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Current Collection in a Magnetic Field

Final Report of E. N. Krivorutsky, Spring 1997

In accordance with the contract, two problems were studied:

1. The upper-bound limit of current collection taking into account the current's magnetic field
2. Preparation toward numerical computation of current collection

Also investigated was a proposed scheme of measuring the location of the end body.

Upper-Bound Limit of Current Collection

The upper-bound limit of current collection in a uniform magnetic field is given by the well-known formula of Parker and Murphy. If the magnetic field is not essential, then the current is 10-20 times less than this. The question under consideration is this: what is the role of the tether current magnetic field? Does it change the upper-bound limit of current collection? Also, is there a dependence on the shape of the current collecting wire?

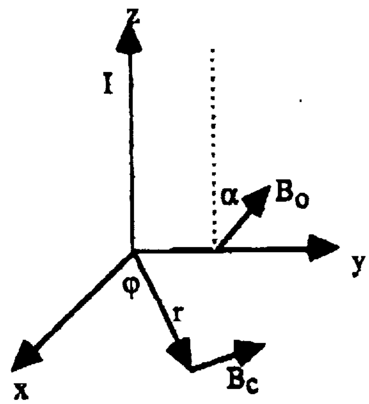
Cylindrical wire

Magnetic field of the current

It is supposed below that the fields and plasma are uniform along the wire. In the coordinate system given in Figure 1 with the current $\vec{I}$ along the z axis, the Earth's magnetic field $\vec{B}_o$ is in the y-z plane and the current magnetic field $\vec{B}_c$ is in the x-y plane. The complete magnetic field $\vec{B}$ is then

$$\vec{B} = a\left[\hat{e}_r \sin\varphi + \hat{e}_\varphi(1/\rho + \cos\varphi) + \hat{e}_z \cot\alpha\right] \qquad (1)$$

$$a = B_o \sin\alpha \qquad \rho = r/r_o \qquad r_o = 0.2I/a$$



Here, I is the current in amperes, B_o is in Gauss, and r and r_o are in cm. From this, the equation of the field lines in the x-y plane is then

$$\ln\rho + \rho\cos\varphi = g = \text{const} \tag{2}$$

The separatrix at g=-1 divides the closed lines around the wire and the unclosed lines farther out.

Close to the wire, B_c becomes greater than B_o. This is true for r<3 cm for I=5 A, and also at r<6 cm for I=10 A. On the surface of the wire, $B_c/B_o \cong 60$ for I=10 A and r_{wire}=0.1 cm. The magnetic force is the same order as the electric force for r<15 cm if I~10 A and the wire potential is .$\phi_w \sim 10^3$ V.

The magnetic field potential can be written

$$\vec{A} = -\hat{e}_r A_r(r,\varphi) - \hat{e}_z g \tag{3}$$

Particle motion and the upper-bound limit of current collection

The Lagrangian of the particle motion in this magnetic field is

$$L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\varphi}^2 + \dot{z}^2\right) + \frac{e}{c}\left(\dot{z}A_z + \dot{r}A_r\right) - e\phi \tag{4}$$

where ϕ is the electrostatic potential. L does not depend on z, so $\partial L/\partial \dot{z} = \text{const}$, or

$$m\dot{z} + \frac{e}{c}A_z = P_z = \text{const} \tag{5}$$

Taking into account that

$$E = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\varphi}^2 + \dot{z}^2\right) + e\phi = \text{const} \tag{6}$$

and neglecting the thermal motion, it can be found that a particle can reach the wire if

$$\frac{e}{mc}A_{zw}-\sqrt{-\frac{2e}{m}\phi_w}\le\frac{e}{mc}A_{z\infty}<\frac{e}{mc}A_{zw}+\sqrt{-\frac{2e}{m}\phi_w} \tag{7}$$

where the subscript w indicates quantities related to the wire. It follows from (7) that on the region of the wire with fixed A_{zw}, particles are coming from the region $y\to\infty$, where $A_z|_{y\to\infty}\sim x$ gives a region of current collection

$$\Delta x=2\frac{mc}{eB_\perp}\sqrt{-\frac{2e}{m}\phi_w} \tag{8}$$

For the cylindrical wire all points on the wire have nearly the same magnetic potential, $A_{zw}\cong$const, and the collected current is then

$$j=2en_o v_T\frac{\sqrt{-\frac{2e}{m}\phi_w}}{\omega_B\sin\alpha} \tag{9}$$

This result is exactly the same as Parker-Murphy. It should be noted that the limit $\alpha\to 0$ can be found only if ϕ_w as a function of α is known.

Wire of arbitrary shape

The inequality (7) is valid for an arbitrary shape of the wire, but in this case different points of the wire surface have a different magnetic potential, A_W. For fixed A_W, the region of current collection is still given by (8). For different points on the wire, A_W is different and the ends of the region $\Delta x=x_2-x_1$ will also be different. The region of current collection will be $\delta+\Delta x$ where

$$\delta=\frac{A_{p,max}-A_{p,min}}{B_o\sin\alpha}\sim\frac{aB_c}{B_o\sin\alpha} \tag{10}$$

Here, a is the characteristic width of the wire and $\Delta A_p/a\sim B_c$.

For $I\sim$5-10 A, $a\sim$0.5-1.0 cm and $\delta/\Delta x\sim$0.1-0.5. Therefore, the upper-bound of the current collection weakly depends on the shape of the wire and can be estimated by the Parker-Murphy formula.

Kinetic Models for a Numerical Calculation of the Current Collection

For a kinetic description of the current collection, a particle simulation code as well as the kinetic calculation can be used. A particle simulation code [such as that of Dr. Singh] can be adopted for current collection by including the magnetic field of the current.

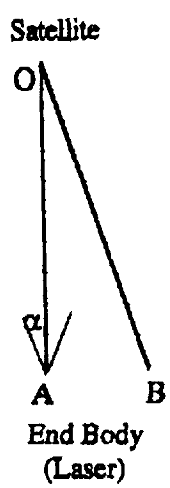
However, inclusion of particle scattering in the code will be difficult. The two characteristic scales, the Debye radius and the mean free path, are 0.2-1.0 cm and 400-600 cm, respectively, are too much different.

In a model based on the kinetic equation, the last problem can be solved in two ways. The first is due to greater time of calculation gives an opportunity to include the smallest length scale. The second method uses different models for the different lengths. The region of current collection can be divided into a region where scattering is essential, a region where it is not important but where the drift equation is still correct, and a region from out near the separatrix in to the wire. An approach for calculations in the second region on an analytical and partly computational level has been developed (together with Drs. G. V. Khazanov and M. W. Liemohn).

Scheme for Measuring the End Body Position

Suppose a source of light (laser) located on the end body spins with some period T at an angle α from the tether (Figure 2). In position A, the time between two signals registered at O (mother body) is the same. If the laser is in point B, however, the time will be different. It is supposed that the laser is oriented in the same way to the gravity field in points A and B. Dr. Stone proposed a hanging laser to achieve this. For $\alpha \sim 10^{\circ}$, l=OA=10^6 cm, Δ=AB=10 cm, and T=60 s, the difference in the time is $\Delta t \sim 10^{-4}$ s. If the source of light gives a circle or an arc with a radius close to the maximum displacement AB at the mother body (horizontal plane containing O), then the position of the end body in the plane perpendicular to the plane of the figure can be found. Two light receivers separated by about 10 cm on the mother body are needed. The 10 cm end body displacement can be registered, comparing the time when the signal is registered by the different receivers. The difference is around 10^{-5} s. Such times are easily measured. The oscillation of the end body around its axis also can be easily registered using a radial dark lane on the light source. As the body rotates, the dark line in the laser point also rotates and can be measured.

The main question is the power of the laser needed during the daytime. It can be lowered using filters and different receiving schemes. During the night, the signal will be well registered.



Summary

1. It is found that the upper-bound limit for current collection in the case of strong magnetic field from the current is close to that given by the Parker-Murphy formula. This conclusion is consistent with the results obtained in laboratory experiments. This limit weakly depends on the shape of the wire. The adiabatic limit in this case will be easily surpassed due to strong magnetic field gradients near the separatrix. The calculations can be done using the kinetic equation in the drift approximation. Analytical results are obtained for the region where the Earth's magnetic field is dominant (item 2 above).
2. The current collection can be calculated (neglecting scattering) using a particle simulation code. Dr. Singh has agreed to collaborate, allowing the use of his particle code. The code can be adapted for the case when the current magnetic field is strong. (The results from item 1 above are valid only for the upper-bound limit, but not for the real current collection.) The needed time for these modifications is 3-4 months.
3. The analytical description and essential part of the program is prepared for the calculation of the current in the region where the adiabatic description can be used. This was completed with the collaboration of Drs. Khazanov and Liemohn.
4. A scheme of measuring the end body position is also proposed. The initial estimations are shown in item 3 above. The scheme was discussed in the laboratory (with Dr. Stone) and it was concluded that it can be proposed for engineering analysis.

The all above mentioned items can be presented in more details if it will be necessary.