EXPERIMENTAL CONTROL OF THERMOCAPILLARY CONVECTION IN A LIQUID BRIDGE

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ABSTRACT

We demonstrate the stabilization of an isolated unstable periodic orbit in a liquid bridge convection experiment. A model independent, nonlinear control algorithm uses temperature measurements near the liquid interface to compute control perturbations which are applied by a thermoelectric element. The algorithm employs a time series reconstruction of a nonlinear control surface in a high dimensional phase space to alter the system dynamics.

INTRODUCTION

Hydrodynamic instabilities arise frequently in materials processing and can significantly impact the quality of the resulting products; these instabilities can be present even in a microgravity environment where buoyancy effects are suppressed. In float-zone refinement, for example, the appearance of time-dependent thermocapillary convective flow induces undesired variation in the chemical composition of crystals processed either on Earth or in microgravity [1]. Recent experimental and theoretical studies have explored hydrodynamic instabilities in model systems such as liquid bridges [2-5]; these studies are an important first step toward devising strategies for improving process outcomes.

There has been much progress in the control of dynamics in low-dimensional nonlinear systems [6]. These efforts, popularly known as "control of chaos", have succeeded in stabilizing various physical systems including lasers [7], chemical reactions [8], and cardiac tissue [9]. Unfortunately, these methods fail when the goal dynamics for the controlled system is distant in phase space from the autonomous system behavior. Recently, however, an algorithm that targets goal dynamics anywhere in the phase space was proposed [10]. The algorithm requires no knowledge of the underlying, possibly nonlinear, governing equations; instead, time series of the system response to random perturbations is used to construct a control law in a time-delay space.

EXPERIMENTS AND CONTROL ALGORITHM

Convection Cell A liquid bridge is formed by a drop of silicone oil (tetradecamethylhexasiloxane with > 95% purity [11]) confined between two coaxial cylindrical stainless steel rods (radius $r = 0.3$ cm) that are separated vertically by a gap $l = 0.3$ cm [Fig. 1(a)]. The rods differ in temperature by $\Delta T > 0$ with the upper rod warmer than the lower; the mean temperature of the bottom rod is 15.0 °C and ΔT is computer-controlled to a precision of ± 0.05 °C. The volume of liquid in the drop is 0.065 cm^3. The dimensionless number that characterizes the surface tension driving is the Marangoni number ($M \equiv \sigma_T \triangle T d/\rho\nu\kappa$, with the liquid surface tension σ, density ρ, kinematic viscosity ν, thermal diffusivity κ, and $\sigma_T \equiv |\frac{d\sigma}{dT}|$).

The flow is measured and controlled by, respectively, a single temperature sensor and single feedback element. The temperature sensor is a 0.03 cm diameter thermistor that is placed approximately $l/2$ above the lower rod and 0.03 cm from the surface of the drop. The sensor yields a measurement that is determined primarily by the local temperature of the nearby liquid interface. The feedback element is a 0.1 cm $\times$ 0.2 cm $\times$ 0.3 cm thermoelectric device which is placed at the same relative location as the temperature sensor on the opposite side of the liquid bridge [Fig 1 (a)]. The temperature of the feedback element can be increased or decreased by applying a voltage of the proper polarity; these temperature changes, in turn, impose a localized temperature perturbation at the surface of the drop, thus altering the thermocapillary forces that drive the flow.

Control Algorithm A representation of the system state is obtained by selected sampling of the temperature sensor output and the applied perturbations. Samples are recorded at local maxima of the sensor output; differences between adjacent maxima are taken to filter out the influence of slow temperature drifts. The resulting sequences of temperature differences x_i and applied perturbations p_i (which are held constant between samples $i-1$ and i) are used to construct a simplified map description of the dynamics through the use of difference equations.

For an m-dimensional system at time step i, the next applied perturbation p_{i+1} is determined from the history (subscript h) and target (subscript t) dynamics:

$$p_{i+1} = C\left[\mathbf{x}_h(i), \mathbf{p}_h(i), \mathbf{x}_t(i+1), \mathbf{p}_t(i)\right]$$

where C is a single-valued mapping into a time-delay space with $\mathbf{x}_h(i) \equiv (x_i, ..., x_{i-m+1})$, $\mathbf{p}_h(i) \equiv (p_{i-d}, ..., p_{i-m-d+2})$, $\mathbf{x}_t(i) \equiv (x_{i+m+d}, ..., x_{i+2m+d-1})$, and $\mathbf{p}_t(i) \equiv (p_{i+m+1}, ..., p_{i+2m-1})$. This formulation assumes that measurements are recorded from a single sensor; therefore, a given system state can be determined by sequence of m measurements and m perturbations. Only one control parameter is available to change the system dynamics; thus, a minimum of m successive perturbations are required to reach the target dynamics, which, for stabilization of an unstable periodic orbit, is given by $\mathbf{x}_t(i+1) = 0$ and $\mathbf{p}_t(i) = 0$. Additionally, d is a delay time which includes the time to compute p_{i+1} and the time before an applied perturbation affects the measurements.

If C is linear, $4m$ unknown coefficients are required to determine p_{i+1}. For the liquid bridge, C is nonlinear and is approximated by a tangential (linear) interpolation in the delay space constructed from time series. Typically, one thousand random perturbations are first applied to the liquid bridge. The applied perturbations and resulting measurements from this "identification stage" are stored as a reference set. When control is activated at time step i, the reference set is searched for $8m$ readings (a minimum of $4m$ are required) that are nearest in the delay space to the history and target dynamics $[\mathbf{x}_h(i), \mathbf{p}_h(i), 0, 0]$. The best linear least squares fit to the $4m$ coefficients of C is obtained by singular value decomposition of the (overdetermined) $8m$ system of selected reference readings and the next applied perturbation for each reading. With the determination of C, the perturbation p_{i+1} is computed and applied. The entire computation procedure takes approximately 0.1 sec on a 120 MHz Pentium PC; this forces a one iteration delay (2.3 s) before the perturbation is applied.

RESULTS

The control scheme is applied for $M = 17750$ where the unperturbed system exhibits a two frequency state. For $M > \sim 14000$, the flow becomes time-dependent with a single fundamental frequency [Fig. 1(b)]; for $M \approx 16500$, a second frequency appears and is locked to the first. When the control algorithm is applied, the second oscillation is rapidly suppressed [Fig. 2(a)]. Initially, the applied perturbations are large; however, once the system approaches the target dynamics, only small perturbations are required to keep the system under control [Fig. 2(b)]. After the control is turned off, the system rapidly returns to the two-frequency dynamics of the unperturbed state.

Effective control is obtained when C is computed for several different values of m and d; however, the fastest convergence is achieved for $m = 4$ and $d = 2$. This result implies the dimensionality of the unperturbed dynamics is $m = 4$. Two dimensions are required to describe the second frequency present in the unperturbed system; the other two dimensions effectively describe the decay of stable modes in the liquid bridge and the thermal relaxation of the feedback element. The delay $d = 2$ is approximately equal to the sum of the calculation delay and the time for waves with azimuthal wavenumber 2 and period of 2.3 s to propagate from the feedback element location to the sensor location.

Figure 3 demonstrates that our control method is effective for stabilizing states which lie far from the unperturbed dynamics in the phase space. Previous methods for controlling periodic orbits relied on the unperturbed dynamics to carry the system close to the target dynamics before the application of control [6]. The target dynamics for our control method can lie in any region of phase space that can be accessed by the random perturbations applied during the "identification" period when the reference set is constructed. The control scheme fails for $M > \sim 19000$ because the dynamics become highly nonlinear, and the one thousand points in our reference set become insufficient for good interpolation. More sophisticated methods developed for nonlinear time series predictions (*e.g.*, neural networks) may help extend the parameter range for control [12].

CONCLUSIONS

Our experiments demonstrate that a single local measurement and feedback perturbation are sufficient to control liquid bridge convection in parameter regimes where the dynamics are sufficiently low-dimensional. For other flow states, however, the spatial structure cannot be ignored. In particular, we have attempted to stabilize unstable time-independent states using the present experiment. Oscillations can be suppressed at the sensor location; nevertheless, infrared imaging reveals the presence of standing waves with antinodes between the feedback element and the sensor. In this case, multiple spatially distributed measurements and perturbations will be required for control; we are presently modifying our control algorithm and experiment to include two sensors and two feedback elements. In this way, liquid bridge convection serves as an ideal proving ground for developing methods for controlling spatially extended nonlinear systems.

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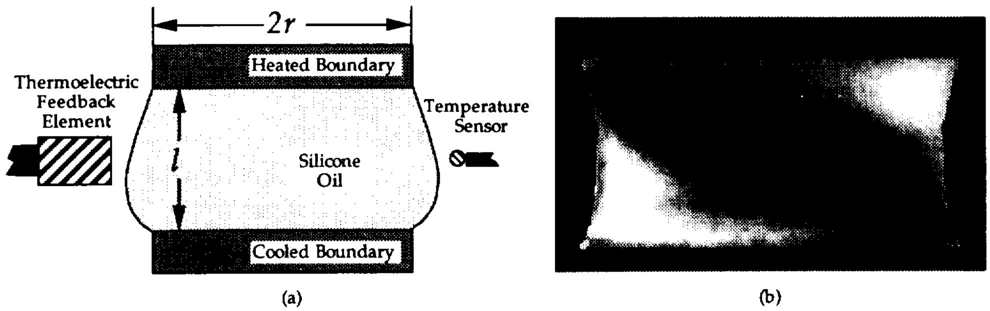

Fig. 1. (a) Sketch of liquid bridge convection experiment. (b) Infrared image of the brightness temperature (darker shading for colder temperatures) for time-periodic liquid bridge convection. The "barber-pole" structure of the temperature corresponds to two waves that propagate azimuthally (right to left in the figure).

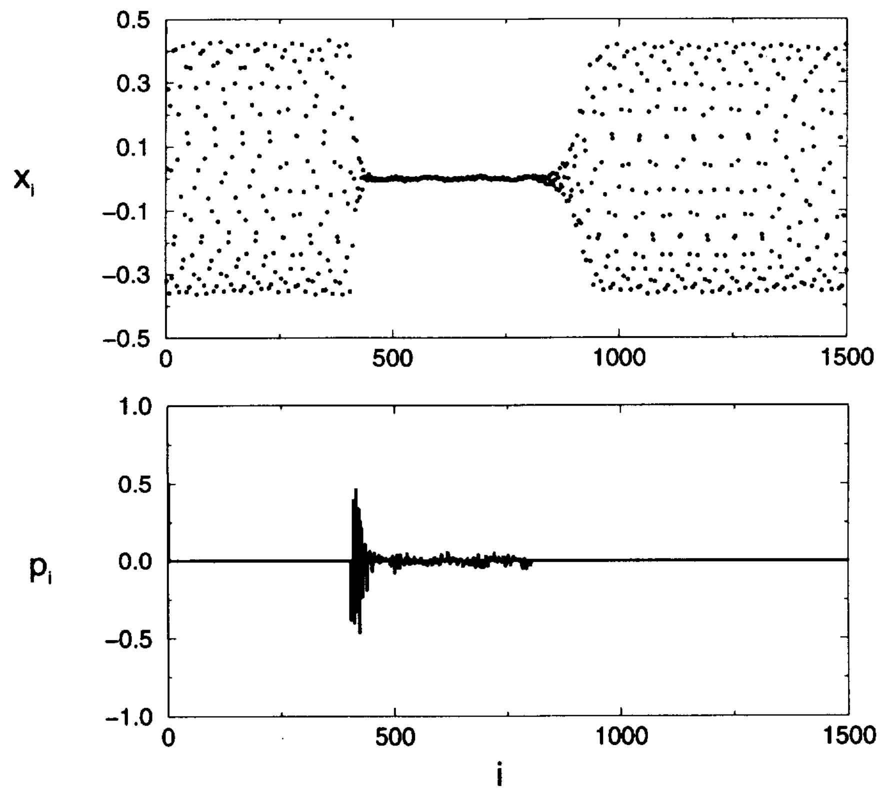

Fig. 2. The application of control is illustrated for discretized dynamics by time series of temperature differences x_i (top) and applied perturbations p_i (bottom). The control is applied at time step $i = 300$ and the second oscillation in the two frequency state is rapidly suppressed. Releasing the control at time step $i = 800$ caused the system to rapidly return to the unperturbed dynamics.

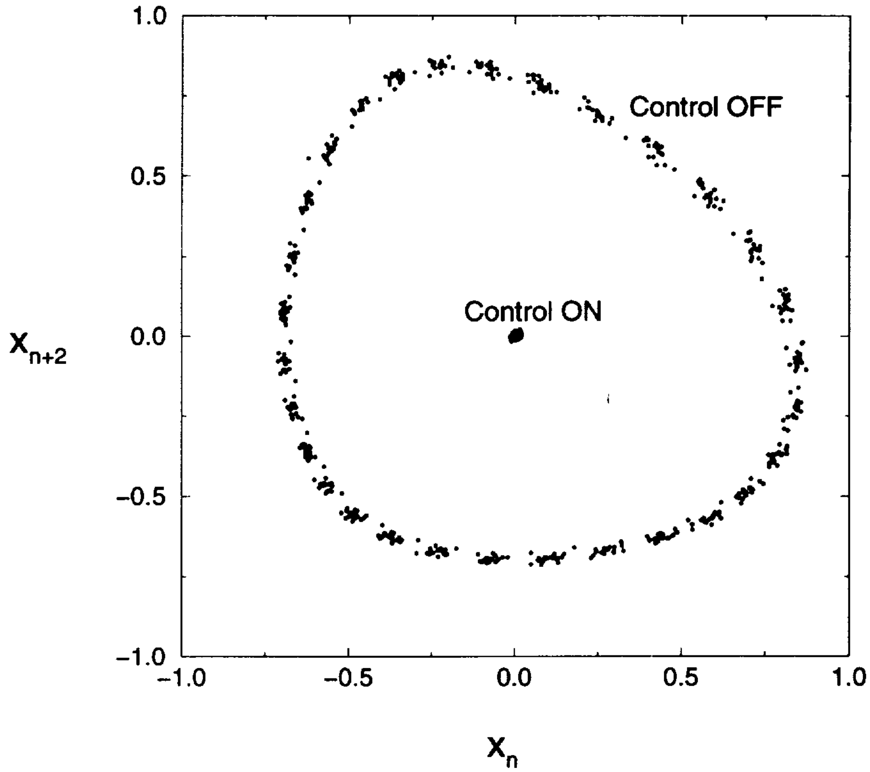

Fig. 3. Second return map constructed from experimental time series illustrating both the torus dynamics (control "off") and the stabilized periodic orbit (control "on").