

US005255349A

United States Patent [19]

[11] Patent Number: **5,255,349**

Thakoor et al.

[45] Date of Patent: **Oct. 19, 1993**

[54] **ELECTRONIC NEURAL NETWORK FOR SOLVING "TRAVELING SALESMAN" AND SIMILAR GLOBAL OPTIMIZATION PROBLEMS**

[75] Inventors: **Anilkumar P. Thakoor**, Pasadena; **Alexander W. Moopenn**, Hawthorne; **Tuan A. Duong**, South Pasadena; **Silvio P. Eberhardt**, Pasadena, all of Calif.

[73] Assignee: **The United States of America as represented by the Administrator of the National Aeronautics and Space Administration**, Washington, D.C.

[21] Appl. No.: **789,567**

[22] Filed: **Nov. 8, 1991**

Related U.S. Application Data

[63] Continuation-in-part of Ser. No. 470,664, Jan. 26, 1990, abandoned.

[51] Int. Cl.⁵ **G06F 15/46; G06G 7/12**

[52] U.S. Cl. **395/27; 395/24**

[58] Field of Search **395/27, 24**

[56] References Cited

U.S. PATENT DOCUMENTS

4,660,166	4/1987	Hopfield	395/24
4,807,168	2/1989	Moopenn et al.	395/27
4,931,763	6/1990	Ramesham et al.	395/27

OTHER PUBLICATIONS

Goldstein, M., N. Toomarian, and J. Barhen, 1988. "A Comparison Study of Optimization Methods of the Bipartite Matching Problem (BMP)". *IEEE conf. Neural Networks II-267*, San Diego, Calif., Jul. 24-27, 1988.

Hopfield, J. and D. Tank, 1985. "Neural Computation of Decisions in Optimization Problems". *Biol. Cybern.* 52:147-152.

Moopenn A., A. Thakoor, and T. Duong, 1988. "A Neural Network for Euclidean Distance Minimization". *Proc. IEEE conf. Neural Network II-349* San Diego, Calif., Jul. 24-27, 1988.

Primary Examiner—Allen R. MacDonald

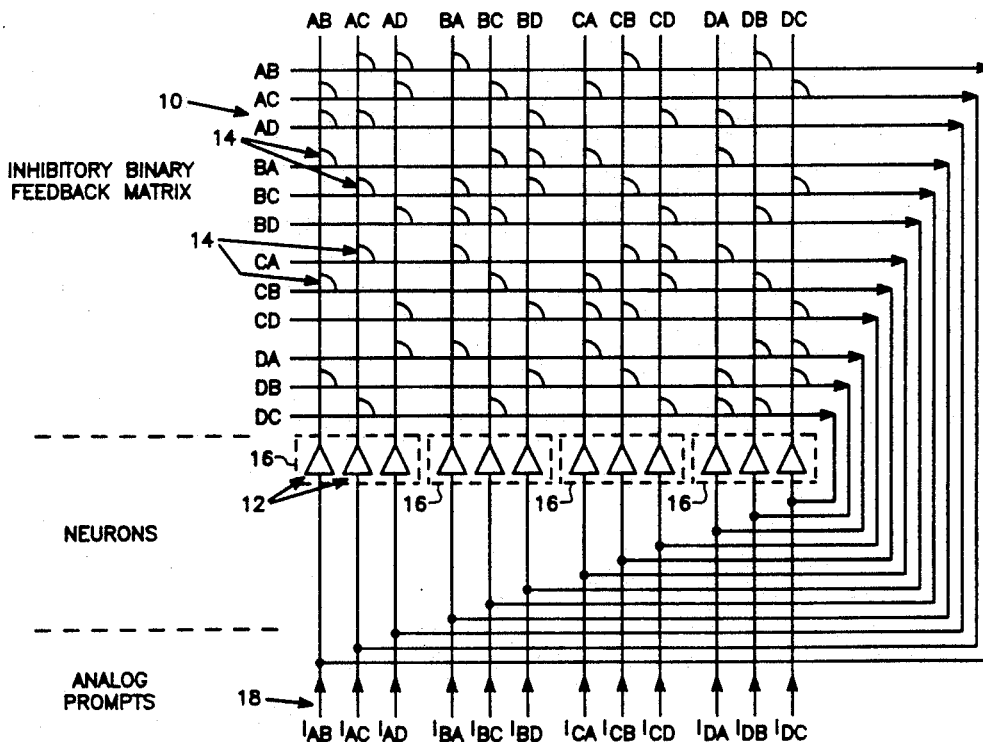
Assistant Examiner—George Davis

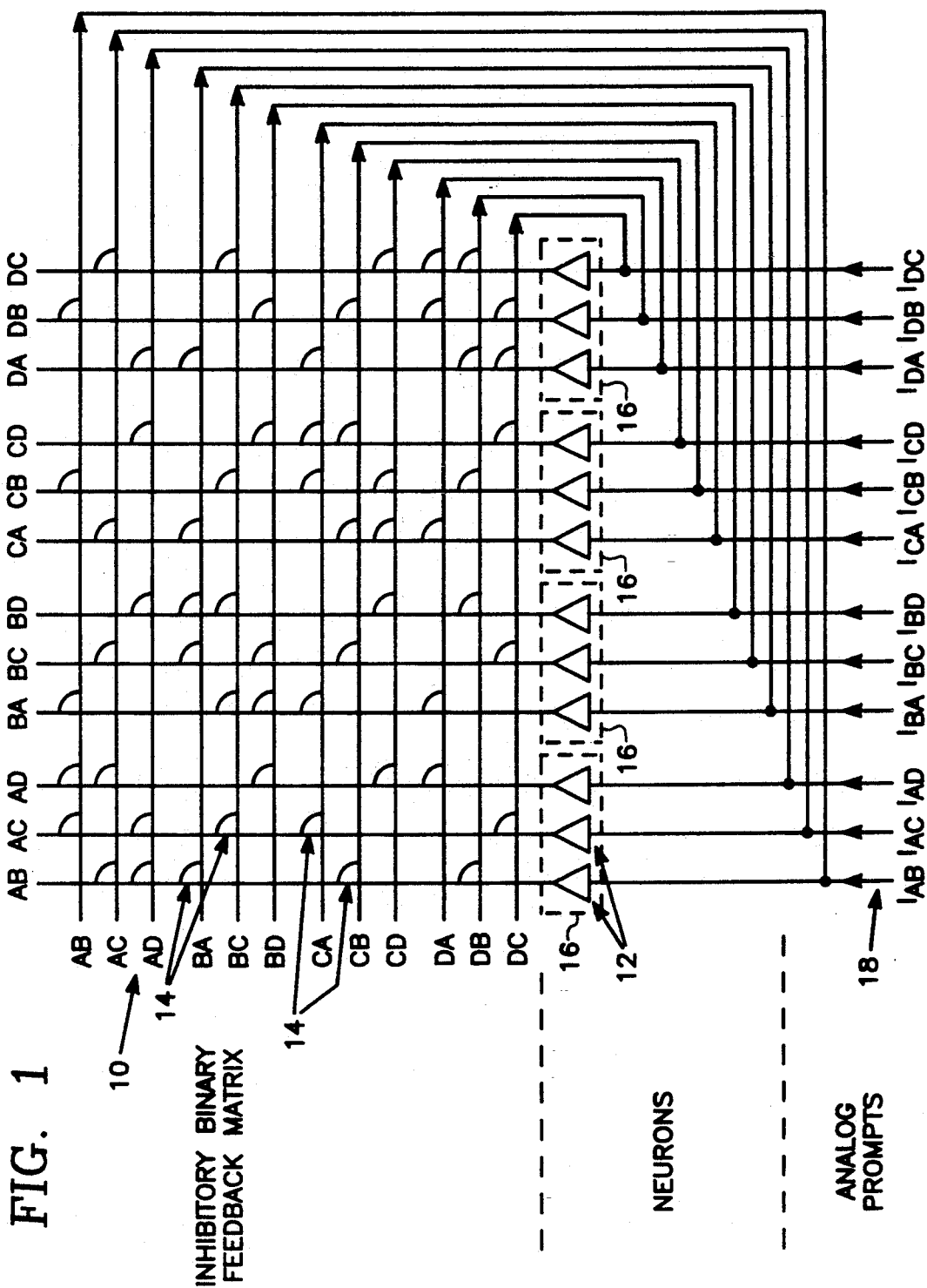
Attorney, Agent, or Firm—John H. Kusmiss; Thomas H. Jones; Guy M. Miller

[57] ABSTRACT

This invention is a novel high-speed neural network based processor for solving the "traveling salesman" and other global optimization problems. It comprises a novel hybrid architecture employing a binary synaptic array whose embodiment incorporates the fixed rules of the problem, such as the number of cities to be visited. The array is prompted by analog voltages representing variables such as distances. The processor incorporates two interconnected feedback networks, each of which solves part of the problem independently and simultaneously, yet which exchange information dynamically.

29 Claims, 6 Drawing Sheets





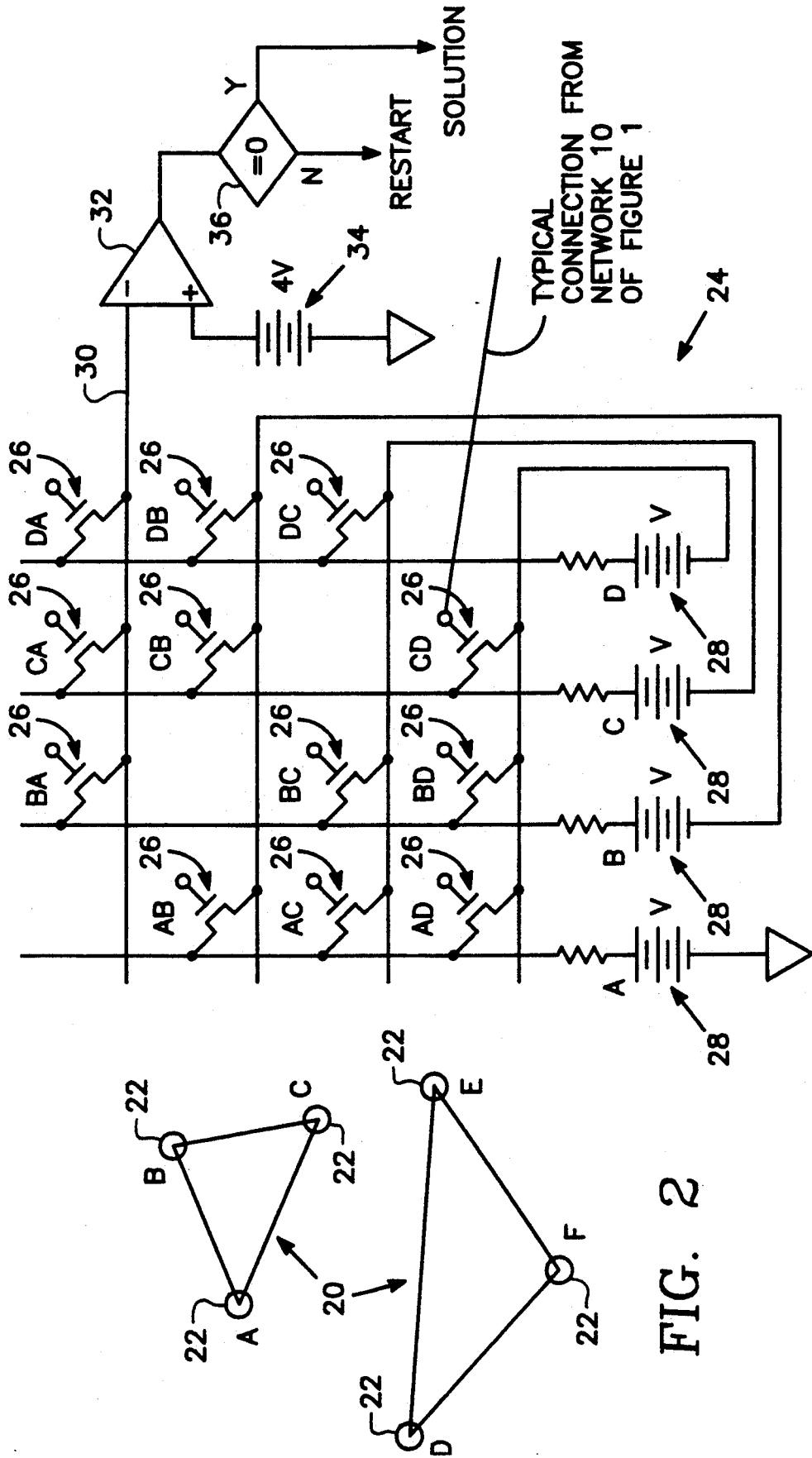
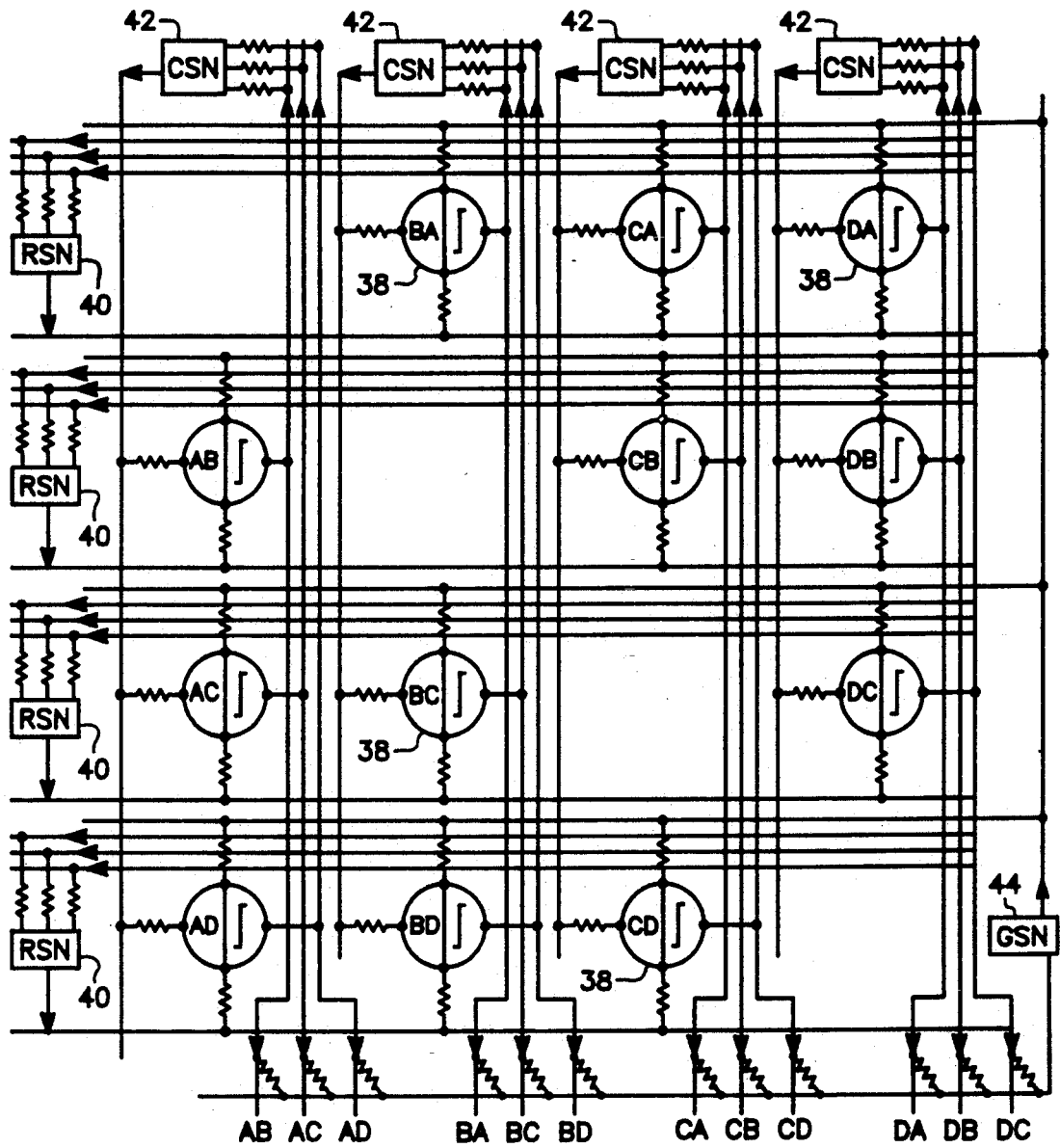


FIG. 3

FIG. 2



RSN
ROW
SUPER
NEURON

CSN
COLUMN
SUPER
NEURON

GSN
GLOBAL
SUPER
NEURON

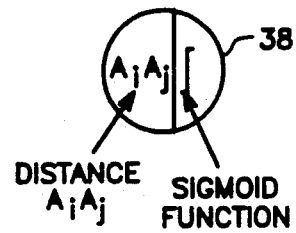


FIG. 4

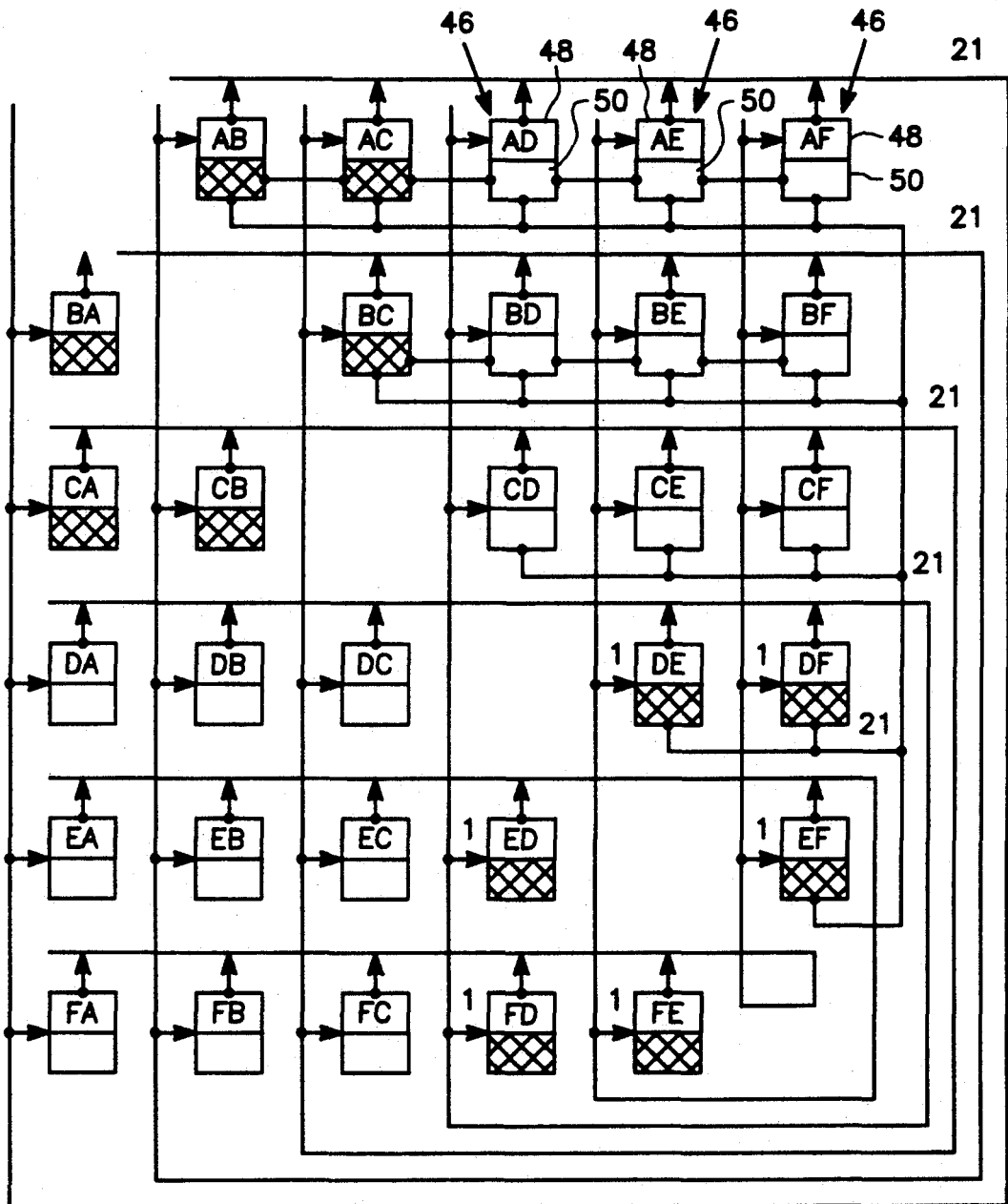
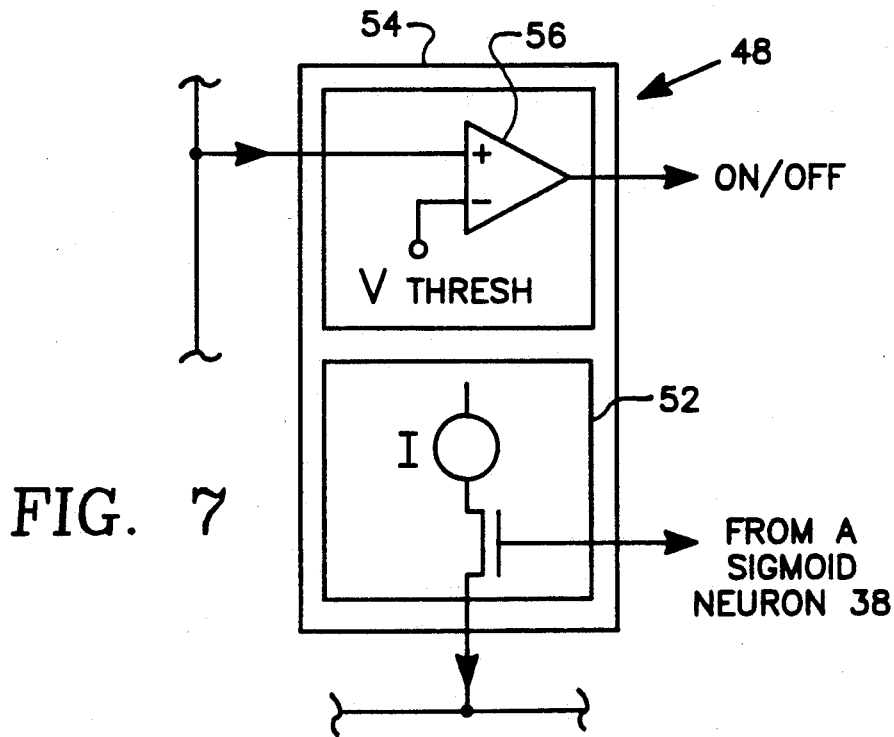
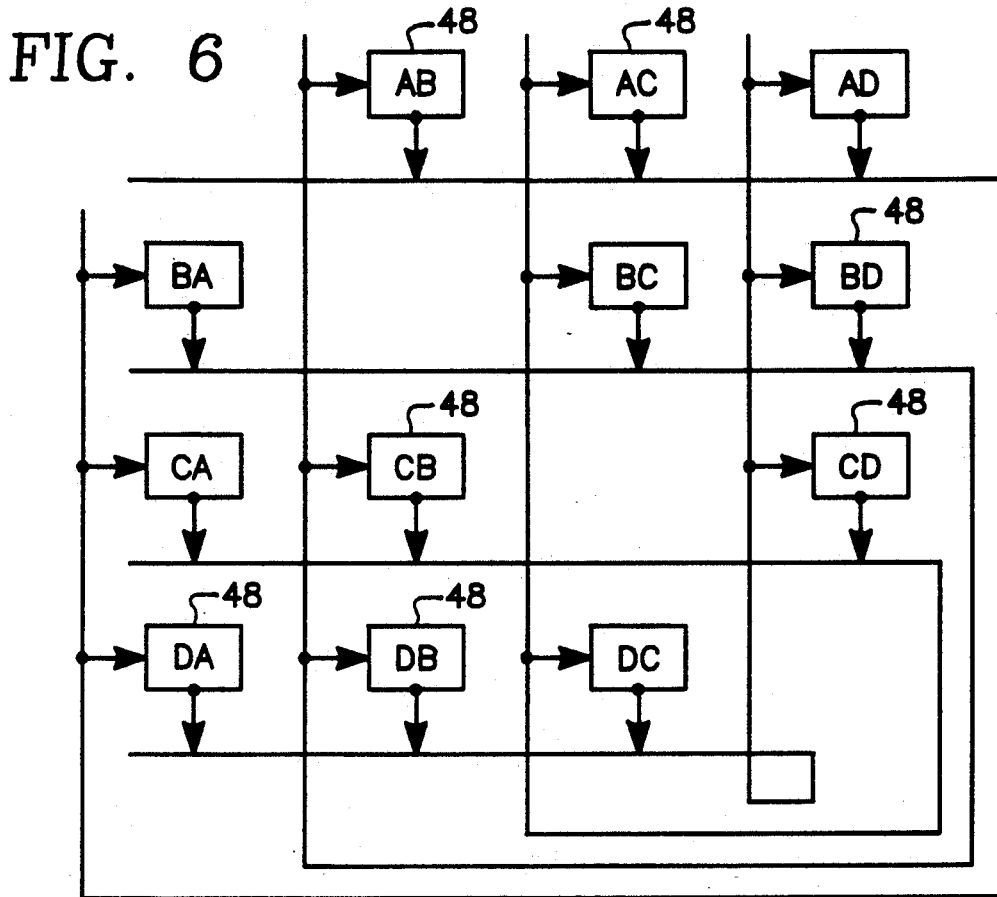


FIG. 5



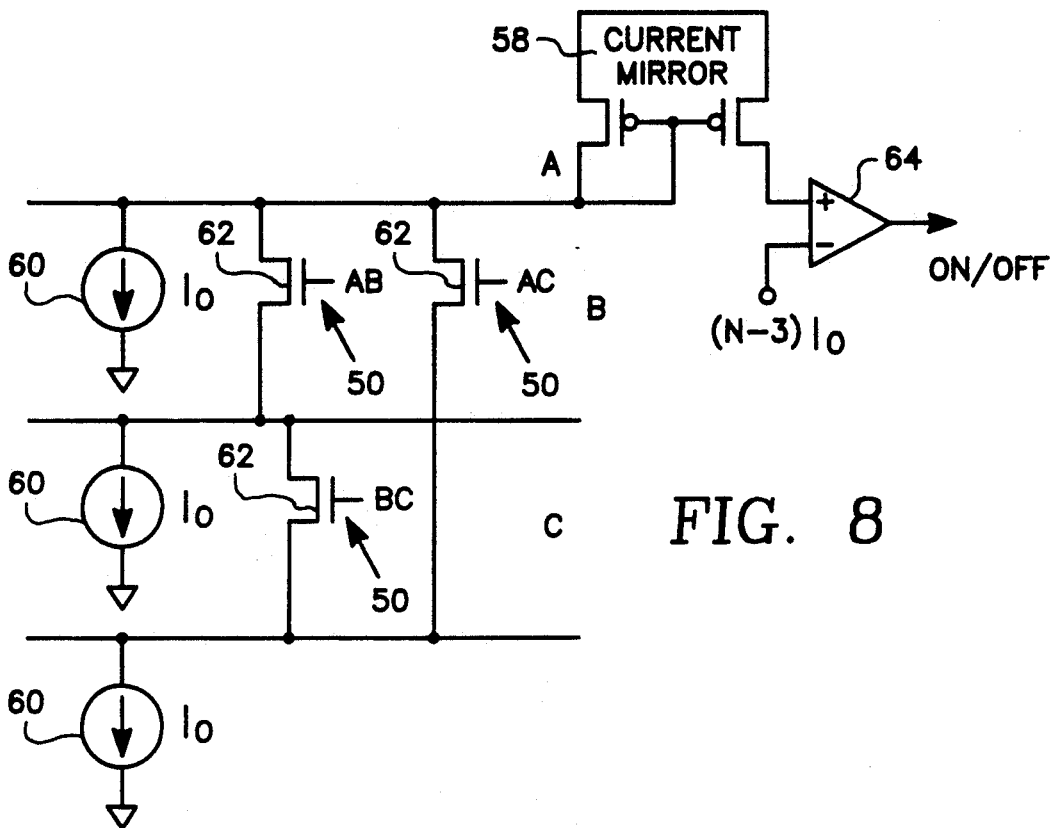


FIG. 8

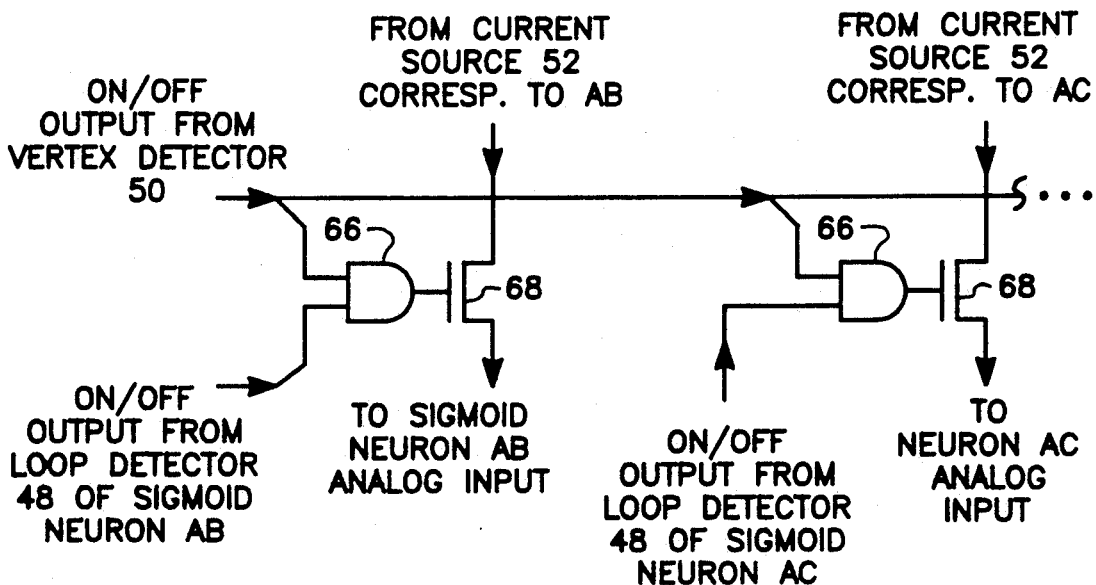


FIG. 9

ELECTRONIC NEURAL NETWORK FOR SOLVING "TRAVELING SALESMAN" AND SIMILAR GLOBAL OPTIMIZATION PROBLEMS

ORIGIN OF THE INVENTION

The invention described herein was made in the performance of work under a NASA contract, and is subject to the provisions of Public Law 96-517 (35 USC 202) in which the Contractor has elected not to retain title.

This is a continuation-in-part application of U.S. patent application Ser. No. 07/470,664 filed Jan. 26, 1990 and now is abandoned.

TECHNICAL FIELD

The invention relates to neural computing networks and, more particularly, to a high-speed neural network based processor for solving global optimization problems involving selecting a single optimum route between a plurality of points comprising, a binary synaptic array incorporating the fixed rules of the problem being solved and having a plurality of inputs thereto and outputs therefrom, the outputs comprising the answer to a problem being solved by the processor; neural connecting means comprising a plurality of neurons each having an input and an output for connecting inputs into the binary synaptic array, the outputs of the neurons being connected to the inputs of the binary synaptic array; analog prompting means connected to the inputs of the neurons for prompting the array with analog voltages representing variables of the problem being solved; and, feedback means for feeding back outputs from the binary synaptic array to the neural connecting means to cause the processor to precipitate an answer to a problem being solved, the feedback means comprising two interconnected feedback networks, each of which solves part of the problem being solved independently and simultaneously, the two interconnected feedback networks being connected to exchange information dynamically, a first one of the two interconnected feedback networks solving a basic part of the problem being solved and a second one of the two interconnected feedback networks monitoring the solution to the basic part of the problem being solved by the first one and adding further limitations to assure that the first one provides one and only one solution to the basic part of the problem being solved which meets all predefined constraints defining the entire problem being solved, the first one of the two interconnected feedback networks employing binary inhibitory synapses in a feedback matrix to enforce constraints associated with the problem to be solved which are binary in nature and feeds analog conditions into the first one of the two interconnected feedback networks using analog prompting whereby a user can dynamically change analog represented variables of the problem to be solved, the binary inhibitory synapses including means for grouping the neurons into groups and for preventing more than one of the neurons in a group from indicating travel between a pair of the plurality of points at any given time, the second one of the two interconnected feedback networks including, a plurality of first voltage sources representing respective ones of the plurality of points, a plurality of binary switch means connected to respective ones of the plurality of voltage sources for providing a feedback control signal to the first one of the two interconnected feedback networks, each of the

plurality of binary switch means being associated with one of the neurons and being connected to receive a control signal therefrom whereby the plurality of binary switch means are turned "ON" and "OFF" by their associated one of the neurons, a single output connected to the plurality of binary switch means for outputting a voltage which is the sum of the voltages from the plurality of voltage sources connected to the output by ones of the binary switch means which have been turned "ON" by an associated one of the neurons, and differential sensing means having a first input connected to the single output and a second input connected to a second voltage source equal to the sum of the plurality of first voltage sources for outputting a RESTART feedback control signal to the first one of the two interconnected feedback networks when the differential sending means finds that voltage values at the first and second inputs of the differential sensing means are not equal.

BACKGROUND ART

In the field of computing hardware, there is a class of global optimization problems which is typified by the so-called "traveling salesman problem" (TSP). In the TSP, a salesman with N cities on his itinerary is to select the round-trip tour that goes through each of the N cities and minimizes the total distance travelled. As those skilled in the art are well aware, such "global optimization problems" are very computation intensive since the time required to find the optimal tour solution by an exhaustive search grows faster than any polynomial function as the size of the problem increases.

In the past, a variety of heuristic search algorithms have been applied to this problem with varying degrees of efficiency. Artificial neural network approaches to TSP have also been proposed in recent years (e.g. Hopfield and Tank in 1985). These prior art neural network approaches to TSP typically require a large number $2N^3$ for N cities of analog synapses. Such prior art neural network solutions have been tested on software digital simulators; however, their hardware implementation as an actual neural network have seldom been attempted due primarily to the involved hardware complexity in the artwork architectures.

Multiparameter optimization problems impose multiple constraints, to be dealt with simultaneously, to select the "best situation" or the global solution under given conditions. With increasing dimensionality, therefore, solutions of such problems become highly computation intensive. A brute force approach in a complex situation requires actual comparison and search through all possible solutions to determine and select the "lowest cost" situation. In the case of the TSP, for example, for the N cities the possible number of distinct tours is given by $T = (N)!/2N$. For four cities, the number of tours is only three. For only eight cities, however, the number grows to 2,520. At only twelve cities, the number of tours leaps to almost 20 million. Clearly, such problems quickly approach a level where a brute force approach is impractical even at modern computer speeds. In portable implementations, for example, where computer power is very limited, a better solution is obviously needed.

The above-mentioned neural network solution of the TSP proposed by Hopfield and Tank consists of an $N \times N$ array of neurons. The rows are labelled by the cities and the columns by the order in which the cities

are visited. Thus, a valid tour corresponds to having only a single active neuron in each row as well as in each column of the neuron array. Lateral inhibition along the rows as well as columns is required to enforce these constraints, in addition to a large number ($\sim 2N^3$) of analog synapses to "store" the inter-city distance information. Although the algorithm does capture all the constraints in the proposed network, the complexity of the network architecture has been the primary constraint in preventing its actual hardware implementation. In other words, it works in simulation; but, is too complex to build in actuality. The promise of high speed solutions to TSP and similar global optimization problems from the neural net approach, therefore, has not been realized so far.

STATEMENT OF THE INVENTION

Accordingly, it is an object of this invention to provide an implementation of the traveling salesman problem in a neural network that lends itself to easy hardware implementation.

It is another object of this invention to provide an approach to the implementation in a neural network of a general class of global optimization problems as typified by the traveling salesman problem.

Other objects and benefits of this invention will become apparent from the detailed description which follows hereinafter when taken in conjunction with the drawing figures which accompany it.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a simplified drawing depicting the first network portion of the present invention.

FIG. 2 is a simplified drawing depicting how the first network portion of FIG. 1 may provide a solution to the problem to be solved not meeting all the constraints of the total problem.

FIG. 3 is a simplified drawing depicting the auxiliary network portion of the present invention which is used in combination with the first portion of FIG. 1 to prevent the situation shown in FIG. 2.

FIG. 4 is a simplified schematic block diagram of a neural network in accordance with an alternative embodiment of the invention.

FIG. 5 is a simplified schematic block diagram of a portion of a cluster buster circuit corresponding to an improvement of the embodiment of FIG. 3.

FIG. 6 is a simplified schematic block diagram illustrating a portion of the circuit of FIG. 5 embodying a set of loop detectors.

FIG. 7 is a simplified schematic diagram of a loop detector circuit corresponding to FIG. 6.

FIG. 8 is a simplified schematic diagram illustrating a portion of the circuit of FIG. 5 embodying a set of vertex detectors forming a vertex detector network.

FIG. 9 is a simplified schematic diagram illustrating a control circuit responsive to the loop and vertex detectors of FIGS. 6 and 9.

DETAILED DESCRIPTION OF THE INVENTION

The present invention as now to be described achieves its stated objectives by employing a hybrid approach in which analog prompting is utilized in association with only binary synapses. This, of course, provides an implementation which is significantly simpler to implement in hardware than the traditional wholly analog approach of the prior art. In the approach taken

by the inventors herein, appropriate partitioning of the basic problem of TSP and the unique scheme of analog prompting make this significant difference in the overall complexity of the network. Constraints such as (1) visit every city only once, (2) do not retrace the path, and (3) do not skip any city, are essentially binary in nature. On the other hand, the requirement to minimize the total distance travelled clearly deals with analog quantities.

This invention uses binary inhibitory synapses in a feedback matrix to enforce those constraints which are binary in nature and feeds analog conditions into the network using analog prompting. The major difference from prior art solutions to TSP, for example, lies in the representation of the problem in the network. The neurons in this invention represent inter-city connections rather than the cities and the sequence in the tour. This allows the user to represent (i.e., feed in) the analog distances or costs of the corresponding travels as analog prompts, or fixed bias inputs, to the neurons in the network.

FIG. 1 shows a schematic of a feedback network 10 according to the present invention implementing a four-city (A, B, C, and D) TSP. The neurons 12 are associated with the inter-city travels (e.g. AB, AC, AD, BC, BD, ... etc.). The sparsely connected feedback matrix of synapses 14 represents the binary constraints. For example, when neuron 12 labelled "AB" is "ON" (i.e., the salesman travels from A to B), he cannot travel from A to C, A to D, or B to A in that tour; therefore, neuron 12 "AB" inhibits, for example, "AC", "AD", and "BA", through the inhibitory binary synapses 14 in the matrix. All such binary connections form the "symmetric" feedback matrix shown for four cities in FIG. 1. Therefore, in each subgroup of neurons 12 (e.g., AB, AC, and AD), lateral excitation/inhibition control is provided, as represented by the dashed boxes 16, such that one and only one neuron 12 among them is "ON", since salesman can go "out" from city A only once. For a given situation, the inter-city distances are used to generate the analog excitatory prompts at 18 to all the neurons 12 (proportional to the corresponding distance-costs subtracted from a fixed, preselected excitation level).

The network 10 of FIG. 1, when prompted at 18 with the appropriate analog values, does generate a tour that goes through all the cities, visiting each city only once and without skipping any city; however, it must be pointed out that in an attempt to minimize the distance cost as much as possible, this particular network often tends to generate smaller disjointed closed loops connecting groups of cities independently, rather than taking a single closed loop, particularly when the cities are clustered in groups farther away from each other. FIG. 2 depicts such a solution for a six city TSP with two closed triangular loops 20 connecting three cities 22 each (i.e., A, B & C and D, E & F) instead of the desired single six city tour. Of course, in that process the network 10 is doing a superb job of enforcing all the implemented constraints and minimizing the total cost. To enforce the remaining condition that there should be only one closed loop tour, therefore, an auxiliary network working in combination with the network 10 is necessary. FIG. 3 shows a schematic of the auxiliary network 24 for use in association with the above-described example of a four city TSP situation depicted in FIG. 1. The 4×4 array of binary switches 26, except for the diagonal, is directly connected to the output activity (with a fixed threshold) of all the neurons 12 of

the previous network 10, as labelled. The voltage sources 28 represent the cities and are labelled accordingly (e.g., A, B, C and D). When a solution starts emerging from the first network 10 of FIG. 1, the switches 26 in the auxiliary network 24 close and the signals from the different city (i.e., voltage) sources 28 start connecting themselves in a series, adding to a signal that goes back to the first network 10. In one possible scenario, the auxiliary network 24 can be used to inhibit several of the set of neurons 12 which are "ON" in a multi-loop solution. This forbids the multi-loop solution (e.g., as depicted in FIG. 2) from being a stable state of the network 10. On the other hand, if all cities are connected in a single tour as desired, the auxiliary circuit 24 gives its "approval" by sending no inhibitory signal to the winning set of neurons 12.

The foregoing operation of the auxiliary network 24 of FIG. 3 works as follows. Each city's voltage source 28 provides a fixed voltage potential (V). The output 30 of the auxiliary network 24 is connected as one input of a differential amplifier 32. The other input of the differential amplifier 32 is connected to a voltage source 34 which is equal to the sum of the Vs for all the cities. In this case, therefore, with four cities the voltage source 34 provides a constant voltage of 4V to the differential amplifier 32. The output of the differential amplifier 32 is connected to thresholding logic 36. If the thresholding logic 36 senses that the output of the differential amplifier 32 is zero, it knows that all the cities are included in the present solution since there is a connection through the switches 26 (controlled by the connections to the network 10 of FIG. 1) to each voltage source 28 (in order for 4V to be output at 30). In such a case, the thresholding logic 36 outputs a "SOLUTION" indication as indicated. If, on the other hand, the thresholding logic 36 senses that the differential amplifier 32 is outputting other than zero, it knows that all cities are not accounted for in the present solution being proposed by the network 10 and, therefore, the thresholding logic 36 outputs a "RESTART" command to the network 10 of FIG. 1. Thus, the auxiliary network 24 of FIG. 3 in combination with the network 10 of FIG. 1 assures that the proper total solution to the TSP is quickly precipitated.

In another embodiment, the large number $O(N^3)$ of binary/analog synapses is replaced by only $2N^2$ binary synapses and $\sim N^2$ analog synapses. This alternative embodiment is distinguished by the addition of $2N+1$ superneurons: one for each row, one for each column, and one for global control. In comparison with the previous method, this approach reduces the hardware complexity at least by a factor of N. As shown in FIG. 4, the outputs of the neurons 38 from each row (column) are fed into the corresponding row- (column-) superneuron, 40 (42) which in turn sends back an appropriate level of excitatory or inhibitory signal to all the neurons in that row (column), to ascertain a valid tour with a fixed number of "ON" neurons 38. The global superneuron 44 similarly keeps a "watch" on the total number of "ON" neurons in the selected solution. The superneurons 40, 42, 44 therefore collectively provide the simultaneous dynamic control over the evolution of the circuit of FIG. 4 during its convergence behavior.

In our network, the usual input resistor rho and input capacitor C (used typically in a fully connected feedback neural network) for each neuron are replaced by simple low pass filter circuits using an "RC" circuit (not shown). Furthermore, the usual current bias is also not

necessary as the superneurons 40, 42, 44 can be very sensitive for taking over the role of bias currents in feedback networks. Our software simulation results show that a single loop or a multi-loop solution is always found by our circuit with superneurons with guarantee. Further, the cluster buster circuit described below is simple enough to implement in analog hardware and promises to find a single loop solution in a very short time.

DESCRIPTION AND EXPLANATION

The proposed feedback network (FIG. 4) utilizes three types of neurons:

1. $N \times (N-1)$ sigmoid neurons 38 associated with individual inter-city travels. These are arranged in a "diagonal-less" $N \times N$ array and are of the type employed in the embodiment of FIG. 1. As indicated in the legend of FIG. 4, each neuron 38 responds to two inputs A_i and A_j , which are analog voltages corresponding to distances or transitions between cities or points. Each neuron 38 generates an output signal which is a sigmoid function of the two inputs A_i and A_j .

2. Local row and column superneurons 40, 42 (RSN and CSN) associated with each row (and column) of the sigmoid neurons 38 described above. There are of course $2N$ of these for the N-city problem. Each of these superneurons 40, 42 only control the corresponding group of neurons in a single row (or column) to satisfy the local constraint (e.g., only two "ON" sigmoid neurons 38 in a given row or column if $N=4$).

3. A single "global" superneuron 44 (GSN) that monitors the total activity of all the $N(N-1)$ sigmoid neurons, 40, 42 to meet the requirement of the constraint that only N "inter-city travels" are allowed globally.

Sigmoid neurons 38 selected as the final solution must therefore satisfy the constraints imposed by all the superneurons 40, 42, 44 simultaneously, in order to obtain a valid solution. The symmetry across the diagonal in the array (FIG. 4) dictates that, each of the N row superneurons 40 (or each of the N column superneurons 42) must ensure two and only two sigmoid neurons "ON" in each row (column) in the case of $N=4$, for example. This is the local constraint which implements the ultimate constraint that every city is visited once and only once in a closed tour.

Since the "diagonal-less" matrix of the sigmoid neurons 38 of FIG. 4 is symmetrical across the (missing) diagonal, the tour can be read directly from only the upper triangle or the lower triangle of the matrix. The duplication of the triangle is only an implementation convenience. After providing all the costs of the inter-city travels to the matrix of sigmoid neurons as analog prompts in the manner of the embodiment of FIG. 1, the matrix of FIG. 4 goes into action. To start with, several of the low cost sigmoid neurons 38 tend to come "ON". If the tour however is not complete, superneurons 40, 42 act based on the shortage or excess in the number of the "ON" neurons in each row and column, providing the desired excitation or inhibition, respectively. The procedure occurs simultaneously at every neuron. Since, in this process, it is difficult for the primary decision to interchange information between rows and columns while interacting with high gain neurons, the global neuron 44 is needed to provoke and bring back the process in a reasonable linear region of the dynamics in order to exchange the information globally. The global superneuron 44 thus checks the $2N$ sigmoid neurons which are "ON" in the whole network. The inhibi-

tion or excitation is applied by the global superneuron 44 to all the sigmoid neurons 38 depending on the excess or shortage in total number of neurons which are "ON".

To understand the dynamics of a complete convergence cycle of the network of FIG. 4, consider a set of analog prompts being applied to the array of sigmoid neurons 38. All sigmoid neuron outputs are initially low. All RSN's 40, CSN's 42, and the GSN 44 that do not get sufficient input from the sigmoid neurons 38 generate excitations for their corresponding rows and columns. As the analog prompt to each sigmoid neuron 38 provides varying activity in the array, some of the sigmoid neurons 38 increase their output faster than others and the associated RSNs 40 and CSNs 42 adjust their excitatory/inhibitory feedback levels to dynamically satisfy the given constraints, until the solution is reached. For example, let the sigmoid neuron AB (and thus BA) be on. The row and column that include those neurons AB and BA would still not be satisfied with their constraints (two sigmoid neurons "ON" per row and column). The excitations are thus applied to stimulate other neurons such as AC, AD, BC, and BD, etc. Although this example is explained in a serial fashion, it should be noted all the components of the network in an implementation would actually behave in a fully parallel fashion.

Unfortunately, the network of FIG. 4 also does not guarantee a single loop tour. Depending on the positions and distribution of the cities, a multi-loop tour is occasionally formed. This is not a negative attribute of the network at all because the network in fact does attempt to always provide the lowest cost solution as it should, which only happens to be a multi-loop solution in some cases, due to the city distribution. Thus, the network provides an additional piece of valuable information: "hire more than one salesman".

FIG. 2 discussed above shows a low cost multi-loop solution for $N=6$ with two closed triangle loops, each of which is connecting three cities, instead of a single six city tour, which would of course be a higher cost solution. To reduce the multi-loop solution to a single loop, a modified cluster buster circuit is proposed in the following. The following cluster buster circuit (FIG. 5) is useful with either of the embodiments of FIG'S. 1 and 4.

In order to achieve a fast response from the cluster buster circuit, $N(N-1)$ sigmoid neuron outputs are made to communicate with the cluster buster circuit in a parallel fashion. The matrix of FIG. 4 of the sigmoid neurons 38 is symmetrical. Therefore, we need to use only $N(N-1)/2$ (upper triangle or lower triangle matrix) outputs to give enough information to the cluster buster circuit. However, in the current version of the cluster buster circuit (schematically shown in FIG. 5) a complete, symmetrical matrix of a cluster buster circuit is used. Based on the provided information, the cluster buster circuit of FIG. 5 detects the presence of a multi-loop situation and generates corrective action to push the network of FIG. 4 towards finding a valid single loop solution. In a nutshell, when the circuit of FIG. 5 detects multiple closed loops, it inputs additional "costs" to selected neurons 38 forming the multi-loop solution of FIG. 4 (in effect expanding the corresponding travel distances involved in those loops). These costs increase gradually until the closed loops are broken, leading to a convergence of the sigmoid neuron array of FIG. 4 in a different direction. The cluster buster circuit of FIG. 5 therefore consists of two sub-circuits at each node 46: a loop detector circuit 48 and

a vertex detector circuit 50. Each of these circuits 48, 50 functions independently but in close cooperation with each other. Both loop and vertex detector circuits 48, 50 must be "ON" simultaneously, to activate the action of breaking the loops. The "ON" cells corresponding to the solution of FIG. 2 are shaded in FIG. 5.

Each node 46 as shown in FIG. 5 has two parts: the upper rectangle represents a loop detector 48 that checks if the corresponding city is contained in any loop, and the lower rectangle represents a vertex detector 50. All of the vertex detectors 50 in FIG. 5 together form a vertex detector network to be described below which detects the presence of a multi-loop solution. For the loop detector 48 (shown separately for clarity in FIG. 6), a simple current gain amplifier cell is used. There are two parts 52, 54 in this cell 48 (see FIG. 7). A constant current source i , if the corresponding sigmoid neuron 38 is "ON", provides an output current from the first part of the cell 52 to the overall output line of each row of the cluster buster circuit of FIG. 6, establishes a certain voltage level for that row. Secondly, the feedback current collected from all the cities in the same row (say i th row) is fed into the feedback input of the second part 54 of the corresponding (i th) column with a gain. The symmetry of the matrix dictates that if any closed loop is formed (which invariably would need nodes from several rows and columns), the voltage level at the corresponding rows would increase continuously due to the constructive feedback, reaching above a certain predetermined threshold V_{thres} . This would be a direct indication of a closed loop formation. If the travels selected at any time do not form a closed loop, then the corresponding row output voltage would decrease exponentially with feedback and would always collapse consistently and remain below the threshold. Therefore, a comparator 56 in the second part 54 produces an "ON" or "OFF" signal depending upon whether the feedback voltage is above or below V_{thres} , respectively.

Returning to FIG. 5, all the vertex detectors 50 form a network partially shown in FIG. 8. A current mirror 58 is arbitrarily assigned to one row of the vertex detector network. In the vertex detector network of FIG. 8, a current source 60 is assigned for every row corresponding to a city. (The rows and columns of the vertex detector network of FIG. 8 correspond to the rows and columns of the neural network of FIG. 1 or FIG. 4. It will be remembered that, in accordance with the embodiment of FIG. 1, each neuron 38 corresponds to an inter-city trip while each row or column corresponds to a city.) In addition, all cities communicate pairwise with each other in the vertex detector network of FIG. 9 through a set of switches (transistors) 62, comprising each vertex detector 50 and having control inputs connected to outputs of corresponding sigmoid neurons 38, as indicated in FIG. 8. If a closed loop is formed, any "ON" sigmoid neurons 38 in the loop would turn on corresponding switches 62 to connect rows (cities) together. If the current mirror 58 happens to be a part of a loop, it would reflect the number of cities in that loop by summing all the current sources 68. If the current output from the current mirror 58 is larger than $(N-3)I_0$, a comparator 64 provides a vertex detector "OFF" output. Otherwise, it generates an "ON" output. With the vertex detector "OFF" and the loop detector "ON", we can imply that a single loop solution is found. If the current mirror 58 does not belong to any loops,

we can imply that the solution is not found. In this case, the vertex detector output is "ON".

In order to check for multi-loops and to bust the closed loops efficiently to each inter-city connection, an AND gate 66 in the controller of FIG. 9 checks whether:

- (a) The loop detector is "ON" for that inter-city or sigmoid neuron;
- (b) it is not a single loop solution with N vertices (vertex detector-ON).

The controller of FIG. 9 operates as follows:

If the output of the AND gate 66 is HIGH, it turns on an N-channel gate 68 to send the amount of current from the current source 52 to the corresponding sigmoid neuron 38 as inhibitory feedback, to increase the distance of the correspond inter-city trip. The additional "cost" between cities in the loops is increased continuously until closed loops no longer exist, then the added "cost" is removed leaving the original analog inter-city distance values. In this embodiment, the additional "cost" is imposed on any "ON" sigmoid neurons 38 in proportion to the population of the loops of which they are members.

As mentioned above, the controller of FIG. 9 breaks up multiple closed loops by imposing added costs on each "ON" neuron 38 in proportion to the number of neurons comprising the closed loop of which it is a member. However, in an alternative embodiment of the controller of FIG. 9, the added cost instead may be proportional to the original cost (inter-city distance) or analog value of the neuron. This would be accomplished by deriving the current source input to the transistor 68 from the initial analog voltage which first establishes the inter-city distance of the neuron, rather than the current source 52. In yet another alternative embodiment of the controller of FIG. 9, the imposed cost would be a uniform constant amount for all "ON" neurons. This would be accomplished by providing the same current source to all of the transistors 68. Finally, the controller of FIG. 9 may embody a combination of all of the foregoing ways of computing the added cost for each "ON" neuron.

In a corollary variation of the controller of FIG. 9, rather than adding cost to the "ON" neurons in the form of inhibitory feedback, as discussed above, cost may be subtracted from the "OFF" neurons in the form of excitatory feedback. In order to accomplish this, the loop detector input to the AND gate 66 would be inverted and the polarity of current through the transistor 68, as applied to the neuron analog input, would be inverted. In this way, and in accordance with the foregoing description, the added "reward" (as opposed to added "cost") would be one of the following or combination thereof: (a) inversely proportional to the number of neurons in the loop including a city corresponding to the "OFF" neuron, (b) inversely proportional to the inter-city distance associated with the "OFF" neuron or (c) a uniform constant amount for all "OFF" neurons.

Wherefore, having thus described our invention, what is claimed is:

1. A high-speed neural network for solving global optimization problem involving selecting routing between a plurality of points comprising:

- a binary synaptic array having a plurality of inputs and outputs and incorporating fixed rules of the problem being solved in a plurality of connecting binary inhibitory synapses;

- b) a neuron array comprising a plurality of non-linear neuron elements each having an input and an output with said outputs of said non-linear neuron elements connected to respective ones of said inputs of said binary synaptic array;
 - c) analog prompting means connected to said inputs of said non-linear neuron elements for prompting said neuron array with analog voltages representing variables of the problem being solved; and,
 - d) feedback means connected from said outputs of said binary synaptic array to respective ones of said inputs of said non-linear neuron elements, said outputs of said binary synaptic array also comprising the output of the neural network based processor.
2. The neural network of claim 1 wherein: the network incorporates two interconnected feedback networks, each of which solves part of the problem independently and simultaneously, said two interconnected feedback networks being connected to exchange information dynamically.
3. The neural network of claim 2 wherein:
- a) a first one of said two interconnected feedback networks solves a part of the problem; and,
 - b) a second one of said two interconnected feedback networks monitors solution of said part of the problem and adds further limitations to assure that said first one provides one and only one solution to said part of the problem which meets all predefined constraints defining the problem being solved.
4. The neural network of claim 3 wherein: said first one of said two interconnected feedback networks employs binary inhibitory synapses in a feedback matrix to enforce constraints associated with the problem to be solved which are binary in nature and feeds analog conditions into said first one of said two interconnected feedback networks using analog prompting whereby a user can dynamically change analog represented variables of the problem to be solved.
5. The neural network of claim 4 wherein: said binary inhibitory synapses are disposed to be associated with a plurality of groups and include means for preventing more than one of said non-linear neuron elements of a group from being "ON" to indicate travel between a pair of the plurality of points at any given time.
6. The neural network of claim 3 wherein said second one of said two interconnected feedback networks includes:
- a) a plurality of first voltage sources representing respective ones of the plurality of points;
 - b) a plurality of binary switch means connected to respective ones of said plurality of first voltage sources for providing a feedback control signal to said first one of said two interconnected feedback networks, each of said plurality of binary switch means being associated with one of said non-linear neuron elements and being connected to receive a control signal therefrom whereby said plurality of binary switch means are turned "ON" and "OFF" by their associated one of said non-linear neuron elements;
 - c) an output connected to said plurality of binary switch means for outputting a voltage which is the sum of the voltages from said plurality of voltage sources connected to said output by ones of said binary switch means which have been turned

"ON" by an associated one of said non-linear neuron elements; and,

- d) differential sensing means having a first input connected to said output and a second input connected to a second voltage source equal to the sum of said plurality of first voltage sources for outputting a RESTART feedback control signal to said first one of said two interconnected feedback networks when said differential sensing means finds that voltage values at said first and second inputs of said differential sensing means are not equal.

7. The neural network of claim 1 wherein:

- a) said binary inhibitory synapses are disposed to be associated with a plurality of groups; and,
 b) said non-linear neuron elements are interconnected in an inhibitory manner to prevent more than one of said non-linear neuron elements associated with a group from connecting inputs into said network to indicate travel between a pair of the plurality of points at any given time.

8. A high-speed neural network for solving a global optimization problem involving selecting a single optimum route between a plurality of points comprising:

- a) a binary synaptic array incorporating fixed rules of the problem and having a plurality of inputs thereto and outputs therefrom, said outputs comprising an answer to said problem;
 b) neural connecting means comprising a plurality of neurons each having an input and an output for connecting inputs into said binary synaptic array, said outputs of said neurons being connected to said inputs of said binary synaptic array;
 c) analog prompting means connected to said inputs of said neurons for prompting said binary synaptic array with analog voltages representing variables of the problem; and,
 d) feedback means for feeding back outputs from said binary synaptic array to said neural connecting means to precipitate an answer to said problem, said feedback means comprising two interconnected feedback networks, each of which solves part of the problem being solved independently and simultaneously, said two interconnected feedback networks being connected to exchange information dynamically, a first one of said two interconnected feedback networks generating a solution to a basic part of the problem and a second one of said two interconnected feedback networks monitoring said solution to said basic part of the problem and adding further limitations to assure that said first one provides one and only one solution to said basic part of the problem being solved which meets all predefined constraints defining the entire problem being solved.

9. The neural network of claim 8 wherein:

said first one of said two interconnected feedback networks employs binary inhibitory synapses in a feedback matrix to enforce constraints associated with the problem to be solved which are binary in nature and feeds analog conditions to said first one of said two interconnected feedback networks using analog prompting whereby a user can dynamically change analog represent variables of the problem to be solved.

10. The neural network of claim 9 wherein:

said binary inhibitory synapses include means for grouping said neurons into groups and for preventing more than one of said neurons in a group from

indicating travel between a pair of the plurality of points at any given time.

11. The neural network of claim 8 wherein said second one of said two interconnected feedback networks includes:

- a) a plurality of first voltage sources representing respective ones of the plurality of points;
 b) a plurality of binary switch means connected to respective ones of said plurality of voltage sources for providing a feedback control signal to said first one of said two interconnected feedback networks, each of said plurality of binary switch means being associated with one of said neurons and being connected to receive a control signal therefrom whereby said plurality of binary switch means are turned "ON" and "OFF" by their associated one of said neurons;
 c) an output connected to said plurality of binary switch means for outputting a voltage which is the sum of the voltages from said plurality of voltage sources connected to said output by ones of said binary switch means which have been turned "ON" by an associated one of said neurons; and,
 d) differential sensing means having a first input connected to said single output and a second input connected to a second voltage source equal to the sum of said plurality of first voltage sources for outputting a RESTART feedback control signal to said first one of said two interconnected feedback networks when said differential sensing means finds that voltage values at said first and second inputs of said differential sensing means are not equal.

12. The neural network of claim 8 wherein:

said neural connecting means are interconnected in an inhibitory manner to prevent more than one of said neurons from connecting an input into said array to indicate travel between a pair of the plurality of points at any given time.

13. A high-speed neural network for solving a global optimization problem involving selecting a single optimum route between a plurality of points comprising:

- a) a binary synaptic array incorporating fixed rules of the problem and having a plurality of inputs thereto and outputs therefrom, said outputs comprising an answer to said problem;
 b) neural connecting means comprising a plurality of neurons each having an input and an output for connecting inputs into said binary synaptic array, said outputs of said neurons being connected to said inputs of said binary synaptic array;
 c) analog prompting means connected to said inputs of said neurons for prompting said binary synaptic array with analog voltages representing variables of the problem; and,
 d) feedback means for feeding back outputs from said binary synaptic array to said neural connecting means to precipitate an answer to said problem said feedback means comprising two interconnected feedback networks, each of which solves part of the problem independently and simultaneously, said two interconnected feedback networks being connected to exchange information dynamically, a first one of said two interconnected feedback networks generating a solution to a basic part of the problem and a second one of said two interconnected feedback networks monitoring the solution to said basic part of the problem and adding further

limitations to assure that said first one provides one and only one solution to said basic part of the problem which meets all predefined constraints defining the problem said first one of said two interconnected feedback networks employing binary inhibitory synapses in a feedback matrix to enforce constraints associated with the problem [to be solved] which are binary in nature and feeds analog conditions into said first one of said two interconnected feedback networks using analog prompting whereby a user can dynamically change analog represented variables of the problem, said binary inhibitory synapses including means for grouping said neurons into groups and for preventing more than one of said neurons in a group from indicating travel between a pair of the plurality of points at any given time, said second one of said two interconnected feedback networks including:

d1) a plurality of first voltage sources representing respective ones of the plurality of points,

d2) a plurality of binary switch means connected to respective ones of said plurality of first voltage sources for providing a feedback control signal to said first one of said two interconnected feedback networks, each of said plurality of binary switch means being associated with one of said neurons and being connected to receive a control signal therefrom whereby said plurality of binary switch means are turned "ON" and "OFF" by their associated one of said neurons,

d3) a single output connected to said plurality of binary switch means for outputting a voltage which is the sum of the voltages from said plurality of first voltage sources connected to said output by ones of said binary switch means which have been turned "ON" by an associated one of said neurons, and

d4) differential sensing means having a first input connected to said single output and a second input connected to a second voltage source equal to the sum of said plurality of first voltage sources for outputting a RESTART feedback control signal to said first one of said two interconnected feedback networks when said differential sensing means finds that voltage values at said first and second inputs of said differential sensing means are not equal.

14. A neural network for solving global optimization problems comprising:

an array of neurons arranged in rows and columns, said neurons being associated with corresponding analog values characteristic of transitions between respective pairs of points, each of said neurons comprising means for outputting the corresponding analog value in an "ON" state and for outputting a lesser value in an "OFF" state;

a column super-neuron associated with each of said columns, said column super-neuron comprising means for applying feedback to each neuron in the corresponding column in accordance with the number of "ON" neurons in said column;

a row super-neuron associated with each of said rows, said row super-neuron comprising means for applying feedback to each neuron in the corresponding row in accordance with the number of "ON" neurons in said row.

15. The neural network of claim 14 further comprising:

a global super-neuron associated with all of said neurons, said global super-neuron comprising means for applying feedback to all of said neurons in accordance with the number of "ON" neurons in said array.

16. The neural network of claim 15 wherein: said global super-neuron comprises means for applying an excitory feedback to the neurons in said array whenever the number "ON" neurons in said array is less than a global constraint: applying inhibitive feedback thereto whenever the number of "ON" neurons in said array is greater than said global constraint: and applying no feedback when the number of "ON" neurons is equal to said global constraint.

17. The neural network of claim 14 wherein: said row super-neuron comprises means for applying an excitory feedback to the neurons in the corresponding row whenever the number "ON" neurons in said row is less than said the specified local constraint; applying the inhibitive feedback thereto whenever the number of "ON" neurons in said row is greater than said the specified local constraint and applying no feedback when the number of "ON" neurons is equal to the specified local constraint;

said global super-neuron comprises means for applying an excitory feedback to the neurons in the corresponding column whenever the number "ON" neurons in said column is less than said the specified local constraint; applying the inhibitive feedback thereto whenever the number of "ON" neurons in said column is greater than said the specified local constraint and applying no feedback when the number of "ON" neurons is equal to the specified local constraint.

18. The neural network of claim 17 wherein said rows and columns correspond to different points and said neurons correspond to transitions between pairs of said points and said analog values correspond to values of said transitions, and wherein said local and global constraints limit the number of transitions between any pair of points to a single transition and participation of each said point is exactly two such transitions in the final stable state solution.

19. The neural network of claim 18 wherein the number of points is 4 and said local constraint requires said row and column super-neurons to apply inhibitive feedback whenever the number of "ON" neurons in any row or column exceeds 2 and to apply excitory feedback whenever the number of "ON" neurons in any row or column is less than 2.

20. The neural network of claim 14 wherein each of said row and column super-neurons receives the outputs of the neurons in the corresponding row and column, respectively, and sends a feedback signal to first and second inputs of each neuron in said row and column, respectively.

21. The neural network of claim 20 wherein said global super-neuron receives the outputs of all of said neurons and sends a feedback signal to a third input of each neuron.

22. The neural network of claim 21 wherein each of said inputs is characterized by resistance which tends to have a stabilizing influence on said network.

23. The neural network of claim 14 further comprising a circuit for breaking multiple loop solutions comprising:

vertex detector means for sensing whenever there is more than one closed loop of said points corresponding to "ON" neurons;

loop detector means for sensing the number of "ON" neurons in a closed loop; and

means for transmitting feedback to selected ones of said neurons in response to said vertex and loop detector means.

24. The neural network of claim 23 wherein said feedback transmitted by said means for transmitting is a function of the number of "ON" neurons in respective closed loops.

25. The neural network of claim 24 wherein said means for transmitting feedback comprises means for transmitting an inhibitory feedback signal to said "ON" neurons proportional to the number of "ON" neurons included in the corresponding closed loop.

26. The neural network of claim 24 wherein said means for transmitting feedback comprises means for

transmitting an excitory feedback signal to an "OFF" neuron inversely proportional to the number of "ON" neurons included in a closed loop which includes a "ON" neuron having a point in common with said "OFF" neuron.

27. The neural network of claim 23 wherein said means for transmitting transmits at least one of:

(a) inhibitive feedback to selected ones of said "ON" neurons, and

(b) excitory feedback to selected ones of said "OFF" neurons.

28. The neural network of claim 27 wherein said feedback is one of (a) proportional and (b) inversely proportional to the corresponding analog value.

29. The neural network of claim 27 wherein said feedback is a uniform amount for one of (a) said "ON" neurons and (b) said "OFF" neurons.

* * * * *

20

25

30

35

40

45

50

55

60

65