NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2250

AN ANALYSIS OF THE APPLICABILITY OF THE HYPERSONIC SIMILARITY LAW TO THE STUDY OF FLOW ABOUT BODIES OF REVOLUTION AT ZERO ANGLE OF ATTACK

By Dorris M. Ehret, Vernon J. Rossow,
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Ames Aeronautical Laboratory
Moffett Field, Calif.

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SUMMARY

The hypersonic similarity law as derived by Tsien has been investigated by comparing the pressure distributions along bodies of revolution at zero angle of attack. In making these comparisons, particular attention was given to determining the limits of Mach number and fineness ratio for which the similarity law applies. For the purpose of this investigation, pressure distributions determined by the method of characteristics for ogive cylinders for values of Mach numbers and fineness ratios varying from 1.5 to 12 were compared. Pressures on various cones and on cone cylinders were also compared in this study.

The pressure distributions presented demonstrate that the hypersonic similarity law is applicable over a wider range of values of Mach numbers and fineness ratios than might be expected from the assumptions made in the derivation. This is significant since within the range of applicability of the law a single pressure distribution exists for all similarly shaped bodies for which the ratio of free-stream Mach number to fineness ratio is constant. Charts are presented for rapid determination of pressure distributions over ogive cylinders for any combination of Mach number and fineness ratio within defined limits.

INTRODUCTION

According to the hypersonic similarity law as derived by Tsien (reference 1) the flow is similar about slender, pointed, similarly shaped bodies at zero angle of attack in high Mach number air streams, provided the ratio of free-stream Mach number to body fineness ratio is a constant. If this law were valid for wide ranges of shape and Mach number, it would be extremely valuable, for not only would it be of aid in the correlation of existing data, but it would also greatly simplify the determination, either theoretically or experimentally, of the characteristics of a family of bodies at hypersonic Mach numbers. The determination of these characteristics would be reduced simply to evaluation of the characteristics for a few representative values of the ratio of free-stream Mach number to body fineness ratio.

Since in the derivation of the similarity law, Tsien necessarily made a number of simplifying assumptions, the question naturally arises as to how serious these assumptions limit the usefulness of the law. In an attempt to answer this question, the pressure distributions on two types of bodies have been determined for several Mach numbers and fineness ratios, and the distributions have been compared in terms of the hypersonic similarity law. The results of this investigation are reported herein.

SYMBOLS

C_D drag coefficient[1] $\left(\frac{\text{drag}}{q_o S_b}\right)$

c_p specific heat of the gas at constant pressure

c_v specific heat of the gas at constant volume

d maximum diameter of body

H total-pressure head

K similarity parameter $\left(\frac{M_o}{l/d}\right)$

l length of nose of body

l/d fineness ratio of nose of body

m slope of pressure-distribution curve $\left[\frac{d(\log p/p_o)}{d(\text{percent of nose length})}\right]$

M Mach number

M* ratio of local speed to critical speed of sound $\left[\frac{\gamma+1}{(\gamma-1)+(2/M^2)}\right]^{1/2}$

p static pressure

q dynamic pressure

S_b maximum cross-sectional area of body

[1] In this report C_D refers only to that part of the drag contributed by the pressures acting on the body nose. The nose of the body is considered to include the section forward of the cylindrical section or forward of the maximum diameter.

x longitudinal coordinate of body measured from vertex

θ local angle of inclination of the surface of the body measured relative to the body axis

γ ratio of specific heats of the gas (c_p/c_v)

φ angle of inclination of shock wave measured relative to the body axis

Subscripts

o free-stream conditions

THEORETICAL CONSIDERATIONS

The hypersonic similarity law derived by Tsien in reference 1 states that similarly shaped bodies of revolution at zero angle of attack create similar flow fields if K, the ratio of free-stream Mach number to body fineness ratio, is constant. Similarly shaped bodies are defined as those having the same thickness distribution along the length; that is, the ordinates of any body in the family are a constant multiple of the ordinates of any other body in the family. Similar flows result in pressure distributions which are the same in terms of p/p_o, ratio of local to free-stream static pressure. This means that either the pressure ratios p/p_o or the pressure coefficients[2] $\frac{p-p_o}{p_o}$ at any given station (same percent of nose length) are identical for a group of similar bodies having the same value of the similarity parameter K. Since under these conditions the pressure coefficients are identical, the drag parameters $C_D \frac{q_o}{p_o}$ for bodies of revolution are also identical if the drag coefficient is referred to the maximum cross-sectional area. In other words, for similarly shaped bodies, both the pressure distribution and the drag parameter are functions of the single parameter K.

[2]It should be noted that pressures are identical for a given value of K only when expressed nondimensionally in terms of the free-stream static pressure p_o. Consequently, throughout this report the term "pressure coefficient" is used to indicate $\frac{p-p_o}{p_o}$. The more common pressure coefficient $\frac{p-p_o}{q_o}$ can be obtained readily from the relation $\frac{p-p_o}{q_o} = \frac{(p-p_o)}{p_o}\frac{p_o}{q_o}$.

In his derivation of the similarity law Tsien assumed that (1) the flow was inviscid, (2) the flow was irrotational, (3) the Mach number was much greater than 1, and (4) the body was slender and pointed. Each of these assumptions could limit the usefulness of the similarity law and therefore should be examined. It is known that practical flows are viscous and that the principal effect of viscosity is the formation of a boundary layer which effectively changes the body shape. In the past it has been found that, insofar as the flow field and the resulting pressure forces are concerned, this effect of boundary layer usually can be taken into account by considering that the lateral dimensions of the body are altered by the amount of the boundary-layer displacement thickness. This scheme should work equally well in the present case. Although Tsien based his derivation on the equations for irrotational flow, Hayes (reference 2) concludes that the hypersonic similarity law should be valid for rotational flow as well as for irrotational flow. The assumptions of high Mach number and of slender bodies would be expected to be more serious restrictions. It is the purpose of this report to determine how seriously these latter assumptions limit the usefulness of the similarity law and to determine the range of values of K for which the law is applicable.

PROCEDURE

From an engineering or design viewpoint, the most important result of similarity in airflow about bodies of revolution is similarity in pressure distribution and drag. Therefore, to check the usefulness of the hypersonic similarity law, it would be desirable to have experimental data on drag and pressure distributions on similarly shaped bodies for a wide range of fineness ratios and Mach numbers. At the time that this investigation was undertaken, however, it was apparent that such data were not available and could not be obtained in a reasonable length of time. The alternative was to utilize a theoretical procedure which could be relied upon to give accurate results for the conditions of interest. Various linear theories which are relatively simple to apply were considered for this investigation but were rejected because they did not meet the accuracy requirements. The method of characteristics which is comparatively difficult and tedious to apply was finally chosen since its inherent accuracy is limited only by the fineness of the Mach net chosen and the accuracy of the computations.

In selecting the bodies and Mach number combinations to be covered in this investigation the following factors were considered:

1. To test the validity of the hypersonic similarity law for a given value of K, a minimum of two solutions for similarly shaped bodies is required.

2. To determine the range of values of K for which the law holds, pairs of solutions are required for various values of K.

3. To check the possible influence of body shape, at least two body shapes should be investigated.

To meet these requirements and to cover the range of Mach numbers desired, cone cylinders and ogive cylinders (bodies with conical- and ogival-nose sections, respectively) were chosen with combinations of Mach number and fineness ratio as follows:

Nose shape	K[3]	M_o	l/d[3]
Conical	1	3	3
	1	6	6
Ogival	.5	3	6
	.5	6	12
	1	3	3
	1	6	6
	1	9	9
	2	6	3
	2	12	6

It should be noted that ogives do not have exactly the same distribution of cross-sectional area along the length and as a result do not meet the requirement of similarity in body shape; nevertheless, they were chosen for study since the configuration is of interest and the deviation from similarity is not significant for the bodies listed.

Two procedures for applying the method of characteristics were used in determining the flow around the bodies. Sauer's graphical method for three-dimensional flow as outlined in reference 3 was used for most of the solutions. A semigraphical method, given as procedure 1A in reference 3, was used for the slender bodies at the higher Mach numbers (M_o=9, l/d=9, and M_o=12, l/d=6). Both methods were carried out for irrotational[4] inviscid flow using a ratio of specific heats γ of 1.4.

A characteristic solution supplies the velocity distribution along the body surface. These velocities, which are in terms of Mach angle or M*, the ratio of the local speed to critical speed of sound, are transformed to Mach numbers. The pressure distribution is then calculated using the following equation:

$$\frac{p-p_o}{p_o} = \frac{(p/H)(H/H_o)}{p_o/H_o} - 1 \qquad (1)$$

[3]Throughout this report K and l/d refer to only the nose section of the body.

[4]The bow shock wave was allowed to curve; however, the rotation term was dropped from the compatibility equations. (See reference 3.)

From the basic equations of supersonic flow

$$p/H = (1 + 0.2\ M^2)^{-3.5};\ p_o/H_o = (1 + 0.2\ M_o^{\ 2})^{-3.5}$$

$$H/H_o = \left(\frac{6M_o^{\ 2}\ \sin^2\varphi}{M_o^{\ 2}\ \sin^2\varphi + 5}\right)^{3.5}\left(\frac{7M_o^{\ 2}\ \sin^2\varphi - 1}{6}\right)^{-2.5}$$

where φ is the shock wave angle. (It should be noted that H/H_o and p_o/H_o are constants for each solution.)

To employ the method of characteristics, the flow conditions must be known along some curve in the flow field. To secure such a starting line of known conditions on the ogive cylinders, the tips of the ogival-nose sections were approximated by cones which were tangent to the ogives at 5 percent of the ogive-nose length. (An exception is the case for $l/d = 6$, $M = 3$ where a 10-percent cone was used.) The effect of this approximation was investigated on a given ogive by comparing a solution for a cone tangent at 1.66 percent to a solution where the cone was taken tangent at 5 percent of the nose length. It was found that downstream of the conical sections there was no discernible difference in Mach number distribution for the two cases. Since the pressure distribution was desired for a true ogive, the total-head ratio across a shock wave due to the vertex angle of the ogive (not of the approximated cone) was used in equation (1) to compute the pressure coefficients. Values of the pressure coefficients at the vertex of the ogives were obtained by graphical interpolation of values given in Kopal's tables (reference 4) for cone angles equal to the true vertex angles of the ogives. This procedure produced values of pressure coefficients which could be faired to smooth curves representing the pressure distributions on pure ogival noses.

Since the hypersonic similarity law was tested by comparing characteristic solutions, a high degree of accuracy was required in the computations. Therefore, the factors determining the accuracy of the computations and the graphical construction involved in the solutions were carefully considered and evaluated. The general criterion followed was that the accuracy was to be such that the scatter of values of Mach angles on the surface of the body was always within 3 percent of the total variation of Mach angles over the body.

RESULTS AND DISCUSSION

Validity of the Hypersonic Similarity Law

The pressure distributions obtained by the method of characteristics as outlined in the preceding section are presented in figure 1 for cone cylinders (K = 1) and in figure 2 for ogive cylinders (K = 0.5, 1, and 2). For each value of K a mean curve is faired through the computed points for all solutions. It is immediately apparent from the results shown in these figures that, over the relatively broad range of Mach numbers and fineness ratios considered, the hypersonic similarity law applies with remarkable accuracy. The pressure coefficients for different cases with the same value of K are very close to the single faired curve, indicating that the pressure distribution is essentially a function of K. It should be noted that on both the ogive cylinder and the cone cylinders the similarity in pressure distributions holds on the cylindrical afterbody as well as on the nose. Since the similarity law holds for both cones and ogives, it is expected that it would also apply to other families of pointed bodies of revolution.

The validity of the hypersonic similarity law for pressure distributions over bodies of revolution has been demonstrated for the cases considered in figures 1 and 2. The limits in Mach number and fineness ratio for which the similarity law applies are not reached for these cases. Some insight into Mach number and fineness-ratio limits can be gained by examining the similarity in pressures on pure cones and then applying these results to other body shapes. This procedure is illustrated in the following paragraphs.

In figure 3 are presented the pressure coefficients on cones as functions of the similarity parameter K for various cone angles (from reference 4). A single curve, favoring the slender cones, has been faired through the data. The spread of the points about this line indicates the degree of dissimilarity in pressures. It is apparent that the similarity in pressure holds for a wide range of values of K for slender cones but that as the cone angle increases this range of K for similarity decreases. For a given cone, deviation from similarity in pressures occurs at low Mach numbers as the point of shock-wave detachment is approached (detachment imminent at lowest value of K plotted for each cone); at high Mach numbers, deviation from similarity occurs as the shock wave approaches the surface of the cone. If, for purpose of illustration, it is assumed that a pressure deviation of ±5 percent from the faired curve can be tolerated in using the similarity law, then the Mach number limits can be determined as a function of fineness ratio of the cones. The limits determined in this way are illustrated in figure 4(a). The shaded area indicates the regions of Mach number and fineness ratio where the similarity law probably will be in error 5 percent or more as far as pressure coefficient on the cone is concerned.

On bodies other than cones, the slope of the body surface at the vertex is usually the largest and thus should provide the most critical test for similarity of pressures. It might be expected, then, that the Mach number and fineness-ratio limits for other body shapes could be estimated as follows:

1. Obtain values of pressure coefficient for the nose semivertex angle θ from tables in reference 4.

2. Plot these pressure coefficients as a function of the similarity parameter for the body shape under consideration and fair the curve as in figure 3(a).

3. Set allowable deviation from this line according to the accuracy required.

4. Determine limits of θ, as fixed by this deviation, for various values of K and convert to limits of M_o and l/d.

This procedure gives the limits of M_o and l/d for similarity of pressure at the nose vertex only. The deviation on the remainder of the body, however, would not be expected to be greater than that at the vertex; this is apparent for ogives in the test solutions presented later.

The limits of Mach number and fineness ratio for ogives were determined in accordance with the foregoing procedure. Figure 3(b) was prepared giving the pressure coefficients at the vertices of the ogives as functions of the similarity parameter of ogives for various vertex angles. Again a single curve was faired favoring slender bodies and the maximum allowed deviation from the curve was ±5 percent. The resulting Mach number limits as functions of fineness ratio of the ogive are presented in figure 4(b) in a manner similar to that for cones. The boundaries marked by the shaded areas in figure 4 should not be considered as sharply defined limits. In general, it is believed that within the requirements of engineering accuracy the hypersonic similarity law should be applicable for cases falling in the unshaded areas.

The solutions included in figure 2 and identified in figure 4 by the circles are well within the unshaded area and therefore do not by themselves provide a test of the boundaries indicated in figure 4(b). To further clarify these boundaries additional solutions which lie near the boundary or within the shaded area were obtained (identified in fig. 4 by the triangles). These solutions are shown in figure 5 with comparative curves from figure 2. The combination with a fineness ratio of 3 and Mach number of 1.5 is just within the shaded area of figure 4(b) and shows a slight deviation from the pressure distribution from figure 2 for a value of K of 0.5. The body with a fineness ratio of 2 at a Mach number of 2 is out of the shaded area and the pressure distribution is nearly the same as that taken from figure 2. The case for a Mach number of 3 and fineness ratio of 1.5 is well within the shaded area and the resulting pressure distribution shows a notable deviation from the distribution given in figure 2 for a value of K of 2. It should be noted, however, that in this case the deviation decreases as the longitudinal coordinate increases. The test solutions of figure 5 show that the unshaded area in figure 4(b) may well be used as a guide in determining the cases for which use of the similarity law for ogives will give acceptable accuracy.

Rapid Method for Determining Pressure Distribution Over Ogive Cylinders

Since within the limitations just discussed the pressure coefficient at each point on similarly shaped bodies is a function of K only, a simple chart can be made which gives the pressures on the bodies for wide ranges of Mach number and fineness ratio. Such a chart (fig. 6) has been prepared for ogive cylinders using the solutions presented in figure 2 and two additional solutions – one at [5]$K = 0.7$ and one at $K = 1.5$. If M_o and l/d are known, K is, of course, fixed and the pressure coefficient $\frac{p-p_o}{p_o}$ for various stations on the ogive cylinder can be read directly from figure 6. The pressure distribution so obtained should be sufficiently accurate for most uses, provided the Mach number and fineness ratio limitations indicated in figure 4(b) are observed.

Pressures read from figure 6 have been compared with pressures obtained from an isolated solution by the method of characteristicss and from wind-tunnel tests. The characteristic solution chosen for comparison was one for a 10-caliber ogive at a Mach number of 2 taken from reference 5. Figure 7 compares the results of this solution with the results from figure 6. The pressure distributions are nearly the same on both the ogival nose and the cylindrical portion – the point of maximum deviation showing a discrepancy of about 5 percent. In general, it may be observed that the discrepancies in figure 7 are of the same magnitude as those which often result from solutions done independently by two people or by two methods for the same conditions. The only experimental data available for comparison were those for the A4V1P presented in reference 6. The experimental and predicted distributions are compared in figure 8 for Mach numbers of 2.47 and 3.24 ($K = 0.71$ and 0.93, respectively). The differences are relatively small on the ogival sections but increase with longitudinal coordinate on the cylindrical portion.

Characteristics of Pressure Distribution Over Ogive Cylinders

In the analysis of the pressure distributions obtained on ogival noses by the method of characteristics the effect of varying either fineness ratio or Mach number while holding the other constant was determined. Some of the effects of such variation, determined from figure 6 and shown in figure 9, are as follows:

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[5]The solution for $K = 0.7$ was obtained by the staff of the Ames 10- by 14-inch supersonic wind-tunnel section using the same procedure as previously outlined.

1. As either the Mach number or bluntness is increased the pressure coefficient at the vertex is increased.

2. The pressure coefficient is zero at approximately the same percent nose length for all fineness ratios and Mach numbers.

3. The negative pressure coefficient at the end of the nose increases in magnitude with increasing Mach number or bluntness.

4. With increasing Mach number or bluntness the rate of pressure recovery along the cylinder decreases.

Further analysis of the pressure distributions showed that the logarithm of the pressure ratio is a linear function of the longitudinal coordinate in percent nose length. Figure 10 shows the resulting linear plots for various values of K. The relationship between the slopes m of these curves and the values of K is also shown in figure 10. It is noticed that the slopes vary nearly linearly with K for values of K greater than 0.5. By making use of these relationships an approximate analytical equation can be readily written for the distribution of pressure ratio over an ogive. The form of the equation is

$$\log \frac{p}{p_o} = m\left(100 \frac{x}{l} - 75\right)$$

or a more exact form is

$$\frac{p}{p_o} = \left(\frac{p}{p_o}\right)_{vertex} 10^{+100\, m \frac{x}{l}}$$

where $\left(\frac{p}{p_o}\right)_{vertex}$ is the pressure ratio for a cone tangent at the vertex and can be obtained from reference 4.

Application of Similarity Law to Pressure Drag

Within the limits of the similarity law the pressure distributions over ogives and cones have been shown to be a function of K only; therefore for these bodies the integrated forebody pressure drag expressed in terms of $C_D \frac{q_o}{p_o}$ should also be a function of K only. This is substantiated by figure 11 which shows the variation of $C_D \frac{q_o}{p_o}$ with K for both cones and ogives. The values given were obtained independently of any previous assumption of similarity.

CONCLUDING REMARKS

The foregoing results illustrate that, in spite of the assumptions made in deriving the hypersonic similarity law, the law is applicable over a relatively wide range of Mach numbers and fineness ratios for bodies with either conical- or ogival-nose sections. This means that there is a single pressure distribution over similarly shaped bodies for each value of the similarity parameter $\frac{M_o}{l/d}$. The law is expected to apply to other pointed bodies of revolution as well as to cones and ogives for which it has been demonstrated.

Since within the limitations of the hypersonic similarity law the pressure coefficient at a given station on similarly shaped bodies is a function of $\frac{M_o}{l/d}$ only, it becomes possible to make a simple chart which gives the pressure distribution over a given family of bodies for wide ranges of Mach number and fineness ratio. Such a chart is presented for the case of the ogive cylinders. The pressure distributions obtained by this method compare very favorably with both experimental data and independent characteristic solutions.

An interesting feature of the pressure distributions on ogives is that, for the Mach numbers and fineness ratios where the similarity law was applied, the logarithm of the pressure ratio p/p_o is a linear function of the longitudinal coordinate.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif., October 23, 1950.

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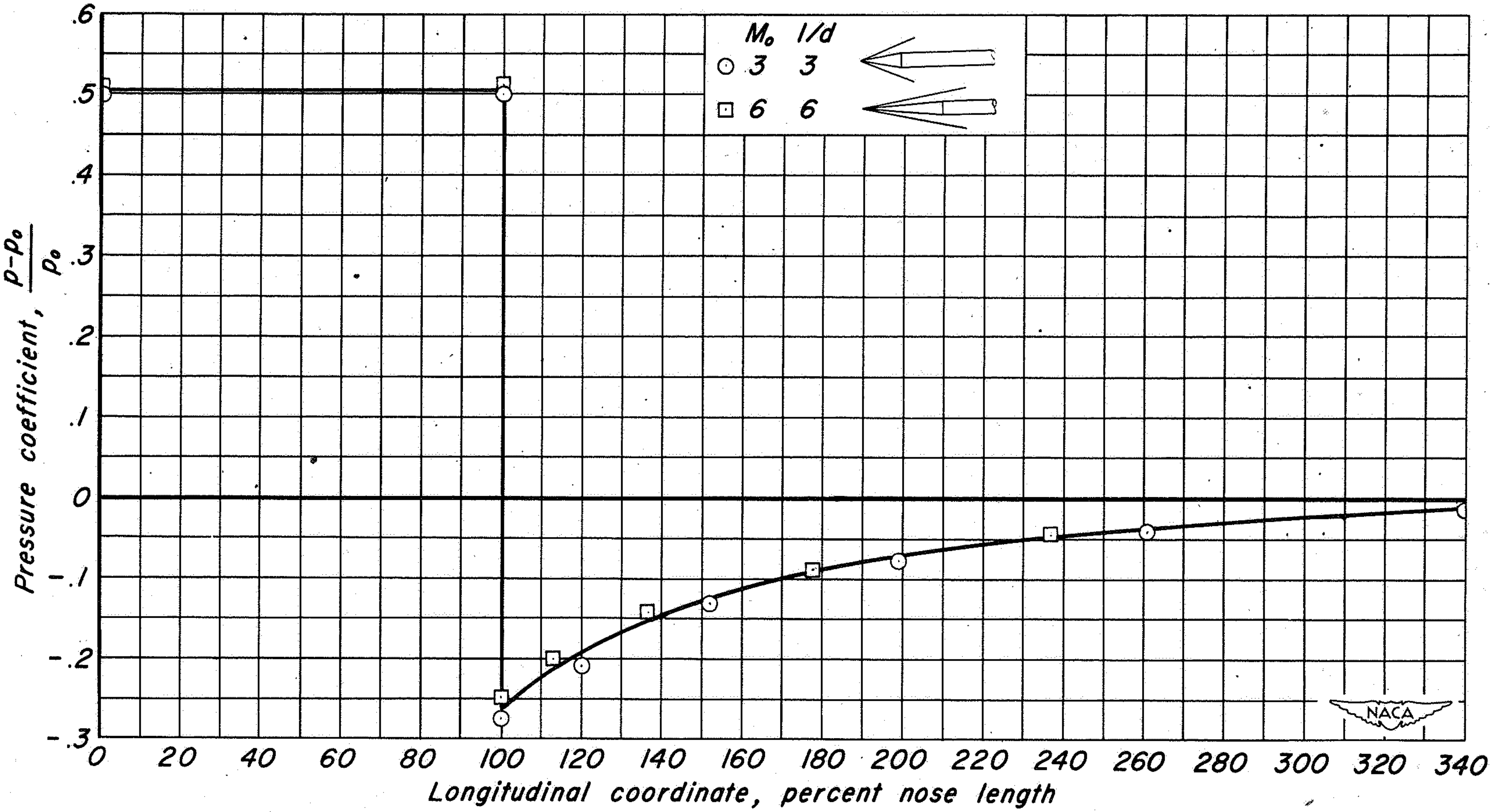
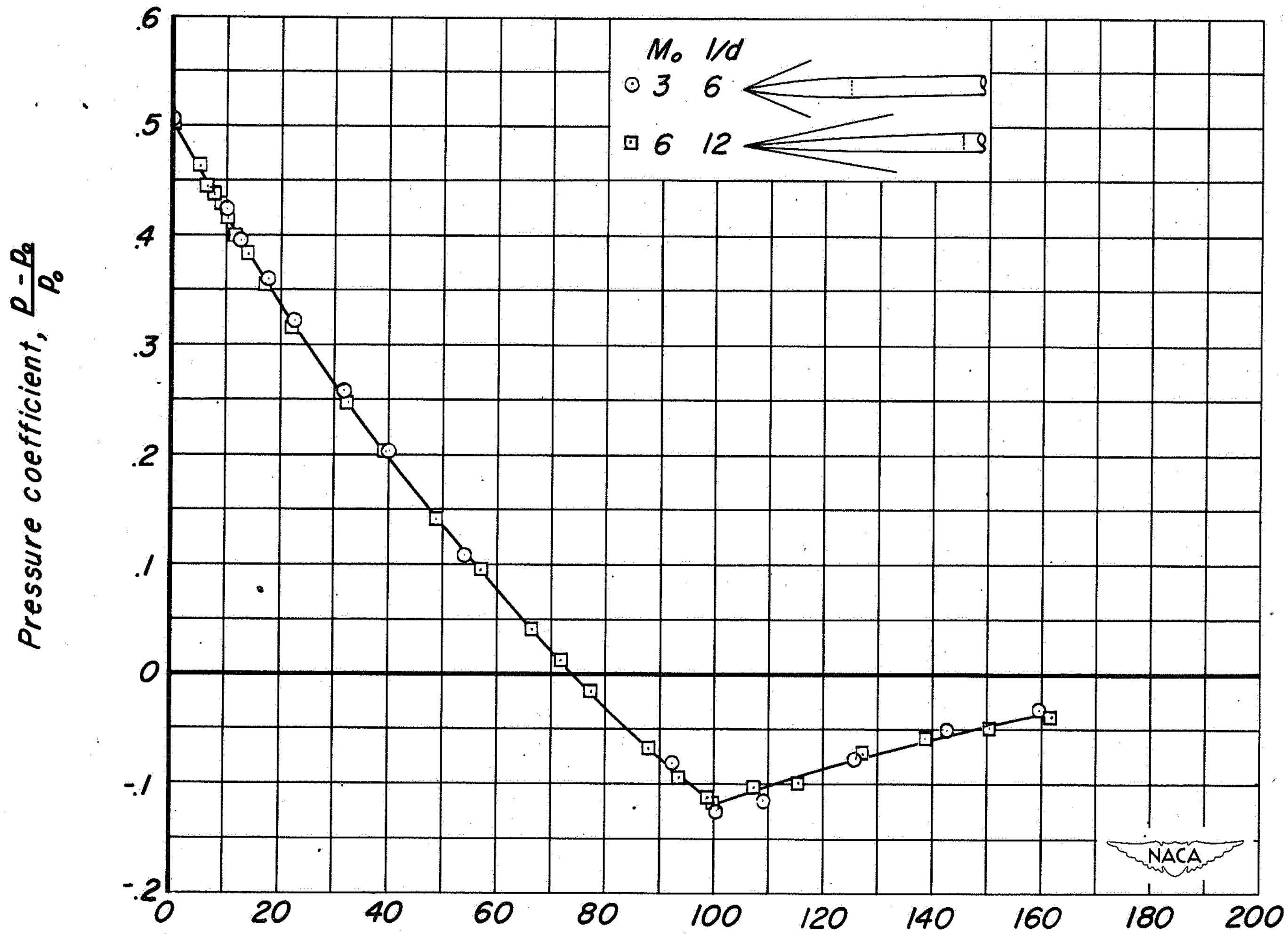


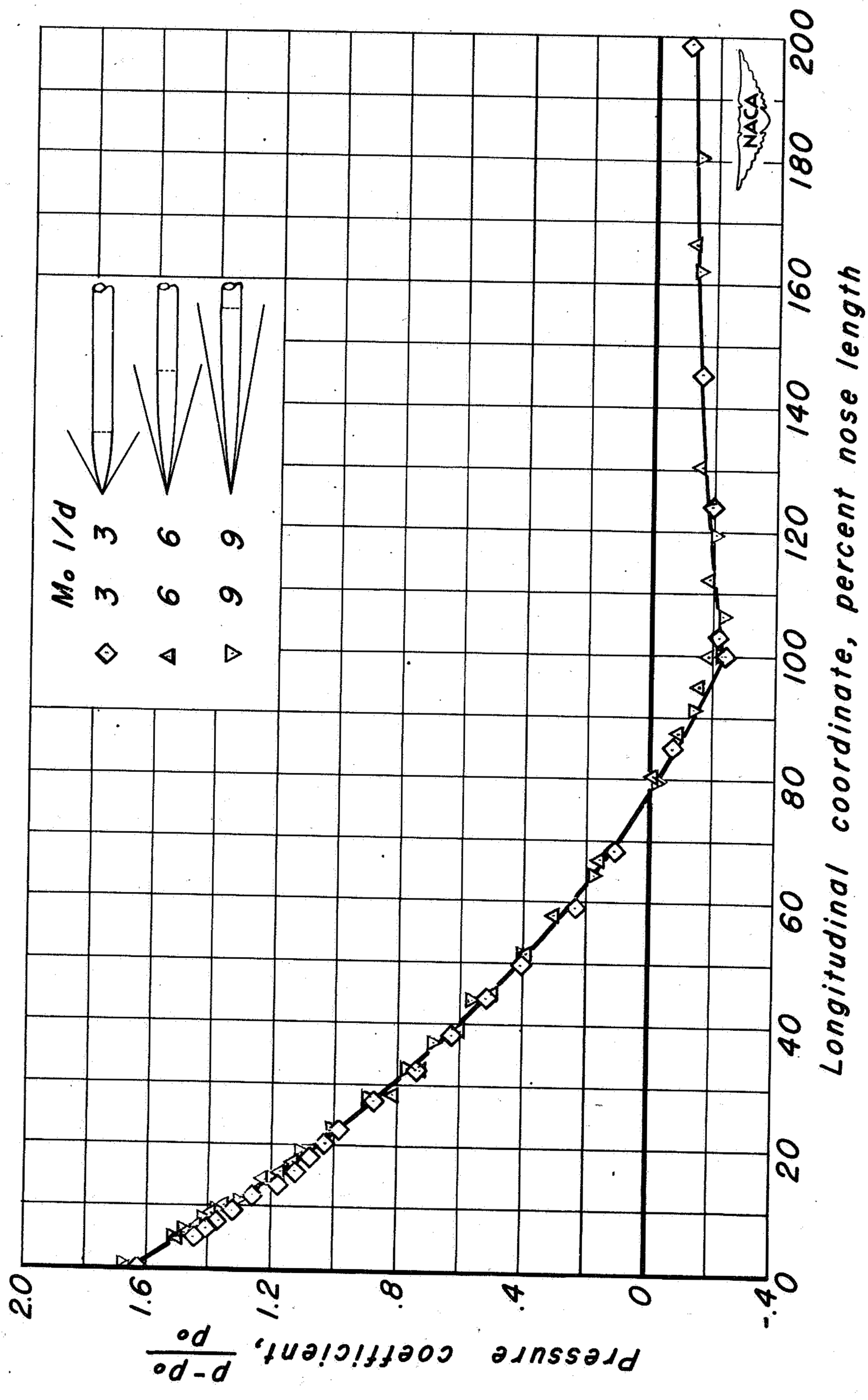
Figure 1.-Variation of pressure coefficient along cone cylinders for a given value of the similarity parameter; K=1.



Longitudinal coordinate, percent nose length

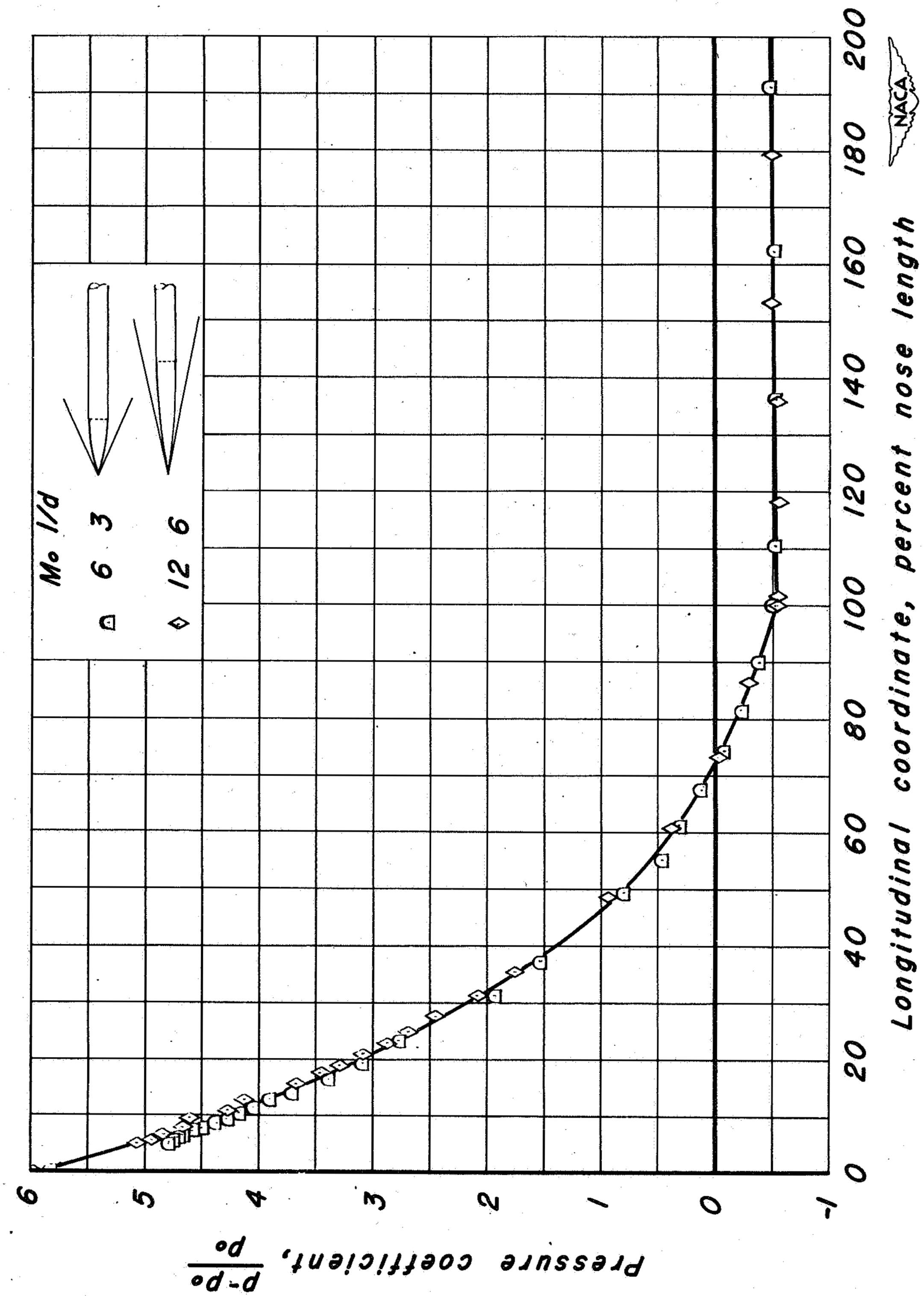
(a) K= 0.5.

Figure 2.-Variation of pressure coefficient along ogive cylinders for given values of the similarity parameter K.



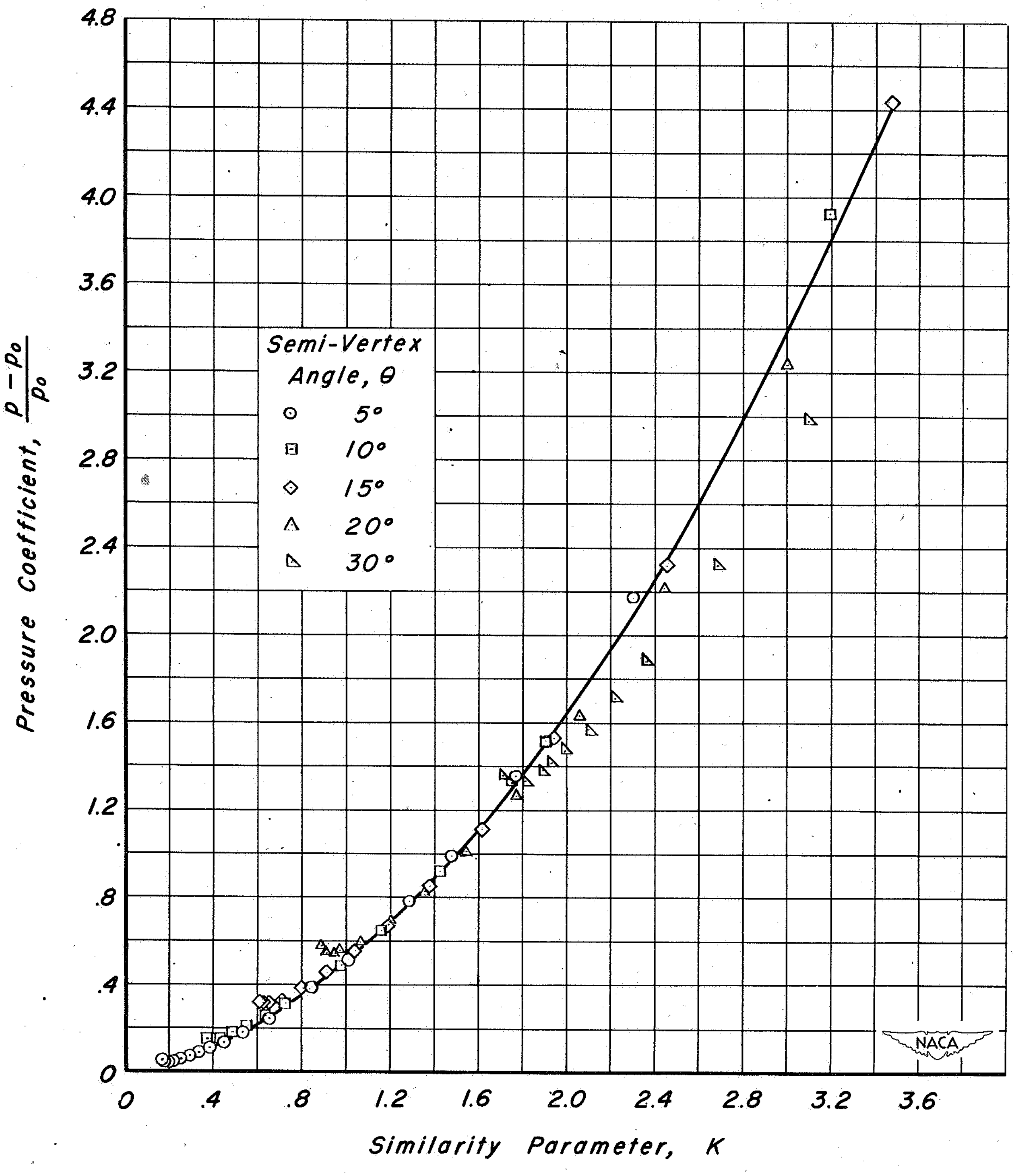
(b) K=1.

Figure 2.- Continued.



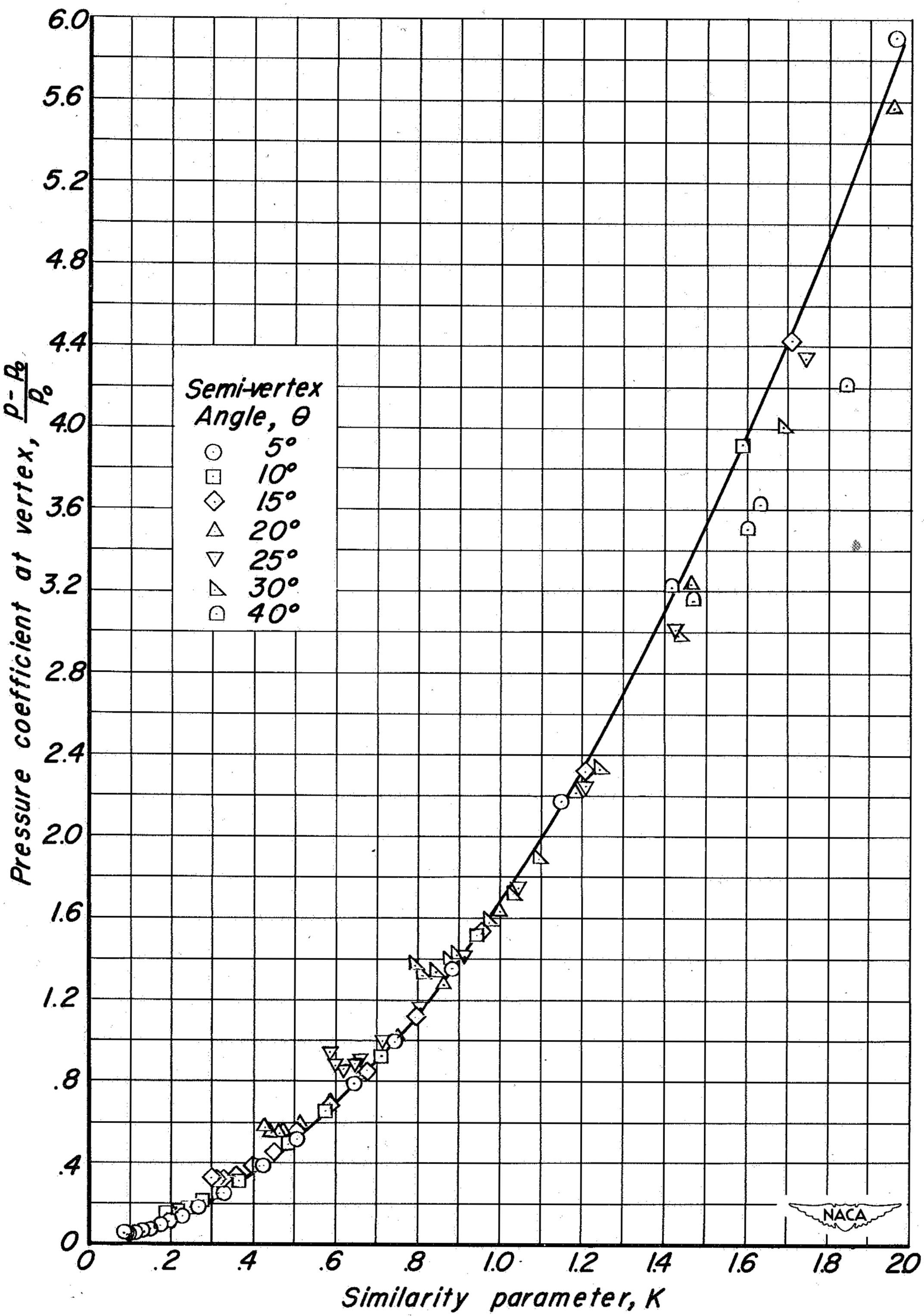
(c) K=2.

Figure 2.- Concluded.



(a) Cones.

Figure 3.—Variation of pressure coefficient at vertex of body with similarity parameter.



(b) Ogives.

Figure 3.- Concluded.

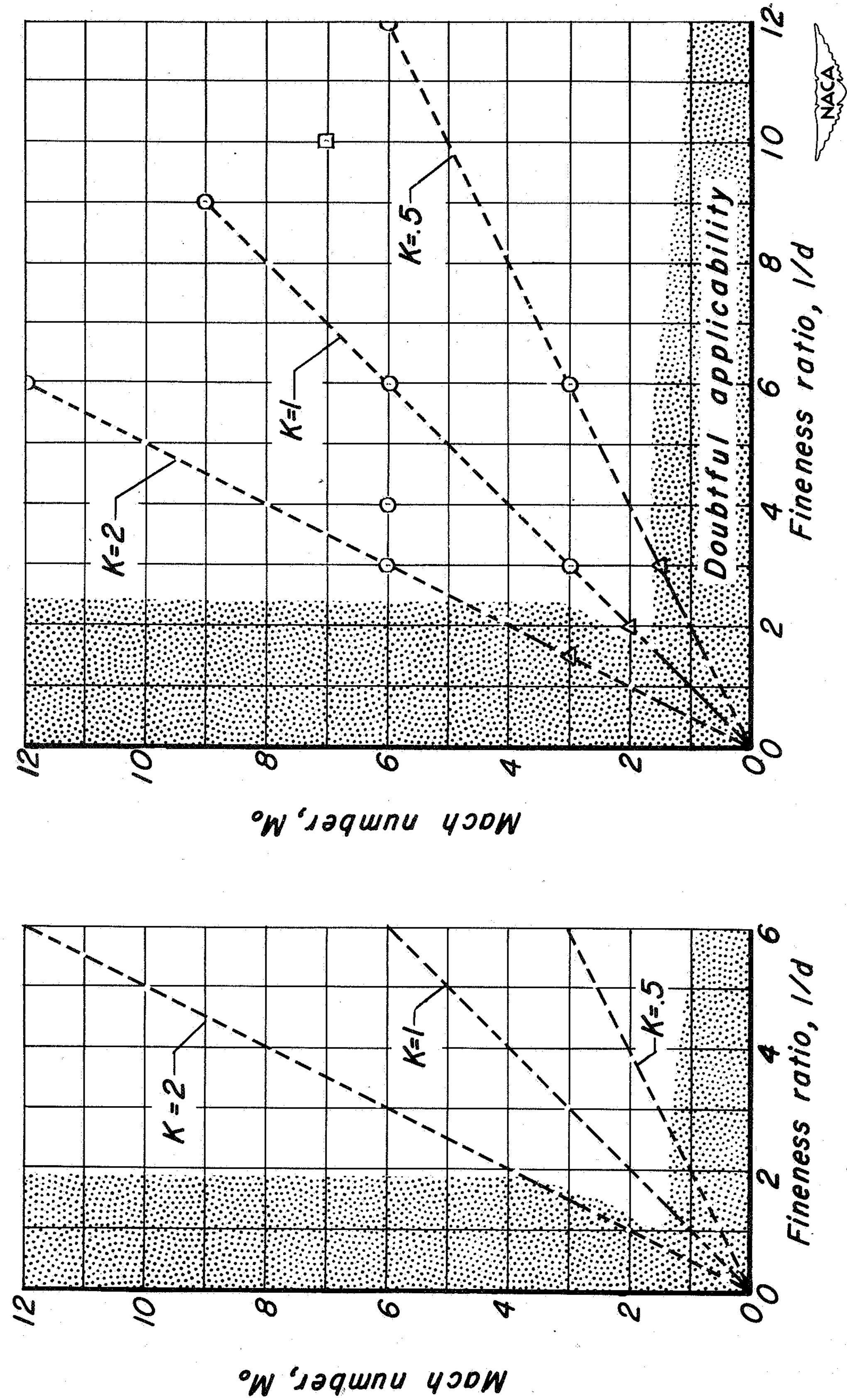


Figure 4.-Range of applicability of similarity law for cones and ogives.

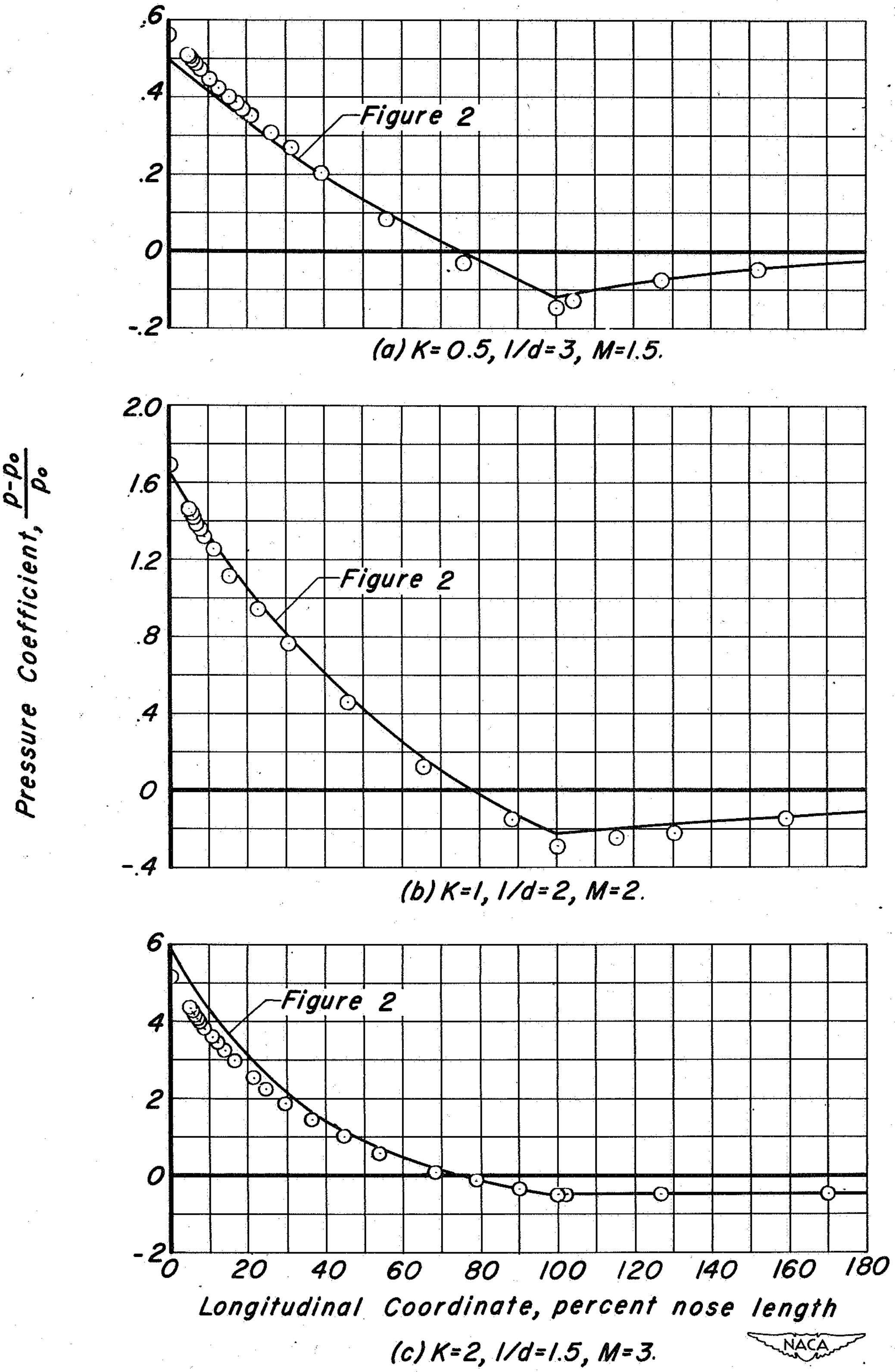
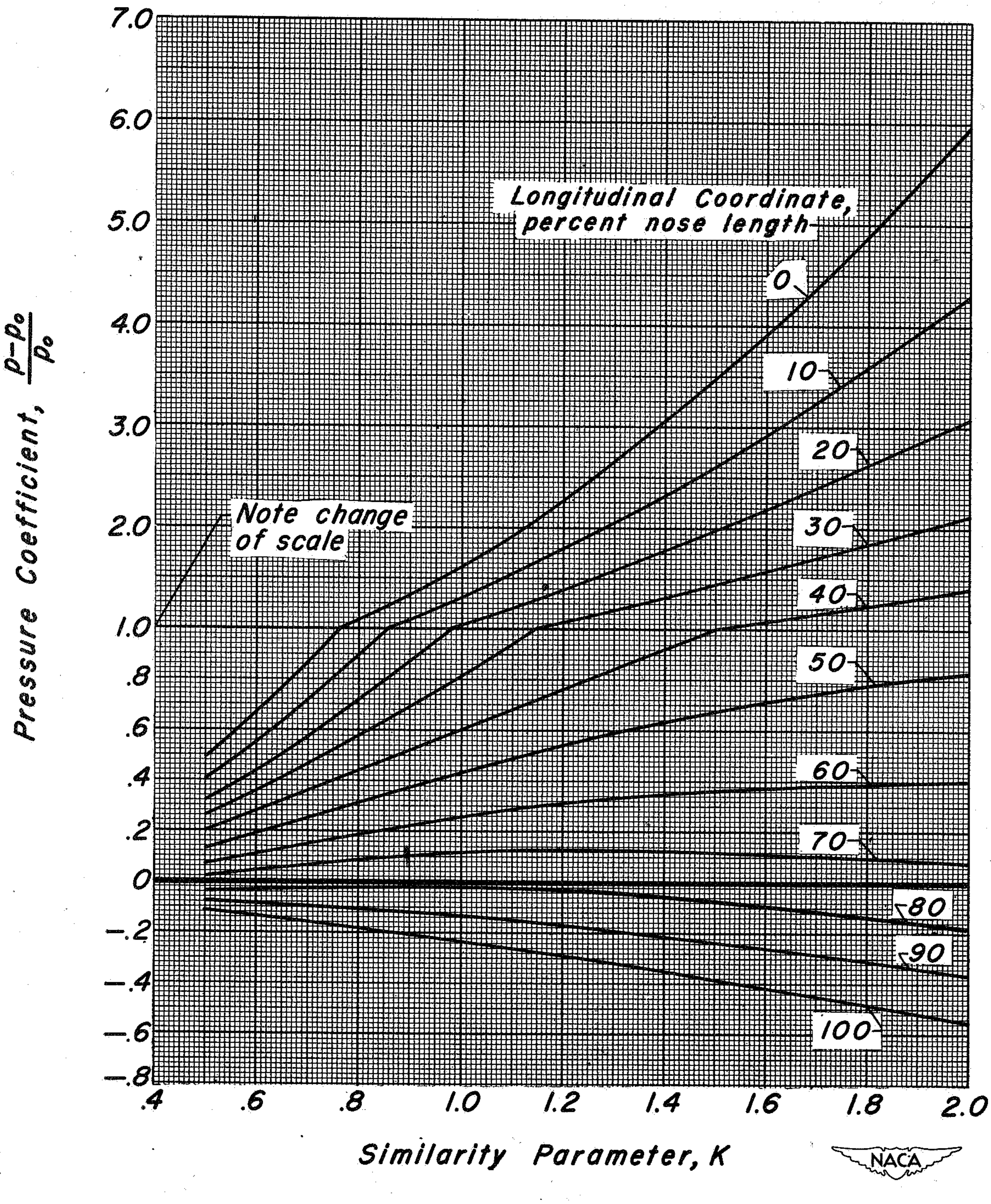
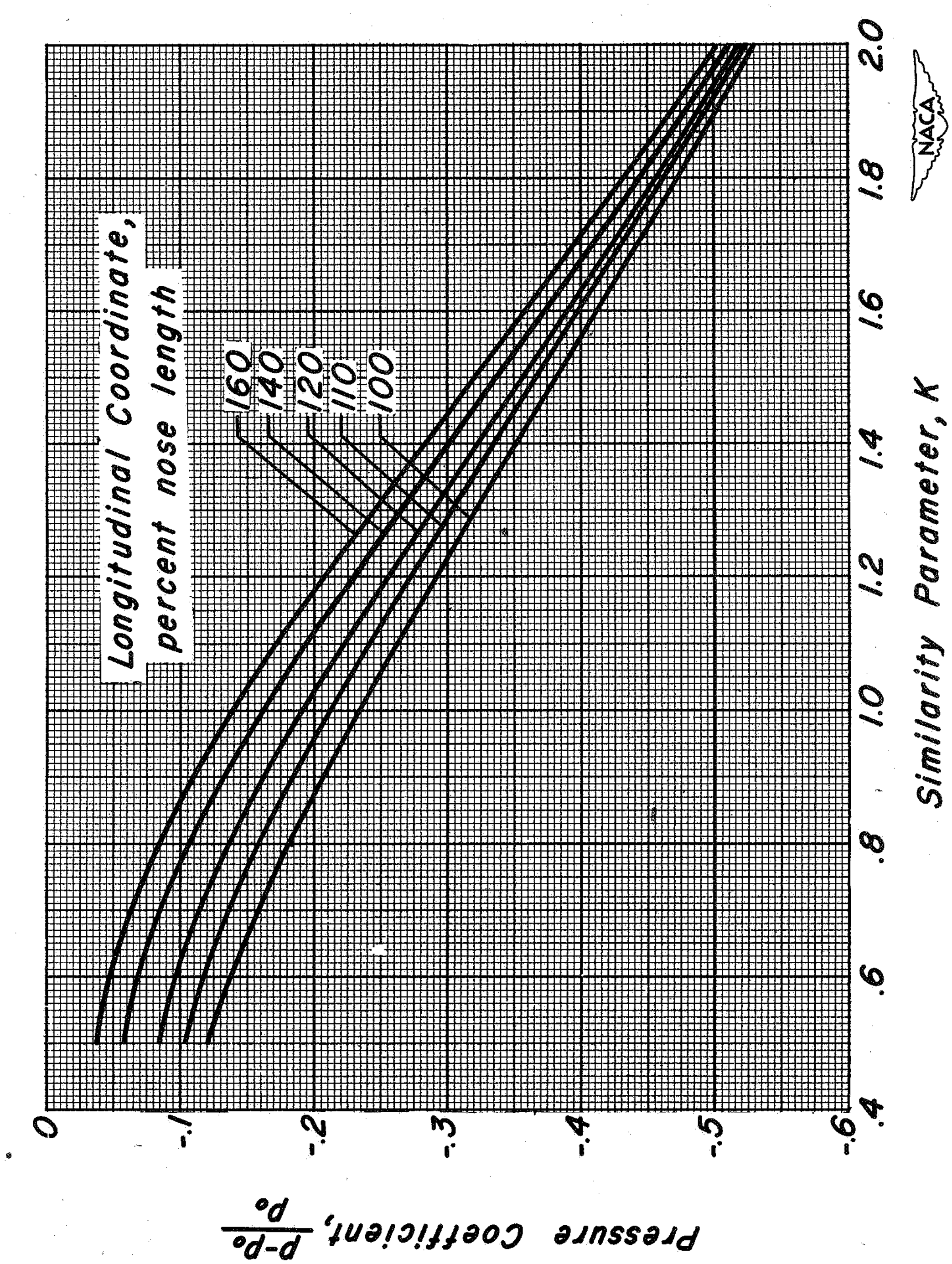


Figure 5.-Solutions to check the lower limits of Mach number and fineness ratio for which the hypersonic similarity law applies.



(a) Ogive.

Figure 6.—Variation of pressure coefficient with K for ogive cylinders.



(b) Cylinder.

Figure 6.-Concluded.

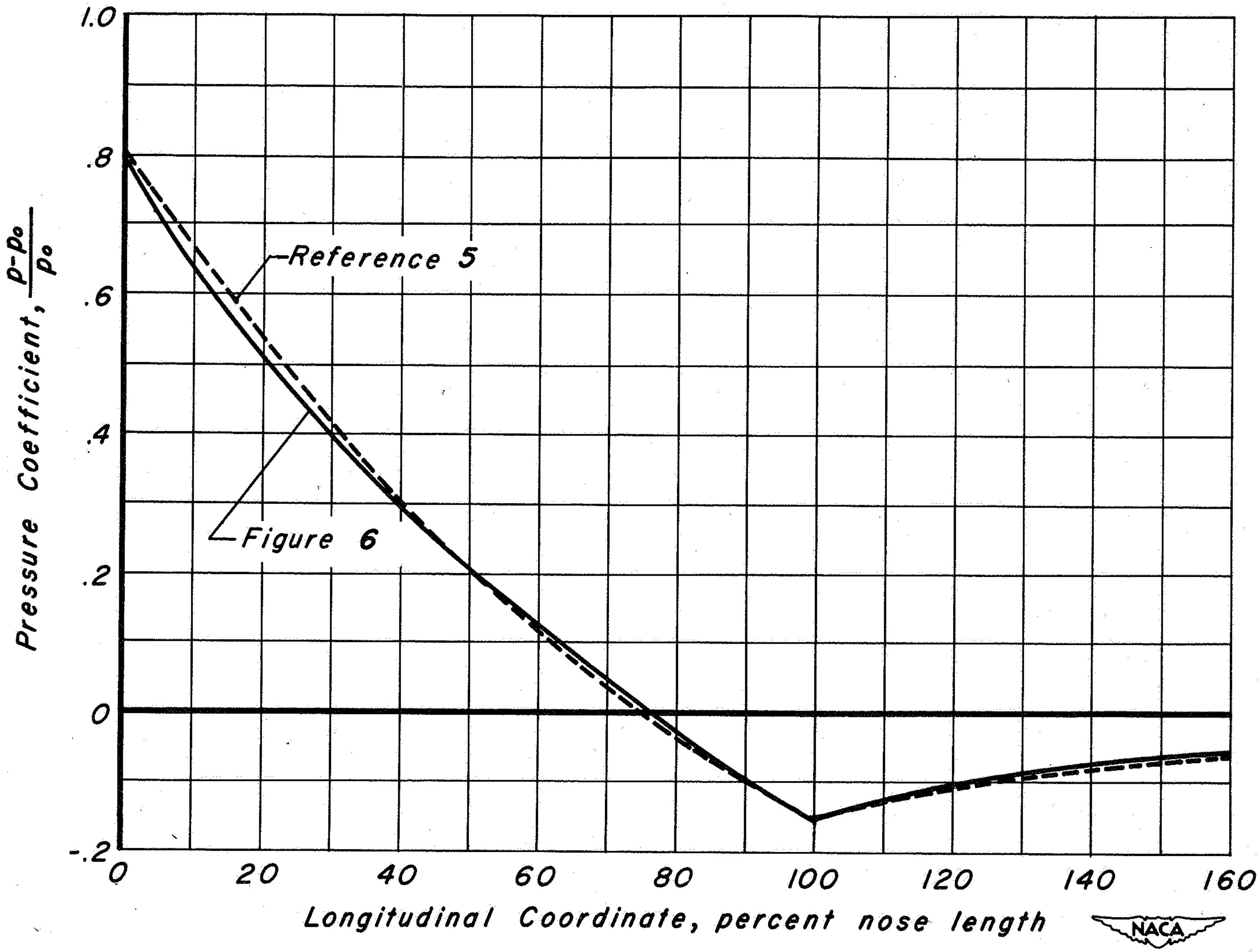
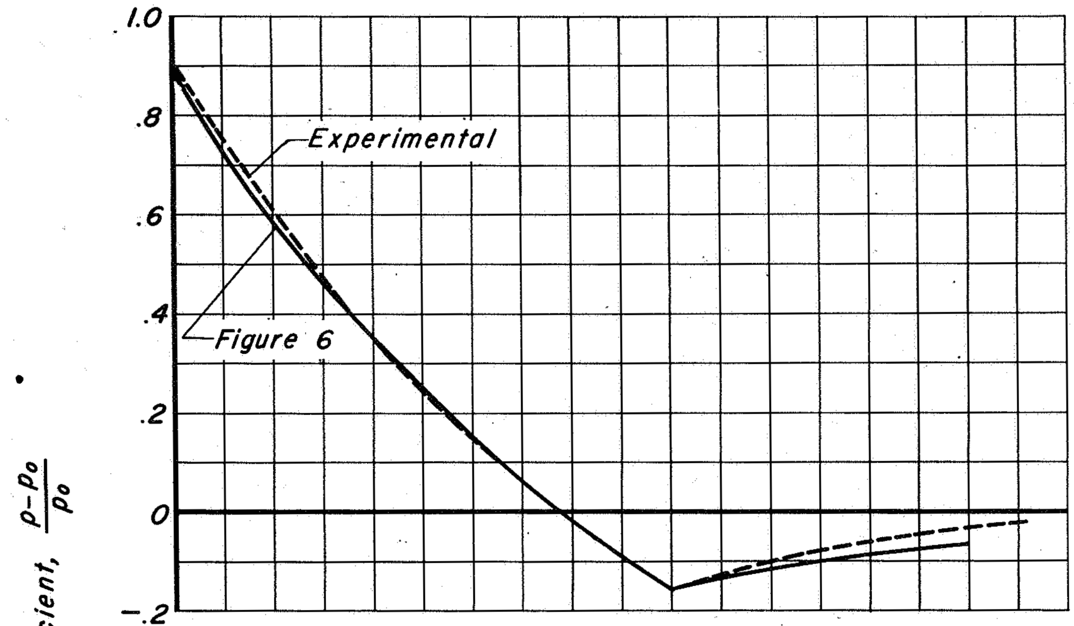
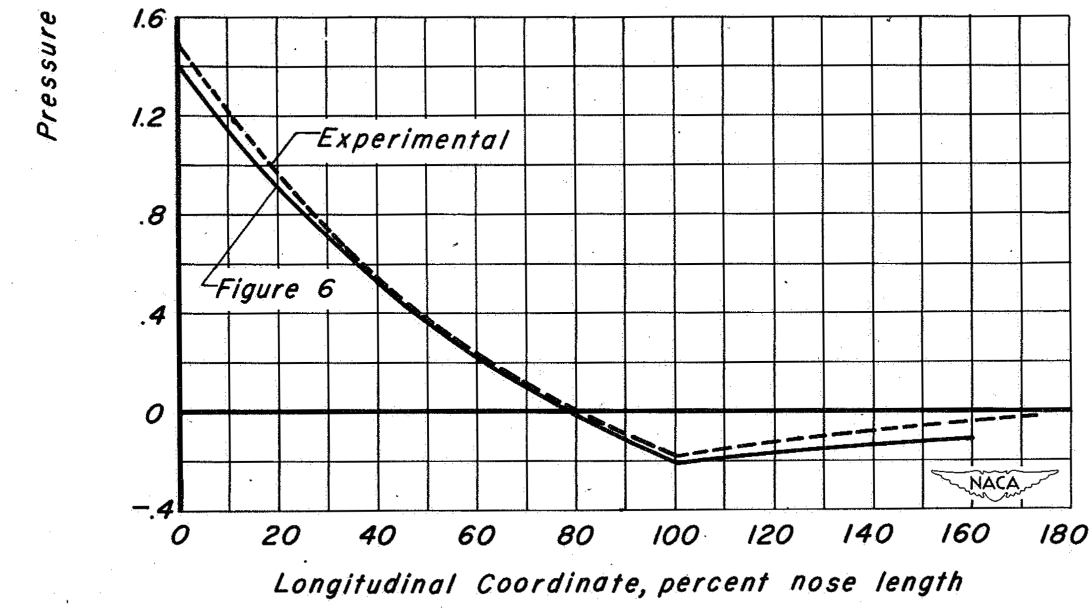


Figure 7.- Comparison of the pressure distribution obtained from figure 6 with the distribution obtained by the method of characteristics for an ogive cylinder; $l/d=3.12$, $M_o=2.0$ (reference 5).

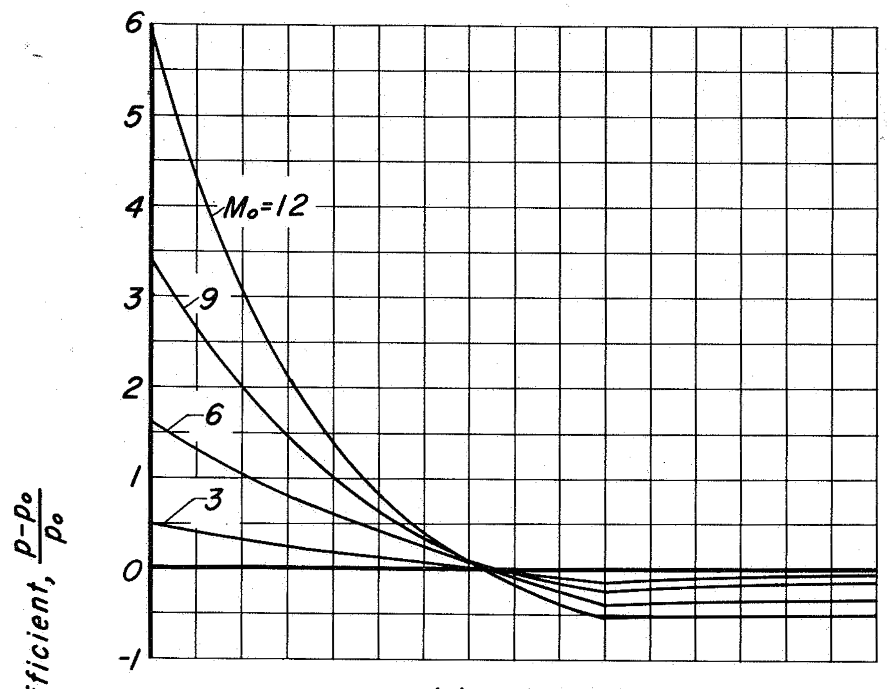


(a) $M_o = 2.47$.

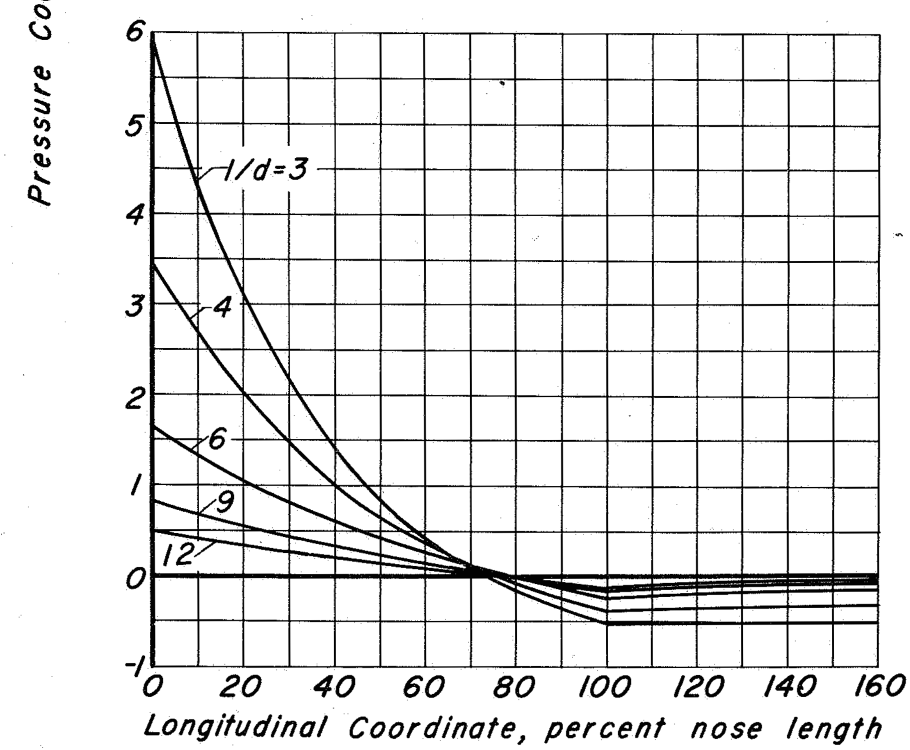


(b) $M_o = 3.24$.

Figure 8.– Comparison of the pressure distributions obtained from figure 6 with those obtained experimentally on the A4V1P; (l/d=3.50) (reference 6).



(a) l/d=6.0.



(b) M_o=6.0.

Figure 9.-Effect of fineness ratio and Mach number on pressure distribution on ogive cylinders.

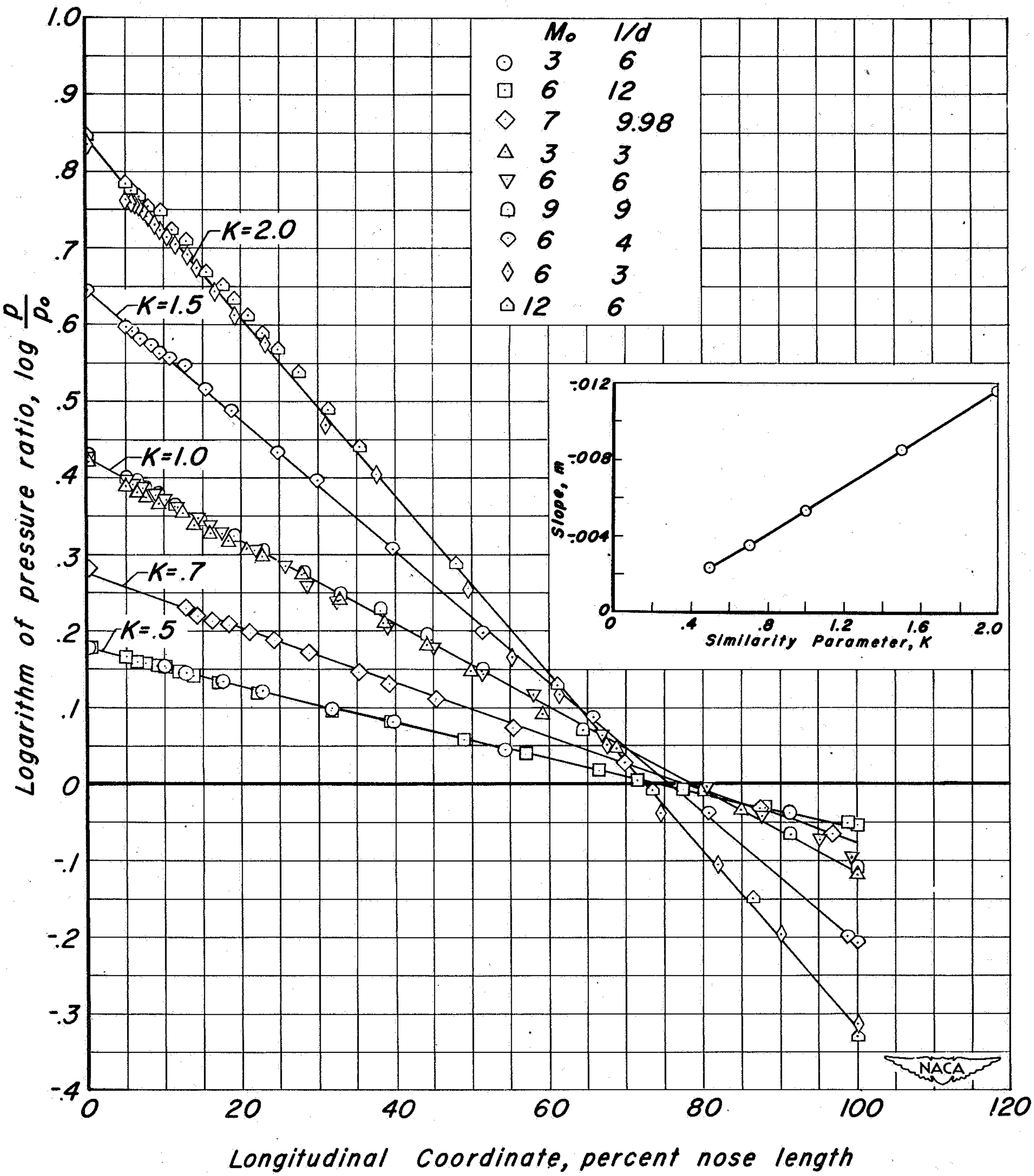


Figure 10.-Variation of the logarithm of the pressure ratio along ogives for various values of the similarity parameter.

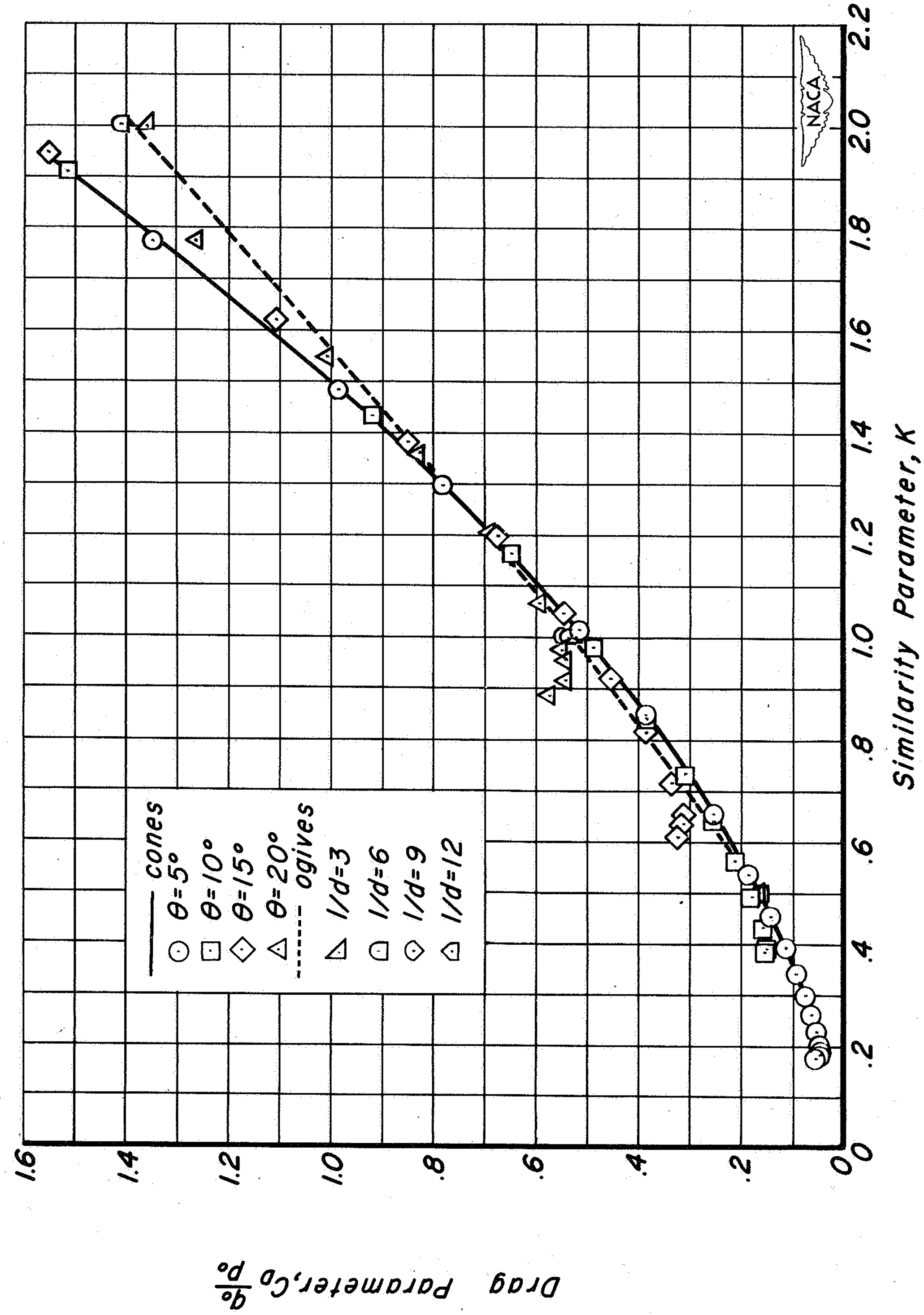


Figure 11.–Variation of drag parameter $C_D \frac{q_o}{p_o}$ with the similarity parameter for ogives and cones.