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# FLUID PHYSICS IN A FLUCTUATING ACCELERATION ENVIRONMENT

J. Ross Thomson, François Drolet and Jorge Viñals
Supercomputer Computations Research Institute
Florida State University
Tallahassee FL 32306-4052

## ABSTRACT

We summarize several aspects of an ongoing investigation of the effects that stochastic residual accelerations ($g$-jitter) onboard spacecraft can have on experiments conducted in a microgravity environment. The residual acceleration field is modeled as a narrow band noise, characterized by three independent parameters: intensity $\langle g^2 \rangle$, dominant angular frequency $\Omega$, and characteristic correlation time $\tau$. Realistic values for these parameters are obtained from an analysis of acceleration data corresponding to the SL-J mission, as recorded by the SAMS instruments. We then use the model to address the random motion of a solid particle suspended in an incompressible fluid subjected to such random accelerations. As an extension, the effect of g-jitter on coarsening of a solid-liquid mixture is briefly discussed, and corrections to diffusion controlled coarsening evaluated. We conclude that $g$-jitter will not be significant in the experiment "Coarsening of solid-liquid mixtures" to be conducted in microgravity. Finally, modifications to the location of onset of instability in systems driven by a random force are discussed by extending the standard reduction to the center manifold to the stochastic case. Results pertaining to time-modulated oscillatory convection are briefly discussed.

## STOCHASTIC MODEL OF G-JITTER

We have introduced a stochastic model [1] to describe in quantitative detail the effect of the high frequency components of the residual accelerations onboard spacecraft (often called $g$-jitter) on fluid motion [2, 3, 4]. Each Cartesian component of the residual acceleration field $\vec{g}(t)$ is modeled as a narrow band noise characterized by three independent parameters: its intensity $< g^2 >$, a dominant frequency $\Omega$, and a characteristic spectral width $\tau^{-1}$. Specifically, $g(t)$ is a Gaussian random process, of zero mean, and autocorrelation

$$< g(t)g(t') > = < g^2 > e^{-|t-t'|/\tau} \cos\left[\Omega(t-t')\right]. \quad (1)$$

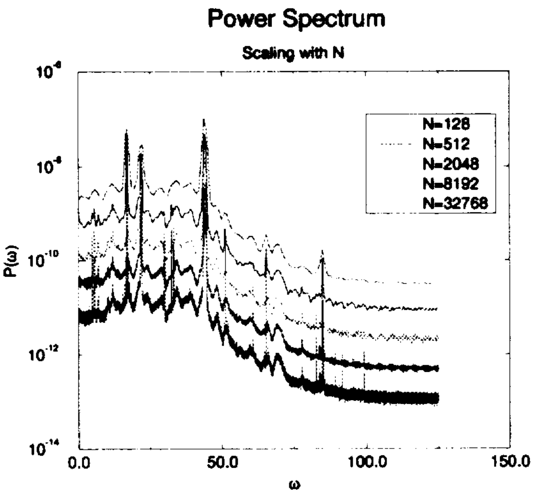
The power spectrum for this autocorrelation function is,

$$P(\omega) = \frac{1}{2\pi} < g^2 > \tau \left( \frac{1}{1+\tau^2(\Omega+\omega)^2} + \frac{1}{1+\tau^2(\Omega-\omega)^2} \right). \quad (2)$$

Each realization of narrow band noise can be viewed as a temporal sequence of periodic functions of angular frequency $\Omega$ with amplitude and phase that remain constant only for a finite amount of time ($\tau$ on average). At random intervals, new values of the amplitude and phase are drawn from prescribed distributions. This model is based on the following mechanism underlying the residual acceleration field: one particular natural frequency of vibration of the spacecraft structure ($\Omega$) is excited by some mechanical disturbance inside the spacecraft, the excitation being of random amplitude and taking place at a sequence of unknown (and essentially random) instants of time.

¿From a theoretical standpoint, narrow band noise provides a convenient way of interpolating between monochromatic noise (akin to more traditional studies involving a deterministic and periodic gravitational field), and white noise (in which no frequency component is preferred). In the limit $\tau \to 0$ with $D = < g^2 > \tau$ finite, narrow band noise reduces to white noise of intensity $D$; whereas, for $\tau \to \infty$ with $< g^2 >$ finite, monochromatic noise is recovered.

In order to ascertain the validity of this model, and to determine the values of the parameters defining the noise, we have analyzed actual $g$-jitter data collected during the SL-J mission (SAMS-258), and studied in detail the time series of head A between MET 0017 and MET 0023, or roughly six hours. First, a scaling analysis has been performed to determine the existence of deterministic or stochastic components in the time series. Figure 1 shows the power spectrum of $g$-jitter calculated over a window of size $N$, and then averaged over the six-hour period (the values of $g$ are sampled at 250 Hz). With the normalization of the



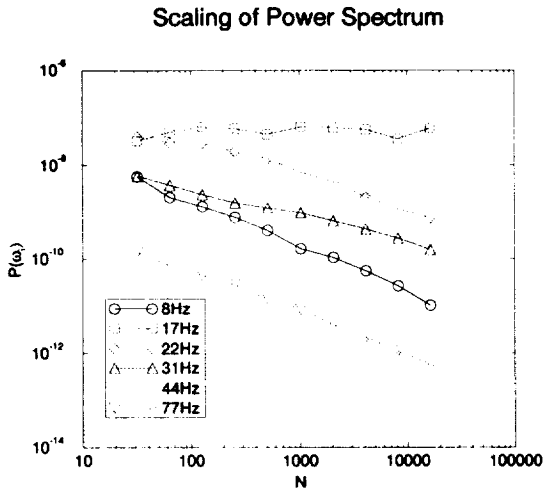




Figure 1: Left, power spectrum as a function of frequency for a six-hour interval during the SL-J mission. The various curves shown correspond to spectra calculated over the window of indicated size. Right, intensity versus window size for a few selected frequencies.

spectrum used, the power spectrum of a deterministic time series ought to be independent of $N$, whereas for a stochastic signal, it should decay as $1/N$ when the window size is much larger than the correlation time $\tau$. Figure 1 (right) shows the dependence of the power of a selected set of frequencies as a function of the window size, displaying its scaling with $N$. During this period of six hours, there appears to be a monochromatic contribution at 17 Hz with an amplitude $\sqrt{< g^2 >} = 3.56 \times 10^{-4} g_E$, where $g_E$ is the intensity of the gravitational field on the Earth's surface. There are two additional components that have a finite correlation time: For the component at 22 Hz we estimate $\sqrt{< g^2 >} = 3.06 \times 10^{-4} g_E$ and $\tau = 1.09$ s, whereas for 44 Hz we find, $\sqrt{< g^2 >} = 5.20 \times 10^{-4} g_E$ and $\tau = 0.91$ s. As an estimate of the white noise background, we obtain from the slope of the intensity of the 8 Hz component versus $N$ the value $D = 8.61 \times 10^{-4} \mathrm{cm}^2/\mathrm{s}^3$.

## RANDOM MOTION OF BUOYANT PARTICLES; COARSENING OF SOLID-LIQUID MIXTURES

We study here the motion of a particle suspended in an incompressible fluid of different density, when the fluid is subjected to an effective acceleration field like the one described above. This type of motion has been termed inertial random walk because of the similarity with Brownian motion [5]. The difference, of course, is that the random motion of the particle is not due to thermally induced collisions with the molecules of the fluid, but results from an effective random buoyant force acting on the particle. The asymptotic mean squared velocity of a particle subjected to narrow band noise is found to be,

$$\langle v^2 \rangle_\infty = \frac{\Delta\rho^2 \langle g^2 \rangle \left(\gamma + \frac{1}{\tau}\right)}{\gamma \left[\left(\gamma + \frac{1}{\tau}\right)^2 + \Omega^2\right]},$$

where $\Delta\rho = (\rho_p - \rho_f)/\rho_p$, $\gamma = 9\pi\eta/2\rho_p R^2$, with $\eta$ the shear viscosity of the fluid, $\rho_p$ and $\rho_f$ the density of the particle and fluid respectively, and $R$ the radius of the particle. The particle undergoes diffusive motion (the mean squared displacement is proportional to time), with an effective diffusion coefficient that is proportional to the intensity of the fluctuating acceleration. On the other hand, if the acceleration field were periodic in time (deterministic case), the mean squared displacement of the particle would remain bounded.

We have also performed an asymptotic analysis away from the deterministic limit ($\tau \to \infty$) to illustrate the emergence of diffusive behavior as the correlation time becomes finite. We have found an effective diffusion coefficient given by,

$$D_{\text{eff}} = \frac{\Delta\rho^2 < g^2 > \gamma}{(\gamma^2 + \Omega^2)^2} \left(2 + \frac{\Omega^2}{\gamma^2} + \frac{\gamma^2}{\Omega^2}\right) \frac{1}{\tau} + \mathcal{O}(\tau^{-2}).$$

Therefore, from a measure of the autocorrelation function of the particle displacement, it should be possible in principle to determine independently the parameters that define $g$-jitter. Knowledge of this sort could conceivably lead to the construction of an instrument that would complement the data set provided by accelerometers.

A residual acceleration field can produce a number of deleterious effects on otherwise purely diffusive controlled coarsening. We have focussed on two such effects: random motion of the suspended particles induced by the effective (random) buoyant force and the concomitant increase in the likelihood of particle coalescence, and additional flow in the fluid phase caused by g-jitter and its effect on solute mass transport. Numerical estimates have been obtained for a solid-liquid mixture of Sn-rich particles in a Pb-Sn eutectic liquid, the system that will be used in a forthcoming microgravity experiment [6, 7].

Neglecting inter-particle interactions, precipitate particles will execute a random motion of the type described above. For the case of monochromatic noise (fixed frequency and random phase), the average quadratic displacement of each particle remains bounded. For values of the parameters appropriate for a a Pb-Sn eutectic liquid and the conditions of the planned microgravity experiment, $\gamma = 260s^{-1}$, and by using the amplitude of the 17 Hz component of the power spectrum, we find that max $\{< x^2 >\} \approx 10^{-8}cm^2$, and hence negligible. At the other extreme, we find that for white noise the mean squared displacement after five hours is $< x^2 > (t = 5 \text{ hr.}) = 8.85 \times 10^{-6}cm$ or $\sqrt{< x^2 >} \simeq 30\mu m$. Clearly the average square displacement induced by the white noise component of the residual acceleration field is much larger than that induced by the monochromatic component, but it is still about one half of the expected average particle size at the end of the coarsening experiment. Therefore Brownian motion induced by g-jitter will not lead to appreciable motion of the precipitate particles relative to their size.

Estimating the effect of g-jitter on mass transport in the fluid phase is far more complex. In the limit of Stokes flow and non-interacting solid particles, (the former is appropriate for the size of the coarsening particles involved), g-jitter induced flow acts to renormalize the solute diffusivity. We find an effective diffusion coefficient given by,

$$D_{eff} = D_s + \frac{\Delta\rho^2 D}{\gamma^2}, \tag{3}$$

where $\Delta\rho$ is the relative difference in density between the liquid and solid phases, and $D_s$ is the solute diffusivity. We find that

$$\frac{\Delta\rho^2 D}{\gamma^2 D_s} = 5 \times 10^{-5} \ll 1, \tag{4}$$

and therefore negligible.

Finally, the effect of particle-particle interaction can be estimated in the overdamped limit. We find that although it would be asymptotically dominant in the limit of large particles (or long coarsening times), the interaction terms also remain small within the range of coarsening times to be explored in the experiment.

## CENTER MANIFOLD REDUCTION FOR STOCHASTICALLY DRIVEN SYSTEMS

The onset of oscillatory instabilities in stochastically driven systems is also being studied. The approach that we follow applies to systems consisting of a "slow" variable $u$ coupled to one (or more) "fast" variable(s) $v$ [8]. In the classical deterministic case, one simplifies the dynamics of the problem close to onset of instability through the adiabatic elimination of the fast variable. The stochastic analog of this reduction scheme consists in assuming a probability distribution of the form $\mathcal{P}(u,v,t) = P(u,t)\delta(v - v_o(u))$, with $v = v_o(u)$ is the center manifold of the associated deterministic problem (in the weak noise limit). The resulting Fokker-Planck equation is then integrated over $v$, yielding an equation for $P(u,t)$. Using this procedure, a generic system of the form

$$\partial_t \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & -\lambda \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} f(u,v) \\ g(u,v) \end{bmatrix} + \xi(t) \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \tag{5}$$

where $\lambda, \alpha > 0$, and $k_{ij}$ are known constants, reduces to the Fokker-Planck equation,

$$\begin{aligned} \partial_t P(u,t) = & -\partial_u[(\alpha u + f(u, v_o(u))P(u,t)] + D\partial_u[(k_{11}u + k_{12}v_o(u)) \\ & (k_{11} + 2k_{12}\partial_u v_o(u))P(u,t) - k_{12}(k_{21}u + k_{22}v_o(u))P(u,t) \\ & +(k_{11}u + k_{12}v_o(u))^2\partial_u P(u,t)]. \end{aligned} \tag{6}$$

The value $\alpha = 0$ corresponds to the deterministic threshold, while the functions $f(u,v)$ and $g(u,v)$ involve terms of the form $u^a v^b$ ($a + b \geq 2$). The random process $\xi(t)$ is assumed to be Gaussian and white, of zero mean and intensity $2D$ ($\langle\xi(t)\xi(t')\rangle = 2D\delta(t-t')$).

As an illustration of the procedure, consider the Van der Pol oscillator

$$\partial_t^2 x = \alpha x - \gamma\partial_t x - a x^3, \tag{7}$$

with the driving force having a stochastic component (i.e, $\alpha \to \alpha + \xi(t)$), $\gamma$ is a constant damping coefficient and $a > 0$ a nonlinear coupling constant. The problem can be mapped to Eq. (5) by first defining $y = \partial_t x$ and then letting $u = x + y/\gamma$ and $v = -y/\gamma$ [8]. Assuming $u, D << 1$, the resulting Fokker-Planck equation yields the stationary distribution

$$P(u) = \mathcal{N}|u|^{-2+\frac{\alpha\gamma}{D}} \exp(-\frac{a\gamma}{2D}u^2), \tag{8}$$

which is normalizable as long as $\alpha \geq D/\gamma$. The theory thus predicts a shift in the threshold from $\alpha = 0$ to $\alpha = D/\gamma$. This result agrees with that obtained by Lücke [9] by other methods, but contradicts earlier work of Knobloch and Wiesenfeld. The probability density can either be unimodal ($\alpha \geq 2D/\gamma$, Fig. 2 (left) or monotone with an infinite peak at zero ($D/\gamma < \alpha < 2D/\gamma$, Fig. 2 (right).

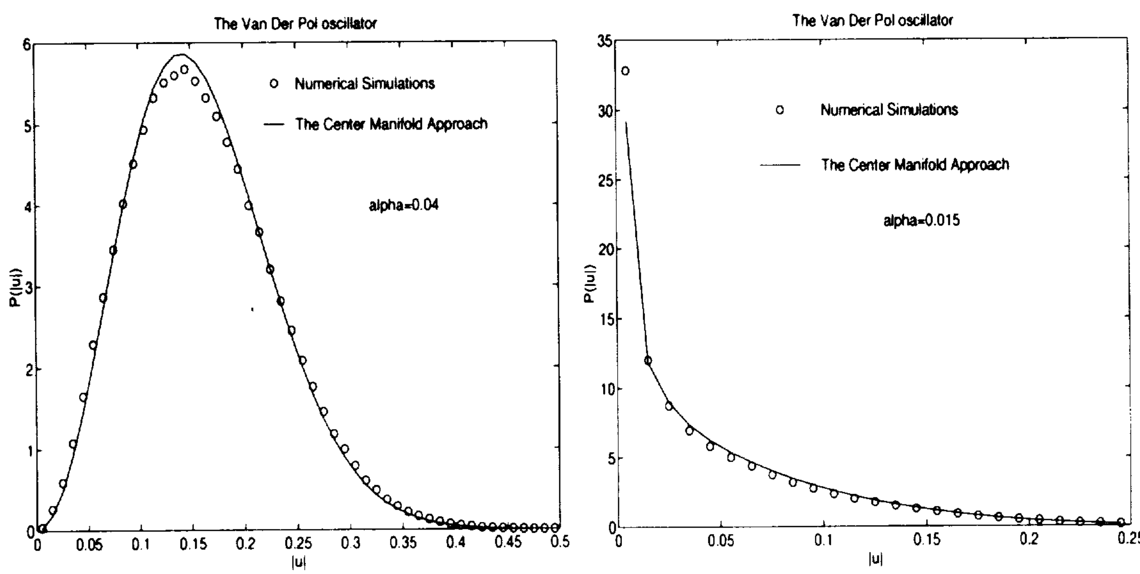



Figure 2: Probability density distribution for the Van der Pol oscillator with noise intensity $D = 0.01(\alpha = \gamma = 1)$.

The procedure can also be used to study the influence of fluctuations on time-modulated oscillatory convection. The left and right traveling waves appearing at large Rayleigh numbers have been modeled by a set of equations for their amplitude ($x$ or $y$) and phase difference ($\chi$) [10]. In the parameter regime in which both waves have equal amplitude (thus corresponding to the emergence of standing waves), the system of equations can be shown to reduce to

$$\partial_t \begin{bmatrix} A \\ \theta \end{bmatrix} = \begin{bmatrix} a_R + \sqrt{b^2 - a_i^2} & 0 \\ 0 & -2\sqrt{b^2 - a_i^2} \end{bmatrix} \begin{bmatrix} A \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{n_R}{4}A^3 - a_i\theta A \\ \frac{n_i}{2}A^2 + a_i\theta^2 \end{bmatrix} \tag{9}$$

close to onset. In Eq. (9), $A = x + y$, $\theta = \chi - \arcsin(a_i/b)$ and the other quantities are constant parameters entering the model. The effect of fluctuations in the Rayleigh number can then be studied by letting $a_R \to a_R + \xi(t)$. The center manifold approach predicts changes in the various moments of $A$, but no shift in the position of the threshold for this instability. This has been verified numerically, as shown in Fig. 3.

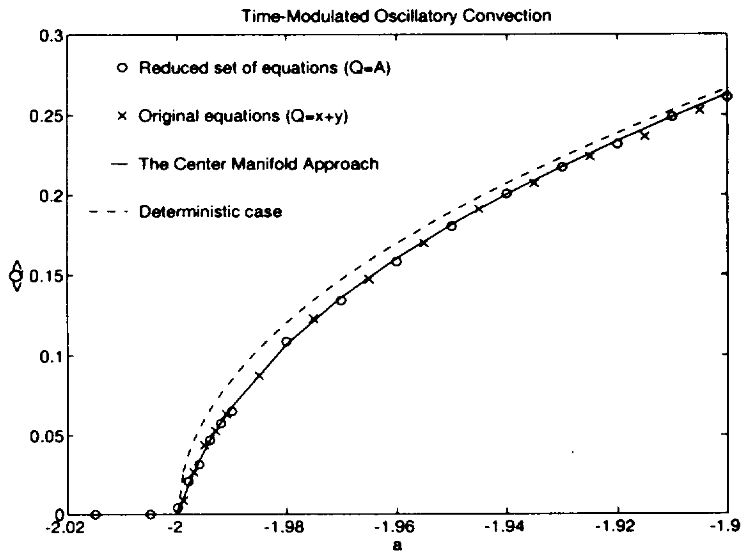



Figure 3: Standing Wave bifurcation in time-modulated oscillatory convection.

## CONCLUSIONS

A stochastic model of the residual acceleration environment onboard spacecraft has been introduced. It provides a realistic description of the various contributions to the acceleration environment, as exemplified by the power spectrum recorded during the SL-J mission. During the time window analyzed, the signal has a deterministic component at 17 Hz (with a correlation time longer than 65 $s$), two other major components at 22 and 44 Hz that can be modeled as narrow band noise of short correlation time (of the order of one second), and a significant white noise background. Since the stochastic model used can smoothly interpolate between the deterministic and white noise limits, it represents a useful tool for numerical analysis and statistical predictions concerning the effect of $g$-jitter on a number of phenomena.

In particular, we have briefly discussed our results concerning three different situations. First, random displacements of buoyant particles result in diffusive motion that could be used in passive accelerometer devices to have an independent measurement of $g$-jitter intensity. This random motion and the concomitant fluid flow have been estimated in connection with the experiment "Coarsening of solid-liquid mixtures". The effect of $g$-jitter has been shown to be of the same order as ordinary Brownian motion and therefore negligible, except possibly for experiments of long duration (over ten hours). Finally, the effect of noise on the onset of oscillatory instabilities, such as in double diffusive convection, is being examined by extending the classical center manifold reduction to the stochastic case. Two examples are briefly discussed that exhibit the main qualitative features of the phenomenon, with particular emphasis being paid to possible shifts in the instability point relative to the deterministic (noiseless) case.

## ACKNOWLEDGMENTS

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