MODELING SELECTIVE AVAILABILITY OF THE NAVSTAR GLOBAL POSITIONING SYSTEM

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SUMMARY

As the development of the NAVSTAR Global Positioning System (GPS) continues, there will increasingly be the need for a software-centered signal model. This model must accurately generate the observed pseudorange which would typically be encountered. The observed pseudorange varies from the true geometric (slant) range due to range measurement errors. Errors in range measurements stem from a variety of hardware and environment factors. These errors are classified as either deterministic or random and, where appropriate, their models are summarized. Of particular interest is the model for Selective Availability (S/A) which was derived from actual GPS data. The procedure for the determination of this model, known as System Identification Theory, is briefly outlined. The synthesis of these error sources into the final signal model is given along with simulation results.

INTRODUCTION

The NAVSTAR Global Positioning System (GPS) is a satellite radionavigation system designed to provide worldwide position, velocity, and time information. The use of GPS may be extended to land, air, sea, and space. For civil air use however, system performance must be proven before regulatory agencies will sanction its use. Strict requirements are imposed and documentation must be produced which demonstrates the system's ability to meet these requirements.

Of these strict requirements, the most difficult to satisfy are low failure probability and high integrity. The role of the regulatory agencies is also seen in the certification of airborne GPS equipment. Before a GPS receiver is certified, documentation must be produced which details the GPS performance. Since the cost of flight testing is high, the majority of this documentation is produced through simulations. One input to these simulations is a model of the navigation system.

This paper presents a signal model which may be used for such purposes. Error source algorithms are generated into one model which computes the observed pseudorange for each satellite in view and determines the perturbed user position. This pseudorange model may be used in the performance analysis of GPS, differential GPS (DGPS), and receiver autonomous integrity monitoring (RAIM) (ref. 1). In addition, the model may be used to determine the required update rate for DGPS (ref. 2).

GPS ERROR SOURCES AND MODELS

The pseudorange observation equation is given by:

$$P_r = P_s + t_{rcvr} - t_{sv} + d_{tropo} + d_{iono} + URE + d_{mp} + d_{HW} + d_{meas} + noise + S/A$$

where:

P_r The observed pseudorange as determined by the receiver, which is the difference between the time the signal was transmitted by the satellite and the time the signal arrived at the receiver, multiplied by the speed of light.

P_s The true geometric (slant) range between the satellite and the receiver.

t_{rcvr} Receiver clock offset from GPS time, as estimated from observed pseudoranges.

t_{sv} Satellite clock offset from GPS time, as decoded from the satellite navigation data.

d_{tropo} Propagation delay caused by the troposphere.

d_{iono} Propagation delay caused by the ionosphere.

URE   User range error.

d_{mp} Specular multipath errors.

d_{HW} Receiver hardware delay.

d_{meas} Receiver measurement bias errors.

noise   Receiver measurement noise, clock noise, and diffuse multipath.

S/A   Selective Availability.

Both the receiver clock offset and hardware bias are essentially identical for each satellite being tracked. As a result, these biases will appear as a clock bias in the user navigation solution and hardly affect the position solution.

The error models for tropospheric delay, ionospheric delay, and URE have been described in detail in the literature and require no further comment. It is the random errors which currently pose the greatest challenge.

Receiver noise, satellite clock noise, and Selective Availability are all stochastic processes and therefore cannot be predicted exactly. While the combination of the receiver and satellite clock noise is basically a white noise process with small variance, S/A is not. Moreover, the exact model which produces S/A is classified for security reasons. Since the actual model which produces S/A is classified, real GPS data from an operational satellite are needed. Once data have been collected which contain S/A, a more rigorous analysis may be performed. In addition, once S/A waveforms have been derived from the data, a technique is required which

determines the model which best fits the process which produced it. This technique is known as System Identification Theory.

SYSTEM IDENTIFICATION THEORY

Following reference 3, the essence of System Identification Theory will be described. The main concept is the idea that the data to be identified can be considered the output of a linear filter whose input is white noise. In general, a process may be described by an Auto-Regressive Moving Average (ARMA) model of a given order.

Collected data are passed through a filter whose coefficients are adjusted until its output is white noise (with variance σ_r^2). The final filter coefficients can then be used to find the inverse of the signal generator filter. Data which are statistically equivalent to the collected data may then be generated by passing white noise (with variance σ_r^2) through the inverse filter. This process is depicted in figure 1.

In summary, the steps which lead to the S/A model are:

1) Obtain S/A sample through data collection.

2) Determine a filter whose output is white noise when the S/A data sample is passed through it. Calculate the variance of the white noise output.

3) Construct the inverse of the filter.

4) Statistically equivalent S/A data may then be obtained by passing white noise (with variance as calculated in step 2) through the filter obtained in step 3.

In addition to the determination of the S/A model, System Identification Theory also offers a significant compression of the S/A data. Once the inverse model is determined, only the model coefficients and the variance of the residuals need to be stored. Based on these few parameters, virtually an infinite amount of statistically equivalent S/A data can be generated.

GPS DATA COLLECTION

As outlined in the above section, the first step in determining a model for S/A is to collect a sample of S/A. Data were collected from the first operational Block II satellite (SV14). Block II satellites are the first satellites capable of introducing S/A into the GPS signal. The satellite was tracked since day 135. On days 189, 190, and 191, error data which are likely to be caused by S/A were collected. As of the time of this writing, the S/A data presented in this paper have not been confirmed by JPO. However, this does not present a major problem as will be discussed later.

Figures 2 and 3 show plots of SV14 pseudorange residuals. Figure 2 shows the pseudorange residuals with S/A turned off. The error is composed mainly of receiver noise. Figure 3 shows what appears to be two different models of S/A.

DETERMINATION OF S/A MODELS

As outlined previously, once a data sample has been collected the next step in the identification process is to determine a proper filter. The proper filter has the following characteristics:

1) Its output is white noise when the data are passed through it.
2) It has a minimum number of coefficients.

Once this filter has been determined, the variance (σ_r^2) of the white noise output is calculated. The S/A generator is then composed of white noise (with variance σ_r^2) input to the inverse of the above described filter.

This procedure was carried out for the four S/A samples collected on days 189, 190, and 191. It is important to note that the data sets analyzed were unfiltered. This was done for two reasons. One was to avoid assumptions imposed on the S/A waveforms by the low-pass filter. The second was to more accurately reproduce the SV14 residuals. The generator generates waveforms which are statistically equivalent to the combination of the input S/A and noise.

The determination of the S/A model is illustrated for the data collected on day 190 (GPS time of the week: 22 - 24 hours). The model chosen was an ARMA model with three poles and two zeros, ARMA(3,2). In order to validate this model, the autocorrelation function of the output is calculated to check for whiteness (figure 4). Since it has a spike at lag 0 and its sidelobes are contained in the 95% confidence limits, it may be considered white noise. An additional test is to plot the power spectrum of the output. White noise is characterized by a flat spectrum. Again, the conclusion is verified (figure 5).

Now that the optimum filter has been determined, the waveform generator results from taking the inverse of the optimum filter. The resulting filter is given by:

$$H(z) = \frac{1 - 1.5047z^{-1} + 0.6428z^{-2}}{1 - 2.0833z^{-1} + 1.2164z^{-2} - 0.1302z^{-3}}$$

As a further check, data are generated and plotted with the original data (figures 6 and 7). Note: the generated data do not look exactly like the original because these data are statistically equivalent. The power spectrum of the collected data and the generated data show the spectral similarity (figure 8).

Table 1 gives the models which result from the identification of the four S/A samples. It is interesting to note that the models for day 189 (166 - 168) and day 190 (22 - 24) are very similar. The same is true for day 190 (0 - 2) and day 191 (24 - 26). It is probable that in

fact two models were used to create the four S/A waveforms.

This brings up an interesting point. As was mentioned in the Data Collection section, the collected waveforms have not been confirmed by JPO as S/A. However, if actual S/A turns out to be different than what has been determined, it is a simple matter to go through the ID process again and change the filter coefficients in the model.

SIGNAL MODEL BLOCK DIAGRAM AND SIMULATION RESULTS

The block diagram of the signal model is given in figure 9. The GPS ephemerides, the true user state, and the error sources combine in a truth model. The truth model outputs satellite positions and perturbed pseudoranges to the navigation solution block. The true user state is then subtracted from the calculated user state resulting in the user state error.

Table 1

MODELS IDENTIFIED FROM THE FOUR S/A SAMPLES

Day (GPS time of week in hours)	Model	σ_r^2
189 (166-168)	$H(z) = \dfrac{1 - 1.6814z^{-1} + 0.8082z^{-2}}{1 - 2.3161z^{-1} + 1.6630z^{-2} - 0.3450z^{-3}}$	20.6
190 (0-2)	$H(z) = \dfrac{1 - 1.5890z^{-1} + 0.6744z^{-2}}{1 - 1.9335z^{-1} + 0.9345z^{-2}}$	17.3
190 (22-24)	$H(z) = \dfrac{1 - 1.5047z^{-1} + 0.6428z^{-2}}{1 - 2.0833z^{-1} + 1.2164z^{-2} - 0.1302z^{-3}}$	22.3
191 (24-26)	$H(z) = \dfrac{1 - 1.5703z^{-1} + 0.6566z^{-2}}{1 - 1.9149z^{-1} + 0.9168z^{-2}}$	16.6

In order to concentrate on the effects of S/A, all errors other than S/A and receiver and satellite clock noise were set to zero. In addition, the user position was constant and a fixed satellite constellation was chosen (figure 10). The horizontal dilution of precision (HDOP) is 2.92, the east dilution of precision is 1.17, and the north dilution of precision is 2.68. Four thousand data points were taken to show the distribution of the horizontal position errors.

The first scenario was designed to verify the simulation model. Pseudorange errors were taken from a normal distribution with zero mean and a standard deviation of 5 meters. As would be expected, the scatter plot of position errors shows them to be approximately Gaussianly distributed (figure 11). It is also comforting to see that the error distribution lies along the line of highest dilution of precision.

With the model thus verified, the two filters derived from the S/A waveforms were used to perturb the pseudoranges. The results using the ARMA(2,2) are given in figure 12 and the results from the ARMA(3,2) are given in figure 13. The position errors again appear to be Gaussianly distributed. This is not surprising. By placing S/A on the four pseudoranges and then computing position, one is essentially convolving four random variables. The central limit theorem states that as N increases, the convolution of N random variables will approach a Gaussian distribution. For short periods of time, on the order of one hour, the effect of S/A on the position error is quite different from the effects of the Gaussian noise, however. The S/A position tends to "wander" around the true user position.

The two dimensional root-mean-squared (2D-rms) position errors for figures 12 and 13 are 93.4 and 157.5 meters, respectively. The average HDOP for a 21-satellite constellation is approximately 1.8 (for an elevation mask angle of 5 degrees). Scaling the simulation results with respect to the average HDOP results in 2D-rms position errors of 57.6 and 97.1 meters, respectively.

SUMMARY AND CONCLUSIONS

A signal model is presented which possesses the capability to generate typical errors in pseudorange measurements. The effects of ionospheric delay, tropospheric delay, and URE are detailed along with the models used to generate them. Two S/A models are derived from actual GPS data using System Identification Theory. The resulting signal model is then used to investigate the effects of S/A on position errors. The results satisfy both theory and intuition.

The short term effect of S/A on the horizontal position error is significantly different from the effect of Gaussian noise. Therefore, it is recommended that the System Identification technique outlined in this paper be implemented in GPS receiver test equipment. One principle advantage, as discussed previously, is that the test engineer is not limited to a few waveforms of collected S/A. Given the properly identified model, virtually an infinite amount of statistically equivalent S/A data may be generated to exercise the GPS receiver navigation algorithms.

REFERENCES

1. Kelly, R. J., van Graas, F., and Kuhl, M. R.: Improved Effectiveness of GPS RAIM through Ridge Regression Signal Processing. Proceedings of the 2nd International Satellite Division Meeting of the ION, Colorado Springs, CO, September 27-29, 1989.

2. Kalafus, R. M., Van Dierendonck, A. J., Pealer, N. A.: Special Committee 104 Recommendations for Differential GPS Service. Navigation: Journal of the Institute of Navigation, Vol. 33, No. 1, Spring, 1986.

3. Braasch, M. S.: A Signal Model for the NAVSTAR Global Positioning System. Proceedings of the 2nd International Satellite Division Meeting of the ION, Student Session, Colorado Springs, CO, September 27-29, 1989.
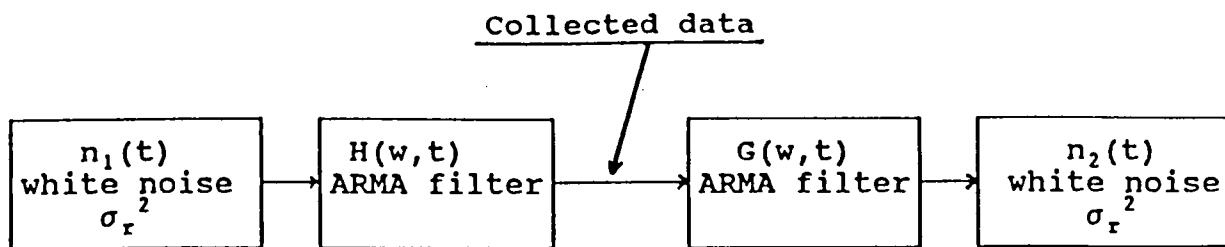
$$H(w,t) = G^{-1}(w,t)$$

$$\sigma_r^2 = \text{variance}$$

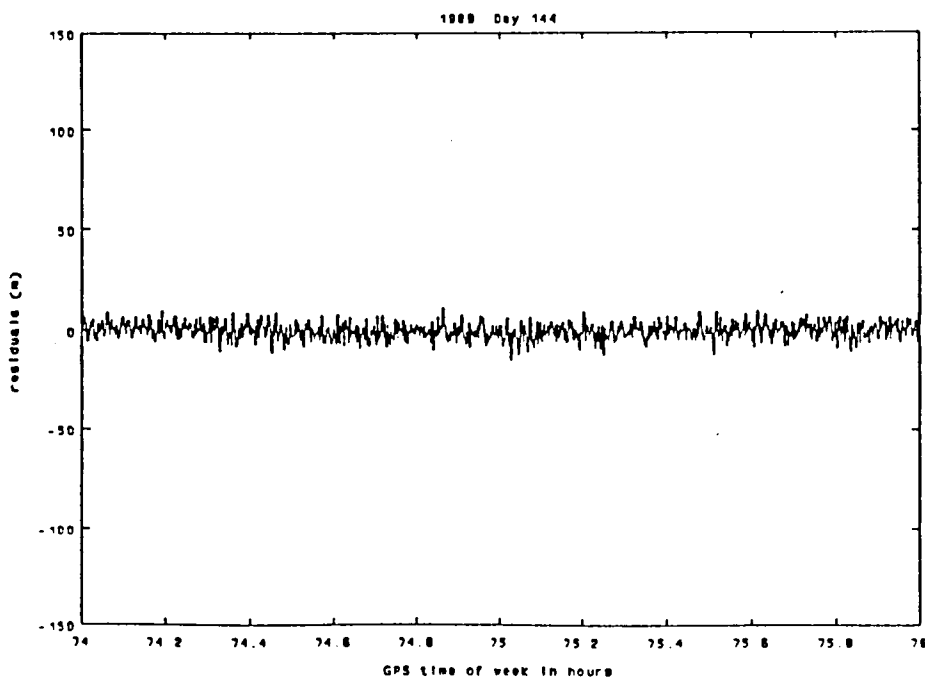Figure 1. Concept behind System Identification Theory.

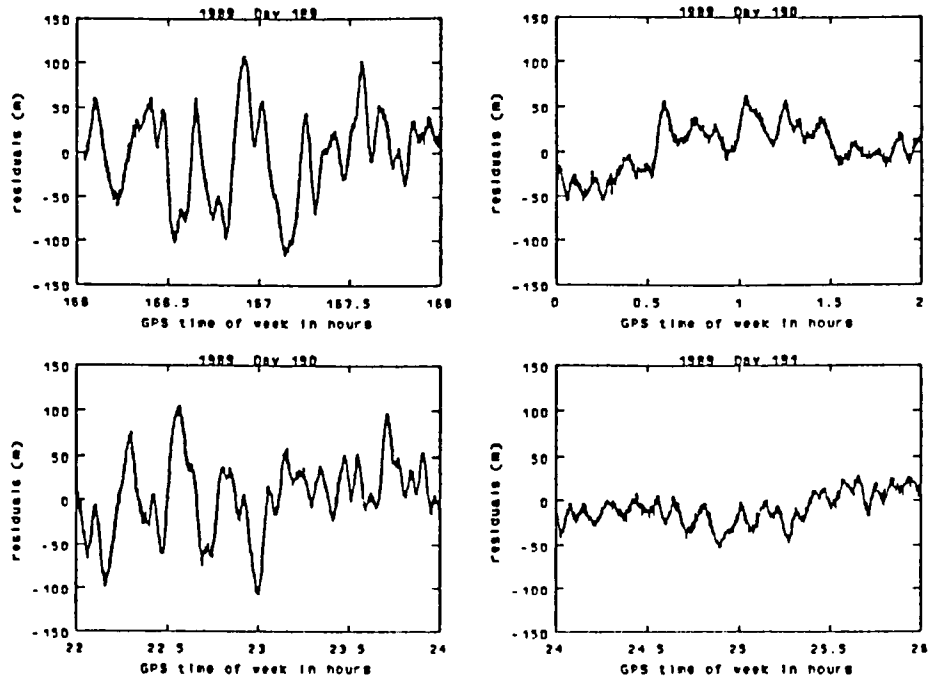Figure 2. Pseudorange residuals (S/A turned off).
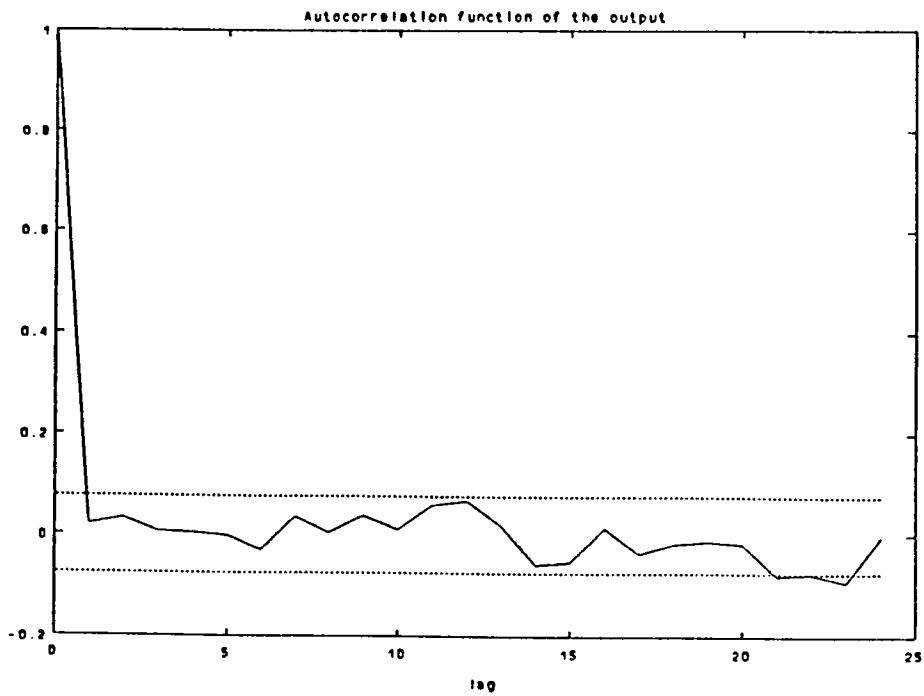
Figure 3. Pseudorange residuals.
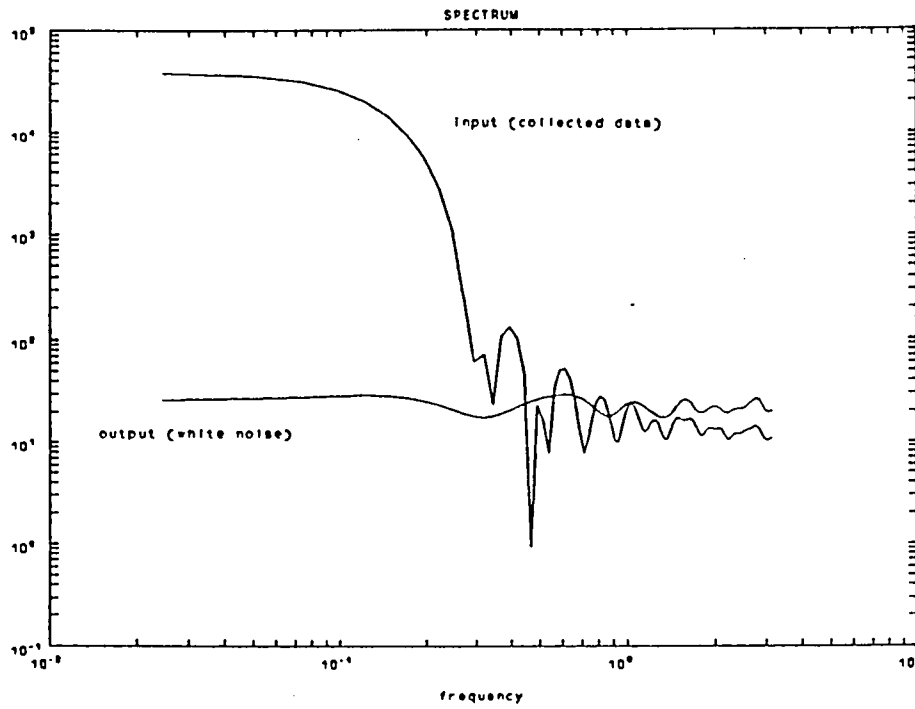

Figure 4. Autocorrelation function of the model output.

SPECTRUM

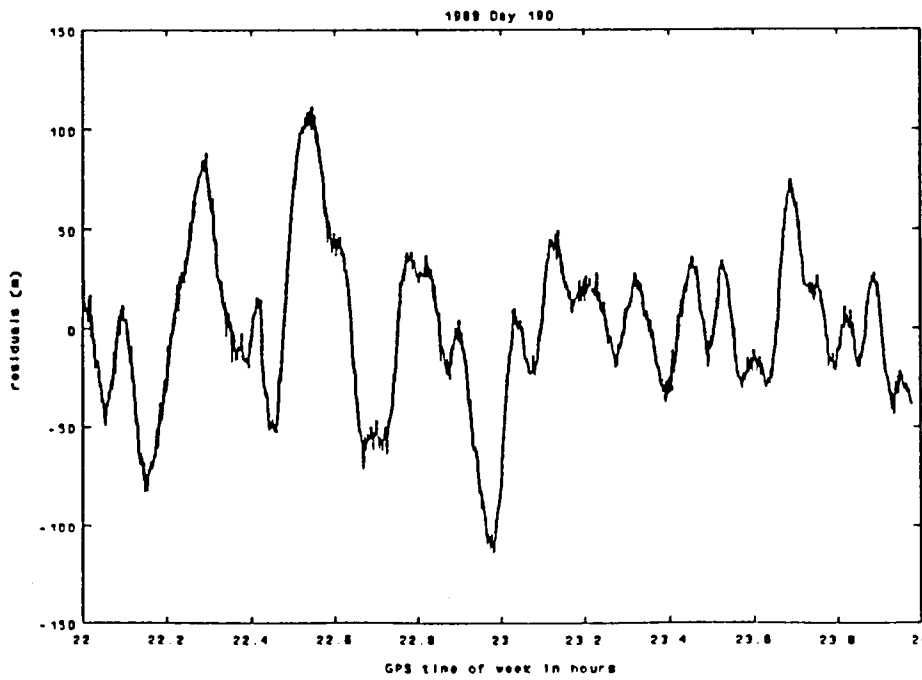Figure 5. Power spectra of model input and output.
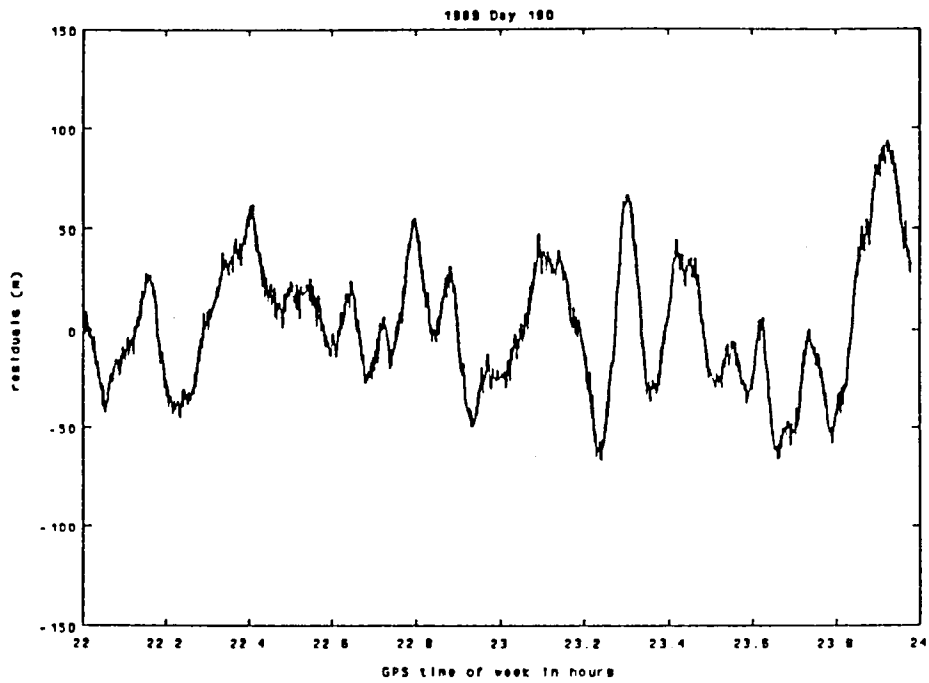
1989 Day 190

Figure 6. Collected S/A data.

Figure 7. Generated S/A data.
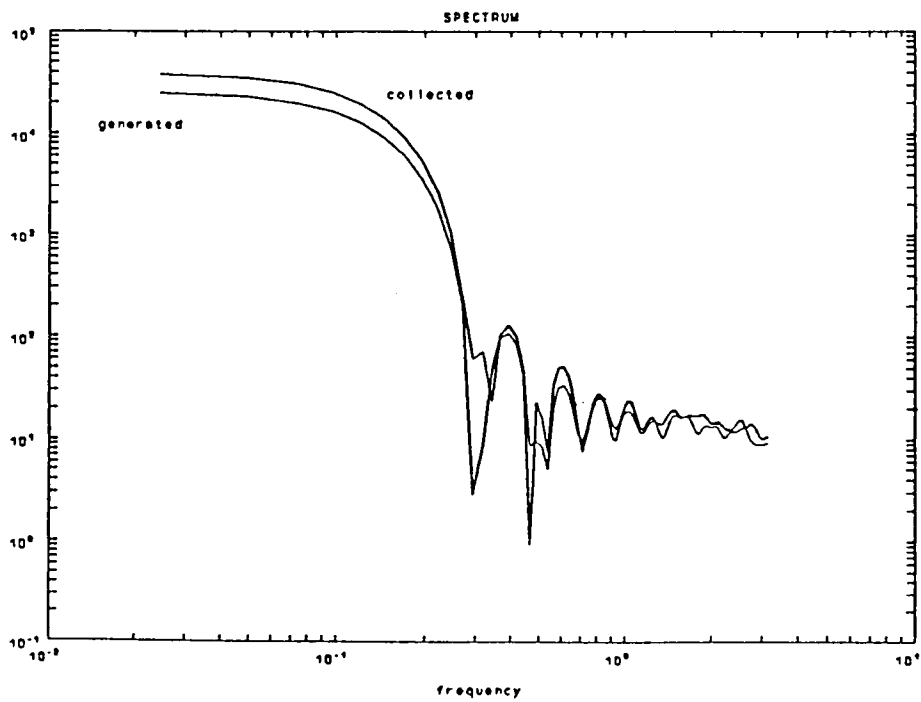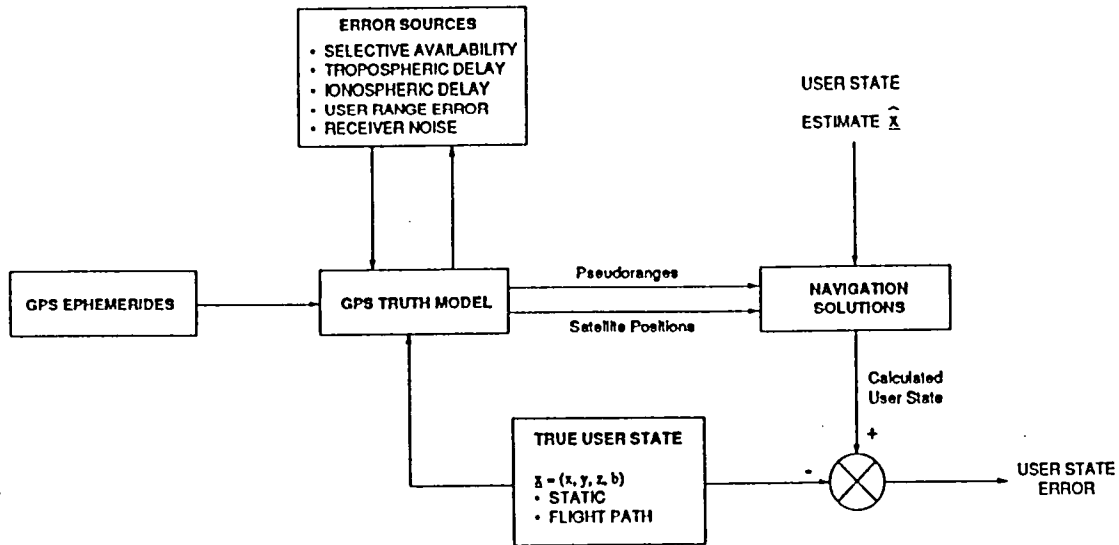

Figure 8. Power spectra of collected and generated data.

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Figure 9. GPS signal model block diagram.
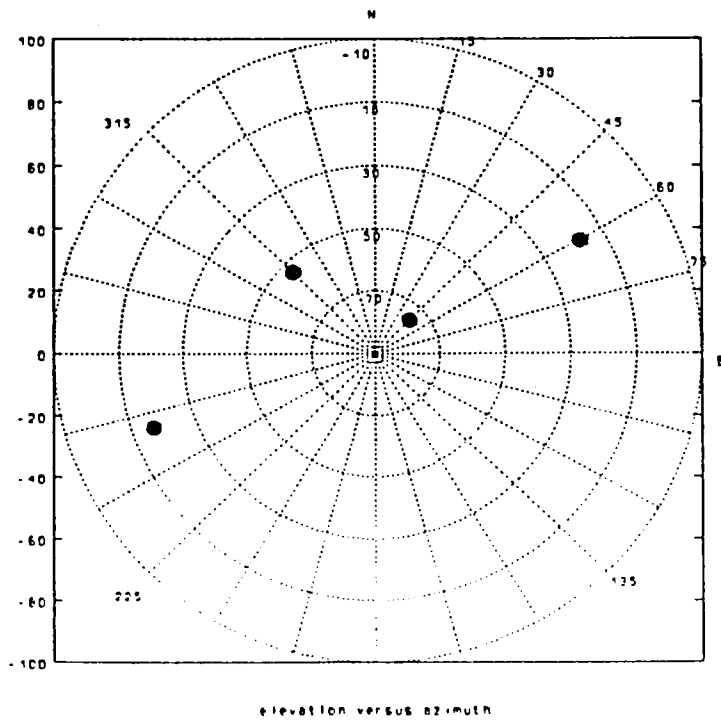
Figure 10. Satellite geometry

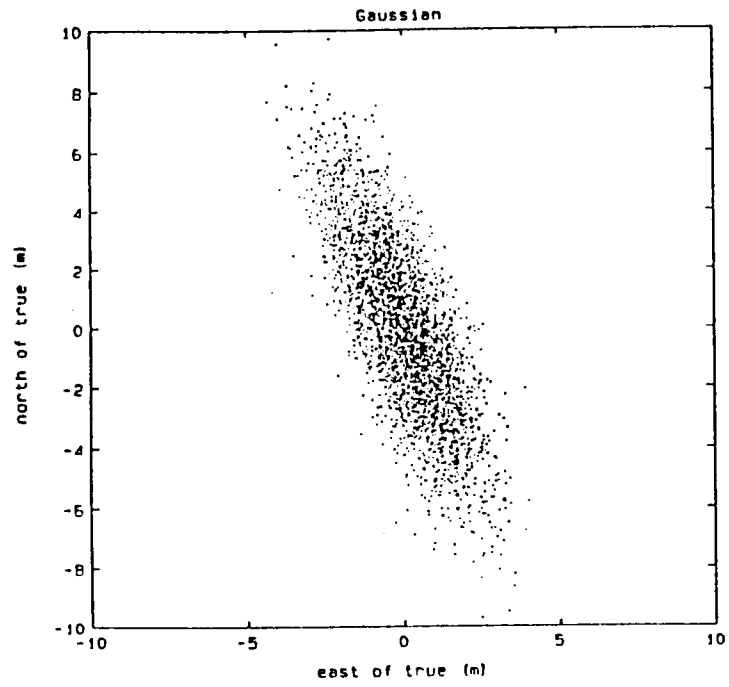Figure 11. Position errors with Gaussian noise on pseudoranges.
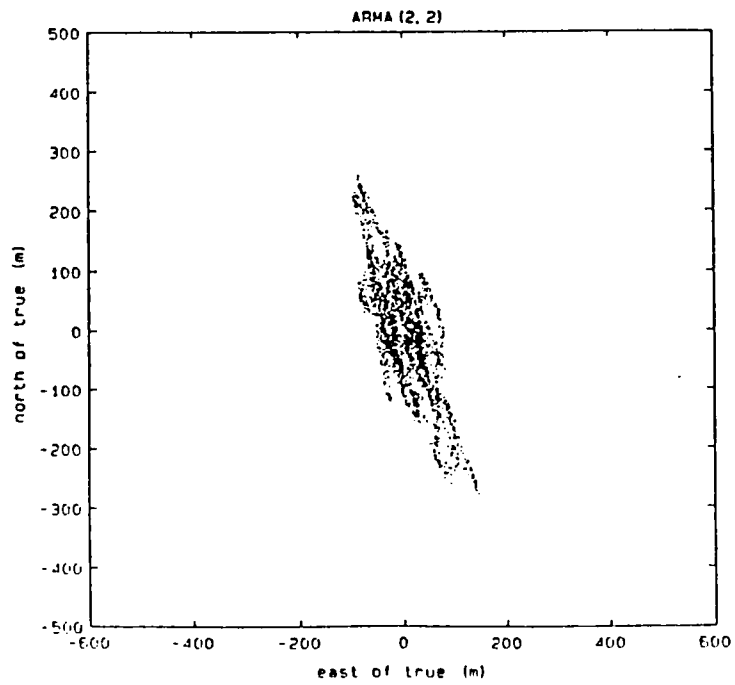

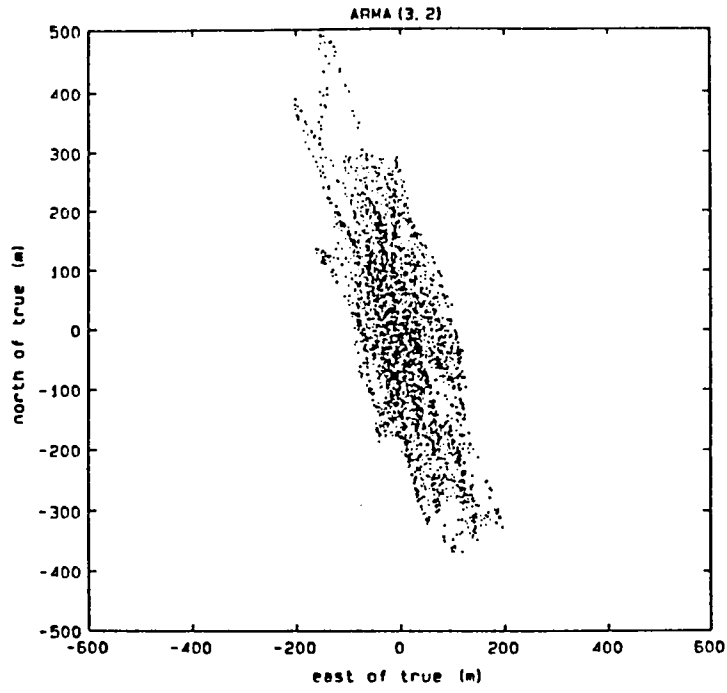Figure 12. Position errors with ARMA(2,2) generated S/A on pseudoranges.

Figure 13. Position errors with ARMA(3,2) generated S/A on pseudoranges.