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The Least-Squares Finite Element Method for Low-Mach-Number Compressible Viscous Flows

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ABSTRACT

The present paper reports the development of the Least-Squares Finite Element Method (LSFEM) for simulating compressible viscous flows at low Mach numbers in which the incompressible flows pose as an extreme. Conventional approach requires special treatments for low-speed flows calculations: finite difference and finite volume methods are based on the use of the staggered grid or the preconditioning technique; and, finite element methods rely on the mixed method and the operator-splitting method. In this paper, however, we show that such difficulty does not exist for the LSFEM and no special treatment is needed. The LSFEM always leads to a symmetric, positive-definite matrix through which the compressible flow equations can be effectively solved. Two numerical examples are included to demonstrate the method: first, driven cavity flows at various Reynolds numbers; and, buoyancy-driven flows with significant density variation. Both examples are calculated by using full compressible flow equations.

INTRODUCTION

In this paper, low-Mach-number, compressible, viscous flows are of interest. Low-speed flows with significant temperature variations are compressible due to the density variation induced by heat addition. For example, a significant heat addition occurs in combustion related flow fields. Inside a chemical vapor deposition reactor, strong heat radiation also results in significant density variation. Although the flow speed is slow, one must employ the compressible flow equations to simulate such flows. However, it is well known that the conventional methods, which can handle high-speed compressible flows easily, fail miserably when applied to these low-Mach-number flows.

In the past, because of wide applications of the low-Mach-number flows, the issue of the efficiency and robustness of the calculations has been investigated. Most of the research, however, utilizes the finite difference and finite volume methods; few attempts have been made using the finite element methods. Conventional finite difference and finite volume methods in solving low-Mach-number flows can be divided into two categories: the pressure-based methods, and the density-based methods. The pressure-based methods have their root in the SIMPLE type algorithm. Essentially, a staggered grid has to be employed, i.e., the pressure and velocities are stored at different nodes. In addition, one usually has to employ a pressure correction equation (or other derived equation) instead of the original continuity

equation when solving the equation set. This approach, to some extent, is similar to the Galerkin mixed finite element methods for incompressible Navier Stokes equations. In the Galerkin mixed method, different elements have to be used to interpolate the velocity and the pressure in order to satisfy the LBB condition for the existence and stability of the discrete solution. Moreover, this approach results in an asymmetric, non-positive-definite coefficient matrix which can not be effectively solved by using iterative methods.

On the other hand, the density-based methods use the same nodes for the velocities and the pressure. Merkle et al. have successfully developed several density based methods for both low-Mach-number flows and all-speed flows. These methods are an extension of the computational schemes for high-speed, compressible flows. All these aerodynamic codes are designed based on the hyperbolic characteristic of the Euler equations; the viscous terms are effective only in a small portion of the domain and are interpreted as the damping of the numerical waves. When simulating low-Mach-number flows, however, the flow field is no longer dominated by the inviscid flow. The conventional aerodynamic codes encounter insurmountable slow-down. As a result, various treatments have been developed to enhance the efficiency. These treatments stem from preconditioning the jacobian matrices of the convective terms in the flow equations to improve their condition numbers. Usually, two steps are involved. First, according to Chorin, one adds a temporal derivative of pressure together with a multiplicative variable β, i.e., the pseudo-compressibility term, to the continuity equation. As a result, numerically viable time derivative terms exist in every equation even for flows at the low-speed (incompressible) limit. Consequently, based on the inviscid terms of the flow equations, the resultant equations become hyperbolic. And, a numerical method for a hyperbolic system can be employed to advance the system in time.

Since the transient solution is not of interest, one can enhance the computational efficiency by tuning up the propagation speed and damping effect of numerical waves so that the calculation can reach steady state faster. This is done by premultiplying a preconditioning matrix to the equation set. The eigenvalues of the convective-term Jacobian matrices are scaled to the same order of magnitude. Therefore, the stability of numerical waves is ensured and the time marching process is under control.

However, it is obvious that when low-Mach-number flows are of interest, the viscous terms play an important role and the flow system is elliptic. When using the preconditioning technique, one fabricates an artificial hyperbolic system in order to employ a time marching scheme to advance the system to a steady state. In other words, the preconditioning method is based on conditioning the inviscid part of the governing equations. For low speed flows, the viscous terms demand an implicit treatment due to the infinite fast characteristic speed. Therefore, as Merkle indicated that the real effect of the preconditioning matrix when using an implicit scheme is to eliminate the numerical stiffness caused by the approximate factorization. For unstructured grid solver, it is largely unclear what will be the effect of this type of preconditioning technique and certainly more study is needed.

In the finite element methods, fewer attempts have been carried out on calculating low-Mach-number

flows. For flow fields inside chemical vapor deposition reactors, Einset and Jensen developed a low-Mach-number formulation which was then solved by a Galerkin mixed method. In developing the low-Mach-number formulation, Einsted et al. proposed a correlation between the density and temperature based on the low-speed condition. The density in the governing equations was then replaced by the temperature. The equation set was solved by a mixed method which results in an asymmetric, non-positive-definite coefficient matrix. Einsted et al. inverted the matrix by the conjugate gradient squared (CGS) method and the generalized minimal residual (GMRES) method.

Because the low-Mach-number flows are closely related to the incompressible flows, it may be worthwhile to briefly review other treatments developed for the incompressible flows. On the finite difference setting, Chorin proposed to use a fractional step procedure to solve the incompressible flow equations. Later on, it was pointed out by Schneider et al. and Kawahara et al. that, by using the fractional step procedure, the restrictions imposed by the LBB condition for mixed formulation no longer apply. Various finite element schemes based on this procedure have been successfully developed and applied to incompressible flows using equal order interpolation. Other approaches, such as the Galerkin Least Squares method proposed by Hughes et al and Sampaio are shown to have similar effects. A wider interpretation of such schemes are described by Zienkiewicz and Wu. In addition, the fractional step procedure have been extended by Zienkiewicz and Wu to high speed compressible Navier-Stokes equations and shallow water equations

In this paper, a new formulation is proposed for the low-Mach number flows, in which the unknowns includes variables such as the vorticity, the pressure variation, and the divergence of velocity. With proper nondimensionalization, the magnitude of each term in the governing equations, which depends on the Mach number of the flow field, can be clearly discerned. As a result, a set of equations suitable for low-Mach-number flows is derived.

We employ the LSFEM as the numerical scheme to solve the low-Mach-number flows. This approach is an extension of the the LSFEM for incompressible flows which has been developed by Jiang et al. The LSFEM always leads to a symmetric, positive-definite matrix which can be efficiently inverted by an iterative scheme such as the conjugate gradient method. In the present paper, however, a direct solver is employed because the formulation and the feasibility of the LSFEM for low-Mach-number flows are of interest instead of the computational efficiency.

We present the governing equations to be solved by the LSFEM. In order to use simple C^o elements, we convert the second-order transport equations to first-order ones by introducing new variables into the equations. Then, the system of equations are nondimensionalized for low-Mach-number flows. In the paper, the implementation of the LSFEM is elaborated in detail. The temporal derivative terms of the flow equations are discretized by the Euler implicit method. Although, the transient solution is not of interest, the temporal derivative terms serves as a relaxation scheme for marching towards a steady state. The nonlinear terms are linearized by Newton's method. The discrete equations are formulated

in an increment form which is then solved by the LSFEM. In the last section, two numerical examples are presented: driven-cavity flows at various Reynolds numbers, and buoyancy-driven flows at various Rayleigh numbers. Both cases are calculated by using the compressible flow formulation.