TDA Progress Report 42-109
May 15, 1992

S14-37
-40
p. 6

N92-29378

Dynamic Modeling of Fluid Transmission Lines of the DSN 70-Meter Antennas by Using a Lumped Parameter Model

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Fluid transmission lines and fittings have been found to significantly affect the dynamic and steady state operation of the engineering system in which they are incorporated. Therefore, a better understanding of the operation of a system can be obtained by including the performance of the transmission lines and fittings within the system model. The most accurate model of a transmission line or fitting is obtained by using a distributed parameter model. However, a distributed parameter model tends to be very complex. This complexity can be avoided without significant loss of model accuracy by using a lumped parameter model when the length of the fluid path through the transmission line or fitting is short. This article develops a lumped parameter model for short fluid transmission lines and fittings, describes the conditions under which the model is valid, and presents the model parameters associated with the servo hydraulic system of the DSN 70-meter antennas.

I. Introduction

All fluid transmission lines and fittings are physically distributed systems because the resistance to fluid flow resulting from the fluid viscosity, inertial, and compressibility effects are spread out over the length of the component. Modeling fluid lines and fittings with distributed parameters generally results in a very complex model. Under certain conditions in which the line or fitting is considered to be short, a lumped parameter model may be used so that the mathematical equations describing the physical behavior are greatly simplified. A lumped parameter model approximates the behavior of the actual line or fitting without considering the distribution of the parameter over the length of the component. This article develops a theoretical model for fluid transmission lines and fittings from a lumped parameter point of view by using the methodology recommended by Meritt [1] and Watton [2]. A description of the conditions under which the lumped parameter model is valid is also included in the article. The model is subsequently used to mathematically describe the behavior of the control lines found within the servo hydraulic system of the DSN 70-meter antennas.

II. Theoretical Model Development

The lumped parameter model developed in this section provides a means of approximating the behavior of short transmission lines and fittings. The model is based on the equation of state for a fluid:

$$\mathrm{d}P = \beta \frac{d\rho}{\rho_0} \tag{1}$$

where P is the fluid pressure, β is the effective bulk modulus, ρ is the density, and ρ_0 is the average density. The effective bulk modulus is given by the relationship

$$\frac{1}{\beta} = \frac{1}{\beta_c} + \frac{1}{\beta_f} \tag{2}$$

where β_f is the bulk modulus of the fluid and β_c is the effective bulk modulus of the container. β_f describes how the fluid density changes with respect to pressure, while β_c describes how the fluid container expands with respect to pressure. The fluid bulk modulus is determined experimentally for a liquid and is given by

$$\beta_f = \gamma \rho_0 \mathrm{R} \theta_0 \tag{3}$$

for an ideal gas, where γ is the specific heat ratio, R is the ideal gas constant, and θ_0 is the average gas temperature. When the container is a circular conduit, the effective bulk modulus of the container, β_c, is given by the equation [2,3]

$$\frac{1}{\beta_c} = \frac{2}{\mathrm{E}\delta} \left[\frac{(1+\nu)(a+\delta)^2 + (1-\nu)a^2}{(2a+\delta)} \right] \tag{4}$$

where a is the internal radius of the conduit, E is the modulus of elasticity, δ is the conduit wall thickness, and ν is Poisson's ratio. Note that the effective bulk modulus will change for different types of fluid containers. When several containers are combined together, as shown in Fig. 1, the system can be represented as a single container by using the equation

$$\frac{V_0}{\beta} = \frac{V_1}{\beta_1} + \frac{V_2}{\beta_2} + \frac{V_3}{\beta_3} + \cdots + \frac{V_i}{\beta_i} \tag{5}$$

where V_0 is the total volume, β is the effective bulk modulus, and β_i and V_i are the effective bulk modulus and volume of the ith container, respectively.

The development of a dynamic model for a short line or fitting using the lumped parameter representation involves expressing Eq. (1) as

$$\frac{\partial P}{\partial t} = \frac{\beta}{\rho_0} \times \frac{\partial \rho}{\partial t} \tag{6}$$

The continuity relationship for a control volume is given by

$$\frac{\partial \rho}{\partial t} = \frac{\rho_0}{V_0} \left[\sum Q_{in} - \sum Q_{out} - \frac{\partial V_0}{\partial t} \right] \tag{7}$$

where V_0 is the total volume, Q_{in} is the flow into the control volume, Q_{out} is flow out of the control volume, and $\partial V_0/\partial t$ is the boundary deformation of the control volume. If the volumetric expansion of the fluid container with respect to pressure is expressed in terms of β_c, then $\partial V_0/\partial t$ is zero. Substituting Eq. (7) into Eq. (6) and setting $\partial V_0/\partial t$ to zero yields

$$\frac{\partial P}{\partial t} = \frac{\beta}{V_0} \left[\sum Q_{in} - \sum Q_{out} \right] \tag{8}$$

The pressure drop across a fitting or pipe where the lumped parameter model is valid can be ignored when the pressure drop across the fluid line is negligible with respect to the system operating pressures. This article considers only fluid lines and fittings with negligible pressure drops. A detailed presentation on modeling pressure drops across fluid conduits can be found in [3].

The conditions under which the lumped parameter model is valid are formulated by considering the speed, c_0, at which pressure waves traverse the fluid component given by the equation

$$c_0 = \sqrt{\frac{\beta}{\rho_0}} \tag{9}$$

Hence, the time it takes a pressure wave to travel down and back along a line or fitting of length L is

$$\tau = \frac{2L}{c_0} \tag{10}$$

Therefore, a lumped parameter model is only valid when the relationship

$$\frac{2L}{c_0} < \frac{1}{f} \tag{11}$$

holds where f is the highest frequency of system oscillation that exists [2].

III. Model Parameters

The parameters of the model of the hydraulic control lines were determined for each control line of the 70-meter antenna servo hydraulic system based on the component layout of the 70-meter antenna servo hydraulic system shown in Figs. 2 and 3. The model parameters are presented in Table 1 and were computed by using Eqs. (2), (4), (5), and (8), along with the physical properties shown in Table 2. The maximum pressure drop between the servovalve and a hydraulic motor due to fluid viscosity is expected to be less than 69,000 Pa (10 PSI) under both tracking and slewing conditions. Hence, the pressure drop due to fluid viscosity is not modeled since it is not considered to be significant. Fluid accelerations in the lines also cause a pressure drop across the lines. However, the pressure drop across a line due to fluid acceleration is believed to have a negligible effect on system performance since the antenna accelerations are both small in magnitude and short in duration under tracking conditions. Therefore, the fluid pressure in each of the control lines is described by

$$\frac{\partial P}{\partial t} = K\left[\sum Q_{in} - \sum Q_{out}\right] \tag{12}$$

where

$$K = 4.756 \times 10^{10}\mathrm{N/m^5}\left(113.0\ \mathrm{lbf/in.^5}\right)\quad \text{[Azimuth control no. 1]} \tag{13}$$

$$K = 4.655 \times 10^{10}\mathrm{N/m^5}\left(110.6\ \mathrm{lbf/in.^5}\right)\quad \text{[Azimuth control no. 2]} \tag{14}$$

$$K = 5.220 \times 10^{10}\mathrm{N/m^5}\left(124.1\ \mathrm{lbf/in.^5}\right)\quad \text{[Elevation control no. 1]} \tag{15}$$

$$K = 5.216 \times 10^{10}\mathrm{N/m^5}\left(124.0\ \mathrm{lbf/in.^5}\right)\quad \text{[Elevation control no. 2]} \tag{16}$$

The summations of the inlet and outlet flow rates into a control line found in Eq. (12) must be specified as boundary conditions by using the servovalve and motor models.

The validity of the model presented in Eq. (12) is determined by using Eqs. (9) and (11). The largest distance between the servovalve and hydraulic motor is approximately 20.83 m (820 in.). From Eq. (9), and Tables 1 and 2, the speed of sound is determined to be approximately 1230 m/sec (48,500 in./sec). Commands are currently sent to the servovalve at a rate of 20 hertz, hence, Eq. (11) becomes

$$\frac{2L}{c_0} = 0.034\ \mathrm{sec} < \frac{1}{f} = 0.05\ \mathrm{sec} \tag{17}$$

which shows that a pressure wave is able to travel from the servovalve to the hydraulic motors and back to the servovalve between the time that two consecutive commands are sent to the servovalve. Therefore, the proposed model is valid.

IV. Conclusion

This article presented a method of approximating the performance of short fluid transmission lines and fittings by using a lumped parameter model and computed the parameters required to model the control lines of the DSN 70-meter antenna servo hydraulic system. It has been found to be more advantageous to use a lumped parameter model than the distributed parameter model because of the mathematical simplicity of the lumped parameter model. This simplicity allows the lumped parameter model to be implemented easily within computer simulation packages.

References

[1] H. E. Merritt, *Hydraulic Control Systems*, New York: John Wiley and Sons, 1967.

[2] J. Watton, *Fluid Power Systems Modeling, Simulation, Analog, and Microcomputer Control*, New York: Prentice Hall, 1989.

[3] F. M. White, *Fluid Mechanics*, 2nd ed., New York: McGraw-Hill, 1986.

Table 1. Transmission line parameters for the control lines of the DSN 70-meter antennas.

Transmission line	Total volume	Effective bulk modulus
Azimuth control no. 1	0.02710 m^3 (1653.6 $in.^3$)	1.2888 x 10^9 Pa (1.8693 x 10^5 psi)
Azimuth control no. 2	0.02770 m^3 (1690.4 $in.^3$)	1.2895 x 10^9 Pa (1.8703 x 10^5 psi)
Elevation control no. 1	0.02455 m^3 (1498.1 $in.^3$)	1.2815 x 10^9 Pa (1.8587 x 10^5 psi)
Elevation control no. 2	0.02457 m^3 (1499.6 $in.^3$)	1.2816 x 10^9 Pa (1.8588 x 10^5 psi)

Table 2. Physical properties.

Fluid bulk modulus	1.379 x 10^9 Pa (2 x 10^5 psi)
Fluid density	850 kg/m^3 (7.95 x 10^{-5} lbf−sec^2/$in.^4$)
Fluid kinematic viscosity	27.7 m^2/sec (1090 $in.^2$/sec)
Inside diameter of 1-in. pipe	2.070 cm (0.815 in.)
Outside diameter of 1-in. pipe	3.340 cm (1.315 in.)
Inside diameter of 1.5-in. pipe	3.399 cm (1.338 in.)
Outside diameter of 1.5-in. pipe	4.826 cm (1.900 in.)
Inside diameter of 2-in. pipe	4.285 cm (1.687 in.)
Outside diameter of 2-in. pipe	6.032 cm (2.375 in.)
Modulus of elasticity of stainless steel pipe	2.0684 x 10^{11} Pa (30 x 10^6 psi)
Poisson's ratio for stainless steel	0.25
Volume of 1.5-in. tee	2.147 x $10^{-4} m^3$(13.1 $in.^3$)
Volume of 2-in. tee	4.015 x $10^{-4} m^3$(24.5 $in.^3$)
Volume of 1-in. hose, 66-in. long	8.495 x $10^{-4} m^3$(51.8 $in.^3$)
Bulk modulus of hose	4.32 x 10^9 Pa (6.26 x10^5 psi)

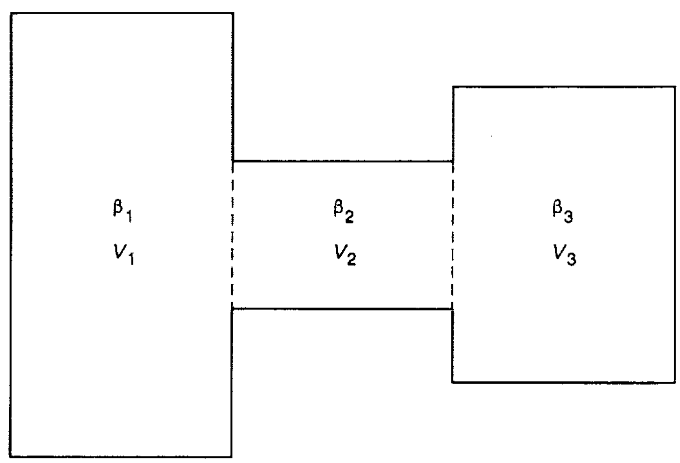

Fig. 1. Combination of fluid containers.

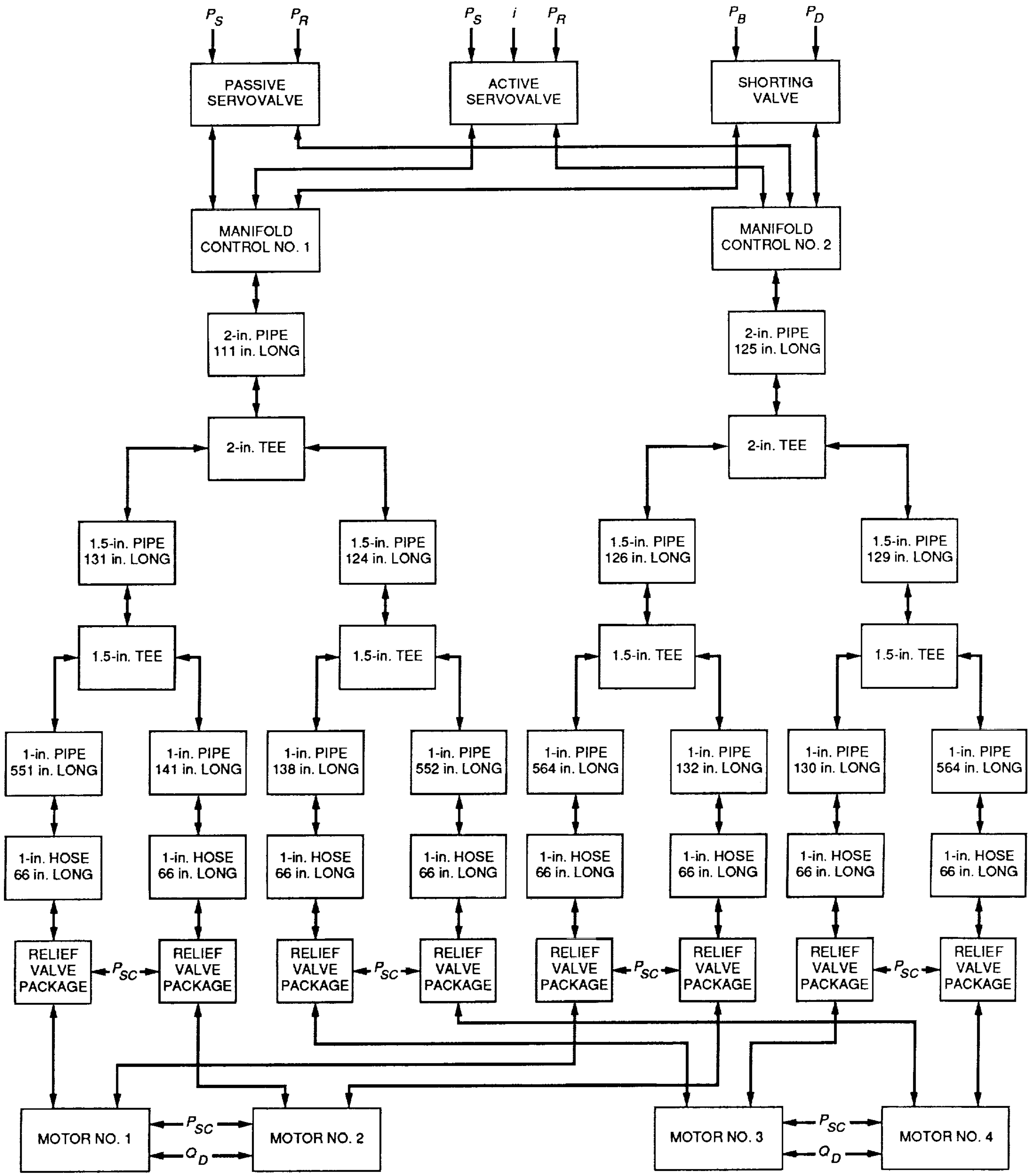

Fig. 2. DSN 70-meter antenna servo hydraulic system azimuth component layout.

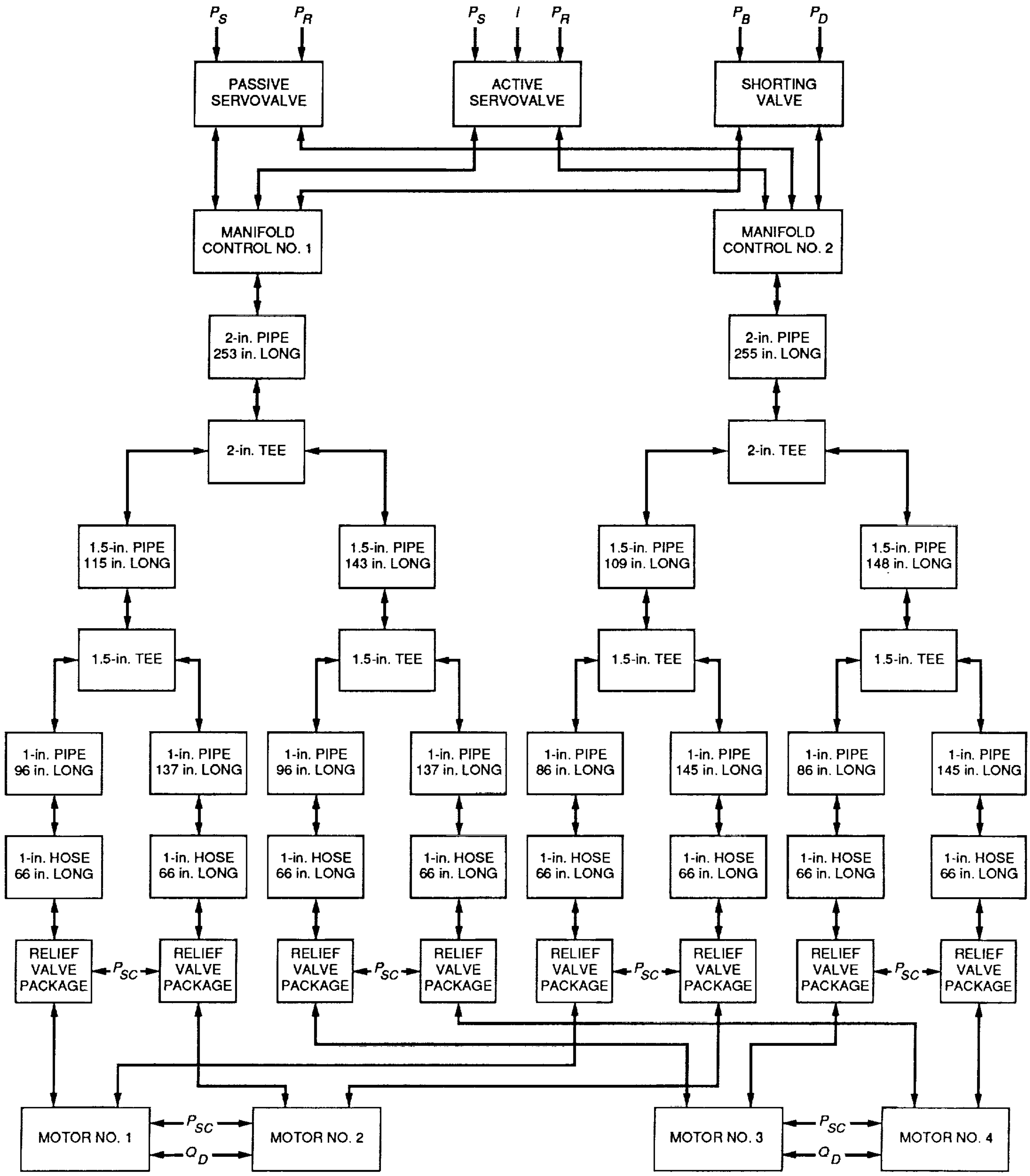

Fig. 3. DSN 70-meter antenna servo hydraulic system elevation component layout.

Appendix

Glossary

a Internal radius of conduit
c_0 Average speed of sound in a fluid
E Modulus of elasticity
f Highest system frequency, in hertz
i Index, current
K Constant associated with a particular fluid line
L Length of fluid transmission line or fitting
P Pressure
P_B Brake pressure
P_D Drain pressure
P_R Return pressure
P_S Supply pressure
P_{SC} Supercharge pressure
Q_D Volumetric flow rate into case drain
Q_{in} Volumetric flow rate into a control volume
Q_{out} Volumetric flow rate out of a control volume
R Constant in ideal gas law
V_i Volume of the ith container
V_0 Total volume of a control volume
β Effective bulk modulus
β_c Effective bulk modulus of the container
β_f Bulk modulus of the fluid
β_i Bulk modulus of the ith container
γ Specific heat ratio
δ Conduit wall thickness
θ_0 Average gas temperature
ν Poisson's ratio
ρ Fluid density
ρ_0 Average fluid density
τ Time delay