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**JOHN F. KENNEDY SPACE CENTER  
UNIVERSITY OF CENTRAL FLORIDA**

**GENERIC TECHNIQUES FOR THE CALIBRATION OF ROBOTS WITH APPLICATION  
OF THE 3-D FIXTURES AND STATISTICAL TECHNIQUE ON THE  
PUMA 500 AND ARID ROBOTS**

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## ABSTRACT

The main objective of this project is two fold: first, to provide the reader with a brief literature survey of the advanced robotics calibration techniques and indicate the advantages, disadvantages, and the suitability of each technique to KSC/RADL's current and future robotics applications. Secondly, to introduce RADL to a relatively simple, inexpensive, and generic technique that could be used in both laboratories and some operation site environments.

In addition, this report gives detailed explanation of the set up procedure, data collection and analysis using this new technique that was developed at the State University of New York at Farmingdale. The technique was used to evaluate the repeatability, accuracy, and overshoot of Unimate Industrial Robot, PUMA 500. The obtained data was statistically analyzed to provide an insight into the performance of the various robotics systems and components. Also, the same technique was used to check the forward kinematics against the inverse kinematics of RADL's PUMA robot.

Recommendations were made for RADL to use this technique for laboratory calibration of the currently existing robots such as the ASEA, high speed controller, Automated Radiator Inspection Device (ARID) etc. Also, recommendations were made to develop and establish other calibration techniques that will be more suitable for site calibration environment and robot certification.

## SUMMARY

Industrial robots are known to facilitate and expedite many maintenance processes, reduce tedious manual tasks, enhance quality and improve accuracy of various engineering applications. Accordingly, the Robotics Applications and Development Laboratory (RADL) at NASA/KSC is currently developing numerous robotics systems to support Shuttle launch operations and preparations.

However, due to various sources of error in the design, manufacturing, and assembly of robotics systems, they experience a lack of accuracy in the positioning of their end effectors. Generally, industrial robots exhibit better repeatability than accuracy. In many cases, current industrial robots show excellent positioning repeatability but relatively poor absolute positioning accuracy. For on-line programming applications where positions are taught and encoder readings are recorded and replayed this will not present a serious accuracy problem as long as the robot control system is capable of repositioning the tool center point within acceptable range. On the other hand, in cases of off-line applications where robots are commanded to locate their end effectors at certain points within the work envelope, robot calibration would play an increasingly important role because most sources of errors will cumulatively influence the robot's accuracy. Therefore, a calibration procedure that is classified as both time-efficient and cost-effective is deemed necessary for RADL/KSC.

Accordingly, this report surveys the available techniques in the literature and examines its suitability for the current and future robotics applications and certification at Kennedy Space Center for both laboratory and on site calibrations. Two systems were recommended for this purpose: the first, is a modified theodolite (Optical telescope) equipped with a laser or ultrasonic generation device and the second is a vision system with a charge couple device (CCD) camera and calibration target. Although these systems provide excellent flexibility and accuracy in an actual robotics operational environment, they are relatively costly.

A simple, inexpensive and straight forward calibration technique that was developed at the State University of New York - Farmingdale is used to establish calibration procedure for RADL. The technique was used to evaluate the accuracy, repeatability, and overshoot of a PUMA 500 robot. Moreover, the report provides step by step statistical analysis that could lead to the source of a problem in a robotics system and help its troubleshooting scheme. RADL can follow the exact similar calibration procedure to calibrate and/or troubleshoot other robots such as ASEA, High speed controller, and the ARID.

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## LIST OF ABBREVIATIONS AND ACRONYMS

RADEL - Robotics Applications And Development Laboratory

GTE - Gear Transmission Error

OPF - Orbit Process Facility

KSC - Kennedy Space Center

ARID - Automatic Radiator Inspection Device

TRE - Total Repeatability Error

USL - Upper Spec Limit

LSL - Lower Spec Limit

UCL - Upper Control Limit

LCL - Lower Control Limit

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# I INTRODUCTION

## 1.1 OVERVIEW OF A ROBOT'S PRECISION

Most of the robotics applications require a considerable degree of positioning accuracy to be maintained by the manipulator's end effectors for a successful continuous operation of the robot. However, the precision and performance of most industrial robots were jeopardized due to inevitable design, manufacturing, environment, and operation cumulative sources of errors that are introduced to the robot's operation. Accordingly, many researchers have addressed the issue of robotic calibration that is mainly conducted using different devices and techniques to close the loop between the robot's tool center point (TCP) and its base coordinate origin point and accurately evaluate this vector. The pose of the end effector with respect to the robot base coordinate system is described in six parameters (three for location and three for orientation).

In general, robotic calibration could be summarized in four steps: measurement data, analysis, identification of key sources of errors, and elimination or compensation for these errors.

## 1.2 SOURCES OF ERRORS IN ROBOTICS SYSTEMS

### (A) Design:

Encoder Resolution, Control System, Flexibility of links and joints

### (B) Manufacturing:

Tolerances, Electronic and Mechanical Zeros not Coinciding

### (C) Environmental:

Temperature Variation

### (D) Operation:

Target Shift (Frame Shift)

In many cases, the sources of error are classified as geometric; affecting the dynamic parameters such as manufacturing tolerances on joints and links etc. and non-geometric; such as, the flexibility of joints and links, gear transmission error etc.



### 1.3 LITERATURE SURVEY

Robot calibration is a process that mainly aims at the improvement of a robot's accuracy by modifying the robot positioning software rather than changing or altering the design of the robot or its control system. Calibration assumes that a nominal relationship between the end-effector position and joint transducer readings is known but that this relationship is not necessarily accurate due to the previously indicated sources of errors.

Calibration procedures vary widely in their complexity and approach. For example, some robot calibration procedures only consider the joint transducer information while others may involve changes in the kinematic and/or dynamic model of the robot [1]. Accordingly, Roth et al. [1] classified calibration in three levels:

Level 1: To ensure that the reading from a joint sensor yields the correct joint displacement

This procedure is usually done as a part of the construction of the robot, and the user may perform such calibration if damage has occurred or if the joint has been disassembled for maintenance. At this level a 3D fixture could be used as indicated later in this report. Also, a different approach [2] involves the use of a polygonal mirror and a theodolite to determine the joints angles accurately Figure (1-1). In this method the operator looks through the theodolite and the joint is rotated until the operator sees his own reflection centered in the theodolite. When this occurs, the mirror surface is almost perfectly normal to the axis of the theodolite Figure (1-2). The particular measurement technique chosen at this level is a trade-off between precision and cost.

In robot calibration the essential issue is not the method but rather the ability to obtain a large number of measurements over a wide area of robot locations [2]. For this technique to be valid it is important to know the location of the theodolite in robot base coordinates which could be laborious and time consuming for the ARID application if not permanently fixed to its base.

Level 2: To improve the accuracy of the kinematic model of the manipulator

A number of different approaches have based on the most popular procedure that was established by Denavit and Hartenberg [3] for developing the kinematic model of a robot manipulator. For example Haung et al [4] of the Florida Atlantic University used the idea of a closed-loop mechanism to measure the actual positions of the manipulator end effector. The measuring device consisted of a passive instrumented mechanical linkage, with its one end fixed to the ground whereas the other end connected to the manipulator end effector through an extension bar

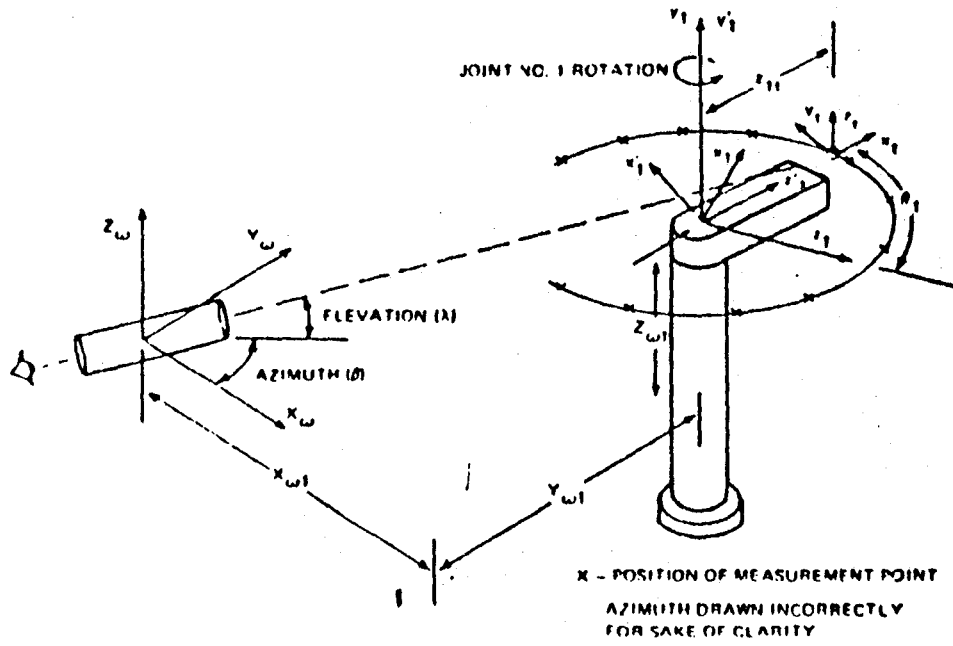


Fig.(1-1) One-degree-of-freedom calibration

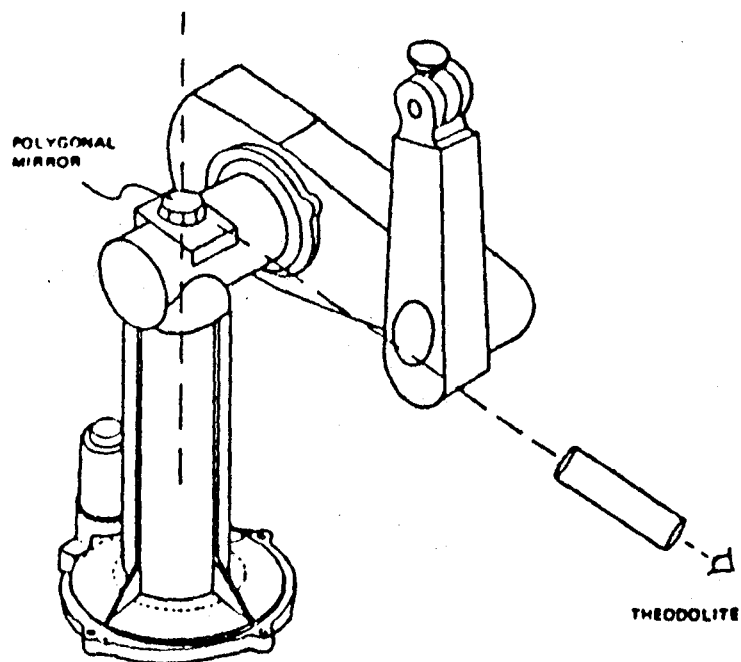


Fig.(1-2) Measuring the gear transmission error in joint 1

Fig(1-3 ). The ground end of the sensing linkage consists of a universal joint instrumented with position feedback devices and a linear scale type axis attached to it. Although this linkage seems inaccurate to be used as a calibration reference, also it is limited to the identification of the manipulator kinematic parameters caused by the geometric errors inherent to the arm. This method assumes rigid joints and links and does not account for their flexibility or the arm non-geometric inherent errors such as links deflection, joint flexibility, gear transmission error (GTE), clearance, backlash, etc. Also, this technique will not be suitable for application in the Orbit Process Facility (OPF) at KSC or with large and heavy arm such as the ARID.

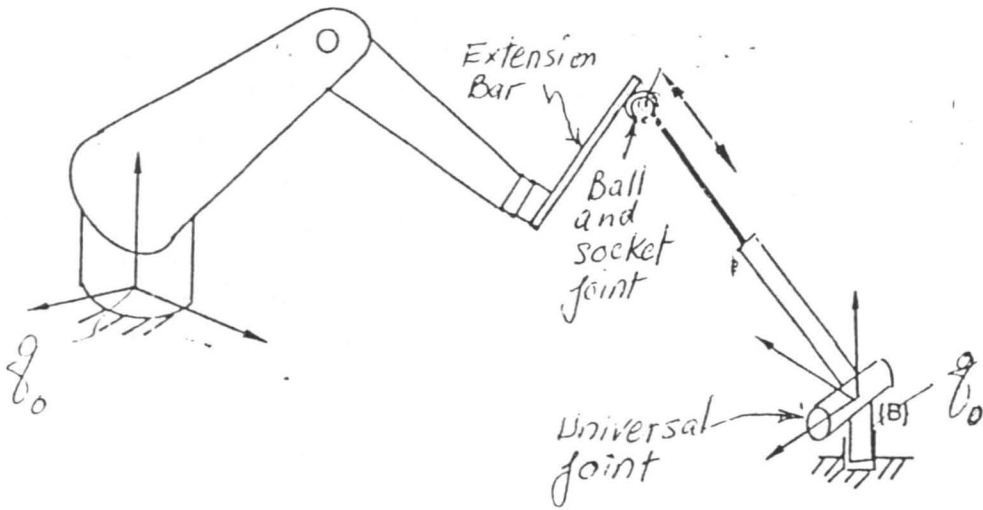
### Level 3: "Non-kinematic" (non-geometric) calibration

Non-kinematic errors in positioning of the end effector of a robot are due to effects, such as joint and link compliance, friction, and clearance. Also, if the robot is under dynamic (rather than kinematic) control, then correction for changes in the dynamic model of the robot constitutes a level 3 calibration.

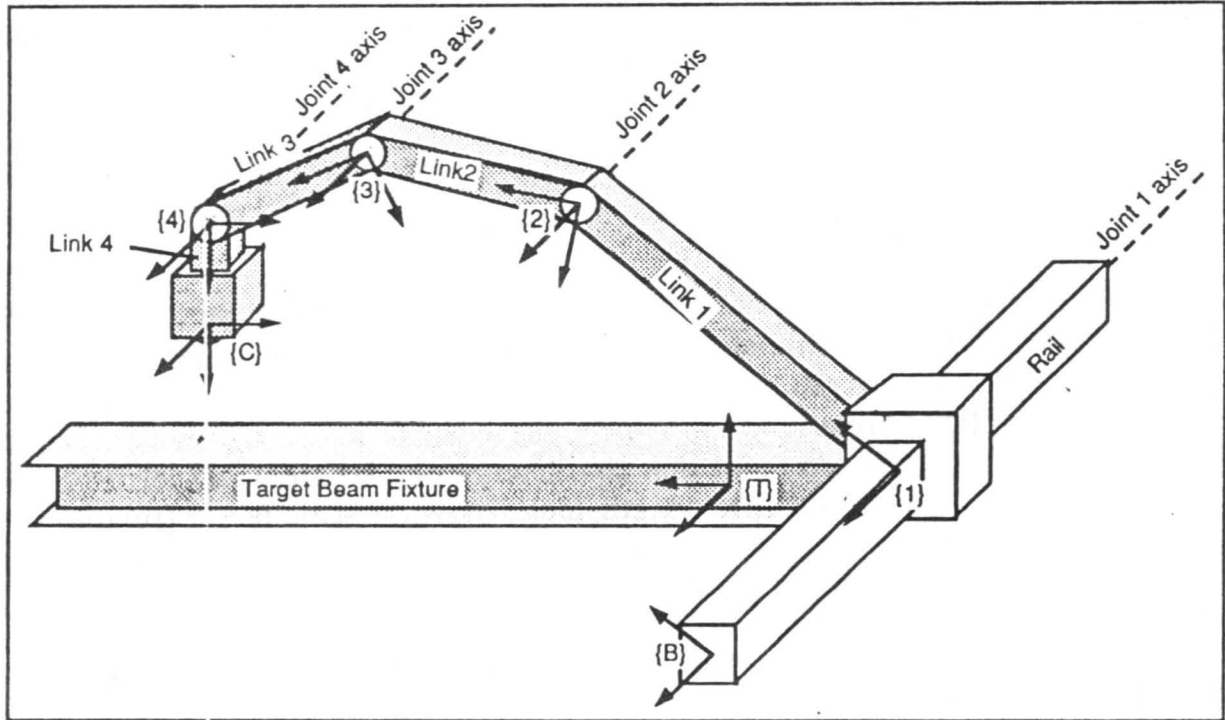
A vision system technique Figure (1-4) is a promising level 3 robotic calibration technique but is not completely developed yet and its accuracy is still questionable particularly in the z-axis perpendicular to the camera lens. However, with a relatively small lens distortion and rather accurate calibration and evaluation of the camera intrinsic and extrinsic parameters, much better accuracy could be achieved and the vision system could be used to its full potential. It is a trade off between precision and cost. The calibration of the vision system consists of camera (eye) calibration, eye-to-hand calibration, and hand calibration see Fig(1-5). The calibration of such system is briefly outlined in the following [5]:

- Camera (eye) calibration

Camera calibration is the problem of determining the elements that govern the relationship or transformation between the 2D image that the camera perceives and the 3D information of the object. There are two kinds of parameters that define this 2D/3D relationship, namely, the intrinsic and extrinsic parameters. The intrinsic parameters are those that characterize the inherent properties of the camera and optics, such as the focal length, lens distortion, scale factor (relationship of the computer image coordinate to the true image coordinate), and image center (point where the optical axis intersects with the image plane). The extrinsic parameters are those defining the position and orientation of the camera with respect to the world coordinate system. There are six of them: three for rotation, and three for translation.



Figure(1-3) System configuration of the passive instrumented Mech. Linkage



Figure(1-4) Naming conventions and coordinate frames for the ARID robot.

- Robot Eye-to-Hand Calibration

3D robotics hand/eye calibration is the task of computing the relative 3D position and orientation between the camera and the robot gripper in an eye-on-hand configuration, meaning that the camera is rigidly connected to the robot gripper. The camera is either grasped by the gripper, or just fastened to it. More specifically, this is the task of computing the relative rotation and translation (homogeneous transformation) between two coordinate frames, one centered at the camera lens center, and the other at the robot gripper. The gripper co-ordinate frame is centered on the last link of the robot manipulator. All 3D measurements made using TV cameras refer to measurement relative to the camera. In order to relate the 3D measurement information to the robot world frame, it is necessary to know the transformation between the robot hand and eye. This is crucial for any robotic vision system.

Whitney et al [2] also generated a model for joint compliance and link bending, the apparatus is shown in Fig.(1-6). Forces were applied to the robot arm first at  $F_1$  and  $F_2$ . Deflections were measured at  $X_i$  ( $i=1,7$ ). The resulting data fit a model of rigid links connected by compliant joints that act as torsional springs. The stiffness at the joints could be evaluated and used to calculate the deflection under any payload. In addition, backlash was evaluated for a horizontal arm where gravity loading forces the backlash to one extreme. A vertical force was applied at  $F_2$ . Displacement is measured at  $X_4$ . When the applied torque is less than the torque due to the arm's weight, the arm's weight keeps the backlash loaded. The arm acts as rigid links on torsional springs. As the applied force increases, the arm is lifted. The arm passes through the backlash dead zone, and is forced against the opposite extreme. Again the arm acts like a linear spring. The backlash dead zone shows up in the force deflection curves drawn in Figure(1-7).

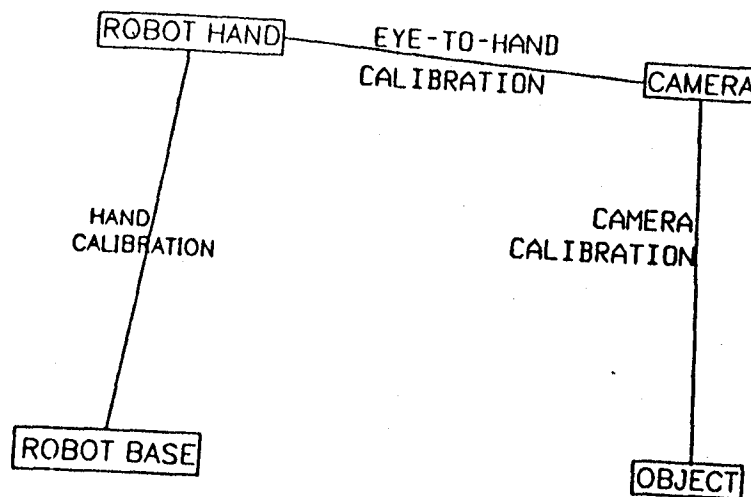
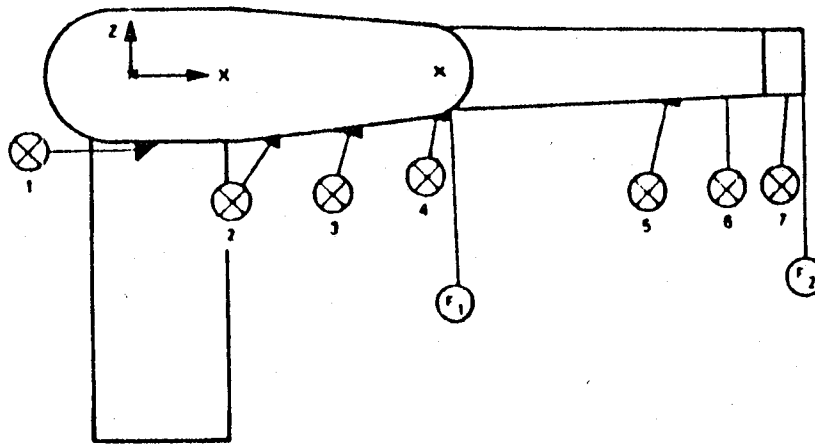


Figure (1-5) To obtain the 3D position and orientation of an object relative to the robot world base, it is necessary to do three calibrations, namely robot hand, eye-to-hand and eye (camera) calibration



F REPRESENTS FORCE MEASUREMENT  
 ⊗ REPRESENTS POSITION MEASUREMENTS

Fig. (1-6) Measurement points for force deflection curves

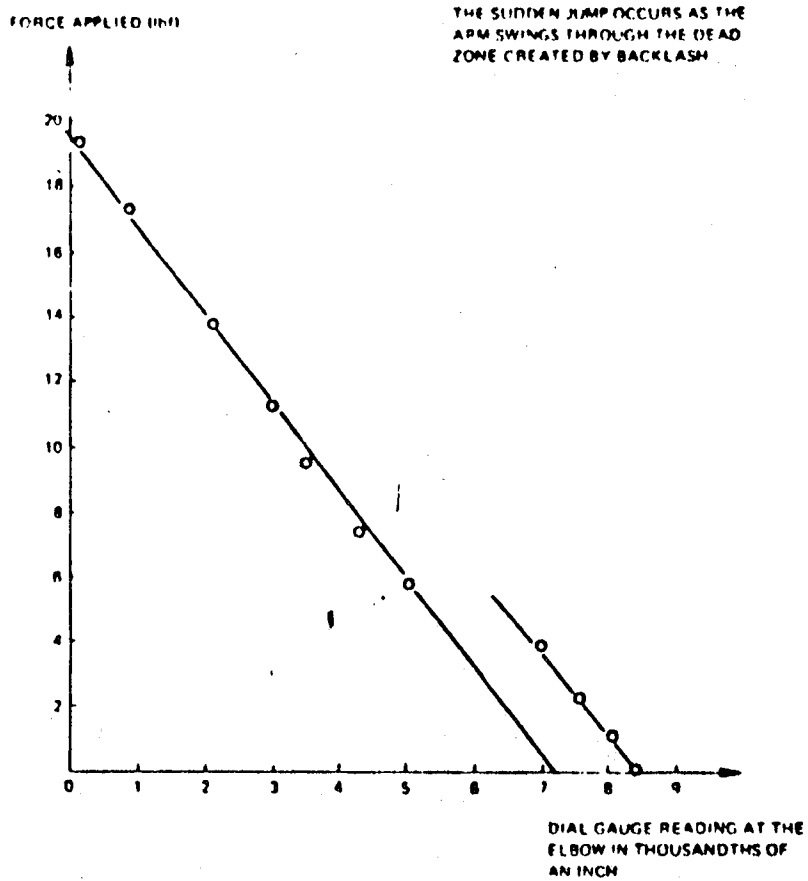


Fig. (1-7) Backlash in Joint 2

## II MEASUREMENT OF ACCURACY AND REPEATABILITY OF UNIMATE INDUSTRIAL ROBOT PUMA 500

### 2.1 OBJECTIVES

The objectives of this task are first: to develop a simple, inexpensive, and straightforward robotic calibration technique to be used for the certification of the high speed controller and the calibration of the ARID. Second: to demonstrate to NASA/KSC robotics group the use of the 3D fixtures and the method of evaluation of the repeatability, overshoot, relative and absolute accuracy of any robot. Third: To provide statistical techniques for the evaluation of these parameters, reduction of noise in the processed data, and the analysis of measured data to examine the performance of a robot. This statistical technique could also be helpful in the troubleshooting of various sources of errors. In addition this technique will have direct application in the certification of the High Speed Controller as well as the calibration procedure of the ARID robot.

### 2.2 EVALUATION OF THE PUMA 500 REPEATABILITY:

#### 2.2.1 SET UP AND PROCEDURE:

The 3D fixture Figure (2-1) fitted with dial indicators in the X,Y, and Z direction was used in the evaluation of the PUMA's repeatability. The fixture was leveled, clamped to the calibration table, and positioned within the work envelope of the PUMA robot. A square cross-section calibration bar 1.5"X1.5"X6" Figure (2-1A) was fitted to the adaptor plate at the end effector of the arm. Each face of the calibration bar is aligned with an axis of the fixture. The alignment was checked by moving the bar in the direction of an axis and no variations were observed in the other two axes. However, if the orientation was of concern to the researcher, a fixture shown in Figure (2-2) will measure the position and orientation of the calibration bar and give necessary information of the end effector pose.

The fixture in Figure (2-1) was used and the robot arm was moved using the teach pendant to bring the calibration bar between the indicators and depress each one 0.250" approximately. At this setting the indicators were zeroed off and this position was "taught" to the robot i.e. all the encoders readings at this position were saved in the computer memory and remembered by the robot as point "A" Figure (2-1). The robot arm and the calibration bar were moved to various locations within the work envelope and the zone of operation, such as points "B","C","D" etc. Move the TCP back to the previously taught point "A". Make sure that the indicator spindles are detracted so they will not be damaged by the returning calibration bar to point "A". Gently return the indicators to touch the calibration bar and record the three

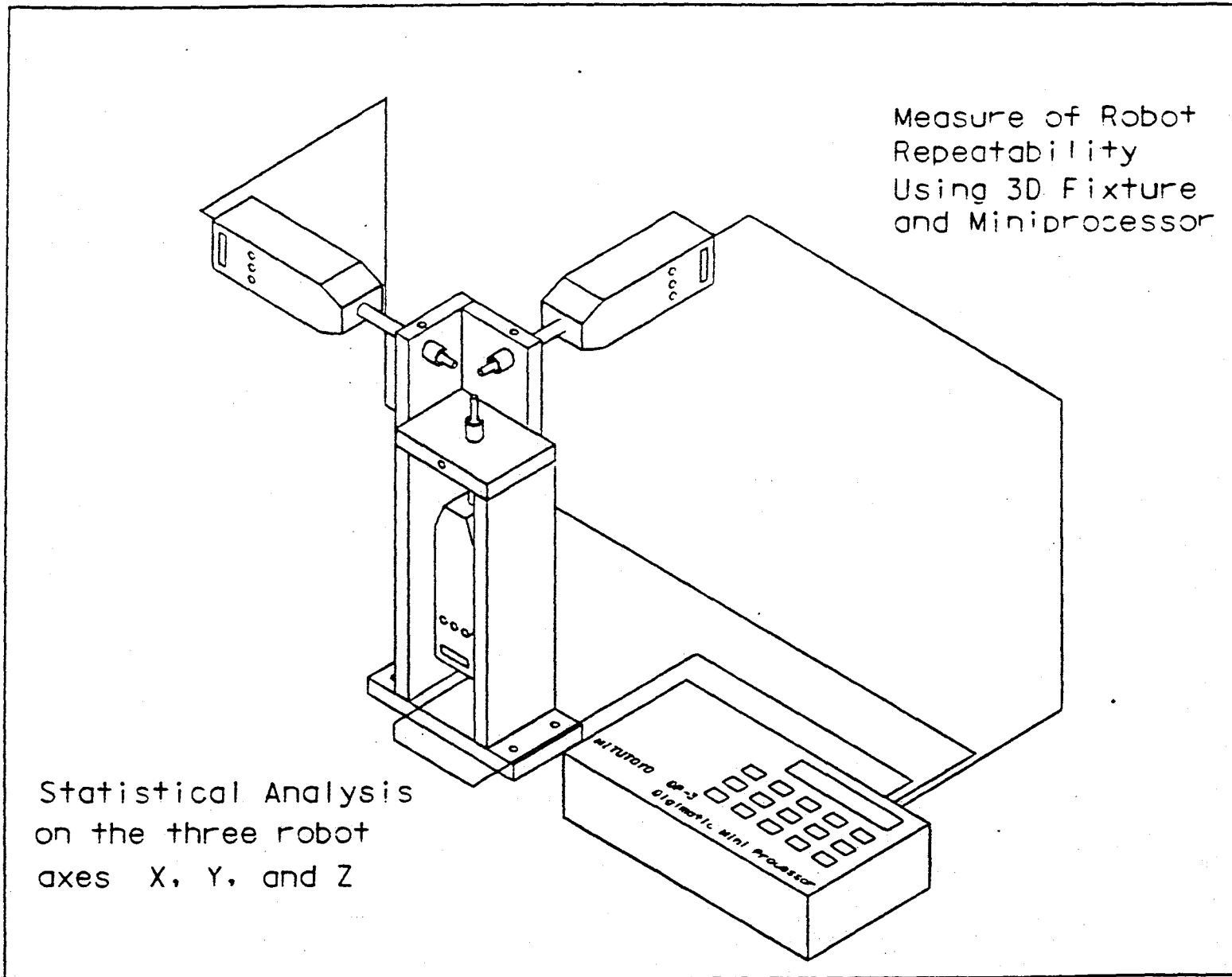


Figure (2-1) 3D Fixture For measuring repeatability



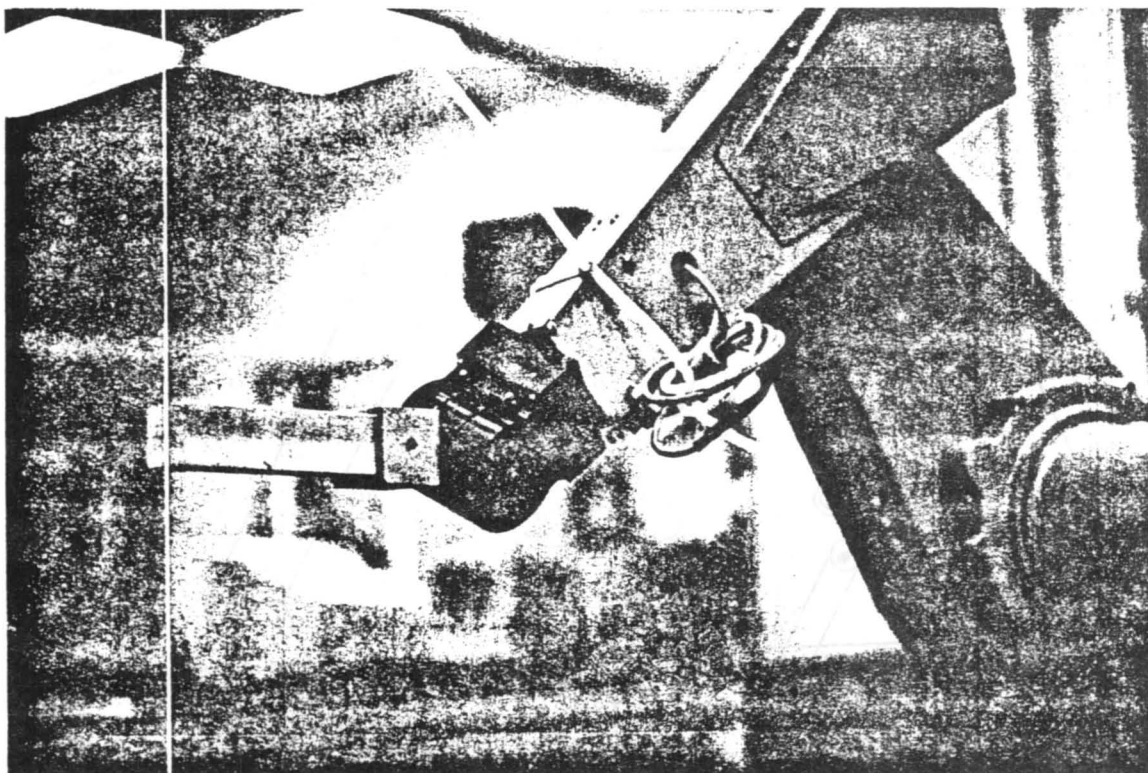
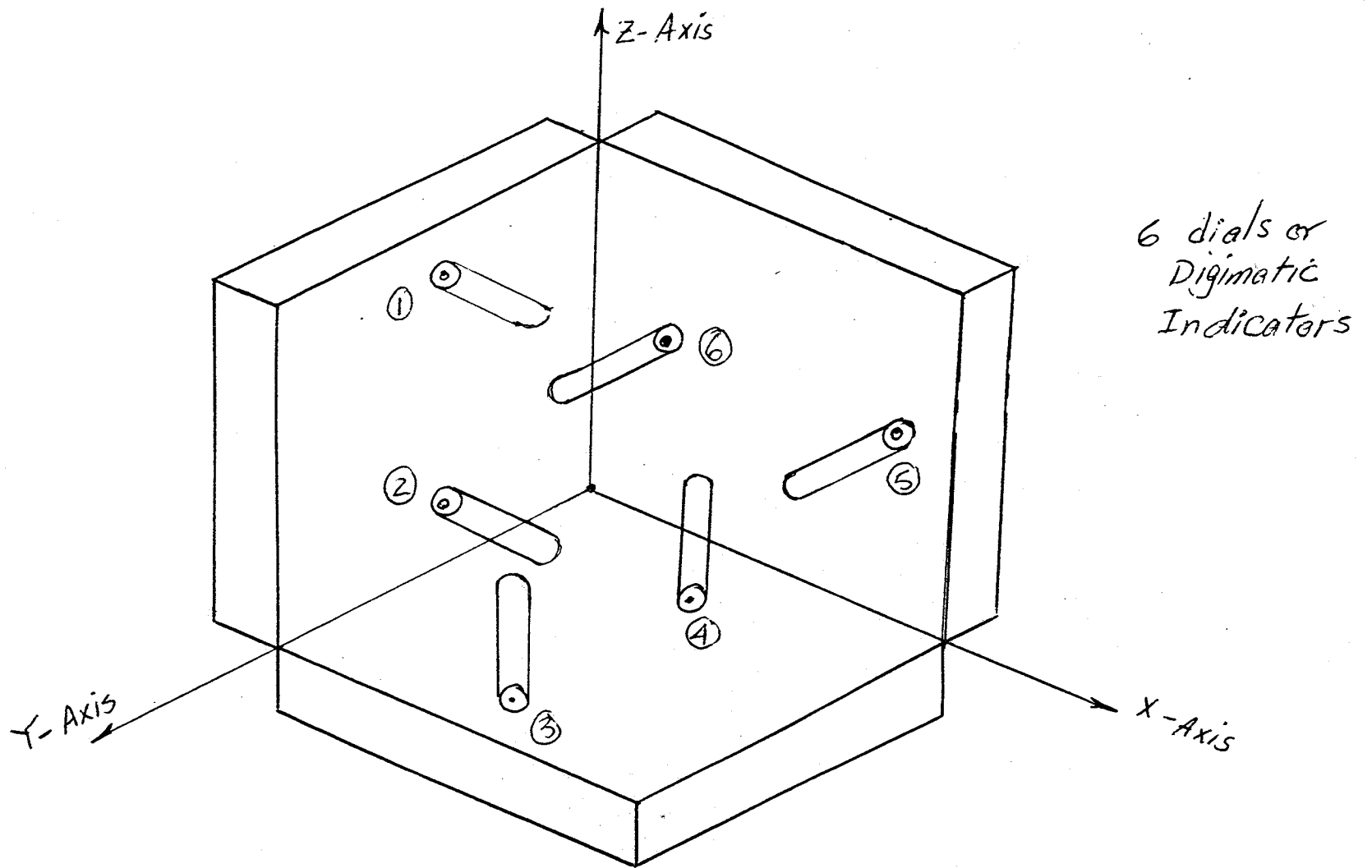


Figure (2-1A) Calibration Bar (1.5" x 1.5" x 6")



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Figure (2-2) 3-D Fixture Provides measurements of the position and orientation errors

components X, Y, and Z of the repeatability error vector for this run. Repeat the same procedure for ten to fifteen times or more. In the current work twelve runs were conducted for the evaluation of the PUMA repeatability and the results were recorded in table (1).

Table (1) X, Y, and Z Components of the Repeatability Error Measurement

RUN No.	X - Axis inch	Y - Axis inch	Z - Axis inch
1	-0.003	+0.001	-0.008
2	+0.001	+0.001	-0.002
3	+0.003	+0.000	-0.003
4	+0.003	-0.003	+0.002
5	+0.004	-0.002	+0.002
6	+0.002	+0.000	+0.000
7	+0.003	+0.000	+0.000
8	+0.003	-0.001	+0.000
9	-0.003	+0.001	+0.001
10	-0.001	-0.001	+0.000
11	-0.001	+0.000	-0.001
12	+0.004	-0.002	-0.003

## 2.2.2 RESULTS AND STATISTICAL ANALYSES:

The plots of the repeatability errors for each axis are shown in Figures (2-3), (2-4), and (2-5). The Mean and the Standard Deviation of each axis positional errors were calculated and the corresponding Histograms were constructed as shown in Figures (2-6), (2-7), and (2-8). In the present analysis it was assumed that the PUMA has to meet positioning requirement of  $\pm 0.010$  inch as a tolerance limits.

The study of these figures will give clear indication of the performance and capability of the robot that was tested. When the points in Figures (2-3), (2-4), and (2-5) appear very close to the control limits or beyond, this will be sufficient sign that calls for the immediate maintenance of the robot.

$$\text{The mean positional error} = \frac{1}{N} \sum_{i=1}^N d_i$$

$$\text{The standard deviation } (\sigma) = \sqrt{\frac{\sum_{i=1}^N (d_i - \text{mean})^2}{N-1}}$$

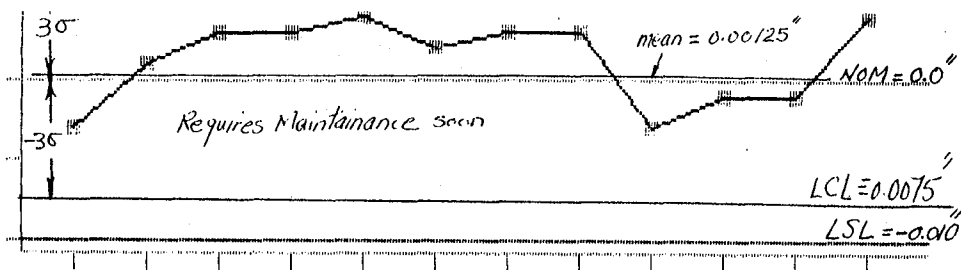
Accordingly, the components average of the positioning error vector for the repeatability of the PUMA could be obtained from the following table:

Table (2) Average Values of The Positioning Error Vector Components

X - Axis	Y - Axis	Z - Axis
0.0012"	-0.0005"	0.001"

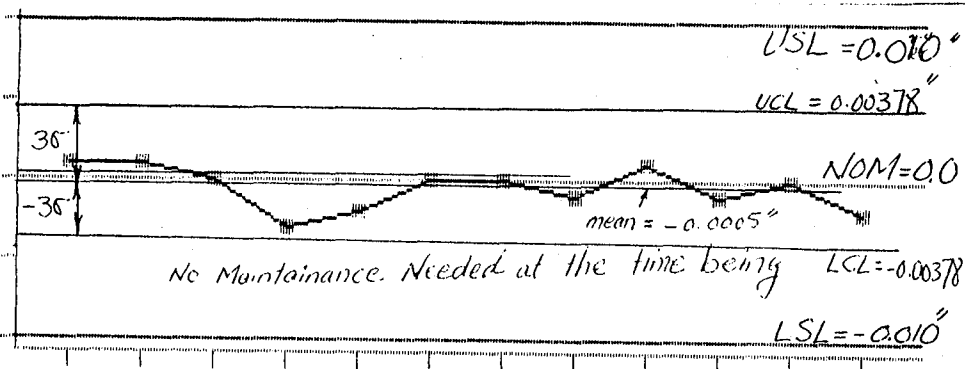
The positioning error vector for the repeatability of the PUMA 500 in the RADL at KSC now reads the following:

$$.0012 \text{ i } + -0.0005 \text{ j } + 0.001 \text{ k}$$



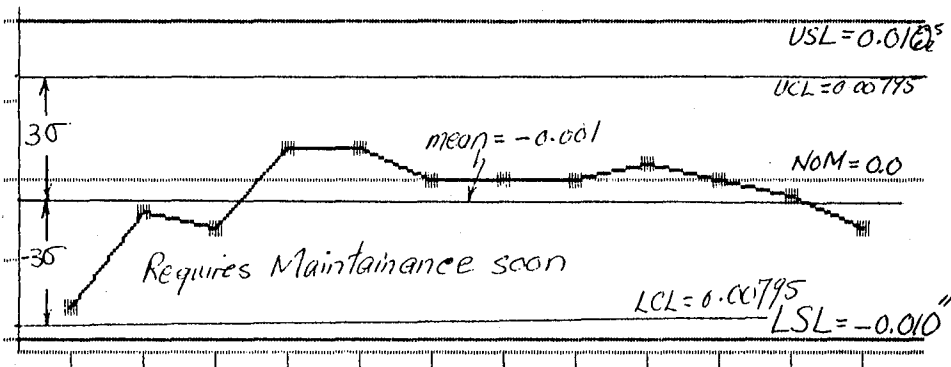
MAX:	+0.00400	USL:	+0.01000
XBAR:	+0.00125	NOM:	+0.00000
MIN:	-0.00300	LSL:	-0.01000
SIG:	+0.00249	RNG:	0.00700
n:	12	NUSL:	0
		NLSL:	0

Figure (2-3) Plot of Repeatability Errors X-Axis



MAX:	+0.00100	USL:	+0.01000
XBAR:	-0.00050	NOM:	+0.00000
MIN:	-0.00300	LSL:	-0.01000
SIG:	+0.00126	RNG:	0.00400
n:	12	NUSL:	0
		NLSL:	0

Figure (2-4) Plot of Repeatability Errors Y-Axis



MAX:	+0.00200	USL:	+0.01000
XBAR:	-0.00100	NOM:	+0.00000
MIN:	-0.00800	LSL:	-0.01000
SIG:	+0.00265	RNG:	0.01000
n:	12	NUSL:	0
		NLSL:	0

Figure (2-5) Plot of Repeatability Errors Z-Axis

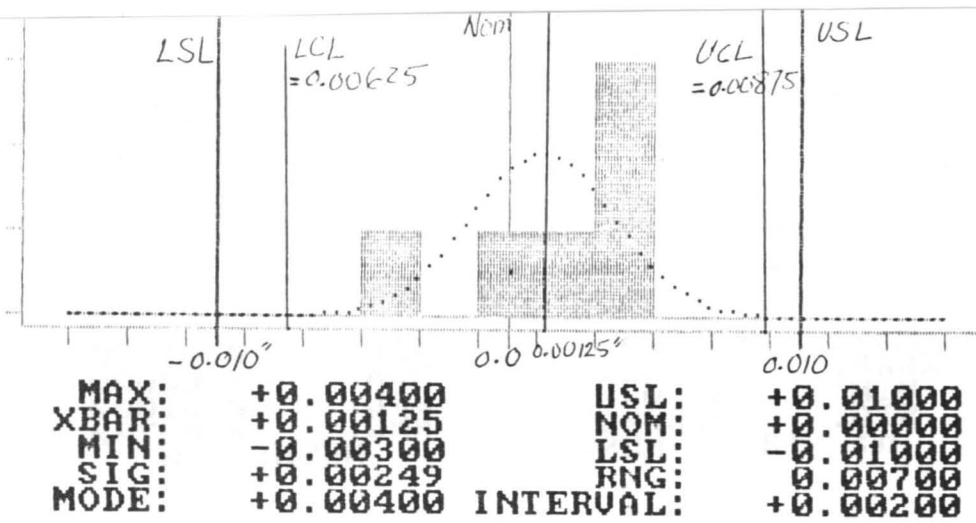


Figure (2-6) Histogram for Repeatability Errors X-Axis

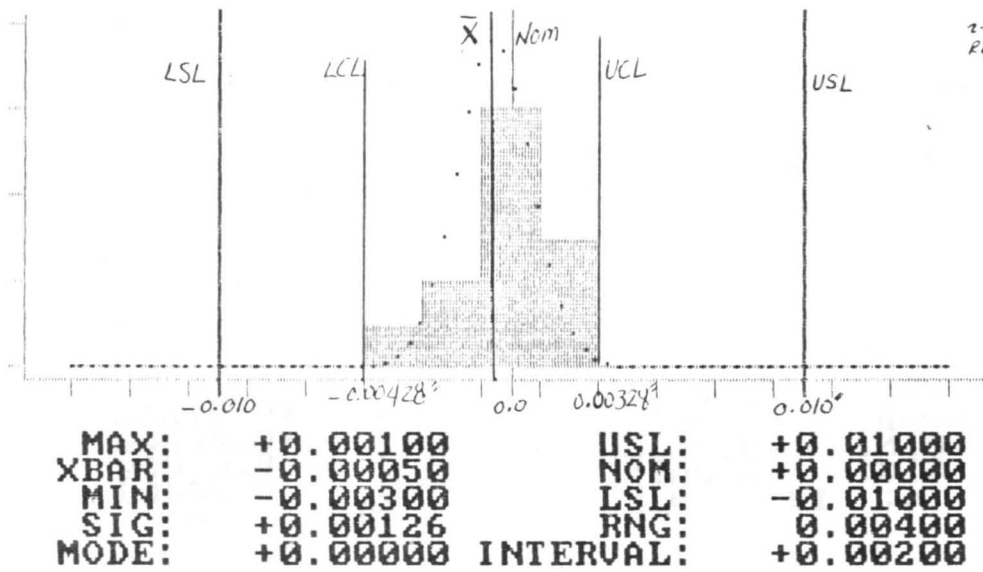


Figure (2-7) Histogram for Repeatability Errors Y-Axis

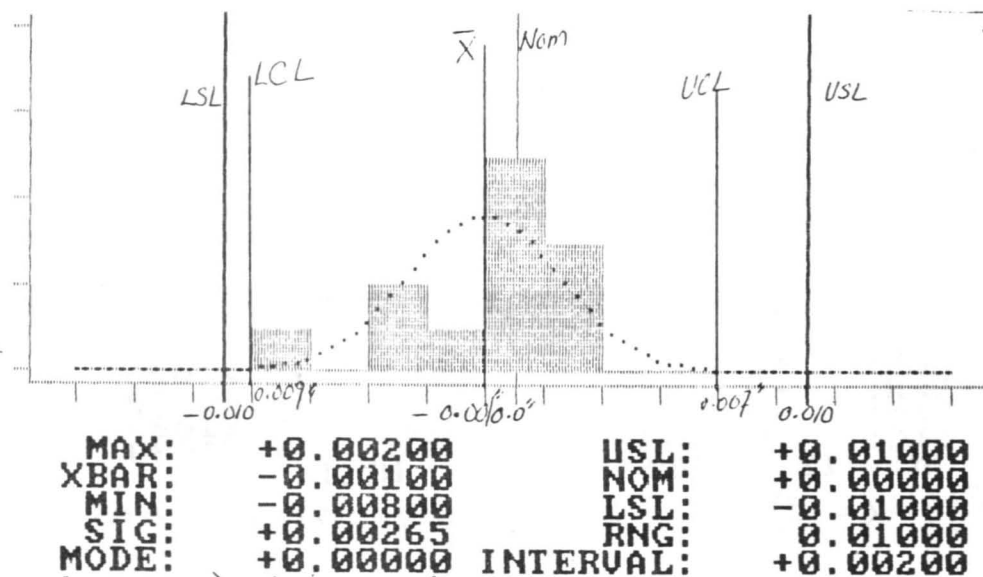


Figure (2-8) Histogram for Repeatability Errors Z-Axis

The above error vector should be compensated for, only, in the forward Kinematics software. After the introduction of this correction model in the software, the calibration procedure should be repeated once more to evaluate the degree of improvement in the robot repeatability that could be measured using the following parameters.

The Total Repeatability Error (TRE):

$$\text{TRE} = \sqrt{(\text{Ave. Error}_x)^2 + (\text{Ave. Error}_y)^2 + (\text{Ave. Error}_z)^2}$$

The PUMA's (TRE) could easily be calculated as follows:

$$\sqrt{\{(0.00125)^2 + (-0.0005)^2 + (0.001)^2\}} = 0.00167 \text{ INCH}$$

The (TRE) represents the magnitude of the repeatability error vector as well as the radius of the repeatability error sphere.

### 2.2.3 IMPORTANT PARAMETERS IN ROBOTICS CALIBRATION

There are important statistical parameters that should be evaluated and used as guide lines to examine the robot's capability of meeting the positioning requirements (upper and lower specification limits) of its end effector to perform a certain task. These parameters are described and evaluated for the PUMA robot as shown in the following:

(A) Process (Robot) Capability Index  $C_p$ :

The robot capability is evaluated by this parameter  $C_p$  (see reference [6] for more details) that is defined as

follows:

$$C_p = \frac{\text{Upper Spec Limit} - \text{Lower Spec Limit}}{6 * \text{Standard Deviation}} = \frac{\text{UCL} - \text{LCL}}{6 * \sigma}$$

(B) The Shift or The Off Set Parameter  $C_{pk}$ :

It is an important parameter to identify if the error measurement is experiencing any major off set or shift from the zero error line. This parameter is defined as follows:

$$C_{pk} = \text{Minimum of } \left\{ \frac{(\text{Average Error} - \text{Lower Spec Limit})}{3 * \text{Standard Deviation}} \right\} \text{ or } \left\{ \frac{(\text{Upper Spec limit} - \text{Average Error})}{3 * \text{Standard Deviation}} \right\}$$

A robot is classified as capable of performing a task that requires certain tolerance limits and specifications on the positioning of its end effector, if the repeatability error data of this robot exhibit  $C_p$  and  $C_{pk}$  values  $\geq 1$ . However, the higher the values of  $C_p$ , and  $C_{pk}$  are, the better the suitability and capability of the robot to perform that task.

In particular, the  $C_{pk}$  parameter lends itself very well to the evaluation for the degree of improvement in a robot's accuracy and repeatability after calibration. The comparison of  $C_{pk}$  values before and after the robot calibration and error compensation will give a good indication of the degree of improvement in a robot's performance.

The previously described statistical parameters were evaluated based on the PUMA repeatability error measurement given in tables (1) and (2) for each axis. The values for  $\sigma$ ,  $C_p$ , and  $C_{pk}$  were calculated based on an assumed allowable positioning tolerance for the PUMA of  $\pm 0.010$ ". The obtained results were tabulated in table (3) below:

Table (3) Statistical Parameters

Parameter	X - Axis	Y - Axis	Z - Axis
$\sigma$	0.00249	0.00126	0.00265
$C_p$	1.338	2.645	1.258
$C_{pk}$	1.17	2.513	1.132



The study of the obtained statistical parameters in table (3) indicates that the PUMA robot is classified as capable of positioning its TCP at a previously recorded point repeatedly within the acceptable tolerance zone of  $\pm 0.010''$ . The probability that the PUMA will fail to meet this tolerance requirement could be calculated as follows:

Area under the curve of the Standard Normal Distribution bounded by the Lower Spec Limit (LSL) and  $(-\infty)$ :

Find the value of  $A_1$  for  $\frac{\text{LSL} - \text{Average Error}}{\text{Standard Deviation } (\sigma)}$  use tables in Appendix

Find the value of  $A_2$  for  $\frac{\text{USL} - \text{Average Error}}{\text{Standard Deviation } (\sigma)}$  use tables in Appendix

$$\text{Probability of failure} = A_1 + A_2$$

The study of table (3) indicated that the most susceptible axis for failure is the PUMA's Z - Axis. Therefore, applying the probability of failure study on that axis shows the following:

$$\frac{\text{LSL} - \text{Average Error}}{\text{Standard Deviation } (\sigma)} = \frac{-0.01 - 0.001}{0.00265} = -4.15$$

From table (1) in the appendix the probability corresponding to - 4.15 is Zero i.e.  $A_1 = 0.0$

$$\frac{\text{USL} - \text{Average Error}}{\text{Standard Deviation } (\sigma)} = \frac{0.01 - 0.001}{0.00265} = 3.39$$

From table (2) in the appendix the probability corresponding to 3.39 is 0.0003 i.e.  $A_2 = 0.0003$

$$\text{The Probability} = A_1 + A_2 = .0003 \text{ i.e } 0.03\%$$

Briefly, using this calibration technique on the PUMA 500 at the RADL indicates that the robot is capable of repeating the recorded position of its TCP within  $\pm 0.010''$  with probability of failure of 0.03%.

## 2.3 MEASUREMENT OF A ROBOT'S RELATIVE ACCURACY (i.e. FORWARD KINEMATICS VERSUS THE INVERSE KINEMATICS)

In most robots the forward kinematic model is usually a closed form accurate mathematical model. The inverse kinematic model however usually has intrinsic inaccuracy due to the linear approximation of the Jacobian matrix that results in accumulation of errors in that model. Thus the relative calibration of the inverse kinematics against the forward kinematics is a useful approach in many six degrees of freedom manipulators. In the ARID both the inverse and forward kinematic models are as accurate because both are closed form solutions.

The calibration of a Robot and the measurements of its relative accuracy requires two 3D fixtures as shown in Figure(2-9).

### 2.3.1 SET UP PROCEDURE:

Place the two fixtures at a reasonable distance from each other within the work envelope of the robot. Align the X,Y, and Z axis of each fixture with respect to the robot axis and with respect to each other following the same technique previously explained in section 2.2. Level the fixtures and clamp them to the calibration table. Move the robot arm with the teach pendant to place the calibration bar between the indicators of the first fixture. Align the fixture coordinates with the robot base or world coordinates such that each indicator is depressed about 0.250".

Zero off all three indicators and record this point as "origin 1" see Figure (2-9). Establish a frame "frame 1" or a coordinate system with its origin at this point "origin 1" for the first fixture. Repeat the same with the other fixture to establish "frame 2" with "origin 2" at the second fixture. Make sure that "origin 1" and "origin 2" are also recorded with respect to the other frame under different name i.e. "origin 1" should be recorded with respect to "frame 2" under a different name such as "origin 3" and do the same for "origin 2" record this point with respect to "frame 1" under a different name such as "origin 4". Display the pose at "origin 1" or "origin 3" in relation to "frame 2" and copy three values for the position and three angles for the orientation on a separate sheet. Note that the 6 values specifying a recorded pose at a certain point such as "origin 1,2,3, or 4" are calculated by the forward kinematics model. Once more, display the pose at "origin 2" and "origin 4" in relation to "frame 1". Make sure that both poses compare very closely before proceeding to the next step.

Locate the calibration bar end at "origin 1" and zero off all the indicators. Do the same for "origin 2". Command the robot to move from "origin 2" to reach "origin 1" by entering the pose six values copied earlier on a separate sheet. Note that the

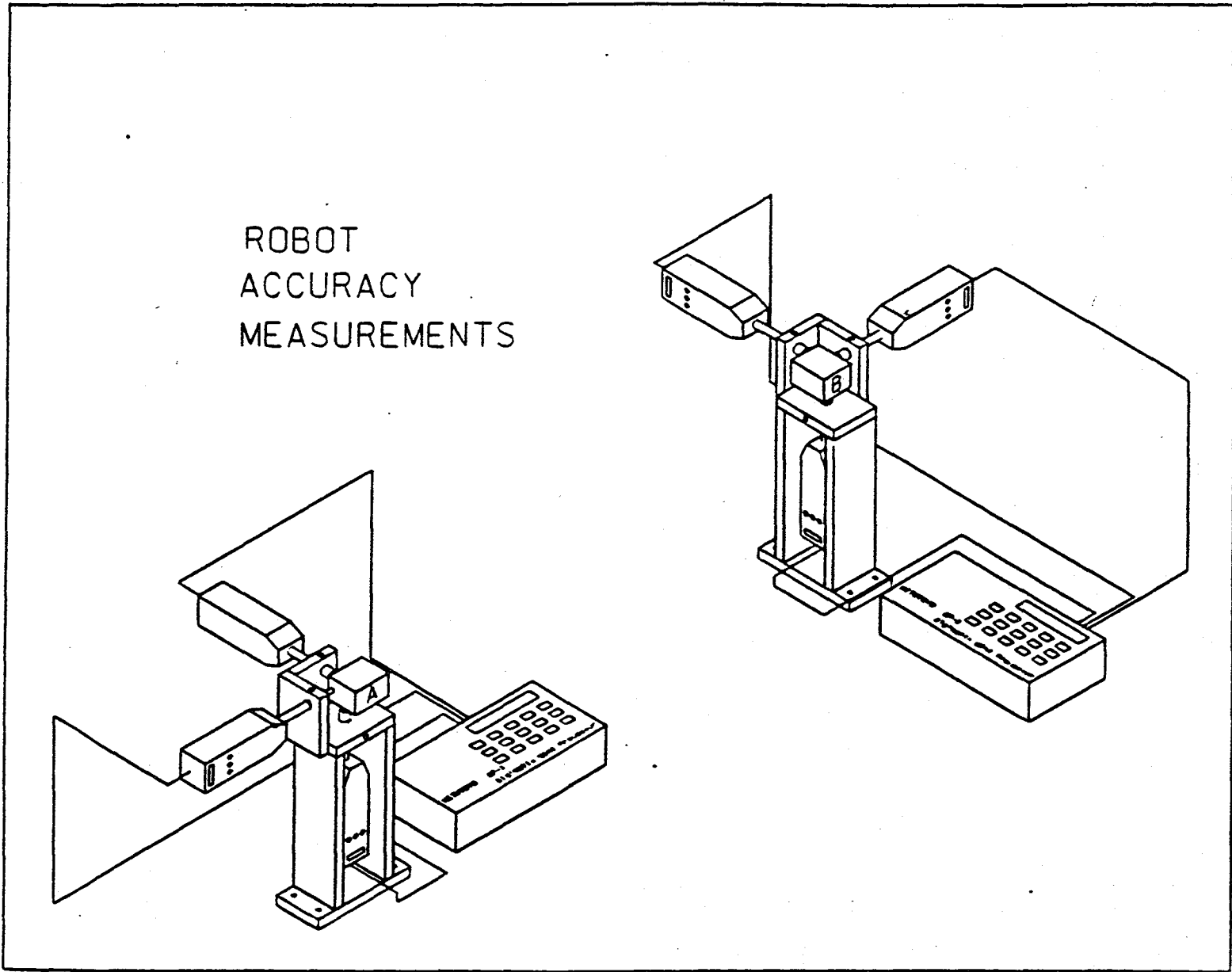


Figure (2-9) 3D Robot Accuracy measurement set up

inverse kinematic model will receive the pose information and calculate the joints angle required to position the robot at that commanded point. Write down the readings of the three indicators. These three readings represent the error vector in the inverse kinematic model relative to the forward kinematic model in that run. Move the robot to the recorded "origin 1" and again zero off the indicators. Repeat the process of shuttling the robot between the two fixtures ten to twenty times and record the error vector for each run.

In the current work ten runs were conducted on the PUMA 500 and the results are given in table (4):

Table (4) Error Vectors of The Inverse Kinematics Relative to the Forward Kinematic Model of the PUMA 500.

RUN No.	X - Axis	Y - Axis	Z - Axis
1	-0.007	-0.001	0.001
2	0.000	-0.005	0.009
3	-0.002	-0.002	0.002
4	-0.003	-0.001	0.004
5	-0.004	-0.001	0.003
6	-0.003	-0.002	0.004
7	0.001	-0.002	0.003
8	0.002	-0.002	0.002
9	-0.004	-0.001	0.004
10	0.001	-0.004	0.003

## 2.3.2 RESULTS AND ANALYSIS

The obtained results for the relative accuracy of the PUMA were analyzed in a similar fashion as the repeatability data were analyzed. The average X, Y, and Z components of the relative error vector are given in the following:

Table (4) The average component of the relative error vector

X - Axis	Y - Axis	Z - Axis
-0.0019"	-0.0021"	0.0035"

Accordingly, the relative error vector of the PUMA 500 in the RADL at KSC now reads the following:

$$- 0.0019 \text{ i} - 0.0021 \text{ j} + 0.001 \text{ k}$$

The previous relative error vector should be compensated for in the inverse kinematics model. After compensation for that error vector the relative calibration procedure should be conducted once more to examine the degree of improvement in the robot performance. The improvement is measured by the following parameters:

The magnitude of the error vector is evaluated using the following formula:

$$\begin{aligned} &= \sqrt{\{( \text{Ave. Error}_x)^2 + (\text{Ave. Error}_y)^2 + (\text{Ave. Error}_z)^2\}} \\ &= 0.0045 \text{ inches} \end{aligned}$$

The comparison of the total repeatability error (0.00167") with the total relative error (0.0045") indicated what always had been expected with industrial robots, they are more repeatable than accurate with an order of magnitude of almost 3. It is expected the ARID robot will not exhibit such discrepancy between its total repeatability error and relative accuracy error because both the ARID's forward and inverse kinematic models are closed form solutions. However it is expected that the ARID's total absolute accuracy will be some orders of magnitudes higher than its (TRE) mainly because of its joints and links flexibility. Therefore, the proposed calibration scheme for the ARID in section 3 is based on the expected high

repeatability of the ARID. This will preclude the need for rigorous calibration techniques for the ARID's absolute accuracy.

The comparison of the values of  $C_p$ ,  $C_{pk}$  in tables (3) and (5) emphasizes the fact that the PUMA's relative accuracy and repeatability errors did not disqualify it from being capable of meeting the specs. The probability that the PUMA will fail to meet the specs is approximately 1%.

Table (5) Statistical Parameters For Relative Accuracy Errors

Parameter	X - Axis	Y - Axis	Z - Axis
$\sigma$	0.0027	0.0013	0.00206
$C_p$	1.234	2.564	1.618
$C_{pk}$	0.999	2.025	1.052

#### 2.4 OVERSHOOT MEASUREMENTS OF THE PUMA

The overshoot is well described in Figure (2-10). The 3D fixtures are also used to measure the overshoot of the PUMA at three different speeds. The measurements shown in table (6) indicated that as the speed increases the overshoot is also increased. Note that the overshoot of the ARID could be evaluated by using the same 3D fixture technique applied to the PUMA.

Table (6) Overshoot for Puma 500

PERCENT. SPEED	OVERSHOOT
SPEED 5	0.0000"
SPEED 50	0.0040"
SPEED 100	0.0090"

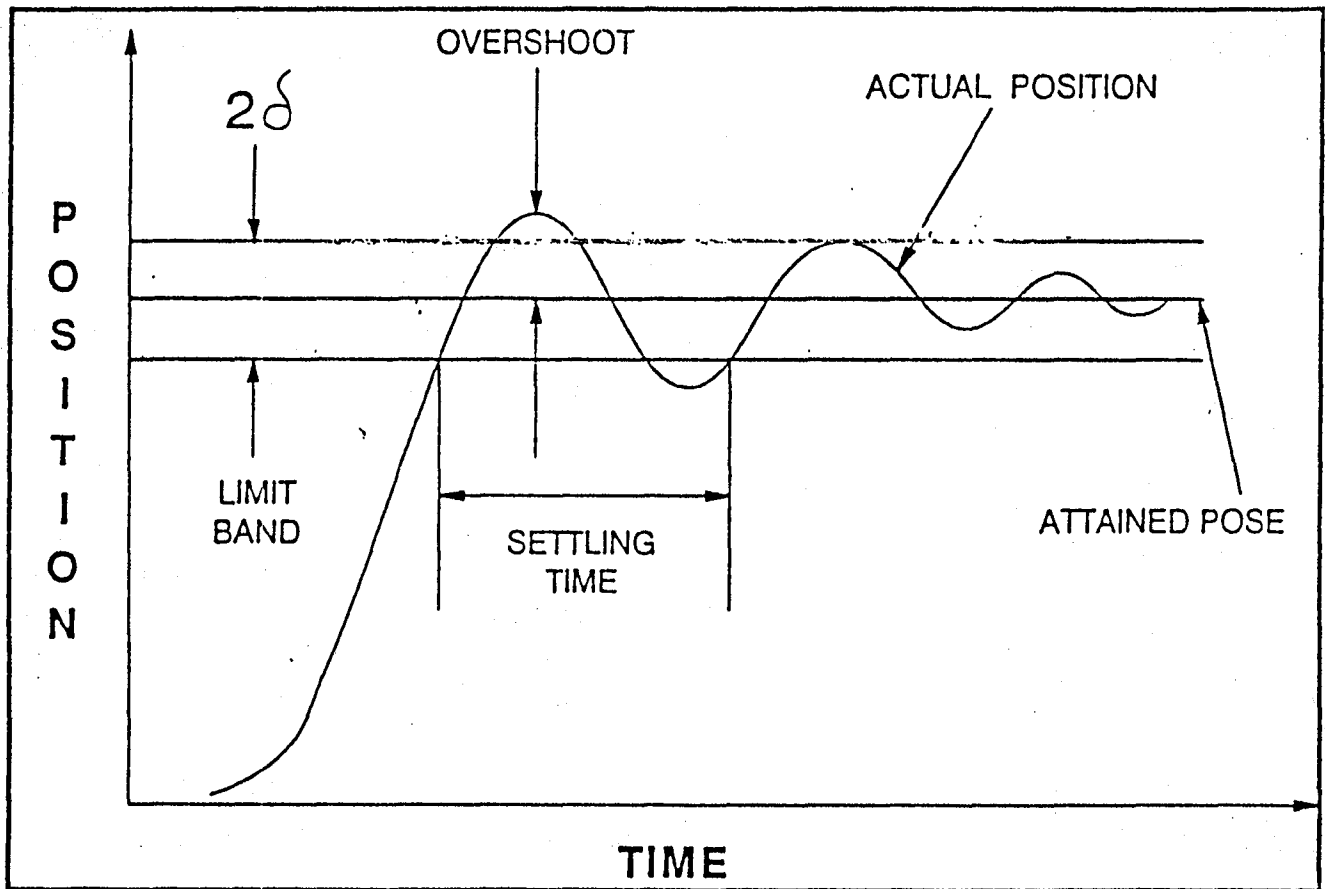


Figure (2-16) Overshoot and Settling Time Concepts

### III CALIBRATION AND TROUBLE SHOOTING PROCEDURES OF THE ARID ROBOT

As mentioned earlier in this report, robot calibration is a process by which robot accuracy can be improved by modifying the robot positioning software rather than changing the design of the robot hardware or its control system. In the literature various procedures and techniques were used to modify the forward and inverse kinematics parameters through an error correction model. Most of these techniques are expensive, labor intensive, and time consuming.

Accordingly, a simple, inexpensive, and straight forward statistical technique and procedures were developed to evaluate and correct for the repeatability and relative accuracy error vectors of the ARID using 3D fixtures. The current design of these fixtures only allows for the calibration of the positioning error. However, if the orientation calibration is also needed the fixture should be modified as shown in Figure (2-2) and a mathematical algorithm should be developed to process the indicators readings and evaluate the pose (position, and orientation).

Because ARID is a dedicated robot to perform an on line inspection, its calibration technique only relays on the manipulator's repeatability and relative accuracy as well as the frame shift of the radiator. Therefore there is no need for the evaluation of the ARID's absolute accuracy and using any of the literature calibration techniques that are expensive, laborious and time consuming. The evaluation of the absolute accuracy is only needed for off line programming which is not the case in the ARID; as ARID only uses on line programming. Thus a tailored calibration technique for the ARID project is proposed to include the radiator frame shift compensation as a part of the calibration package and is described as follows:

#### 3.1 CALIBRATION PROCEDURE FOR THE (ARID) ROBOT AT KSC:

The calibration and troubleshooting procedure using this statistical technique is explained in the following steps:

##### STEP 1: EVALUATION OF ROBOT REPEATABILITY

Evaluate the repeatability of the ARID robot at several points selected randomly within a zone that encompasses the radiator surface where the 4x4 photo frames are expected to cover. Note that the control system was designed to control the free end of a two feet long imaginary fourth link. This was considered to simulate the camera and the length of its axis at the point of intersection with the radiator surface. Therefore, it is recommended to use a calibration bar with a length of 24 inches + the length of the camera. Note that the bar material should be selected such that the weight of the bar will approximately match the weight of the camera. Evaluate the



repeatability error vector as previously described in section 2 of this report and correct for it in the ARID kinematic model (if necessary).

## STEP 2: EVALUATION OF ARID's STATISTICAL CAPABILITY PARAMETERS

If correction is performed in step 1, recollect a few error measurements or use the same error measurement in Step 1 and calculate the statistical parameters  $C_p$ ,  $C_{pk}$  - base the calculation on Upper Spec Limit = 0.125 in, and Lower Spec limit = -0.125 in. If both parameters are in the acceptable range i.e. both are greater than unity (at least) then proceed to the next step. If the value(s) of one or both parameter(s) is (are) not in the acceptable zone i.e. less than unity conduct the trouble shooting procedure described later in this section.

## STEP 3: EVALUATION OF THE ARID RELATIVE ACCURACY

Evaluate the relative accuracy error vector as described earlier in section 2.3 of this report. Compensate for the error vector in the inverse kinematic model if needed. Recollect a few error measurements or use the same data obtained before correction to evaluate the  $C_p$ ,  $C_{pk}$  for this mode of calibration. If both parameters have values greater than one proceed to the next step. If one parameter or both are less than one conduct the troubleshooting procedure (1) described later in this section.

## STEP 4: CAMERA SET UP AND ADJUSTMENT

Replace the calibration bar by the camera. Consider that the radiator has four or six fiducial points. Move the camera to the first fiducial point and locate the cross hair to the center of the fiducial point image. This could be achieved by coinciding the circular image with a circle drawn on the camera screen that has its center right on the cross hair. Also, this could be achieved by image processing and the evaluation of the number of picture cells need for the camera to be moved to get the image exactly in the center of the screen. After centering the image in the exact middle of the screen, use the sonar to set the camera perpendicular to the radiator and at two feet away from it. Re-center the image to the middle of screen that may have be shifted during the adjustment of the camera's perpendicularity and distance relative to the radiator. Keep on refining these three parameters, namely, perpendicularity, centering, and distance (24") with respect to the radiator until acceptable positioning accuracy is achieved. Record this position i.e the readings of the encoders and the corresponding values of the tool center point X,Y, and Z coordinates in the memory of the computer under "fiducial 1" for example.

## STEP 5: FORMATION OF A REFERENCE DATA BASE OF ARID'S JOINT READINGS

Repeat step 4 for all the fiducial points and all the inspection points and generate a permanently stored reference data base that contains reference joints readings end tool center point location coordinates for all these fiducial and inspection points. Now the ARID will be ready to inspect any radiator using this data bank as a reference.

## STEP 6: ACTUAL INSPECTION PROCESS

For the ARID to inspect any radiator the reference data base has to be corrected by the amount of frame shift between the radiator under inspection and the reference data. The correction model for the frame shift will be based on comparison between the fiducial points of the radiator under inspection and the reference points. The frame shift model should correct for the TCP location and the perpendicularity to the radiator for each inspection point. The corrected inspection points will specify the scanning path for ARID to follow. It is strongly recommended that ARID scans on the fly to avoid possible and harmful vibration.

### 3.2 ARID TROUBLESHOOTING PROCEDURE USING THIS STATISTICAL TECHNIQUE

#### 3.2.1 TROUBLESHOOTING PROCEDURE (1); REPEATABILITY ERROR:

If repeatability error data in any of the axes showed a value less than unity for the  $C_p$  parameter; examine the repeatability error of each joint one at a time and evaluate the  $C_p$  parameters for each. The joint that experiences less than unity value for  $C_p$  should be examined for hardware damage and/or encoder or resolver problem(s)

If all joints consistently exhibited repeatability errors that give less than unity for the statistical parameter  $C_p$ ; the control system design has to be rechecked.

#### 3.2.2 TROUBLESHOOTING PROCEDURE (2), RELATIVE ACCURACY CHECK

Examine the inverse kinematics model and software

#### IV CONCLUSIONS

- (1) THE LITERATURE SURVEY EXHIBITED THE RECENT DEVELOPMENT OF ROBOTICS CALIBRATION TECHNIQUES; MOST OF WHICH CAN NOT BE CLASSIFIED AS TIME-EFFICIENT AND/OR COST EFFECTIVE.
- (2) THE 3D FIXTURE IS A SIMPLE, INEXPENSIVE, AND STRAIGHTFORWARD TECHNIQUE THAT COULD BE USED TO CALIBRATE AND TROUBLESHOOT ANY ROBOT SUCH AS THE ARID, ASEA, HIGH SPEED CONTROLLER etc.
- (3) THE 3D FIXTURE AND THE STATISTICAL TECHNIQUE WERE USED TO MEASURE THE PUMA 500 REPEATABILITY AND RELATIVE ACCURACY ERROR VECTORS; THEY READ:  
.0012 I - 0.0005 J + 0.001 K  
AND -0.0019 I - 0.0021 J + 0.001 K  
RESPECTIVELY
- (4) PUMA SHOWED TOTAL REPEATABILITY AND RELATIVE ACCURACY ERROR OF 0.00167" AND 0.0045" RESPECTIVELY
- (5) ARID'S REPEATABILITY, RELATIVE ACCURACY, AND FRAME SHIFT ARE THE MAIN ASPECTS IN THE RECOMMENDED STEP BY STEP CALIBRATION PROCEDURE FOR THE ARID
- (6) THE 3D FIXTURE AND THE STATISTICAL TECHNIQUE ARE RECOMMENDED FOR THE ARID CALIBRATION AND TO EVALUATE ITS REPEATABILITY AND RELATIVE ACCURACY AS WELL AS TROUBLESHOOTING ITS SOURCES OF ERRORS
- (7) IT IS RECOMMENDED THAT THE SAME TECHNIQUE BE USED FOR THE CERTIFICATION OF ASEA'S HIGH SPEED CONTROLLER

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## V APPENDIX

### Appendix

TABLE 1. Values of the Standard Normal Distribution Function\*

$$\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du = P(Z \leq z)$$

z	0	1	2	3	4	5	6	7	8	9
-3.0	0.0013	0.0010	0.0007	0.0005	0.0003	0.0002	0.0002	0.0001	0.0001	0.0000
-2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0126	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0238	0.0233
-1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0300	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0570	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

\*B. W. Lindgren, *Statistical Theory*, The Macmillan Company, 1960.

Appendix

TABLE 1 (Continued)

$$\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du = P(Z \leq z)$$

<i>z</i>	0	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9430	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9648	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9700	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9762	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000