SH4-34
82212

OBSERVATIONS OF TIME-DEPENDENT BEHAVIOR IN THE TWO-LAYER RAYLEIGH-BÉNARD SYSTEM

C. David Andereck, Peter W. Colovas and Michael M. Degen
Department of Physics
The Ohio State University
Columbus, OH 43210

ABSTRACT

In this paper we present results from experiments with a system consisting of two immiscible fluid layers in rectangular and annular geometries, driven by a vertical temperature gradient. Time-dependent variations in the type of coupling observed between the two layers are described and characterized.

INTRODUCTION

Rayleigh–Bénard convection is one of the oldest and most thoroughly studied of fluid dynamical systems. The basic system consists of a single fluid confined between parallel horizontal plates held at a temperature difference ΔT, usually with the higher temperature being on the lower plate. If ΔT is large enough, convection begins in the form of parallel rolls, the result of the destabilizing effect of the buoyancy force overcoming the stabilizing influences of viscous drag and thermal diffusion. The onset of convection in the one fluid Rayleigh–Bénard problem is simply characterized by the Rayleigh number

$$Ra = \frac{g\alpha\Delta T d^3}{\kappa\nu}, \tag{1}$$

the dimensionless temperature difference, and a ratio of the destabilizing force to the stabilizing forces. In equation (1), g is the acceleration of gravity, α is the volume coefficient of expansion, d is the depth of the fluid layer, κ is the thermal diffusivity, and ν is the kinematic viscosity of the fluid. Another nondimensional parameter, the Prandtl number $P = \nu/\kappa$, does not enter into the determination of the onset of convection. Thorough introductions to the stability analysis of the one fluid problem have been presented in the books by Chandrasekhar[1], and Drazin and Reid[2]. One result of these analyses is the fact that the equations are self-adjoint, and all the eigenvalues are real, leading to the "exchange of stability," *i.e.* the growth of small disturbances leads to steady cellular convection, there being no oscillatory modes at criticality. Fluids from water to liquid helium, and gases like air and CO_2 have been used in experiments which explore the one fluid Rayleigh–Bénard problem.

If the upper surface is free, or bounded by another fluid, then surface tension may play a role in determining the onset of convection[3]. Various theoretical analyses have shown the possible existence of oscillatory modes at the onset of convection, for cases of both deformable and nondeformable interfaces[4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]

Gershuni and Zhukovitskii [9] described a possible mechanism for oscillations which can appear in the coupling between two convecting fluid layers. The basic types of coupling are *thermal* and *mechanical*. Thermal coupling is the alignment of the hot rising fluid in both layers, causing rolls that are aligned one above the other to turn with the same sense, either clockwise or counterclockwise. Mechanical coupling is the alignment of the cold falling fluid in one layer with the hot rising fluid in the other layer, with the result that the rolls turn with opposite senses, in a gear-like fashion. The oscillations were predicted to be periodic variations between thermal and mechanical coupling. Experimental work on the two-layer problem has been reported by a number of groups [17, 10, 18, 19, 20], but none have found time-dependence in the coupling at the onset of convection. In this paper we show the results of our experimental investigation of a Rayleigh–Bénard system consisting of two immiscible fluids, concentrating on the search for oscillatory states.

EXPERIMENTAL SYSTEM

The experimental system consists of two fluids confined between horizontal parallel plates and bounded on the sides by transparent walls. The lower fluid, here designated fluid 1, occupies a depth fraction $l_1 = d_1/d$, where d_1 is the actual depth of fluid 1 and d is the total thickness of the cell. Fluid 2, the upper fluid, similiarly occupies $l_2 = d_2/d$. Two different pairs of fluids were selected for use in our experiments. The first pair consists of a silicone oil, Rhone-Poulenc Rhodorsil 47v10, over a perfluorinated hydrocarbon, Fluorinert FC-70, from 3M. They were chosen because theey are nearly immiscible, and because their properties are similiar enough that convection in both layers should start at the same overall ΔT for nearly equal depths. The other pair of fluids is 2 cs silicone oil, Rhodorsil 47v2, over water.

In the rectangular geometry, the fluids are confined within 9.5 *mm* thick glass walls. The interior horizontal dimensions of the cell are 78 *mm* x 21 *mm*, and the total height of the cell $d = 12\,mm$. The dimensions of the cell were chosen so that the onset of convection leads to a nearly one dimensional set of rolls in each layer. These rolls form parallel to the short dimension of the cell, as in single layer convection. The top and bottom of the cell are aluminum blocks. The blocks are thick to assure uniformity of temperature across the surfaces which contact the fluids. The top and bottom blocks are cooled and heated, respectively, by water which is conditioned by temperature controlled baths. In each circulation loop is a thermal low pass filter, which removes the short term fluctuations present in the bath. The system is shown in Figure 1. The annular cell has inner radius $r_i = 44\,mm$ and outer radius $r_o = 64\,mm$. It has the same height as the rectangular cell, and is heated and cooled in the same manner.

The temperature difference between the plates is measured by thermistors embedded in the center of each block, approximately 1.2 *cm* from the contact surface. The long term stability ΔT, is $\pm.02\,C$, while the horizontal variation in plate temperature is less than $\pm.01\,C$.

The thermal patterns in the rectangular system were observed using Schlieren and shadowgraph techniques, while only shadowgraph could be used for the annular cell. The measurement, while local along the length of the cell, is an integral over the width of the fluid layer. A representative Schlieren pattern is shown in Figure 2. The dark regions indicate rolls turning in one direction, while the light regions indicate rolls turning in the other direction. To analyze the space-time behavior of the system, a single horizontal line through the center of each layer was chosen, and the intensity recorded over a period of time encompassing several oscillations. A representative plot is shown for the rectangular cell in Figure 3, and for the annular cell in Figure 4. The resulting data can then be analyzed to find periods, wavelengths, wave speeds, etc. Our analysis comes primarily from this data.

PROCEDURE

While the parameter space of the two-layer Rayleigh-Bénard problem is large[21], practical considerations simplify the experimental process considerably. Upon choosing the fluids, the only experimental variables are the depths of the layers, l_1 and l_2, and the temperature difference, ΔT, between the top and bottom plates. We proceed by filling the cell with the appropriate amount of each fluid to achieve the desired l_1 and l_2. The depth of the lower fluid is measured precisely by an optical telescope, accurate to $\pm.02mm$. The full system is left to equilibrate for a period of 12 hours before the temperature ramping begins. The temperature is then ramped approximately $0.1°C$ at a time, starting from $\Delta T = 0$, and the system is left for at least three hours before data taking begins. As the period of oscillations is on the order of one hour, a data run consists of one "frame" of a single data line in each layer, taken every 30 seconds over a period of 6 to 8 hours. This gives us an effective ramping rate of $\approx 0.2^o C/day$.

Wavelengths and frequencies are computed from power spectra in space and time, respectively. A time average at each spatial point is first subtracted from the data, to remove non-uniformities in lighting due to the system optics. A one-dimensional FFT is then performed on the resulting data to compute the desired spectrum. We report our results in terms of ΔT, the temperature difference across the total height d.

RESULTS AND DISCUSSION

We began our experiments with 47v10 over FC-70, at a depth fraction $l_1 = 0.50$, where calculations predicted the onset of convection to occur at nearly the same ΔT in both layers. At this fraction, we found only mechanical coupling, at ΔT values up to $2.0\,C$. From here, we systematically decreased l_1 until we reached the value $l_1 = 0.39$, where we discovered oscillations slightly above the onset of convection. A plot of our results is shown in Figure 6. Oscillations near onset continue at lower fractions, until the value $l_1 = .345$ is reached, where no oscillations are observed, and the coupling is predominantly thermal. We have calculated wavenumbers and frequencies for the oscillations in both the top and bottom layers from the power spectra. The results are close to those predicted by the numerical analysis of Y. Renardy (unpublished). Figure 6 shows that in order for time-dependent behavior to develop with this combination of fluids, the coupling at the onset of convection must be mechanical.

Our experiments with the combination of fluids 47v2 over water show different behavior at the onset of convection. Calculations predict simultaneous onset of convection for $l_1 = 0.69$ in this system. Our work near this depth fraction shows that in the range $0.71 \geq l_1 \geq 0.60$ the critical mode at onset is time dependent. A space-time plot of this behavior is shown in Figure 5. This is consistent with numerical work done by Y. Renardy, who describes a parameter

$$Y = \frac{\kappa_1 \alpha_1 \rho_1}{\kappa_2 \alpha_2 \rho_2}, \qquad (2)$$

which should be far from one for the possibility of oscillations to be large. The 47v10/FC-70 system has $Y = .776$, while the 47v2/water system has $Y = 0.374$, implying that the range of depths where oscillations can be expected is much larger.

CONCLUSIONS

Numerical studies done on the two-layer Rayleigh-Bénard problem predict that the coupling between the two layers at onset should be predominantly thermal for values of l_1 just below the range where the onset should have oscillatory behavior, and predominantly mechanical for l_1, just above the oscillatory range. The size of the window of oscillations is determined by the parameter Y. Our work in the 47v10/FC-70 shows thermal coupling at low l_1 and mechanical coupling at high l_1. In addition, this system shows oscillatory behavior above onset for a range $0.39 \geq l_1 \geq 0.345$. Further work with the 47v2/water system shows the occurrence of oscillations at the onset of convection for $0.71 \geq l_1 \geq 0.60$, supporting the usefulness of Y as a qualitative predictor for the range of l_1 where oscillatory behavior may be expected.

ACKNOWLEDGEMENTS

We would like to thank Y. Renardy for her discussions on the nature of the system and her numerical predictions. We would also like to acknowledge discussions with P. Kolodner and M. Schatz, and D. A. Campbell & Co. for supplying the silicone oils. This work was supported by NASA grant NAG3-1612.

REFERENCES

[1] S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability* (Oxford University Press, Oxford, England, 1961).

[2] P. G. Drazin and W. H. Reid, *Hydrodynamic Stability* (Cambridge University Press, Cambridge, England, 1981).

[3] J. R. A. Pearson, J. Fluid Mech. **4**, 489 (1958).

[4] F. M. Richter and C. E. Johnson, J. Geophys. Res. **79**, 1635 (1974).

[5] L. Cserepes and M. Rabinowicz, Earth Planet. Sci. Lett. **76**, 193 (1985).

[6] L. Cserepes, M. Rabinowicz, and C. Rosemberg-Borot, J. Geophys. Res. **93**, 12009 (1988).

[7] F. M. Richter and D. P. McKenzie, J. Geophys. Res. **86**, 6133 (1981).

[8] F. H. Busse, Phys. Earth Planet. Inter. **24**, 320 (1981).

[9] G. Z. Gershuni and E. M. Zhukovitskii, Sov. Phys. Dokl. **27**, 531 (1982).

[10] S. Rasenat, F. H. Busse, and I. Rehberg, J. Fluid Mech. **199**, 519 (1989).

[11] Y. Renardy and D. D. Joseph, Phys. Fluids **28**, 788 (1985).

[12] Y. Renardy and M. Renardy, Phys. Fluids **28**, 2699 (1985).

[13] Y. Renardy, Phys. Fluids **29**, 788 (1986).

[14] M. Renardy and Y. Renardy, Physica (Amsterdam) D **32**, 227 (1988).

[15] P. Colinet and J. C. Legros, Phys. Fluids **6**, 2631 (1994).

[16] Y. Renardy and D. D. Joseph, *Fundamentals of Two Fluid Dynamics*, Vol. 1 of *Interdisciplinary Applied Mathematics* (Springer-Verlag, Reading, Massachusetts, 1993).

[17] R. W. Zeren and W. C. Reynolds, J. Fluid Mech. **53**, 305 (1972).

[18] H. C. Nataf, S. Moreno, and P. Cardin, J. Phys. France **49**, 1707 (1988).

[19] P. Cardin and H. C. Nataf, Europhys. Lett. **14**, 665 (1991).

[20] P. Cardin, H. C. Nataf, and P. Dewost, J. Phys. II **1**, 599 (1991).

[21] F. H. Busse, Geophys. Res. Lett. **9**, 519 (1982).

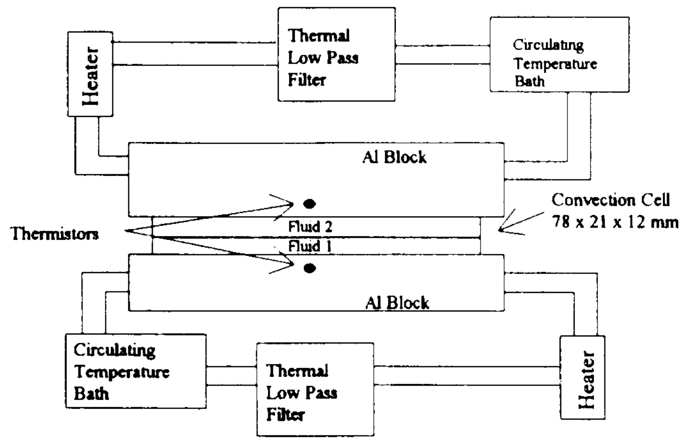
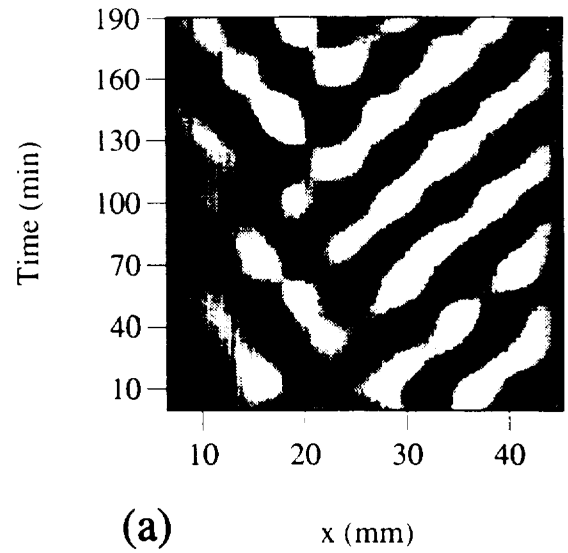


Figure 1: Schematic of the convection cell and temperature control, including thermal low pass filters and in-line active heaters



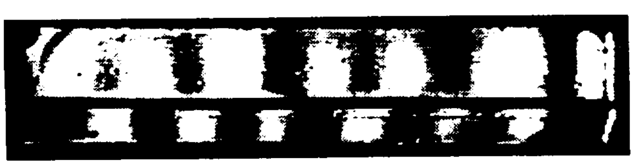


Figure 2: Schlieren image of the 47v10/FC-70 system. Light regions are rolls turning in one direction, Dark rolls turn in the opposite sense. ΔT=1.6 C, l_1=0.38.

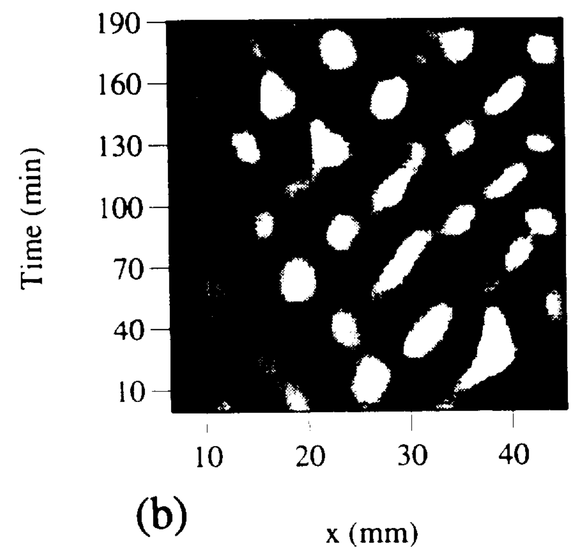
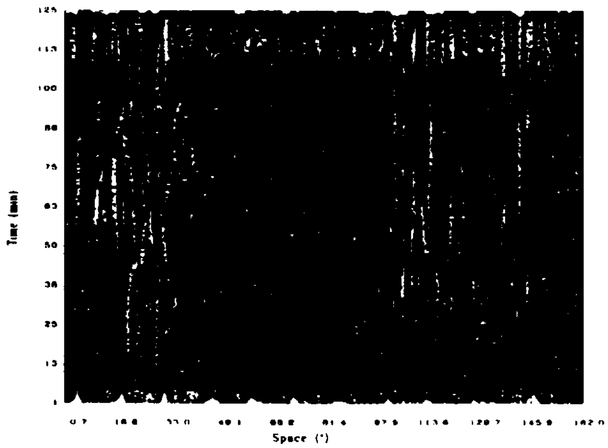
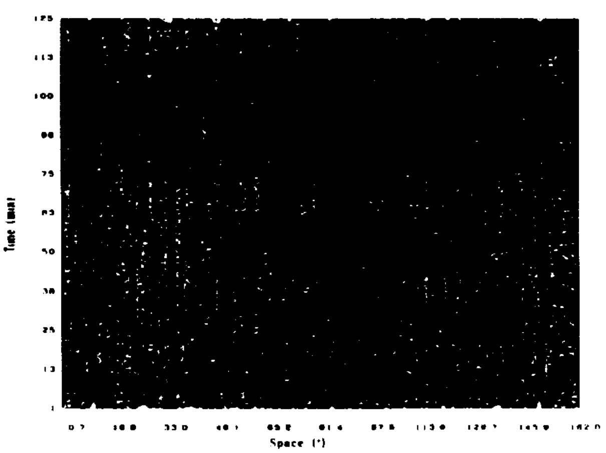


Figure 3: Space-time plot of the (a) top and (b) bottom layer in the rectangular cell, for the state shown in Fig. 2. Time advances up the vertical axis, and a time-averaged background has been subtracted.



(a)



(b)

Figure 4: Background subtracted space-time plot of the (a) top and (b) bottom layer in the annular cell, for $\Delta T=2.0$ C, $l_1=0.37$. The horizontal axis units are degrees around the azimuthal dimension.

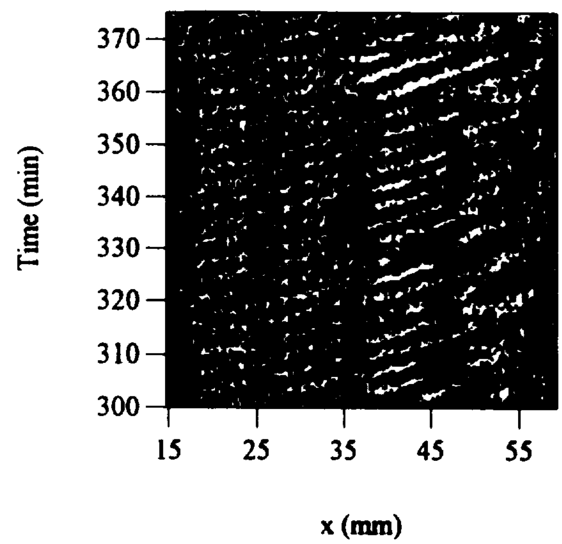


Figure 5: Background subtracted space-time plot of the upper layer in the 47v2/water system near the onset of convection, showing the time-dependent behavior of the rolls. $\Delta T=0.74$C, $l_1=0.6$.

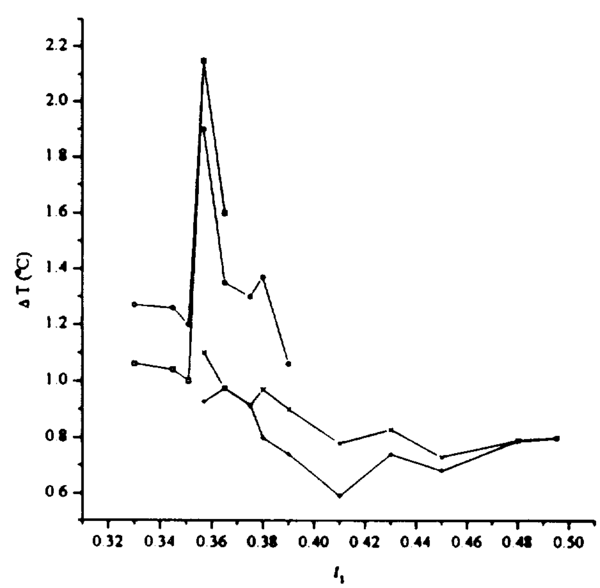


Figure 6: Plot of ΔT vs. l_1 showing the range of coupling and the onset of oscillations in the 47v10/FC-70 system. (X) and (+) indicate the onset of mechanically coupled convection in the top and bottom layers, respectively. (○) and (□) indicate the onset of thermally coupled convection in the top and bottom layers. (●) indicates the onset of oscillations.