# PHOTOGRAPHIC IMAGE RESTORATION

Final Report

NASA/ASEE Summer Faculty Fellowship Program-1991

Johnson Space Center

Prepared by: Gerald E. Hite

Academic Rank: Associate Professor

University & Department: Texas A&M Univ. at Galveston
Dept. of Marine Science
Galveston, Texas 77553-1675

NASA/JSC

Directorate: Space and Life Sciences

Division: Solar System Exploration

Branch: Space Shuttle Earth Observation Office

JSC Colleague: David E. Pitts

Date Submitted: August 10, 1991

Contract Number: NGT-44-001-800

# ABSTRACT

Deblurring capabilities would significantly improve the Flight Science Support Office's ability to monitor the effects of lift-off on the shuttle and landing on the orbiter. This summer a deblurring program was written and implemented to extract information from blurred images containing a straight line or edge and to use that information to deblur the image. The program was successfully applied to an image blurred by improper focussing and two blurred by different amounts of motion blurring. In all cases the reconstructed modulation transfer function not only had the same zero contours as the Fourier transform of the blurred image but the associated point spread function also had structure not easily described by simple parameterizations. The difficulties posed by the presence of noise in the blurred image necessitated special consideration. An amplitude modification technique was developed for the zero contours of the modulation transfer function at low to moderate frequencies and a smooth filter was used to suppress high frequency noise.

## I. Introduction

Photographs of lift-off and landing of the shuttle are taken by the Flight Science Suppórt Office. These are used to provide immediate identification of any potential damage to the orbiter as well as to identify sources and effects of any debris created during these crucial periods. The Space Shuttle Earth Observation Office is the repository of photographs taken by astronauts of environmental, geological, meteorological and oceanographic phenomena observed in orbit. Inadvertently many photographs are blurred by improper focusing, relative camera-object motion etc. To gain access to important information contained in such photographs, it is necessary to apply image processing techniques. The more sophisticated of these involve digitization of the raw images and subsequent computer processing of the digitized images or their Fourier transforms.

The research this summer involved developing and implementing a program that would extract information on the point spread function (psf) directly from any blurred image containing a straight line or edge. Although the technique would work for more general blurrings it was especially useful for the two most common types of blurring: improper focusing and motion blurring. In the case of motion blurring the intensity at a point is spread along a line. Although it need not be uniform along the line, for simplicity it was assumed to be along a straight line. In the case of improper focusing the intensity of a point is spread to neighboring points with the relative intensity depending only on the distance from the initial point. The distribution of intensities of each point in the "sharp" image to the points in the blurred image is given by a convolution sum of the "sharp"image and the point spread function. The advantage of a Fourier Transformation (FT) is that it reduces this sum to a product. If there were no noise, the FT of the desired image could be obtained by simply dividing that of the blurred image by that of the psf. However, for the cases of interest the Fourier transform of the psf (MTF) has contours of zero amplitude and becomes small at high frequency. Consequently noise in frequency regions where the MTF is small would result in serious degradation of the signal to noise ratio. Two techniques were applied to minimize such degradation. The first is the implementation of a two parameter high frequency filter. The second is the imposition of a maximum to minimum ratio limit for the amplitude of the filter. This is important at low to moderate frequencies. There are many excellent tests dealing with digital processing (1,2,3) and discrete Fourier transforms (4,5) that provide background for the work described in this report.

## II General Approach and Theory

### II.1 Convolution of the Point Spread Function and Noise

In general a blurred image is the result of light that would have arrived at a particular point being spread over a number of points. If the distribution is the same for all points in the image and if the light is incoherent then in the absence of noise the final intensity at a point (pixel) located by $\vec{m} = [m,n]$ would be given by

(1a)
$$f^b_{\vec{m}} = \sum_{\vec{m}'} p_{\vec{m}-\vec{m}'} f^s_{\vec{m}'}$$

where p is the point spread function, i.e. the image of an ideal point source in the object plane, $f^s_{\vec{m}}$ is the sharp image that would have formed in the absence of blurring and noise.

For the images of interest the digitization process yields an image made up of NxN pixels (N=512). due to the blurring bringing light rays into the image region that they would have otherwise not have entered it, the summation in this expression should include m's outside the range of 0 to N-1. Nevertheless, the sum is usually assumed to be limited to this range to avoid having to deal with an"underdetermined problem"(6).

Noise is always present in the image. It can be the result of electronics, inhomogeneous chemical processes in the original film, etc. Noise can be divided into two contributions: one independent and the other dependent on the signal, i.e. $f^s$(7). In the latter case it could be incorporated into the point spread function. Since we have little knowledge of the nature of the noise, except that it is usually dominant at high frequencies, we will describe it by an unknown function $n_{\vec{m}}$ and write the expression for the blurred image as

(1b)
$$f^b_{\vec{m}} = \sum_{\vec{m}'} p_{\vec{m}-\vec{m}'} f^s_{\vec{m}'} + n_{\vec{m}}$$

The next step is to invert the sum in equation 1 and solve for $f^s$. Since the summation is usually referred to as convolution, the inverse process is referred to as deconvolution. The basic technique is to expand all functions in terms of a complete set of basis functions which will turn the summation into a simple product of the expansion coefficients.

## II.2 Discrete Fourier Transormation

For simplicity we use the traditional discrete Fourier expansion

(2a)
$$f_{\vec{m}} = \sum_{\vec{k}} F_{\vec{k}} z^{\vec{k}\vec{m}}$$

where $z = \exp(i2\pi/N)$.The inverse Fourier transformation is then given by

$$F_{\vec{k}} = \frac{1}{N^2} \sum_{\vec{m}} f_{\vec{m}} z^{\vec{k}\vec{m}}$$

In the following we will use lower case letters, e.g. f, for the spatial images and upper case letters, e.g. F, for the Fourier transformed image. The Fourier transform of the point function, psf, is called the modulation transfer function, MTF.
In terms of the Fourier tranformed function Eq. 1 takes the form:

(3a) $$F_k^b = P_k F_k^s + N_k$$

Deleting the common subscripts and solving this equation for the Fourier transform of the sharp image function yields

(3b) $$F^s = F^b/P - N/P$$

If the modulation transfer function and the noise, N, were both known then equation 3b could be used to find $F^s$ and a subsequent inverse Fourier transformation would yield a sharp image. For the sake of discussion let us assume that we have a reasonably good understanding of the blurring process and have a good approximation to psf and thus to the M.T.F. Most likely the blurring extends over a few spatial pixels and MTF will be small at high frequencies, i.e. large values of k. Blurring due to improper focusing, relative motion, atmospheric turbulence are in general of this type. The first two sources of blurring have MTF's that vanish on contours in k-space. Where ever the MTF is small the contribution from noise, N/P, in equation 3b can be significant. Consequently, approximating $F^s$ by $F^b/P$ can result in large noise contributions. The normal procedure to avoid this problem is to multiply $F^b$ by a "filter" function which uses information about the noise to approximate 1/P where noise is unimportant and avoids contributions from regions where noise dominates. The Wiener filter is the most common filter discussed in the literature (2,3,4,6):

(4) $$W^{-1} = P(1 + |N/F^sP|^2)$$

This expression requires knowledge of the amplitude of the function to be calculated and the unknown noise contribution and cannot be used without further approximation. A simple method to avoid over emphasis of noise is present in section IV.

## II.3 Point-Spread-Function for Improper Focusing and Motion Blurring

The point spread function describes how the intensity of any point of the "sharp" image is spread among neighboring points in the blurred image. In the case of improper focusing, the point spread function is angle independent and depends only on the distance from the initial point to neighboring points,

In the simplest parameterization p is constant up to a given radius and zero for larger radii. An annulus of a given thickness could be added to allow the intensity to decrease

(5a) $$0 \leq P^{(R)}_{m} = P(1\vec{m}l)$$

smoothly to zero.

In the case of linear blurring in the direction $\vec{m}_B = [m_B, n_B]$, one must follow the path of a square pixel containing the original point in the sharp image. Since the pixel has finite dimensions pixels whose centers lie near but not necessarily on the line of blurring will receive contributions. In general these points will be located by a vector $\vec{m}_q$ which is the sum of a multiple of $\vec{m}_B$ and a vector of at most unit length perpendicular to $\vec{m}_B$. Thus

(5b) $$P^{(L)}_{m} = \sum_q l_q \delta_{\vec{m}, \vec{m}_q}$$

where q in the sum labels each such pixel center, and $l_q$ ($\geq 0$) is its intensity, and $\delta_{a,b}$ is the Dirac delta function.

## III Construction of the MTF from Blurred Images

### III.1 Use of Zero Contours to Parameterize MTF's

In the most common types of blurring, i.e. due to improper focusing and/or motion blurring, the Fourier transform of the point spread function, the so-called modulation-transfer-function MTF, has contours in the two dimensional frequency space along which it vanishes. In the cases where the blurring is uniform, i.e. the intensity in each square pixel is constant or zero, the MTF's for the cases of interest is the Airy function, $J_1(\alpha f)/\alpha f$ or the sinc-function $\sin(\alpha f) / \alpha f$, i.e. the diffraction patterns for a circular aperture and a slit. Clearly these functions oscillate in sign and have their first zeroes at $1.22\pi$ and $\pi$ respectively. Usually the FT of the blurred image, $F^B$, allows visual identification of the nearest and perhaps other zero contours resulting from the blurring. In the case of motion blurring those parallel contours are more useful at determining the direction of blurring $m_B$ than the spatial image.

### III.2. Direct Computation of the MTF from the Blurred Image.

If the blurred image has a straight edge or line, it can be used to determine the MTF directly without recourse to oversimplified assumptions about the psf. The basic idea results from considering the blurring of a feature in the sharp image. If the feature of interest were the only feature in the image, then equations 1 through 3 would still hold with f and F receiving contributions only from the feature. The MTF would of course not change. Consequently, if noise in the blurred feature can be neglected or suppressed then the MTF would be the ratio of the FT of the blurred feature to the FT of the suppressed sharp feature. Edges and lines are especially appealing since they are geometrically simple. For simplicity, we will limit the discussion to them. In addition it should be realized that the profile of a blurred line can be trivially obtained from the profile of a blurred edge.

In the following the edge or line of interest will be enclosed in rectangle whose sides are parallel or perpendicular to the supposed sharp line. Let vector $\vec{m}_s = [m_s, n_s]$ describe the direction and distance between pixels on the sharp line. Then the vectors

$$\vec{\Upsilon} = [m_s, n_s] \quad \text{and} \quad \vec{\jmath} = [-n_s, m_s]$$

are respectively parallel and perpendicular to the line. Each point (m,n) located by the vector from the origin $\vec{m} = [m,n]$ is simultaneously on a j-line which is parallel to the original line and characterized by

$$j = \vec{m}\vec{\jmath} = -mn_s + nm_s$$

and or an i-line which is perpendicular to the original line and characterized by

$$i = \vec{m}\vec{\Upsilon} = mm_s + nn_s$$

It is easily seen that $\Delta i = 1$ between neighboring perpendicular lines, $\Delta j = 1$ between neighboring parallel lines, that the distance between lines is $1/k_s = 1/\sqrt{m_s^2 + n_s^2}$ and the number of lines crossing between any adjacent points on a perpendicular line is $k_s^2 - 1$.

Neglecting end effects and assuming intensities are constant in the j-direction, then the intensity function for a sharp line, characterized by $j_1$ can be written as

$$l^s_{\vec{m}} = \delta_{j,j_1}$$

and that of its blurred image as

(6) $$l_B(j) = \sum_{\vec{m}'} p_{\vec{m}-\vec{m}'} \cdot \delta_{j', j_1}$$

where $j = \vec{\jmath} \cdot \vec{m}$ and $j' = \vec{\jmath} \cdot \vec{m}'$.

By replacing the sums in the FT over m′ and n′ by sums over i and j, one obtains the MTF

(7a) $$P_{\vec{k}} = \sum_j l^B(j)\, z^{\vec{k}(\vec{m}_j - \vec{m}_{j_1})}$$

where the vectors $m_j$ are to a single point on each of the j-lines. These vectors are to be selected such that the i sums in the ratio of $L^b$ and $L^s$ cancel. In general they are points lying closest to some line crossing the j-lines. In the case of improper focusing they can be taken to be closest to a particular i-line. In the case of motion blurring it would be those closest to a line parallel to $m_B$.

This problem is easily circumvented by considering the MTF on the line $\vec{k} = \kappa\vec{j}$ ($\kappa$=integer). Then

(7b) $$P_{\kappa\vec{j}} = \sum_j l^B(j)\, z^{\kappa(j-j_1)}$$

To find the MTF for improper focusing one utilizes the fact that the appropriate MTF depends only on $|\vec{k}|$ and replaces $\kappa$ by $|\vec{k}|/|\vec{j}| = |\vec{k}|/k_1$;

(7c) $$P^{(D)}_{\vec{k}} = \sum_j l^B(j)\, z^{|\vec{k}|(j-j_1)/k_1}$$

To find the MTF for linear motion blurring one can utilize the fact that the appropriate MTF to first approximation varies only in the $\vec{m}_B$ direction. Projection arguments easily then give

(7d) $$P^{(L)}_{\vec{k}} = \sum_j l^B(j)\, z^{\vec{k}\cdot\vec{m}_B(j-j_1)/(\vec{j}\cdot\vec{m}_B\, k_1^2)}$$

This can also be derived by writing the psf as

$$P_{\vec{n}} = \sum_q l_q\, \delta_{\vec{n},\vec{m}_q}$$

then with $l_{\vec{n}}^B = \delta_{\vec{j}\cdot\vec{n},\, j_1}$ and

$$l^B(j) = \sum_q l_q\, \delta_{j_q,\, j-j_1} ,$$

$$P^{(L)}_{\vec{k}} = \sum_q l_q\, z^{\vec{k}\cdot\vec{m}_q}$$

Using $\hat{m}_q = \hat{m}_B$, $k_1^2\, \vec{k}\cdot\vec{m}_q \propto (j-j_1)\, \vec{k}\cdot\vec{j}/\vec{m}_B\cdot\vec{j}$ one obtains the expression for $P^{(L)}$.

It should be realized that nothing can be learned about the MTF if the motion is parallel to the edge or line. This manifests itself in the $\vec{m}_B\cdot\vec{j}$ factor in the denominator of the exponent.

IV Restoration and Noise

All digitized images are subject to noise. The noise originates from the granular nature of the original photograph, from electronic noise and round-off error in the digitization process etc. Such noise can be both signal dependent and signal independent, spatially correlated and not, it can occur as "sparkle" or spikes in the F.T. of the image. Whatever the source or nature of noise, it complicates the deblurring or deconvolution process. This is especially true in the cases under consideration, improper focusing and motion-blurring where the MTF alternates in sign. Special care must be taken to avoid emphasis of noise in regions where the MTF changes sign, i.e. along the zero-contours.

The standard approach to noise abatement is to use the MTF to construct a Wiener filter. Unfortunately such construction requires information on the noise that is not available. Also as pointed out by Castleman (2) such filters cannot handle image having large flat areas separated by sharp edges which is true of photographs of the shuttle. Consequently, a two-pronged approach to noise suppression was developed. The first involves using a two parameter high frequency component of the form

$$S^{-1} = 1 + \exp \alpha (f - f_c)$$

where $f = |k| = \sqrt{k^2 + l^2}$, $f_c$ is a variable cut-off frequency and $\alpha^{-1} = -(N/2 - f_c)/4$. The function S thus is vanishingly small at low frequencies, ½ at $f = f_c$ and approaches unity at high frequencies.

The preliminary filter, $H_p$ is then defined as

$$H_p^{-1} = P(1-S) + \gamma S$$

where $\gamma$ is the second variable parameter. Since the MTF is normalized to unity at zero frequency, $H^{-1}_p$, starts at unity follows the MTF until $f = f_c$ where it then follows $\gamma S$ and approaches $\gamma S$ and approaches $\gamma$ at high frequencies. The parameter $\gamma$ allows one to emphasize or de-emphasize the high frequency region relative to the low frequency region.

This filter will clearly handle sparkle etc. noise at high frequencies, but it does not remove large values of $H_p$ due to the vanishing of P at small and moderate frequencies. This is accomplished by introducing a third parameter, R, the maximum to minimum allowed values for the magnitude of H. Thus at each frequency point $|H_p|$ is tested and replaced by $R\, H_p/|H_p|$ if it exceeds R. This allows one to tune out or at least avoid over emphasis of noise near the zero contours of P.

## V. Results

The main thrust of the research effort this summer was the designing and implementation of software to extract P from a blurred image. As a preliminary test of this software, it was applied to a Gaussian (5 x 5 pixel)-blurred image. Last summer's work showed that the 5 x 5 pixel windows on the Gaussian was of consequence only at high frequencies. Thus the assumption that the psf was azimuthally symmetric would be reasonable since most of the signal is in the low frequence range. The resulting restored image was not quite as good as that obtained last summer where the 5 x 5 window was taken into account, however the exercise clearly demonstrated that even in the computer generated blurring information needed for constructing the MTF could be extracted from the blurred image.

V.1. Deconvolution of An Improperly Focused Image

Last summer an image was purposely blurred by defocusing the digitizing camera. Although there is no evidence in Figure 1a, the top 1/16 of the image is missing as a result of the digitization process. Such spatial-domaion truncation causes over shooting or ringing in subsequently filtered images (p.26 ref.1). Application of the newly developed software resulted in the MTF shown i Figure 1b and gave a somewhat better restored image than that obtained last year. Last year the psf was approximated by a disc of constant amplitude (4 pixels in radius) and surrounded by a fuzzy annulus 2 pixels in width (see Fig. 2a). As shown in Figure 2b, the psf derived from an edge in the blurred image (see Fig. 1a) is similar in size but has an intensity minimum in its center. Shown in Figure 3a,b are the Fourier transforms of the blurred and the restored image.

As observed last summer, the Fourier transforms are much more sensitive to the restoration process than the spatial images. It is suggestive that fine tuning on the restoration process is much easier to observe on the Fourier image than on the image itself. The expected ringing is again present in the spatial image. The rectangular form of the Fourier transformed image is apparently due to the digitization process since it occurs in all images digitized with its video digitization system.

V.2 Restoration of Two Images Blurred by Relative Motion

Two images were blurred by moving them during the digitization process. The first was quite severely blurred in that the motion extended over about thirty pixels. The second was more moderate in that the blurring was over about a dozen pixels. In both cases the same edge circled in Fig. 5a was used in extracting the psf from the blurred image. Shown in Figure 3b,c, and 4b,c are the Fourier transforms of the blurred and the restored images. The original sharp image is essentially that shown in Figure 1a. In both cases the "zero contours" are evident in the F.T. of the blurred image (see Fig. 3b and 3c). These are easy to recognize with the eye. The number of pixels to the first divided into N gives a good estimate of the biurring distance in the spacial domain. Similarly the Fourier transformed image can give a better determination of the direction of blurring than can be obtained from the spatial image. One simply uses PROFIL to define a line parallel to a maximum contour passing through k=0 and finds its slope parameters. $\Delta k$, $\Delta l$, then $m_B:n_B = \Delta l:\Delta k$, since the direction of blurring is perpendicular to extreme contours of P. This was used in Eq. 7d for determining P.

Clearly the software can be used to restore images blurred by improper focus or by linear motion blurring if the blurred image contains a straight line or edge.

VI Future Research

In last summer's report in addition to the program to construct MTF's directly from blurred images there were two other areas of potential research. The first of these involved the development of filters to reduce noise. Although some work has bene done along this line (see Section IV), many of the suggestions remain relevant. The use of windowing or image completion is important for incomplete images digitized by the Sony TV camera. However, these problems may be circumventted by using the Eikonix system.

The main thrust of future work should be the development of a user-friendly-interactive-restorative system. The essential programs are now in place, it is simply a matter of working them into a "cookbook" that can guide the inexperienced user to construct for any blurred image of interests the appropriate MTF and restoration filter. Such a program would use a decision-true format shown in Fig. 8. Photographs of the Space Shuttle Vehicle during launch and landing would contain straight edges and would be deblurred using an MTF extracted directly from the blurred images. Photographs taken by the Space Shuttle crews of agricultural, desert and ocean features would require parameterized MTF's since they do not in general contain straight edges or lines. In such cases one would extract information needed to determine the parameters from their Fourier transforms. One would use a roster of the zeroes of the appropriate MTF, i.e. Airy or sinc function. WARP would then be used to rotate the sinc roster, and it stretch the roster until it overlaped the "zero" contours of $F^B$. This would generate the appropriate parameters, e.g. the radius of the Airy disc or the length and direction of the motion blurring.

The next step is to select the high frequency portion of the restoration filter. As presently envisioned this would have two parameters one a frequency, $f_c$, value which would be highest frequency where $F^B$ is free of "sparkle" noise, e.g. N/6 or N/4 depending on the degree of blurring. The second parameter $\gamma$ which determines the relative emphasis of the high frequency region with respect to the very low frequency region, could be set equal to one or varied slightly to produce a restored $F_k$ where large amplitudes are mostly confined to the central region.

The final step would be to select the maximum to minimum amplitude ratio, R, in the restoration filter. This would be done by taking a value like 10 and seeing if the "zero" contours persist or have been replaced by contours of large amplitudes in the restored $F_k$. One would then raise or lower R until the amplutudes in the regions of the contours are about the same size as their neighbors.

After a restored $F_k$ is obtained which is similar to ones of less blurred images one would then take its inverse Fourier transform and obtain the restored image. Other features such as "sparkle" removal could be added later if they show promise of further improving the process. This should be a user-friendly-interactive program which guides the use along until a reasonable restored image is obtained.

VII References


1. Moik, J. G., Digital Processing of Remotely Sensed Images, NASA SP-431, 1980.

2. Castleman, K. R., Digital Image Processing, ISBN 0-13-21365-7 Prentice-Hall Inc. 1979.

3. Gonzelez, Wintz, Digital Image Processing.

4. Press, W. H., B. P. Flannery, S. A. Teukolsky, W. T. Vetterling, Numerical Recipes, ISB 0-521-30811-9, Cambridge Univ. Press, 1986.

5. Reynolds, G. O., J. B. DeVelis, G. B. Parrent Jr., B. J. Thompson, Physical Optics Notebook: Tutorial in Fourier Optics, ISBN 0-8194-0130-7, 1989.

6. Menke, W., Geophysical Data Analysis: Discrete Inverse Theory", ISBN 0-12-490921-3, AP, 1989.

7. Ward, R. K., B.E.A. Saleh, Restoration of images Distorted by System of Random Impulse Resonances, J. Opt. Soc. Am., A2,pp 1254-1259, 1985.

8. Tescher, A. G., H. C. Andrews, Data Compression and Enhancement of Sample Images, Applied Opts 11, #4, 919-925, 1972.

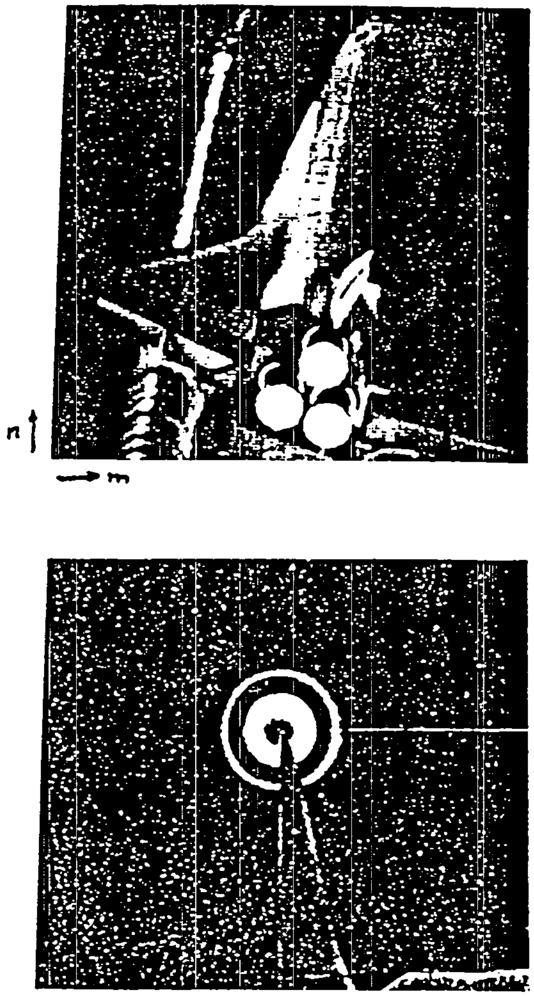


Figure 1a, b

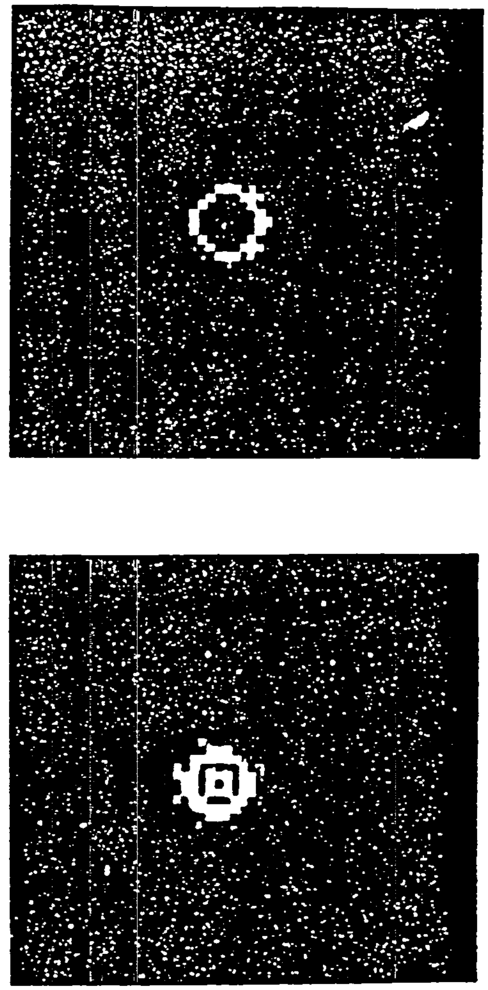

Figure 2a, b

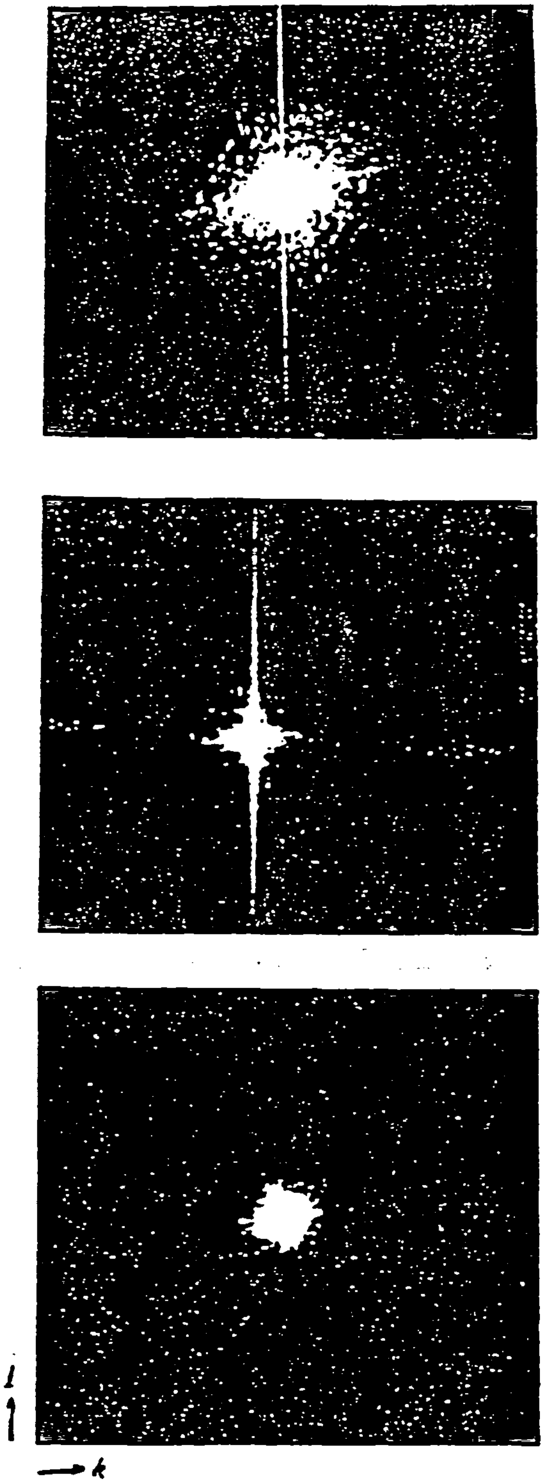


Figure 3a, b, c

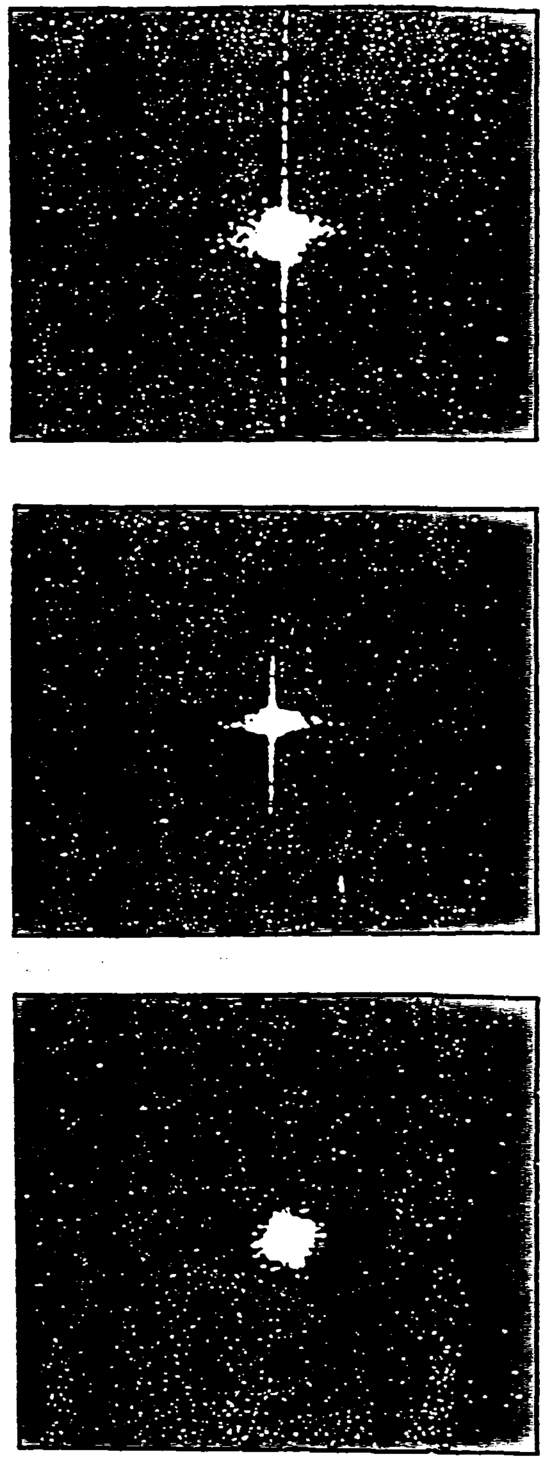

Figure 4a, b, c

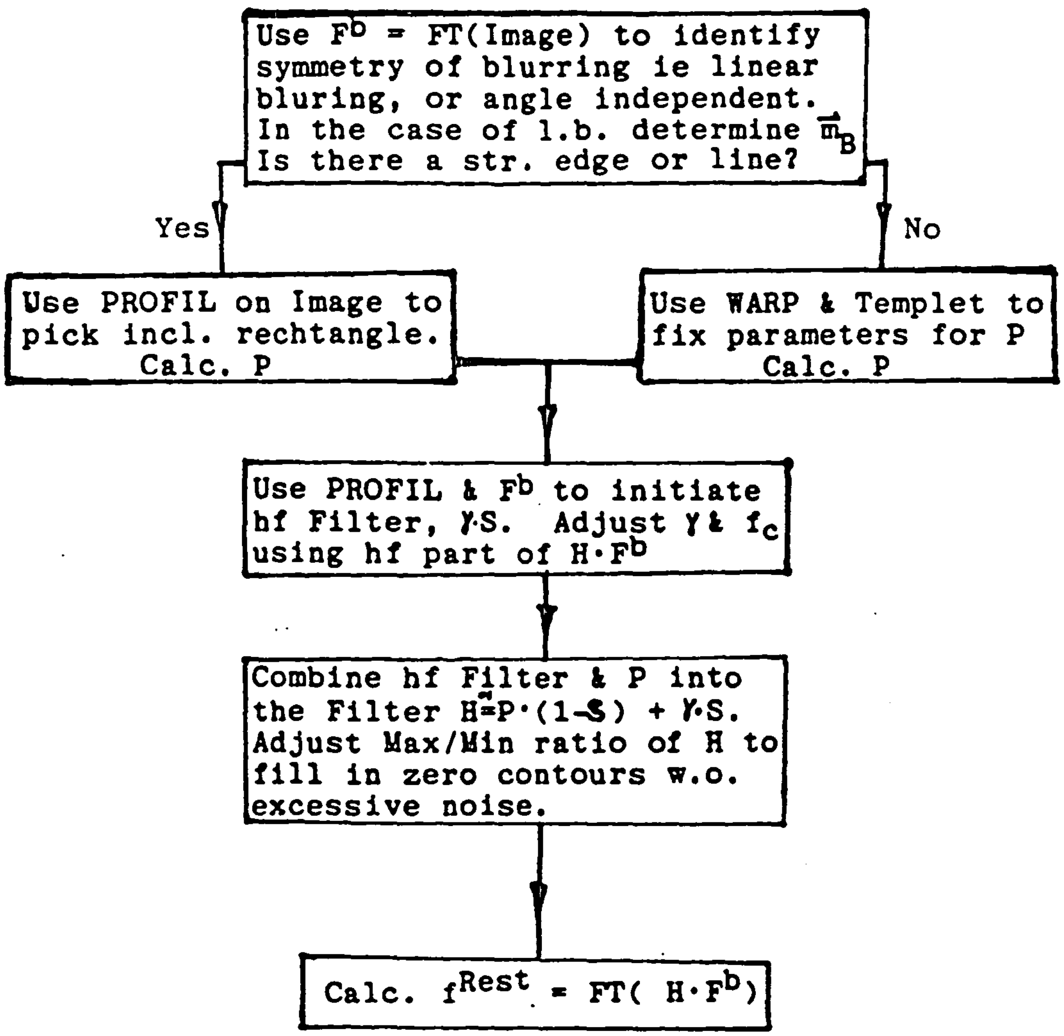
Use Fb = FT(Image) to identify symmetry of blurring ie linear bluring, or angle independent. In the case of l.b. determine mB Is there a str. edge or line?
Yes
No
Use PROFIL on Image to pick incl. rechtangle. Calc. P
Use WARP & Templet to fix parameters for P Calc. P
Use PROFIL & Fb to initiate hf Filter, γ·S. Adjust γ & fc using hf part of H·Fb
Combine hf Filter & P into the Filter H=P·(1-S) + γ·S. Adjust Max/Min ratio of H to fill in zero contours w.o. excessive noise.
Calc. fRest = FT( H·Fb)

Figure 5