A Local Condensation Analysis Representing Two-Phase Annular Flow in Condenser/Radiator Capillary Tubes

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ABSTRACT

NASA's current effort for the thermal environmental control of the Space Station Freedom is directed towards the design, analysis, and development of an Active Thermal Control System (ATCS). A two-phase, flow-through condenser/radiator concept has been baselined, as a component of the ATCS, for the dissipation of space station thermal load into space.

The proposed condenser concept is similar to the single-phase deployable photovoltaic radiator. Here the heat rejection occurs through direct condensation of ATCS working fluid (ammonia) in the small diameter radiator tubes. Analysis of the condensation process and design of condenser tubes are based on the available two-phase flow models for the prediction of flow regimes, heat transfer and pressure drops. The prediction formulas employ the existing empirical relationships of friction factor at gas-liquid interface.

A recent preliminary analysis of condensation in the radiator tubes at Johnson Space Center revealed that annular flow covers over 90% of the tube length. This analysis uses prediction formulas for pressure drop which are, in general, based on interfacial friction factor model for steam-water flow in large tubes. The contributions made by capillary waves and transverse radius of curvature to the instability of interface are ignored in the current models.

The prediction of the interfacial friction factor and the major uncertainties associated with it play a vital role in modelling the annular flow regime. The condensation process in the proposed ATCS radiator includes two-phase flow in small tubes. The capillary waves present in the annular flow regime contribute to the liquid-vapor interfacial friction and hence have a major influence on the pressure drop and heat transfer. We have reviewed the present models of micro condensation in two-phase annular flow regime. The majority of formulations, describing the annular condensation process, are based on either on homogeneous or one dimensional separated flow models.

This study is an attempt to investigate the stability of interfacial waves in two-phase annular flow. It contains the formulation of a stability problem in cylindrical coordinates. The contribution of fluid viscosity, surface tension and transverse radius of curvature to the interfacial surface is included in this formulation. A solution is obtained for Kelvin-Helmholtz instability problem which can be used to determine the critical and "most dangerous" wavelengths for the interfacial wave. The annular condensation is examined in this study and the general equations describing the condensation process is included.

INTRODUCTION

Two-phase condensing phenomena occur in a variety of industrial applications including condenser systems associated with nuclear and conventional power plants, space power generations, thermal energy conversion systems, vapor compression refrigeration cycles, and chemical processing. The ability to understand and to model the principal physical mechanism associated with the condensing flow processes is of considerable importance.

Design and optimization of operating conditions of condensers under space environment conditions require qualitative information including pressure drop, void fraction distribution, quality distribution, flow stability. The behavior of condensation flow phenomena and the transition between flow regimes must be analyzed and understood before condenser systems may be designed for space applications. The approach adopted by the majority of the investigators has been to first focus on the changes that could occur in the flow regimes and their boundaries and also at the pressure drop at reduced-g environment. Rezkallah [1, 2] suggests that in two phase gas-liquid flow under microgravity conditions there exist a significant shift in the transition boundaries when compared with flow regime maps of the normal gravity. Under those conditions there is a pronounced influence of surface tension.

The dynamic characteristics of two-phase condensing flow have received very little attention compared to their boiling flow processes. Commonly, the flow analysis of condensation process is based on existing prediction formulas for two-phase flow models [3]. These prediction relations, in general, have been developed for two-phase, two-component systems (e.g. air-water or steam-water), which have often been obtained in rather large diameter tubes under earth's normal gravity field. The most widely used technique for determining two-phase flow regimes in tubes is that of Martinelli [4], which assumes that the pressure drop of the separate liquid and gas phases are equal, and makes no assumption regarding the specific flow configuration.

Research on condensation heat transfer in two phase flow under microgravity conditions is very limited, and experimental data for heat transfer coefficients under those conditions are sparse. Our understanding of many important physical phenomena in two phase flow remains incomplete. The microgravity, high-vacuum, low-temperature environment of space cannot be duplicated on earth , and it is too costly to place more than a few large-scale tests in space. Therefore, the heat transfer coefficient correlations for single or two-phase flow, and the pressure drop multiplier relations for two-phase flow under low gravity conditions must be determined analytically.

Surface tension forces play a significant role in determining flow regimes in small diameter tubes even at 1-g conditions. The presence of an interface between two fluid phases can influence the motion of fluids and the pressure drop when the interface has a finite curvature (transverse curvature) that is different from that at equilibrium [5]. In most studies of two-phase flow the effect of transverse curvature on the interface has been ignored and the interface has been assumed to be flat. However, the pressure drop in two-phase flow is principally due to the influence of the interfacial roughness elements. Small capillary-type waves on the interface (ripples and disturbance waves) which act as surface roughness give rise to an increased pressure drop. Since gravitational forces at 1-g are nearly dominant in such flows, surface tension has been assumed insignificant, and

therefore given little or no consideration. But, under reduced gravity conditions there is a pronounced influence of surface tension on the flow condition. These waves vary in length and in amplitude. Some amplitudes are several times greater than the mean film thickness. In two-phase flow in small diameter tubes, the interface is curved and deformed as a result of the forces acting on it. Therefore, the shape of the interface must be determined as part of the solution of the problem of two-phase flow.

The condensation process inside tubes may include several flow regimes. The most common flow regimes are annular flow (with liquid film distributed essentially uniformly along the tube wall), annular-stratified flow (with a preponderance of liquid flow along the tube bottom due to the gravitational action), wavy annular flow (in which the shearing action of higher vapor flow initiates the growth of large amplitude waves, occasionally reaching the top of the tube), slug flow (in which the regular and periodic bridging of the tube diameter results in separated slugs of vapor moving the liquid).

In a space environment there is no gravitational force acting on the liquid film. Consequently a stratified flow regime is not expected to exist. The surface wetting characteristics typical of refrigerant fluids causes uniform distribution of the condensate film along the periphery of the tube. That is, the flow is annular type. In annular condensing flow, the liquid film remains relatively thin, and has increasing velocity in the direction of flow, so that at high qualities one expects good uniform heat transfer along the entire tube periphery. The vapor velocity decreases as the vapor mass flow rate decreases more rapidly than the vapor flow area. Eventually the vapor reaches a blunt interface where all the vapor is condensed. At this point the liquid velocity is quite low since it now occupies the entire flow channel.

NASA's current effort for the thermal environmental control of the Space Station Freedom is directed towards the design, analysis, and development of an Active Thermal Control System (ATCS) to remove all the electrical loads, plus any biological or chemical generated heat loads [6]. A two-phase, flow-through condenser/radiator concept has been baselined, as a component of the ATCS, for the dissipation of the thermal load into space. Here the heat rejection occurs through direct condensation of the ATCS working fluid (ammonia) in the condenser/radiator tubes (each 10.08 ft. long with a nominal diameter of 0.09 inches). Design requirements for the condenser tubes are: 98% quality ammonia at the inlet and a minimum of 10°F subcooling at the outlet of the condenser tubes with the constraint that the pressure drop in the condenser panel should not exceed 1.4 Psid.

The pre-integrated truss heat rejection system consists of two radiator arrays: one array rejects heat from the 58°F ATCS loop and the other rejects heat from the 35°F starboard loops. Each array consists of three Orbital Replacement Unit (ORU) radiators connected in parallel. Each ORU contains eight radiator panels and has two parallel condensation paths.

A preliminary analysis of condensation in the ATCS radiator tubes at Johnson Space Center [7] revealed that annular flow covers over 90% of the tube length. The analysis were based on modified prediction formulas in [3] which contains a compilation of the existing empirical relationships for two-phase flow models for the prediction of flow regimes, heat transfer, gas-liquid interfacial friction factor pressure drops. These analysis revealed that the fluid flow is predominantly laminar in the liquid film and turbulent in the vapor core.

11-4

OBJECTIVES

The basic objective of this effort is to study the microgravity condensation process in small diameter tubes to support NASA's current mission on analysis of the two-phase flow-through condenser concept for the Space Station Freedom. The specific goals of this work to establish the forces that contribute to the friction factor at the liquid-vapor interface, to predict pressure drop and heat transfer, and to examine the interfacial instabilities contributing to the transition of flow from annular to slug flow regime. These will require a detailed analysis of the capillary waves at the liquid-vapor.

This study will be limited to examination of condensation process in the annular flow regime of the condenser tube and the examination of the capillary waves at the liquid-vapor interface and their contribution to the interfacial friction factor. In the current investigation, the modeling of the capillary waves at the liquid-vapor interface is based on potential flow theory (inviscid fluid flow). A more detailed and complete analysis requires the consideration of shear forces at the liquid-vapor interface.

We propose to use a separated two-phase flow model to study the condensing process in small diameter tubes. This requires solving the continuity, momentum and energy equations for each phase and using appropriate boundary conditions at the interface to couple the two phases together. Thus, the shape of the interface and proper formulation of tangential and normal stresses at the interface will be the focus of this research. Oscillating pressure difference component across the interface (due to surface tension and transverse curvature) which contributes to flow instability will be included in the proposed analysis.

Most studies conducted on two-phase flow condensation assume that the liquid-vapor interface is flat and ignore the influence of interfacial waves on pressure gradient. It is important to examine the behavior of the interfacial waves in two-phase flow and their contribution to pressure drop. The stability of interfacial waves is also important in determination of flow regimes in two-phase flow situations. The majority of investigations on interfacial wave behavior are based on inviscid fluid models. These works usually do not consider the effect of surface tension on interface. Most analysis of the effect of surface tension on the interface consider two-dimensional waves in Cartesian coordinate and hence do not include the contribution of the transverse radius of curvature; this is an important parameter in modeling two-phase flow in cylindrical pipes, especially in small diameter tubes.

The analytical treatment of surface waves originated with Helmholtz [8], Kelvin [9], and Rayleigh [10]. Other researchers have attempted to extend these studies. For example, Taylor focused his investigation on inertial instability of ideal homogeneous incompressible fluid. The instability of the horizontal interface between two ideal incompressible fluids dates back to G.I. Taylor [11]. The modern extension of viscid flows has been reviewed by a number of researchers who have suggested ways to include interfacial surface tension and the fluid viscosity. In recent years few studies [12,13] have considered the contribution of fluid viscosity on the " most susceptible" or "most dangerous" wavelength and the corresponding frequency.

Lienhard and Dhir [12,13] studied the viscous hydrodynamics instability of peak pool boiling problem. The formulation of their instability problem was based on a model in Cartesian coordinate, which considered a vapor jet flowing in a pool of liquid. However, in their analysis they included the contribution of transverse curvature of vapor jet to instability as a form of an additional oscillating pressure difference component across the interface. They suggested that the pressure transverse pressure effect at the liquid-vapor interface may be presented as:

$$\Delta p_{tr} = \frac{\sigma \eta}{2 R_j^2}$$

(1)

where σ is the surface tension, R_j is the radius of the jet, and η (z, t) is the height of the the propagating interfacial wave.

ANALYSIS

In this work we will attempt to formulate the instability of interfacial wave in annular two-phase flow. In general, formulation of the problem will include the fluid viscosity and interfacial pressure gradient resulting from the surface tension and transverse radius of curvature. We will solve this problem for inviscid flow and make recommendations on the approach to solve the problem for viscous fluid flow.

Formulation of Stability Problem:

In formulating an interfacial wave model of annular flow we take an approach similar to that taken by Lienhard and Dhir [12,13]. However, we will use cylindrical coordinates to model our instability problem. The assumptions used in modeling the instability problem are:

1)   the liquid and vapor are incompressible
2)   both fluids are Newtonian
3)   the base flow is steady, axisymeteric and one dimensional
4)   The slope of interface is small everywhere
5)   the nonlinear effects are negligible
6)   the gravitational forces on the flow are negligible.

A schematic model of interfacial wave in two-phase annular flow is shown in Fig. 1. In this model vapor flows in the core of the cylinder and the liquid film flows adjacent to the wall. For this configuration we imagine a sinusoidal wave form at the interface and attempt to determine the propagation speed of the wave. The disturbance wave is assumed to oscillate as

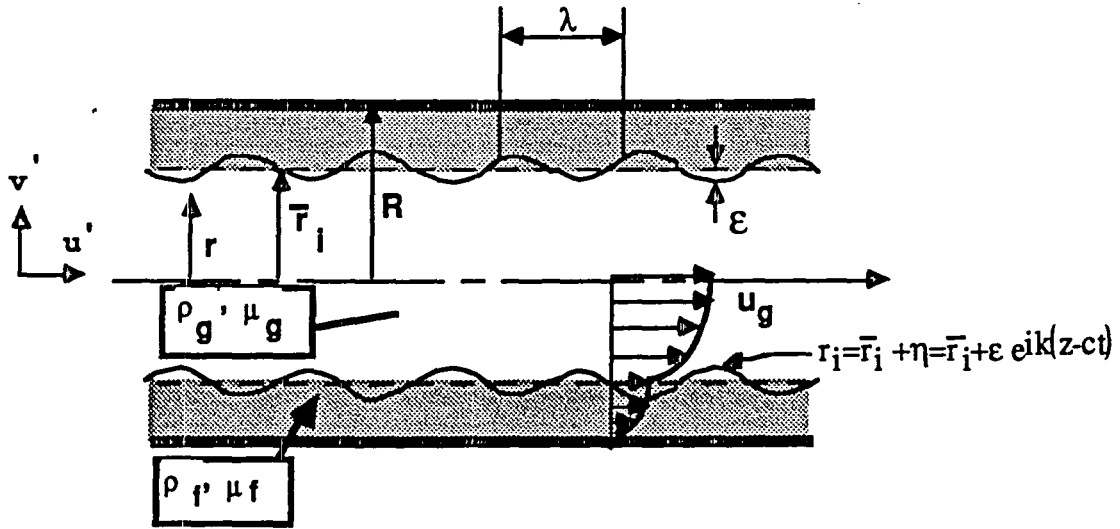$$\eta = \varepsilon \, e^{i k (z - c t)}$$

(2)

Figure 1. - Wave-shaped interface separating liquid film and vapor core.

where ε is the wave amplitude, $k = 2\pi/\lambda$ is the wave number describing the disturbance periodicity in the axial direction, λ is the wavelength $c = k\omega$ is the velocity of propagating wave and ω is the frequency of wave. The equation of interface is

$$r_i = \overline{r_i} + \eta = \overline{r_i} + \varepsilon \, e^{ik(z-ct)} \tag{3}$$

where $\overline{r_i}$ is the mean position of the interface. The criteria for the wave to be stable requires c to be real. If c is imaginary then the wave is unstable. The case $c/i < 0$ represents a decaying wave and the situation $c/i > 0$ denotes a growing wave.

Modeling the annular two-phase flow as inviscid and axisymetric, the continuity and momentum equations reduce to:

$$\frac{\partial u_i}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\,v_i) = 0 \tag{4}$$

$$\frac{\partial u_i}{\partial t} + u_i\frac{\partial u_i}{\partial z} + v_i\frac{\partial u_i}{\partial r} = -\frac{1}{\rho_i}\frac{\partial p_i}{\partial z} + v_i\left(\frac{\partial^2 u_i}{\partial z^2} + \frac{\partial^2 u_i}{\partial r^2} + \frac{1}{r}\frac{\partial u_i}{\partial r}\right) + G_z \tag{5}$$

$$\frac{\partial v_i}{\partial t} + u_i\frac{\partial v_i}{\partial z} + v_i\frac{\partial v_i}{\partial r} = -\frac{1}{\rho_i}\frac{\partial p_i}{\partial r} + v_i\left(\frac{\partial^2 v_i}{\partial z^2} + \frac{\partial^2 v_i}{\partial r^2} + \frac{1}{r}\frac{\partial v_i}{\partial r} - \frac{v_i}{r^2}\right) + G_r \tag{6}$$

For the convenience and the space economy we have used the subscript "i" to represent the governing equations of both the liquid and vapor with a single set of equations (substitution of g and f for i gives relations for the vapor core liquid film region, respectively). Unless otherwise specified, we will use this subscript to represent the physical variables for both fluids with a single expression.

Now let's assume that steady annular flow U_i (r) is disturbed slightly such that

$$u_i = U_i(r) + u'_i(r,z,t) \tag{7}$$

$$v_i = v'_i(r,z,t) \tag{8}$$

where the perturbed components u' and v' are infinitesimally small compared with the average velocity of the base flow (note that in condensation problem $U_i = U_i$ (r, z) and one dimensional flow assumptions might not give accurate results). The pressure can be presented as

$$p_i = p_i(z) + p'_i(r,z,t) \tag{9}$$

where

$$p' = \varepsilon \; p'(r) \; e^{i \, k \, (z - c \, t)} \tag{10}$$

We would like to determine whether the flow is unstable relative to the postulated disturbance. Substituting equations (7) through (10) into the governing equations (5) through (6), eliminating the steady state solution, and linearizing the results (ie. neglecting the terms of second order in u'_i and v'_i) the following equations can be written for the perturbed flow

$$\frac{\partial u'_i}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r \, v'_i) = 0 \tag{11}$$

$$\frac{\partial u'_i}{\partial t} + U_i\frac{\partial u'_i}{\partial z} + v'_i\frac{\partial U_i}{\partial r} = -\frac{1}{\rho_i}\frac{\partial p'_i}{\partial z} + \nu_i\left(\frac{\partial^2 u'_i}{\partial z^2} + \frac{\partial^2 u'_i}{\partial r^2} + \frac{1}{r}\frac{\partial u'_i}{\partial r}\right) \tag{12}$$

$$\frac{\partial v'_i}{\partial t} + U_i\frac{\partial v'_i}{\partial z} + = -\frac{1}{\rho_i}\frac{\partial p'_i}{\partial r} + \nu_i\left(\frac{\partial^2 v'_i}{\partial z^2} + \frac{\partial^2 v'_i}{\partial r^2} + \frac{1}{r}\frac{\partial v'_i}{\partial r} - \frac{v'_i}{r^2}\right) \tag{13}$$

These equations must satisfy appropriate boundary conditions and interfacial requirements. For the vapor core the boundary conditions are

$$v'_g = 0 \qquad \text{at} \qquad r = 0 \tag{14}$$

$$\frac{\partial u'_g}{\partial r} = 0 \qquad \text{at} \qquad r = 0 \tag{15}$$
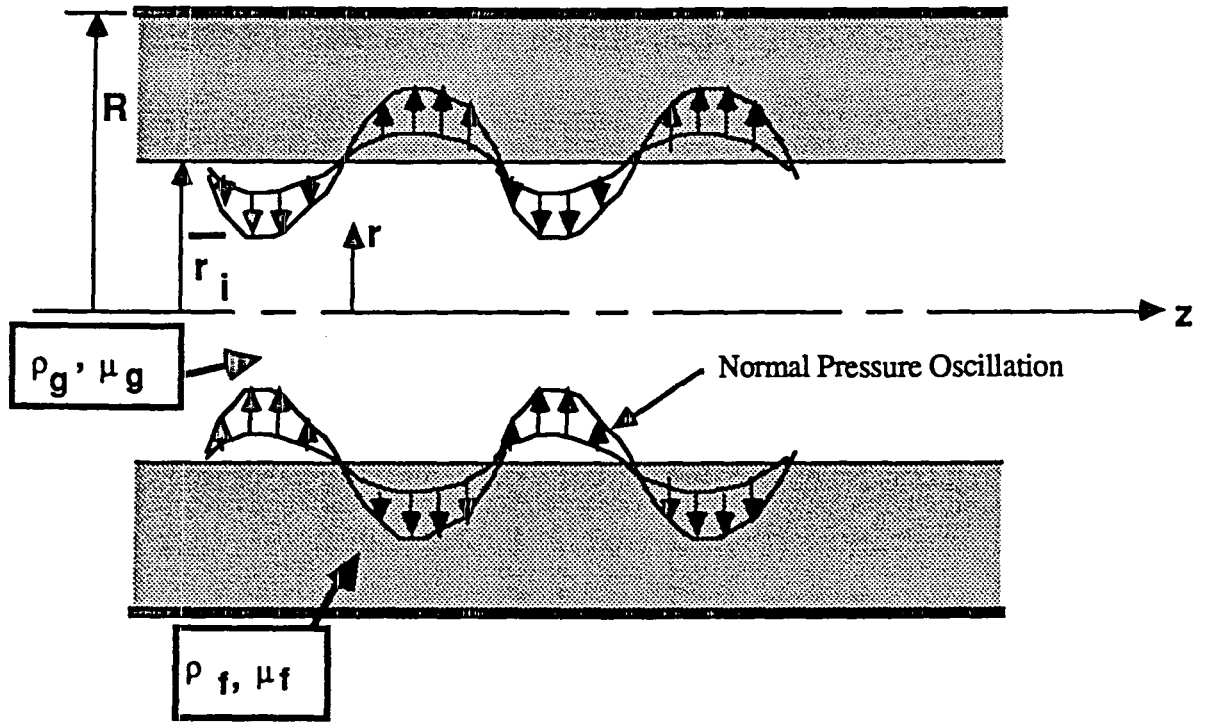
Figure 2. Pressure oscillation due to the transverse radius of the curvature of the interface.

The boundary conditions for the liquid film flow are

$$v'_f = 0 \qquad \text{at} \qquad r = R \qquad (16)$$

$$u'_f = 0 \qquad \text{at} \qquad r = R \qquad (17)$$

The following conditions are the result of force balance at the liquid-vapor interface. At the interface the normal stress exerted by the vapor is equal to those of the liquid film.

$$p_g - 2\mu_g \frac{\partial v'_g}{\partial r} + \sigma \frac{\partial^2 \eta}{\partial z^2} + \frac{\sigma \eta}{2 \bar{r_i}^2} = p_f - 2\mu_f \frac{\partial v'_f}{\partial r} \qquad \text{at} \qquad r = \bar{r_i} \qquad (18)$$

This force balance includes the surface tension contribution of pressure oscillation due to the transverse radius of the curvature of the interface as shown in Fig. 2. Note that the last term on the right hand side of equation (18) is the local time average of this pressure oscillation. A balance of tangential forces due to shear stresses of the liquid and vapor on the interface will yield

$$\mu_g \left(\frac{\partial u'_g}{\partial r} + \frac{\partial v'_g}{\partial r}\right) = \mu_f \left(\frac{\partial u'_f}{\partial r} + \frac{\partial v'_f}{\partial r}\right) \qquad \text{at} \qquad r = \overline{r_i} \qquad (19)$$

The linearized Kinematic condition at the interface requires that

$$v'_i = \frac{\partial r_i}{\partial t} + U_i \frac{\partial r_i}{\partial z} \qquad \text{at} \qquad r = \overline{r_i} \qquad (20)$$

Now we introduce the perturbation stream function as

$$\psi = \varepsilon \, F(r) \, e^{i k (z - c t)} \qquad (21)$$

such that

$$u' = \frac{1}{r} \frac{\partial \psi}{\partial r} \qquad \text{and} \qquad v' = -\frac{1}{r} \frac{\partial \psi}{\partial z} \qquad (22)$$

Substituting equations (21) through (22) and (10) into equations (12) and (13) we get the following relations for the equation of motion

$$\frac{1}{r}(U_i - c)\frac{\partial F_i}{\partial r} - \frac{1}{r}F\frac{\partial U_i}{\partial r} = -\frac{1}{\rho_i} p'_i + \frac{v_i}{i\,k}\left(\frac{1}{r}\frac{\partial^3 F_i}{\partial r^3} - \frac{1}{r^2}\frac{\partial^2 F_i}{\partial r^2} + \frac{1}{r^3}\frac{\partial F_i}{\partial r} - \frac{k^2}{r}\frac{\partial F_i}{\partial r}\right) \qquad (23)$$

$$\frac{k^2}{r}(U_i - c)F = -\frac{1}{\rho_i}\frac{\partial p'_i}{\partial r} - i\,k v_i\left(\frac{1}{r}\frac{\partial^2 F_i}{\partial r^2} - \frac{1}{r^2}\frac{\partial F_i}{\partial r} - \frac{k^2}{r}F_i\right) \qquad (24)$$

Eliminating p'_i by combining equations (23) and (24) the following relationship will result.

$$(U_i - c)\left(\frac{\partial^2 F_i}{\partial r^2} - \frac{1}{r}\frac{\partial F_i}{\partial r} - k^2 F_i\right) + \left(\frac{1}{r}F_i\frac{\partial U_i}{\partial r} - F_i\frac{\partial^2 U_i}{\partial r^2}\right) =$$
$$\frac{v_i}{i\,k}\left(\frac{\partial^4 F_i}{\partial r^4} - \frac{2}{r}\frac{\partial^3 F_i}{\partial r^3} + \frac{3}{r^2}\frac{\partial^2 F_i}{\partial r^2} - \frac{3}{r^3}\frac{\partial F_i}{\partial r} + 2\frac{k^2}{r}\frac{\partial F_i}{\partial r} - 2k^2\frac{\partial^2 F_i}{\partial r^2} + k^4 F\right) \qquad (25)$$

Equation (25) represents the "Orr- Sommerfeld" equation in cylindrical coordinates. Equations (23) through (25) are similar to those developed in [14]. In fact the left hand side of equations (23) through (25) are identical to the corresponding equations in [14]. However, [14] omits terms in the left hand side of those equations corresponding to equations (23) and (25).

It is more practical and useful for analysis of the instability problem to obtain a solution for equation (25) in dimensionless form . Let us define the

following dimensionless variables for the problem (for convenience we make dimensionless symbols identical to those used for the original physical variables).

$$z = \frac{z}{R} ; \qquad r = \frac{r}{R} ; \qquad \varepsilon = \frac{\varepsilon}{R} ; \qquad \eta = \frac{\eta}{R} ; \qquad \lambda = \frac{\lambda}{R} ; \qquad r_i = \frac{r_i}{R}$$

$$u'_i = \frac{u'_i}{U_0} ; \qquad U_i = \frac{U_i}{U_0} ; \qquad v'_i = \frac{v'_i}{U_0} ; \qquad v'_i = \frac{v'_i}{U_0} ; \qquad p'_i = \frac{p'_i}{\rho_i U_0^2} \qquad c = \frac{c}{U_0}$$

Where U_0 is an average fluid velocity (e.g., the superficial vapor velocity). Substituting these dimensionless variables in equations (23) through (25) results in the following relations:

$$\frac{1}{r}(U_i - c)\frac{\partial F_i}{\partial r} - \frac{1}{r} F\frac{\partial U_i}{\partial r} = - p'_i + \frac{1}{i\,Re_i}\left(\frac{1}{r}\frac{\partial^3 F_i}{\partial r^3} - \frac{1}{r^2}\frac{\partial^2 F_i}{\partial r^2} + \frac{1}{r^3}\frac{\partial F_i}{\partial r} - \frac{\alpha^2}{r}\frac{\partial F_i}{\partial r}\right) \qquad (26)$$

$$\frac{\alpha^2}{r}(U_i - c)F = - \frac{\partial p'_i}{\partial r} - \frac{i}{Re_i}\left(\frac{1}{r}\frac{\partial^2 F_i}{\partial r^2} - \frac{1}{r^2}\frac{\partial F_i}{\partial r} - \frac{\alpha^2}{r}F_i\right) \qquad (27)$$

$$(U_i - c)\left(\frac{\partial^2 F_i}{\partial r^2} - \frac{1}{r}\frac{\partial F_i}{\partial r} - \alpha^2 F_i\right) + \left(\frac{1}{r} F_i\frac{\partial U_i}{\partial r} - F_i\frac{\partial^2 U_i}{\partial r^2}\right) =$$

$$\frac{1}{i\,\alpha Re_i}\left(\frac{\partial^4 F_i}{\partial r^4} - \frac{2}{r}\frac{\partial^3 F_i}{\partial r^3} + \frac{3}{r^2}\frac{\partial^2 F_i}{\partial r^2} - \frac{3}{r^3}\frac{\partial F_i}{\partial r} + \frac{2\alpha^2}{r}\frac{\partial F_i}{\partial r} - 2\,\alpha^2\frac{\partial^2 F_i}{\partial r^2} + \alpha^4 F\right) \qquad (28)$$

where $\alpha = kR$ is the dimensionless wave number and Re_i is the Reynold number defined as

$$Re_i = \frac{U_0 R}{v_i} \qquad (29)$$

The non-dimensional form of the disturbance wave, the equation of interface and the perturbation stream function are expressed by the following relations:

$$\eta = \varepsilon\ e^{i\alpha(z-ct)} \qquad (30)$$

$$r_i = \overline{r_i} + \eta = \overline{r_i} + \varepsilon\ e^{i\alpha(z-ct)} \qquad (31)$$

$$\psi = \varepsilon\ F(r)\ e^{i\alpha(z-ct)} \qquad (32)$$

To analyze the stability of the interfacial waves we must solve equation (27), using relevant boundary and interfacial conditions. Note that for inviscid flow or high Reynold numbers the left hand side of the equation vanishes and it reduces to

11-11

$$(U_i - c)\left(\frac{\partial^2 F_i}{\partial r^2} - \frac{1}{r}\frac{\partial F_i}{\partial r} - \alpha^2 F_i\right) + \left(\frac{1}{r}F_i\frac{\partial U_i}{\partial r} - F_i\frac{\partial^2 U_i}{\partial r^2}\right) = 0 \tag{33}$$

The contribution of the radial variation of the base velocity to the instability problem is represented by the last bracket in equation (33). For uniform base flow this term also vanishes and equation reduces to Helmholtz instability problem in cylindrical coordinates:

$$(U_i - c)\left(\frac{\partial^2 F_i}{\partial r^2} - \frac{1}{r}\frac{\partial F_i}{\partial r} - \alpha^2 F_i\right) = 0 \tag{34}$$

The relevant boundary and interfacial condition for the "Orr- Sommerfeld" problem, equation (27), can be obtained from equations (14) through (20). They are

$$F_g = \frac{\partial F_g}{\partial r} = \frac{\partial^2 F_g}{\partial r^2} = 0 \qquad \text{at} \qquad r = 0 \tag{35}$$

$$F_f = \frac{\partial F_f}{\partial r} = 0 \qquad \text{at} \qquad r = 1 \tag{36}$$

$$p_g - \frac{2}{Re_g}\frac{\partial v'_g}{\partial r} + \frac{\sigma}{\rho_g R U_0^2}\frac{\partial^2 \eta}{\partial z^2} + \frac{\sigma \eta R}{2 \rho_g U_0^2 \bar{r_i}^2} = \left(p_g - \frac{2}{Re_g}\frac{\partial v'_g}{\partial r}\right)\left(\frac{\rho_f}{\rho_g}\right) \quad \text{at } r = \bar{r_i} \tag{37}$$

$$\left(\frac{\partial u'_g}{\partial r} + \frac{\partial v'_g}{\partial r}\right) = \left(\frac{\partial u'_f}{\partial r} + \frac{\partial v'_f}{\partial r}\right)\left(\frac{\mu_f}{\mu_g}\right) \qquad \text{at} \qquad r = \bar{r_i} \tag{38}$$

$$v'_i = \frac{\partial r_i}{\partial t} + U_i\frac{\partial r_i}{\partial z} \qquad \text{at} \qquad r = \bar{r_i} \tag{39}$$

There is no complete solution available for equation (27) at this time. Most existing solutions are for the special cases of the "Orr- Sommerfeld" equation. Other solutions treat the "Orr- Sommerfeld" equation in Cartesian coordinates. The existing treatment of the instability problems are aimed at flow conditions at low Reynold number. For example, the hydrodynamic instability studies conducted by Lienhard and Dhir [12,13] are based on the formulation of problem in Cartesian coordinates. Their perturbation method solves the instability problem for a case where Re << 1 and the wave number α is close to one.

Solution of Kelvin-Helmholtz Instability Equation:

Our next attempt is to determine the most "dangerous wavelength" by solving equation (34). For the vapor core we use the boundary condition, equation (35), and the kinematic condition at the interface, equation (39). For the liquid film we employ the

boundary condition, equation (36) and the kinematic condition at the interface, equation (37).

The general solution for equation (34) is

$$F_i = C_1 \, r \, I_1(\alpha r) + C_2 \, r \, K_1(\alpha r) \tag{40}$$

where I_1 and K_1 are modified first order Bessel functions. Applying the boundary and kinematic conditions we obtain a solution for the vapor flow, expressed as

$$F_g = -(U_g - c)\left[\frac{r \, I_1(\alpha r)}{I_1(\alpha \, \bar{r}_i)}\right] \tag{41}$$

The solution for the liquid film is

$$F_f = -(U_f - c)\left[\frac{K_1(\alpha r) \, I_1(\alpha) - I_1(\alpha r) \, K_1(\alpha)}{(I_1(\alpha \, \bar{r}_i))(K_1(\alpha)) - (K_1(\alpha \, \bar{r}_i))(I_1(\alpha))}\right] \tag{42}$$

From equation (26) we get the following expression for p'_i

$$p'_i = -\frac{1}{r}(U_i - c)\frac{\partial F_i}{\partial r} \tag{43}$$

Substituting this relation into equation (37), the continuity condition of normal stress at the interface can be presented as

$$-(U_g - c)\frac{\partial F_g}{\partial r} + \frac{\sigma}{\rho_g \, R \, U_0^2} \frac{\partial^2 \eta}{\partial z^2} + \frac{\sigma \, \eta \, R}{2 \, \rho_g \, U_0^2 \, \bar{r}_i^2} = -(U_f - c)\left(\frac{\partial F_f}{\partial r}\right)\left(\frac{\rho_f}{\rho_g}\right) \tag{44}$$

Substituting equations (30), (41), and (42) and solving for c, equation (44) will result in a complex number, where

$$c = c_R + c_I \tag{45}$$

where c_R and c_I are the real and the imaginary part of the wave velocity . The stability condition requires that

$$c_I = 0 \tag{46}$$

The critical wave number, α_c, and wavelength, $\lambda_c = 2 \, \pi / \, \alpha_c$, can be determined from this condition. The most dangerous wavelength can be evaluated using the criteria

$$\frac{\partial(-i \, \alpha c)}{\partial \alpha} = 0 \tag{47}$$

CONCLUSIONS AND RECOMMENDATIONS

The condensation process in annular flow has been discussed. The major forces influencing the pressure drop have been identified. It has been noted that under micro gravity conditions, the forces that influence the flow include inertial, viscous, and surface tension.

The stability of capillary waves in small tubes has been formulated. The general formulation of stability considers both viscous and surface tension effects. In addition, pressure oscillation due to the transverse radius of curvature of the interface has been included in this study. Kelvin-Helmholtz's Instability Problem has been solved. Equations (44) through (47) provide the most dangerous wavelength.

A more complete solution of equation (28) is needed to determine the viscous effect on the stability of the interface. Most models for evaluation of friction factor for condensation process in the annular flow regime are based on one-dimensional, two-phase flow in large tubes under normal gravity conditions. These models ignore the effect of capillary waves on pressure drop. These models require a friction factor multiplier based on an experimental value. All our attempts of modeling the condensation process resulted in a need for an experimental value of friction factor multiplier to determine the pressure drop due to interaction at the interface.

There is need for further study of problems considered in this work. This includes more in depth solutions to the instability problem and attempts to solve the pressure drop problem in two-dimensional flow. A more detailed and complete analysis is required to determine the influence of shear forces on the instability of the interfacial waves.

A complete analysis of the condensation problem requires solving the continuity, momentum and energy equations for each phase and using appropriate boundary conditions at the interface to couple the two phases.

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