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INFLUENCE OF ANALYSIS AND DESIGN MODELS ON MINIMUM WEIGHT DESIGN

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BACKGROUND

Many practical structural design problems that can truly benefit from a formal optimization procedure typically involve very large number of degrees of freedom, design variables, and behavioral constraints which are computationally burdensome. While this large dimensionality of the analysis-design models presents no significant computational difficulties from the point of view of achieving an optimum design, it places real limitations on the economic advantages of using optimization methods as routine design tools.

As a means of accommodating the minimum weight design of large problems within reasonable costs, it has been an accepted practice in most structural optimization computer programs to employ a number of approximations that lead to reducing the problem dimensions during various phases in the optimization process. Thus, in addition to reductions in the number of degrees of freedom implied in selecting a particular finite element analysis model, dimensionality during the design phase may be reduced further by imposing certain preselected relationships between the design variables (linking and/or basis reduction), and by temporary deletion of constraints that are not potentially critical.

In this paper, we examine the results of numerical experiments designed to illustrate how the minimum weight design, accuracy, and cost can be influenced by (a) refinement of the finite element analysis model and associated load path problems and (b) refinement of the design variable linking model. The numerical experiments range from simple structures where the modelling decisions are relatively obvious and less costly to the more complex structures where such decisions are less obvious and more costly. All numerical experiments used in this paper employ the dual formulation in ACCESS-3 computer program (1,2).

Guidelines are suggested for creating analysis and design models that predict a minimum weight structure with greater accuracy and less cost. These guidelines can be useful in an interactive optimization environment and in the design of heuristic rules for the development of knowledge-based expert optimization systems.

EXPERIMENT 1

UNIFORMLY LOADED CANTILEVER BEAM

In the first numerical experiment, we consider the optimum weight dependence on the number of design variables (D.V.) and degrees of freedom (D.O.F.) for the cantilever beam of figure 1. The properties are: elastic modulus = 10×10^6 lb/in^2, Poissons's ratio = 0.3, and weight density 0.1 lb/in^3. In the successively refined design and analysis models shown, the design variables are taken as the bar areas and shear panel thicknesses.

The design constraints are:
Displacement upper/lower bound = ± 0.3 in. at the free end
Upper and lower bound on the conbined Von Mises stresses = ± 25000 lb/in^2
Minimum gage = 0.1 in^2 for bar area, and 0.01 in. for panel thickness
The bottom model shows the 24-D.V. and 48-D.O.F. combination

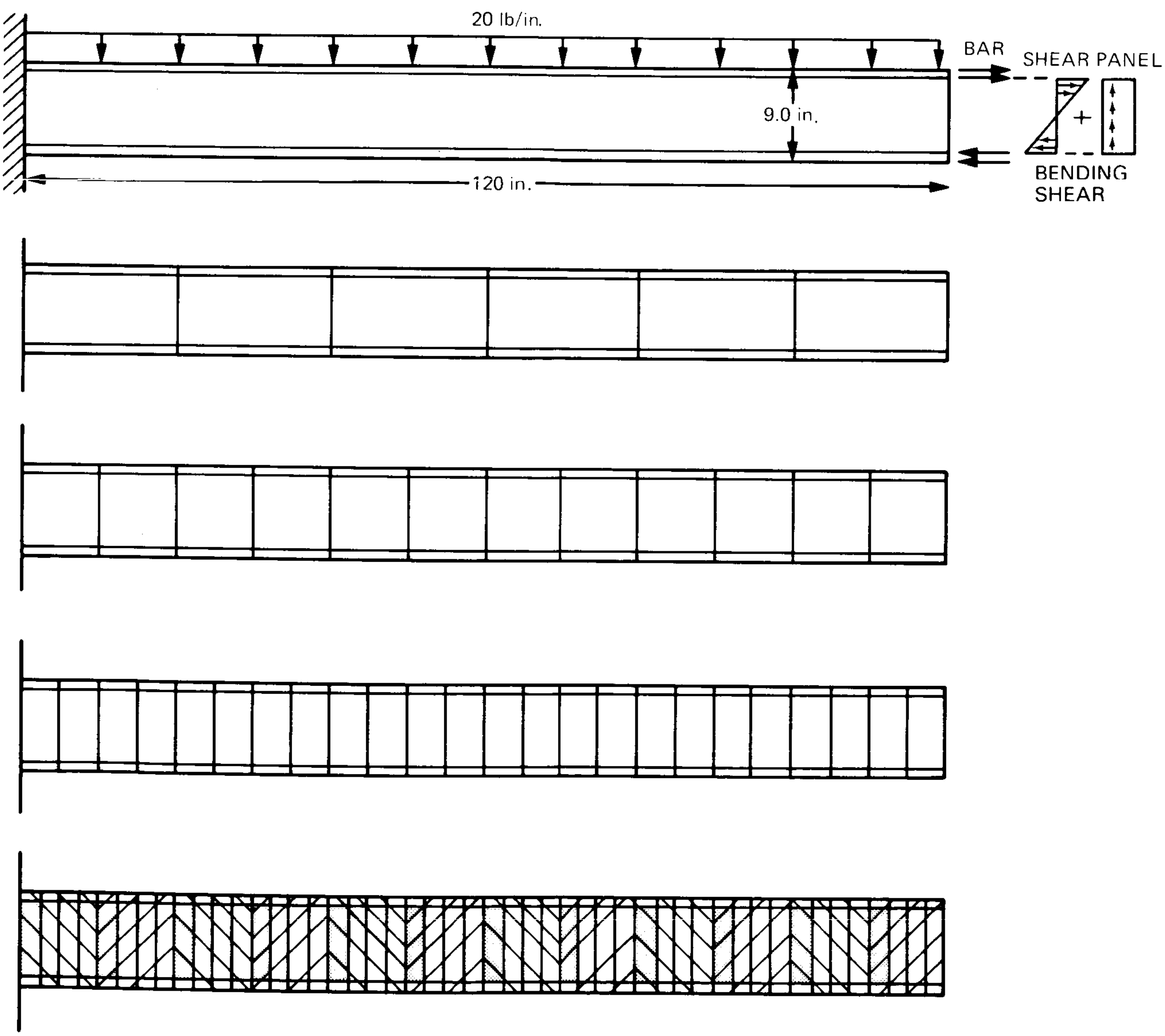


Figure 1

DEFORMATION-CRITICAL CANTILEVER BEAM

In figure 2 below, the optimum weight ($W_{opt.}$) is displayed in figure 2a against changes in the number of D.V., with the number of D.O.F. held constant, and against changes in the number of D.O.F. in figure 2b while holding the D.V. constant.

These results suggest the following observations.

1. A segment of the bar elements of length α from the free end was designed by minimum gage. The distance α increased with refined D.V. and D.O.F. models. Stresses were well below their limits throughout.

2. A greater number of D.O.F. results in higher optimum weight (more flexible structure), while a greater number of D.V. results in lower optimum weight.

3. In an evolving optimization process, it is expedient to start with the practically most crude design and analysis models, then refine both models simultaneously along the dotted line paths (figure 2b), limited by manufacturability constraints and computational cost. This is much less expensive than the analysis-driven alternative of starting with a highly refined analysis model and a crude D.V. model that may be successively refined.

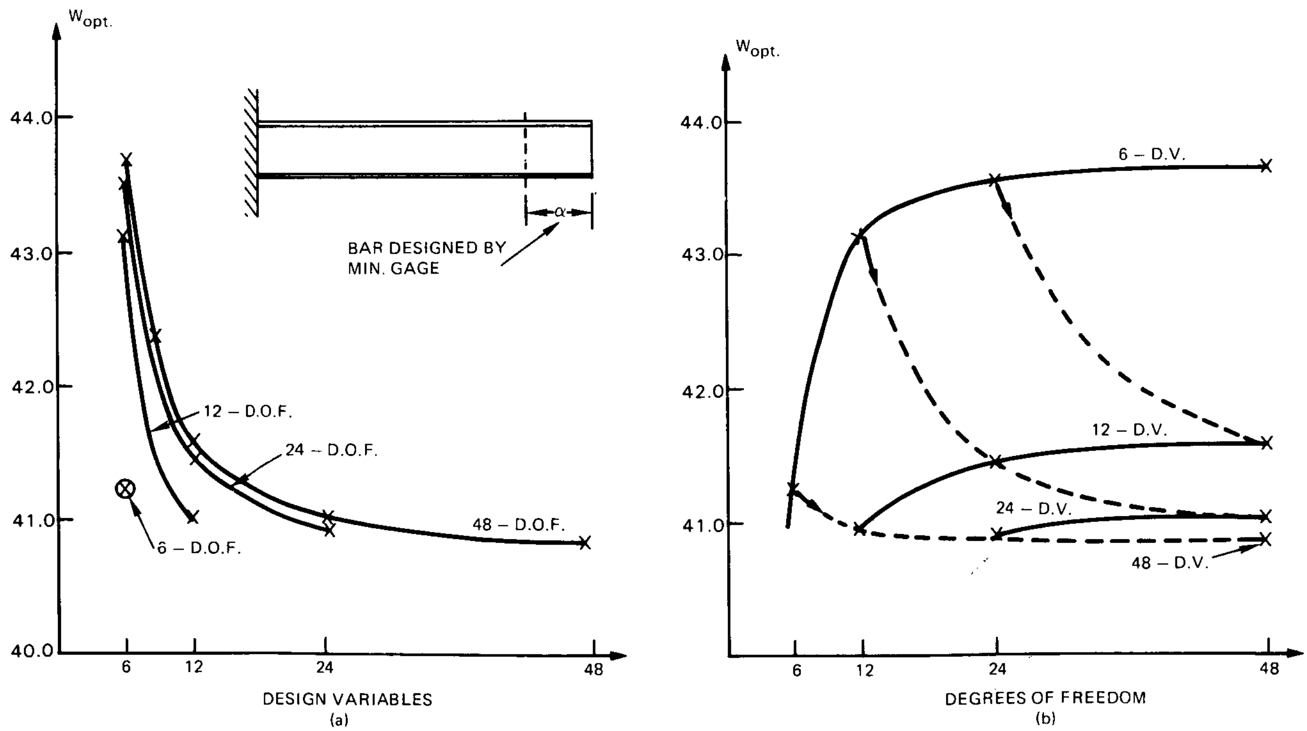


Figure 2

STRESS-CRITICAL CANTILEVER BEAM

Stress criticality was enforced by removing the bound on the free end deformation. From figure 3 below, only the shaded shear panel region of length β was stress-critical, with the bar in that region stressed up to ∿88% of its capacity. The portion α from free end was designed by minimum gage. The sizes of the regions designated by α and β were D.V. and D.O.F. dependent. Of course, had the panel been able to carry only shear, the bar would have been stressed to its fullest.

The same observations (2) and (3) made for the deformation-critical design apply as well in the stress-critical case to a greater extent. Comparison of figures 2 and 3 reveals greater sensitivity of the stress-critical design (over the deformation-critical design) to variations in D.V. and D.O.F. model refinements. This is because stress constraints must be satisfied locally by the linked group, while tip deformation is satisfied globally by contributions of all D.V. groups.

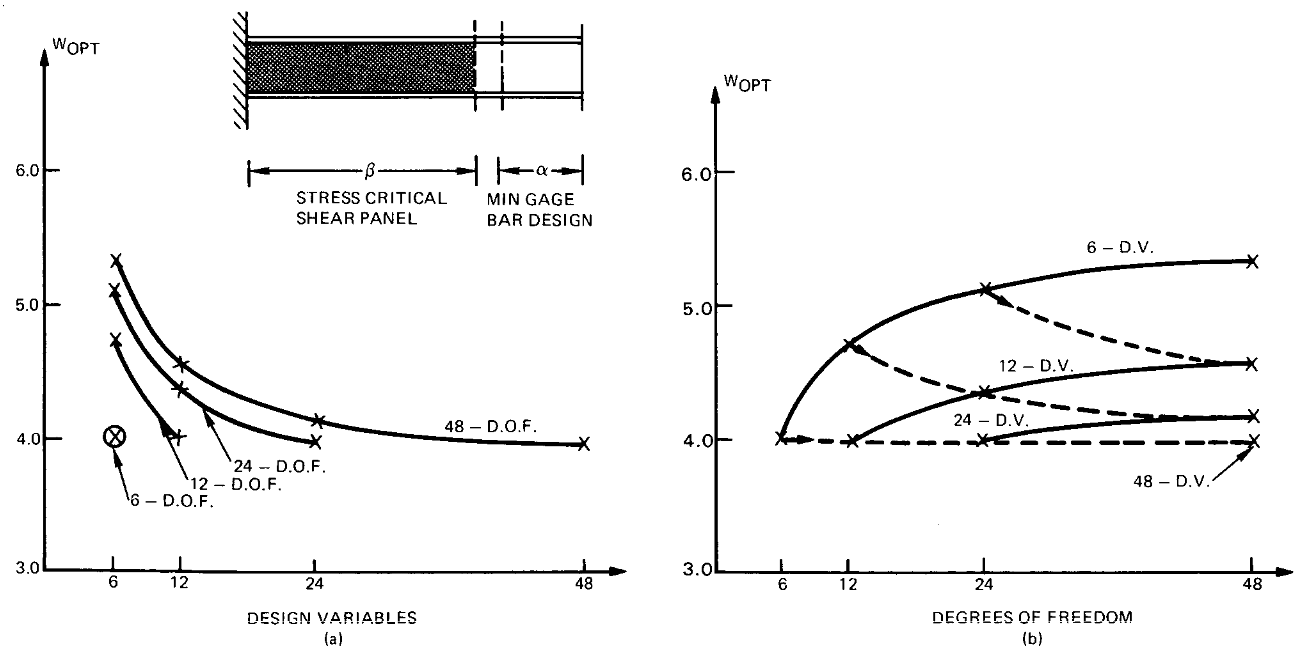


Figure 3

OPTIMIZATION COST FOR STRESS-CRITICAL CANTILEVER

The two figures below display the relative cost of optimizing the cantilever beam as a function of the number of D.V. while holding the number of D.O.F. constant (figure 4a) and as a function of D.O.F. while holding the number of D.V. constant (figure 4b). The cost values are normalized to the smallest (2.2 CPU seconds used for 6 D.O.F. 6 D.V.), and include both analysis and optimization costs. All curves are for 10 analysis/optimization stages that start with the same initial uniform design. Thus figure 4 does not reflect any convergence related costs. The following comments can be made.

1. The relative costs below confirm observation (3) made in connection with figure 2.

2. The total optimization cost includes the analysis cost, which depends upon the number of D.O.F. in the model, and the optimization cost, which depends on the number of potentially active constraints and D.V. Gradient computations, in the present case by the pseudo load method, constitute a large percentage of the optimization costs. This explains the relative insensitivity of cost for a fixed number of D.V. and variable D.O.F., figure 4b, over the cost of a fixed number of D.O.F. and variable D.V., figure 4a.

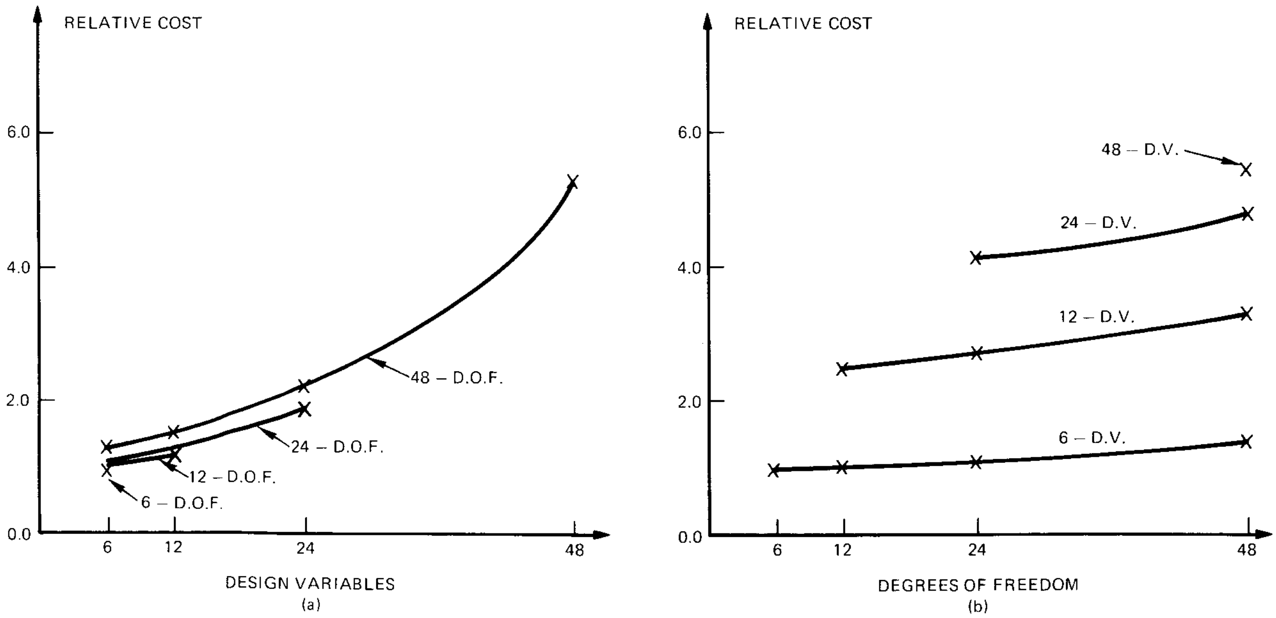


Figure 4

EXPERIMENT 2

LOAD INTRODUCTION IN A FIXED-FIXED BEAM

In complex geometries it is not always possible nor desirable to use spatially uniform analysis and/or design models for optimization. Further, the relative degree of refinement necessary in various regions of the same model is frequently not obvious. Unintended model refinement inaccuracies lead to unbalanced internal loads and incorrect optimum design. This is illustrated here by the symmetric fixed-fixed beam carrying a single load at the center. Thus, the obviously correct symmetric analysis and design models are replaced in the present experiment by the incorrect models of figure 5.

The initially symmetric analysis and design model is successively refined in the number of D.O.F. in the right hand region only, while keeping the design model constant (always with 8-D.V.). The beam properties are the same as in the previous cases.

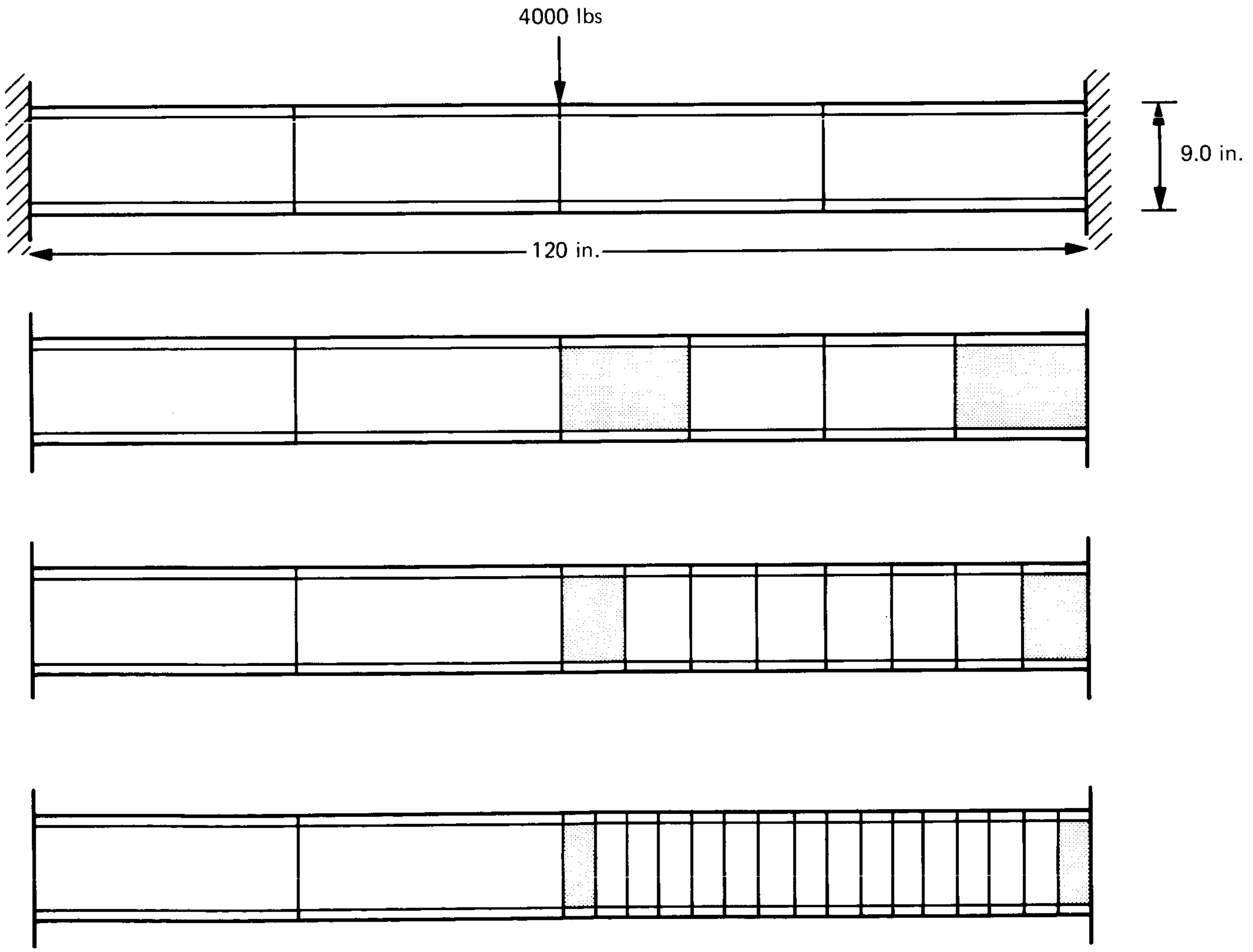


Figure 5

FIXED-FIXED BEAM

The results of figure 6 show how the optimum weight prediction is influenced by incorrect load distribution borne by unbalanced analysis model refinement. Figure 6 suggests the following conclusions.

1. An unbalanced model causes the load to shift toward the stiffer (less refined) region. Consequently larger model imbalance results in a larger increase in weight of the less refined region over the more refined one. In fact, the displacement-critical design exhibited a weight decrease of the refined region with greater D.O.F. refinements, while the weight of the less refined region continued to increase. Reactions computed for the 34-D.O.F. model were:

   Displacement-critical; left = 2530. lb., right = 1470. lb.
   Stress-critical; left = 2470. lb., right = 1550. lb.

2. As in the previous example, the stress-critical design is more sensitive to D.O.F. refinement than the displacement-critical design.

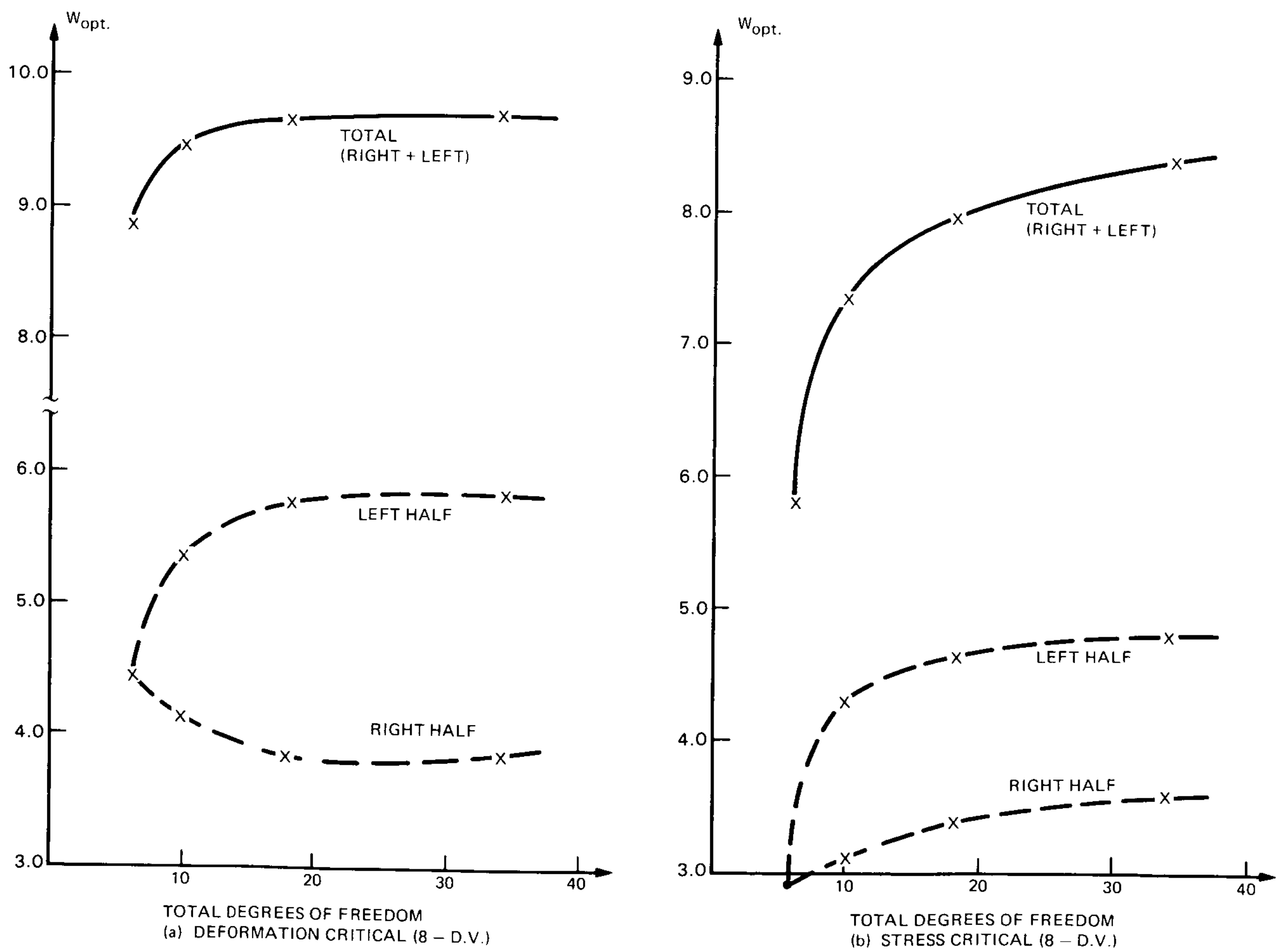
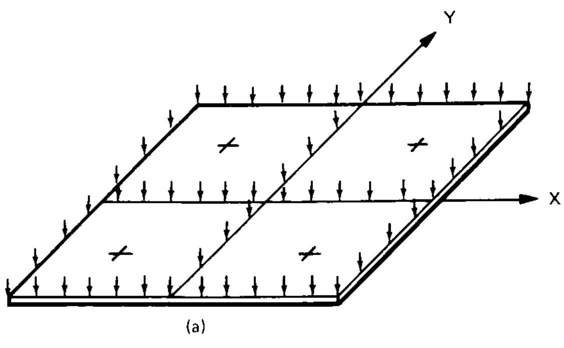


Figure 6

EXPERIMENT 3

4-POINT SUPPORTED RECTANGULAR PLATE

Optimization of the 10 x 12 in., 4-point supported plate of figure 7a is investigated by varying the number of D.O.F. in the model. Some aspects of the same plate problem have been addressed in refs. 2 and 3. The external load and material properties are those of ref. 2. In figure 7b, two models are shown for the symmetric 1/4 plate; a 64-node model (solid lines) with 175 unrestrained D.O.F., and a 225-node model (broken lines) with 644 unrestrained D.O.F. In both cases, the imposed constraints include: minimum plate thickness = .02 in., upper and lower bounds on the out-of-plane displacement = ± .02 in. at the center node and at the exterior corner node, and maximum allowable Von Mises stress σ_v = 25,000. psi for all elements. All elements are triangular plate bending elements linked in 32-D.V. groups as indicated by underlined numbers in the table in figure 8.



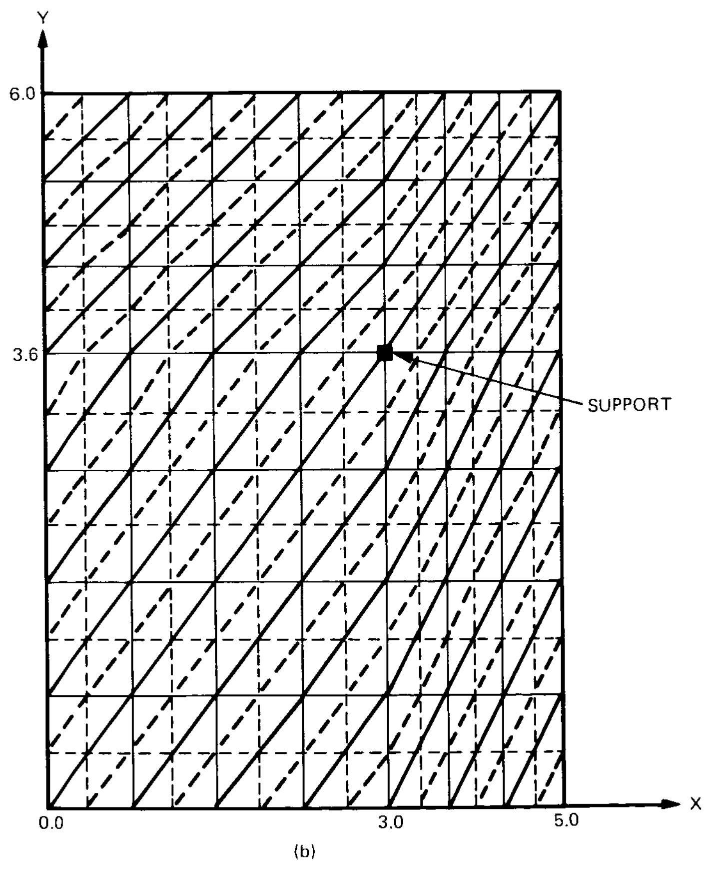


Figure 7

OPTIMIZATION RESULTS FOR PLATE EXAMPLE

Optimization started with the same initial design of 0.71 in uniform thickness and 5.282 lb weight for both the 64-node and the 225-node models. After 12 analysis/design stages, the 64-node model converged relatively smoothly to 2.386 lb, figure 8a, with the displacement at the exterior corner node about 90% critical and the stresses either below critical or up to 0.3% infeasible in the "starred" groups 3, 28, and 30, figure 8b. Optimization for the refined 225-node model proved to be more difficult. Although the corner node displacement near criticality is reduced to only 60%, all designs produced by stages 3 through 15 oscillated in stress infeasibility from a high of ~120% to a low of 18.5% at the 15th stage. The move limit used was 100%. Further iteration stages with 10% move limit did not result in reducing the infeasible stresses. This indicates that the infeasibility is more likely to be due to errors in the refined model stress calculations rather than being due to errors in generation of the approximate dual optimization problem. In such cases, it may be desirable to use D.V. values of the least infeasible design after scaling by the infeasibility. In figure 8b, the quantities given in parentheses are the thicknesses produced by the 15th stage (refined model) after scaling by $\sqrt{1.185}$. The corresponding weight is 2.81 lb.

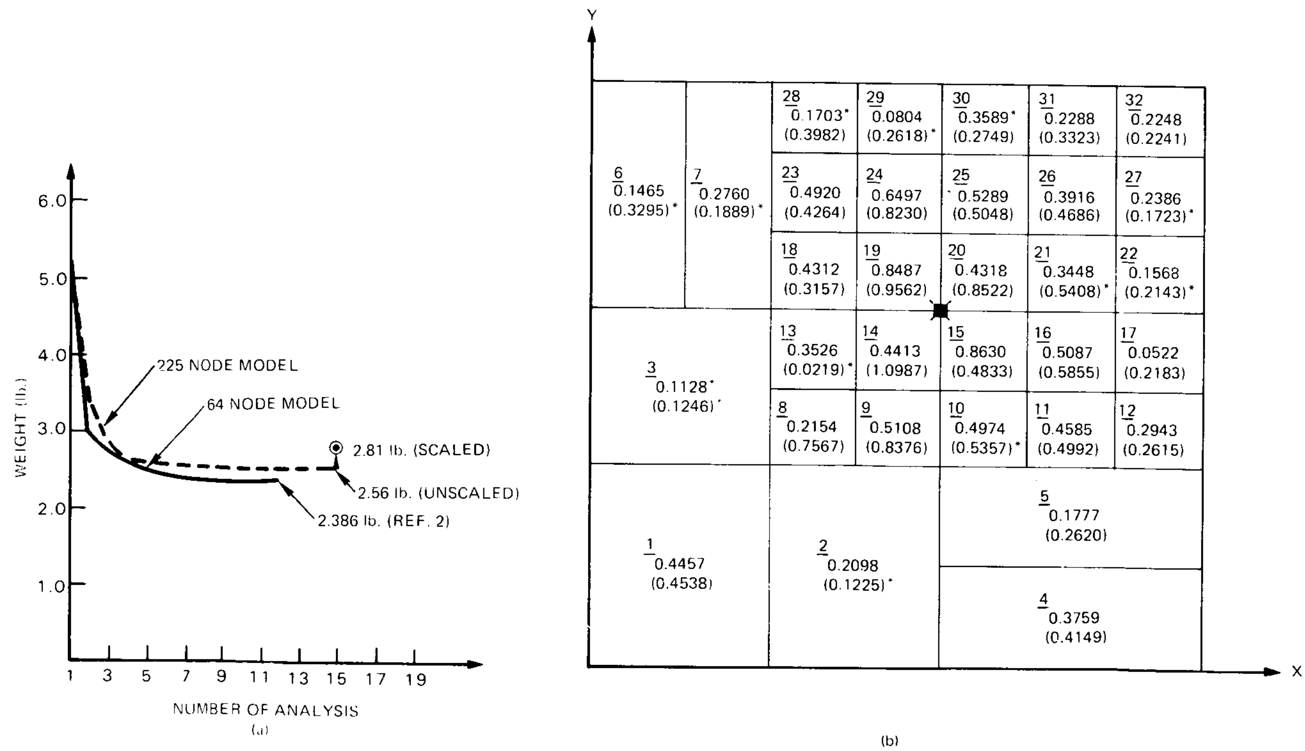


Figure 8

EXPERIMENT 4

ANTENNA STRUCTURE

The optimization of the antenna structure of figure 9 has been addressed in refs. 2 and 4. However, attention is focused here on exploring design model refinement in an interactive environment. The optimization begins with a 60-D.V. model, then continues with a 90-D.V. model and finally a 125-D.V. model. All three models consist of 340 nodes connected by 1149 axial members for the symmetric half of the antenna under symmetric wind loading of ref. 2. The structures consist of radial rib trusses R1, R2, ..., R4 and interconnecting hoop trusses C0, C1, ..., C9. The design constraints are: 33 displacement constraints limiting the Z-deformation at the outer circumference C9 to ±1.0 in., 18 slope constraints along the rib truss coinciding with the X-axis to limit the slope in the XZ plane to ±.0075, stress constraints on all members (not to exceed ±25,000. psi), and minimum gage = 0.2 in.^{2} on the area of all D.V. groups.

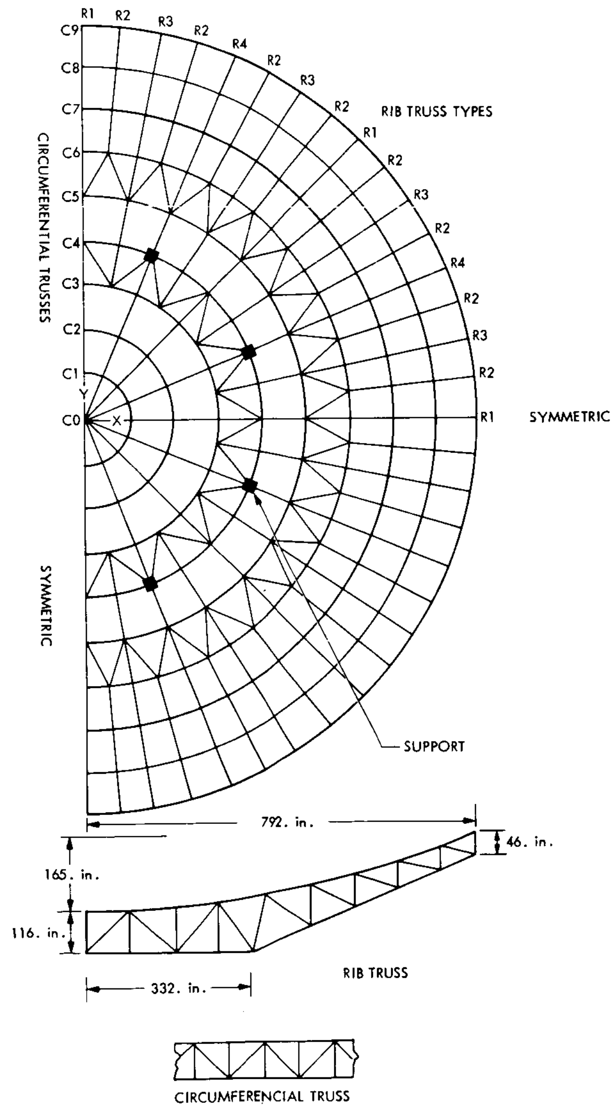


Figure 9

SUCCESSIVE OPTIMIZATION OF ANTENNA

Several strategies may be used for a systematic D.V. model refinement of a complex problem. For example, one may concentrate on refining only those D.V. with values larger than a certain threshold; i.e., where weight reduction has the highest potential. Alternatively, one may consider refining all D.V. that are near critical with respect to either all behavior and side constraints or only a selected set of these constraints. Of course, any strategy used may have to include other problem-peculiar possibilities, and should be tempered with practical manufacturing variability limitations. In the present antenna example, refinement from the 60-D.V. model to the 90-D.V. model is achieved by breaking down all D.V. not at the minimum gage while keeping the top and bottom members identical. The rib horizontals are excepted. Further refinement from the 90-D.V. model to the 125-D.V. model is accomplished by breaking the linking between the top and bottom members. Table 1 gives the D.V. values resulting from the three successive optimizations. As can be seen, the refinements allowed for a redistribution of internal loads, and consequently for redistribution of the structural weight. This is also evident from figure 10a which shows that the total weight is reduced from 9250. lb. (at the end of the 60-D.V. optimization) to 6614. lb. (at the end of the 125-D.V. optimization). In figure 10b, the cumulative optimization cost is displayed.

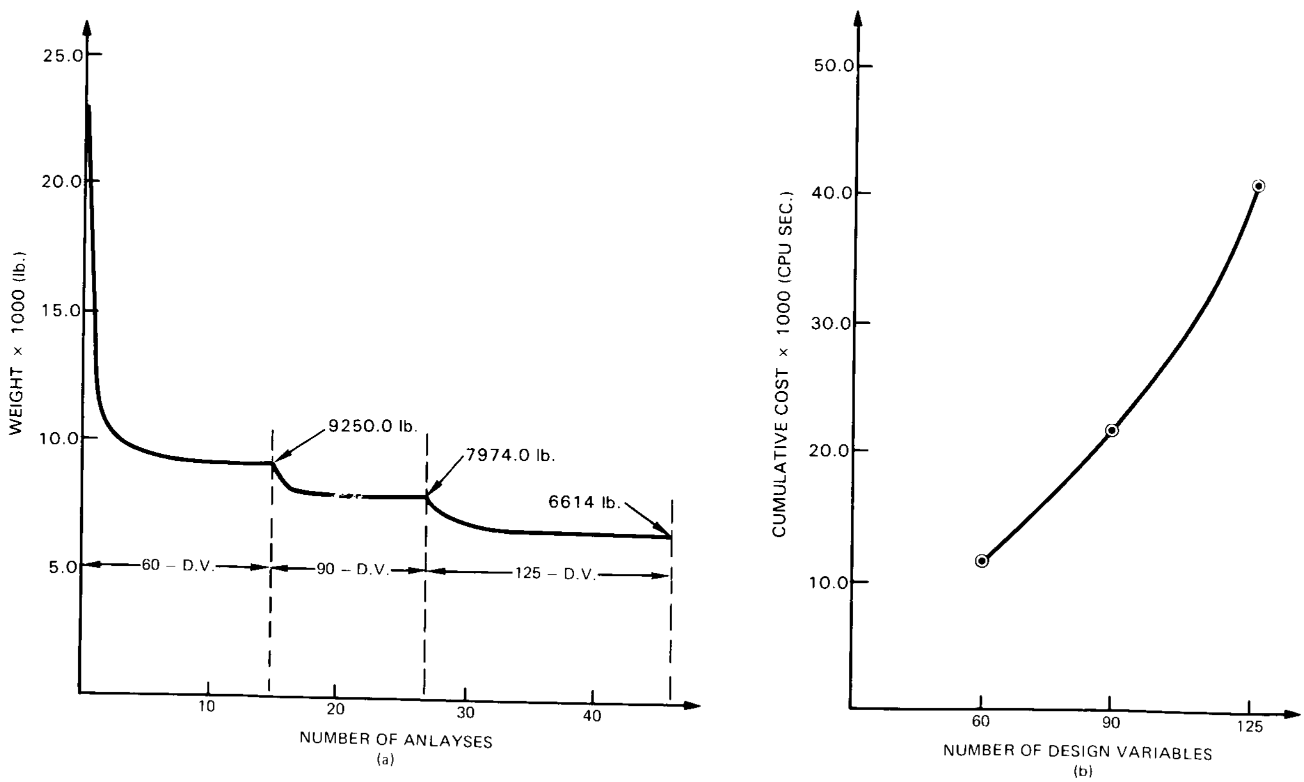


Figure 10

TABLE 1
OPTIMAL CROSS SECTIONAL AREAS FOR ANTENNA

Designation	OPTIMUM (in^2) 60 D.V.	90 D.V.	125 D.V.
Rib Horizontal Type 1			
C0-C4	.200	.200	.200
			.200
C4-C5	.636	.452	.238
			.270
C5-C6	.552	.476	.384
			.200
C6-C7	.774	.652	.200
			.490
C7-C8	.542	.447	.327
			.409
C8-C9	.396	.338	.469
			.200
Rib Horizontals Type 2			
C6-C7	.483	.483	.483
C7-C8	.486	.485	.483
C8-C9	.514	.500	.536
Rib Horizontals Type 3			
C3-C4	.495	.387	.200
			.200
C4-C5	.852	.582	.200
			.240
C5-C6	.664	.676	.325
			.231
C6-C7	.983	.825	.204
			.863
C7-C8	.753	.637	.515
			.484
C8-C9	.468	.430	.560
			.200
Rib Horizontals Type 4			
C0-C4	.532	.213	1.761
			.200
		.389	1.892
			.200
		.452	1.934
			.200
		.771	2.117
			.200
C4-C5	2.352	3.221	.672
			3.674
C5-C6	1.245	1.665	.253
			1.501
C6-C7	1.321	1.353	.200
			1.440
Rib Horizontals Type 4 (Cont'd)			
C7-C8	.958	.926	.534
			.823
C8-C9	.519	.495	.567
			.200
Horizontal Diagonals			
C3-C4	.623	.495	.200
			.200
C5-C6	.410	.534	.359
Rib Diagonals			
C0-C1	.200	.200	.200
		.200	.200
C1-C2	.200	.200	.200
		.252	.200
C2-C3	.300	.200	.200
		.498	.200
C3-C4	.252	.200	.200
		.200	.200
		.548	.200
C4-C5	1.142	.200	.377
		.200	.200
		2.887	3.292
C5-C6	1.817	.641	.859
		1.207	.973
		3.473	2.735
C6-C7	.200	.200	.200
		.200	.200
		.200	.200
C7-C8	.367	.200	.200
		.259	.330
		.649	.609
C8-C9	.200	.200	.200
		.200	.200
		.200	.200
Verticals			
C0	.200	.200	.200
C1	.200	.200	.200
C2	.200	.200	.200
C3	.200	.200	.200
C4 (R1)	2.575	.200	.200
(R3)		.200	.200
(R4)		4.112	3.578
C5 (R1)	2.146	.445	.354
(R3)		1.344	1.087
(R4)		5.200	4.836
Verticals (Cont'd)			
C6	.200	.200	.200
C7 (R1)	.350	.200	.200
(R3)		.246	.275
(R4)		.486	.432
C8	.200	.200	.200
C9	.200	.200	.200
Hoop Horizontals			
C1	.200	.200	.200
			.200
C2	.200	.200	.200
			.200
		.200	.219
			.200
C3	.500	.583	.200
			.306
		.200	.200
			.200
C4	.507	.337	.200
			.200
C5	.200	.284	.200
			.434
C6	3.323	2.163	.200
			.334
		3.296	4.346
			4.417
C7	.200	.200	.200
			.200
C8	.235	.200	.481
			.200
C9	4.348	4.306	5.268
			5.348
		4.047	.490
			.280
Hoop Diagonals			
C1	.200	.200	.200
C2	.200	.200	.200
C3	.200	.200	.200
C4	1.256	.200	.200
		.808	.300
C5	.326	.200	.200
		.977	1.275
C6	.200	.200	.200
C7	.200	.200	.200
C8	.200	.200	.200
C9	.289	.283	.200

CONCLUSIONS

In the optimization of complex structures, selection of the most suitable analysis and design models is frequently not obvious. Accuracy and cost are always limiting and competing factors. Further, design optimization usually is not a one-time process, but is one that evolves with evolution of the design details.

In the preceding pages, results of numerical experiments were discussed to show trends that may be employed in devising cost effective optimization procedures consistent with an evolutionary optimization philosophy, whether it is carried out in the context of a man-machine interactive environment or in the context of an automated expert system. Specifically, the results deal with (1) optimum weight accuracy and associated cost advantages of starting the optimization process with the practically most crude D.V. and D.O.F. models, then simultaneously refining both models in subsequent optimizations; (2) effect of model imbalance on the resulting optimal weight; (3) design infeasibility as a typical difficulty in large problems optimization, and how it may be dealt with; (4) a criterion for selection of successive D.V. model refinements.

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