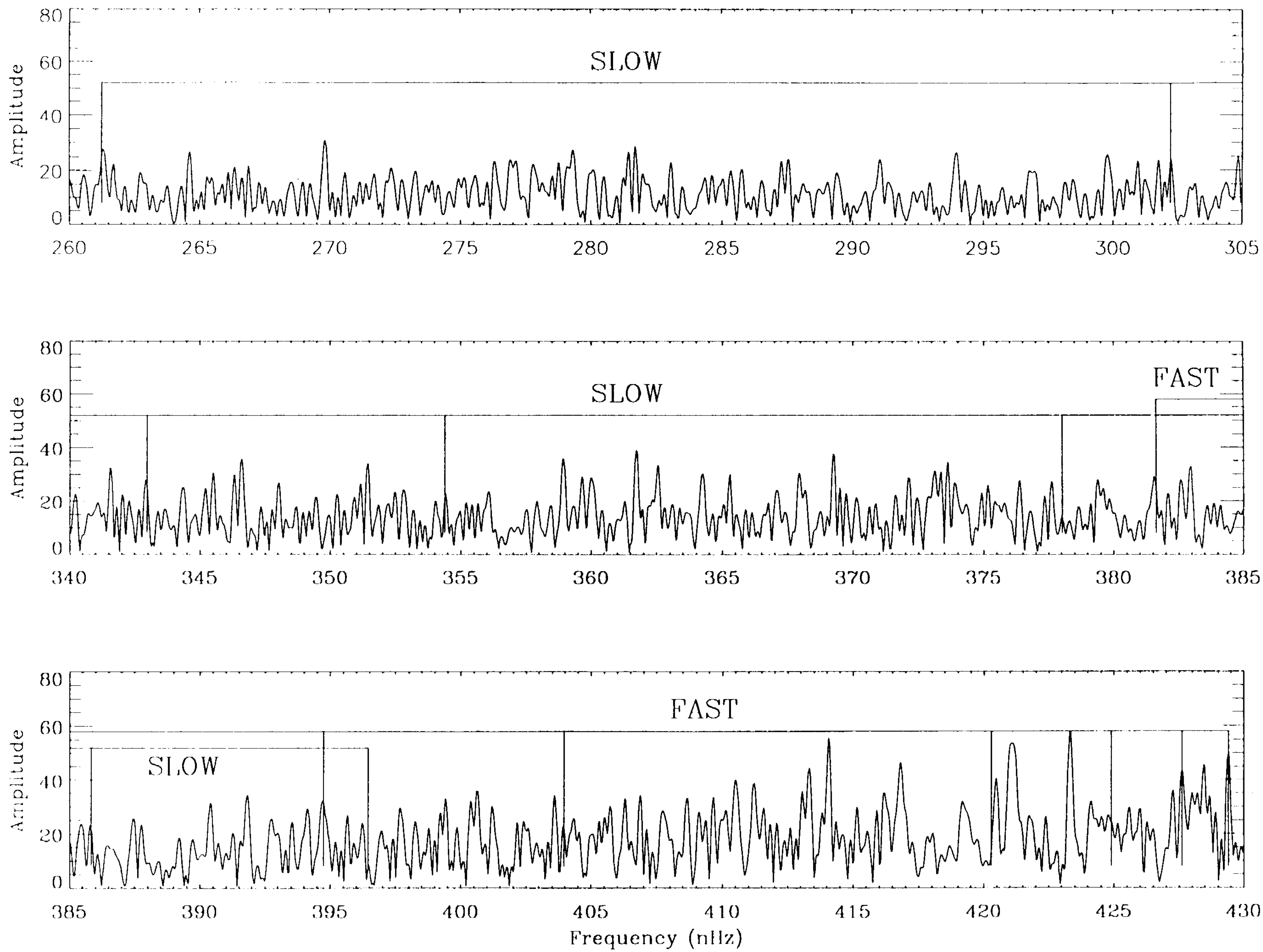

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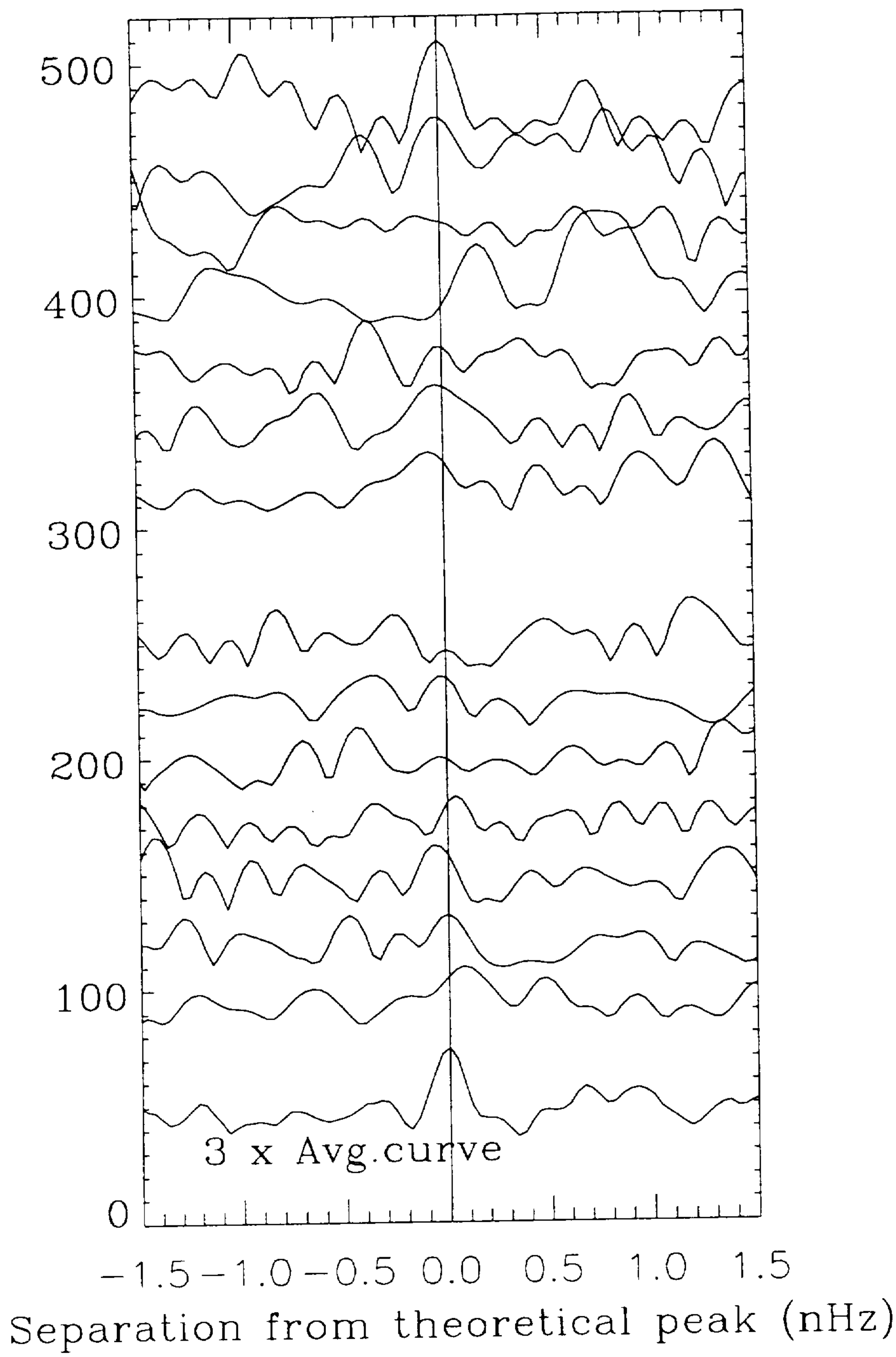

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