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# APPLICATIONS OF CONMIN TO WING DESIGN OPTIMIZATION WITH VORTEX FLOW EFFECT

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## AERODYNAMIC ANALYSIS OF SLENDER WINGS

Slender wings on supersonic cruise configurations are expected to be thin and highly swept. As a result, edge-separated vortex flow is inevitable and must be accounted for in aerodynamic analysis and design. The present method is based on the method of suction analogy (ref. 1) to calculate the total aerodynamic characteristics. The method requires the solution of the attached flow problem, the latter being solved by a low-order panel method in subsonic and supersonic flow (ref. 2). In essence, the lifting pressure is calculated by using a pressure-doublet distribution satisfying the Prandtl-Glauert equation. From the pressure distribution, the leading-edge suction is calculated. The latter is assumed to be the vortex lift through the method of suction analogy. For a cambered wing, the location of vortex-lift action point is important in predicting the aerodynamic characteristics (ref. 2). It is also seen that the effect of camber shape appears nonlinearly in all aerodynamic expressions. (See fig. 1.)

Prandlt-Glauert Equation: $$(1-M^2)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0$$

Boundary Condition: $$[A]\{\Delta C_p\}=\left\{\frac{\partial z_c}{\partial x}\cos\alpha-\sin\alpha\right\}$$

Lifting Pressure: $$\Delta C_p'=\Delta C_p\cos\alpha-2\gamma_x\sin\alpha\sin\left(\frac{\partial z_c}{\partial y}\right)$$

Leading-Edge Suction: $$c_s=c_t\,\frac{\sqrt{1+\left(\frac{\partial z_c}{\partial x}\right)^2+\left(\frac{\partial z_c}{\partial y}\right)^2}\,\left|\frac{d\vec{r}}{dy}\right|}{\left[\left(\frac{\partial z_c}{\partial x}\frac{dz_\ell}{dy}+1\right)^2+\left(-\frac{\partial z_c}{\partial x}+\frac{\partial z_c}{\partial y}\tan\Lambda\right)^2\right]^{1/2}}$$

$$\left|\frac{d\vec{r}}{dy}\right|=\sqrt{1+\tan^2\Lambda+\left(\frac{dz_\ell}{dy}\right)^2}$$

Sectional Characteristics: $$c_{\ell,p}=\frac{1}{c}\int_{x_{\ell e}}^{x_{te}}\Delta C_p'\left(\frac{\partial z_c}{\partial x}\sin\alpha+\cos\alpha\right)\Big/\sqrt{1+\left(\frac{\partial z_c}{\partial x}\right)^2+\left(\frac{\partial z_c}{\partial y}\right)^2}\,dx$$

$$c_{d,p}=\frac{1}{c}\int_{x_{\ell e}}^{x_{te}}\Delta C_p'\left(-\frac{\partial z_c}{\partial x}\cos\alpha+\sin\alpha\right)\Big/\sqrt{1+\left(\frac{\partial z_c}{\partial x}\right)^2+\left(\frac{\partial z_c}{\partial y}\right)^2}\,dx$$

$$c_{\ell,v\ell e}=c_s\left(\frac{\partial z_c}{\partial x}\sin\alpha+\cos\alpha\right)\Big/\sqrt{1+\left(\frac{\partial z_c}{\partial x}\right)^2+\left(\frac{\partial z_c}{\partial y}\right)^2}$$

$$c_{d,v\ell e}=c_s\left(-\frac{\partial z_c}{\partial x}\cos\alpha+\sin\alpha\right)\Big/\sqrt{1+\left(\frac{\partial z_c}{\partial x}\right)^2+\left(\frac{\partial z_c}{\partial y}\right)^2}$$

Action Point of Vortex Lift: $r=c_s c$

## DESIGN PROBLEMS

To design the camber shape, the camber slope is represented by a cosine Fourier series at each of several spanwise stations. The Fourier coefficients are the design variables. To design a leading-edge flap in the vortex flow (i.e. a vortex flap), the coordinates of corner points and the deflection angle are the design variables. The process of wing design is to determine the camber shape and twist distribution such that an objective function, typically the drag, is minimized, subject to various constraints (fig. 2). The latter may include the lift, the magnitude of maximum geometric twist and the magnitude of vortex lift. Other types of constraint are possible (ref. 3). The design of a vortex flap can be described in a similar manner.

Camber Representation for Each Spanwise Station:

$$\left(\frac{\partial z_c}{\partial x}\right)_k = \sum_{j=1}^{N} a_j \cos (j-1)\,\theta_k$$

$$\theta_k = \frac{(2k-1)\pi}{2N}$$

Design Variables: $a_j$

Optimization Problem:

Minimize $F = -\dfrac{C_L}{C_{D_0} + C_{D_1}}$

Subject to Constraints ($G_j$) of

(1) A Given Lift Coefficient

(2) A Given Ratio of Vortex Lift to Total Lift

(3) Magnitude of Local Angles of Attack (Twist)

Figure 2

WING DESIGN OPTIMIZATION BY "CONMIN"

To achieve the wing design optimization, the aerodynamic analysis method is coupled with CONMIN - Constrained Function Minimization Program (refs. 4 and 5). In a typical wing design problem, as many as 70 design variables may be employed. The solution is determined iteratively.

The process starts with the calculation of values of objective and constraint functions for the input design variables. Gradients of these functions are then calculated and CONMIN will determine the best way of changing the design variables to achieve the minimum drag design without violating constraints. (See fig. 3.)

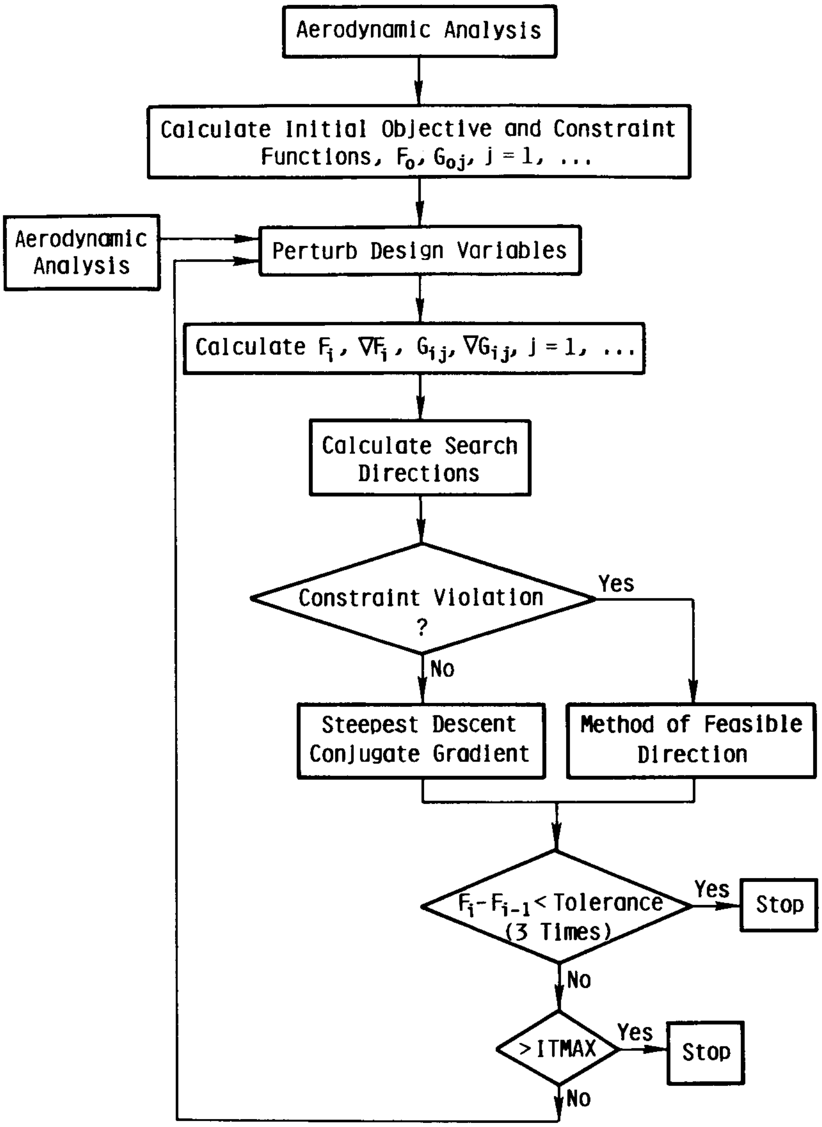


Figure 3

## RESULTS OF VORTEX FLOW AND ATTACHED-FLOW DESIGN FROM AN INITIAL FLAT SURFACE $C_{L_{des}} = 0.3$, M = 0, TWIST ⩽ 8 DEG

Design results very much depend on the imposed constraints. For simplicity, only the constraint of having the lift coefficient, twist and vortex lift be greater than 5% of total lift in the case of the vortex flow design will be imposed in the present study. Results show that starting from an initial flat surface, the final camber shape designed with the vortex flow concept (called the VF design) is similar to that designed with the attached-flow concept (called the AF design) except near the root and the tip. In the mid-semispan region, the surface has large aft camber. Note that if the effect of $\partial z_c/\partial_y$ is ignored and the twist constraint is not imposed, as is usually done in a conventional method, the resulting shapes would involve larger forward camber and unrealistically large twist (ref. 3 and 6). (See fig. 4.)

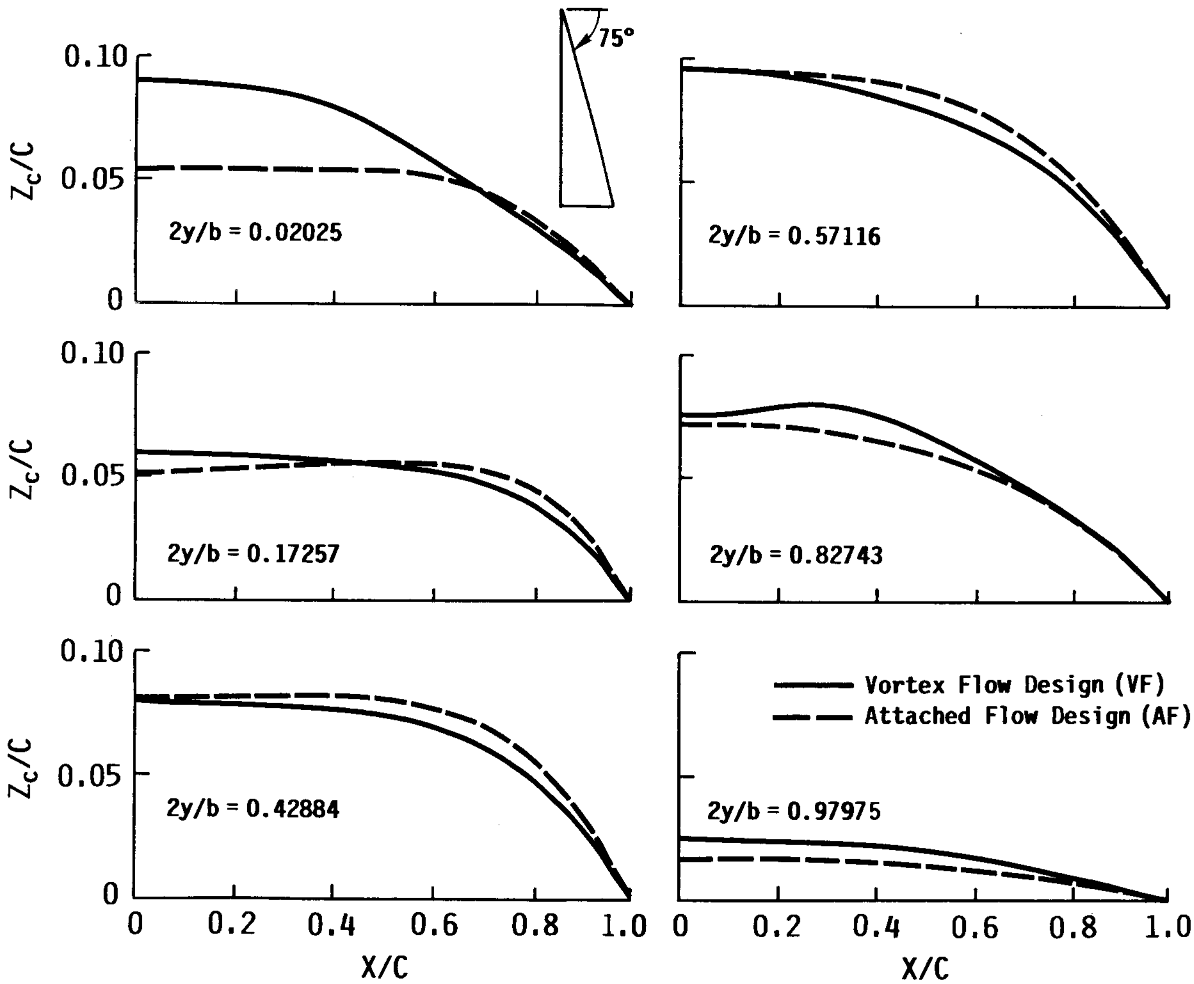


Figure 4

VORTEX FLOW DESIGN FROM DIFFERENT STARTING SHAPES
$C_{L_{des}} = 0.3$, M = 0, TWIST ≤ 8 DEG

In the present nonlinear optimization problem with a large number of design variables, the final solution depends on the initial input. In other words, there are many relative minima in the design space. To show this, the vortex flow design with the same constraints is calculated from three input shapes - a flat surface, the camber shape of the attached-flow design in figure 4 reduced by 10%, and the original camber shape of the attached-flow design. As shown in figure 5, the results are all different. To determine a solution close to a global minimum, Vanderplaats et al. (ref. 7) suggested beginning the optimization from several different initial designs.

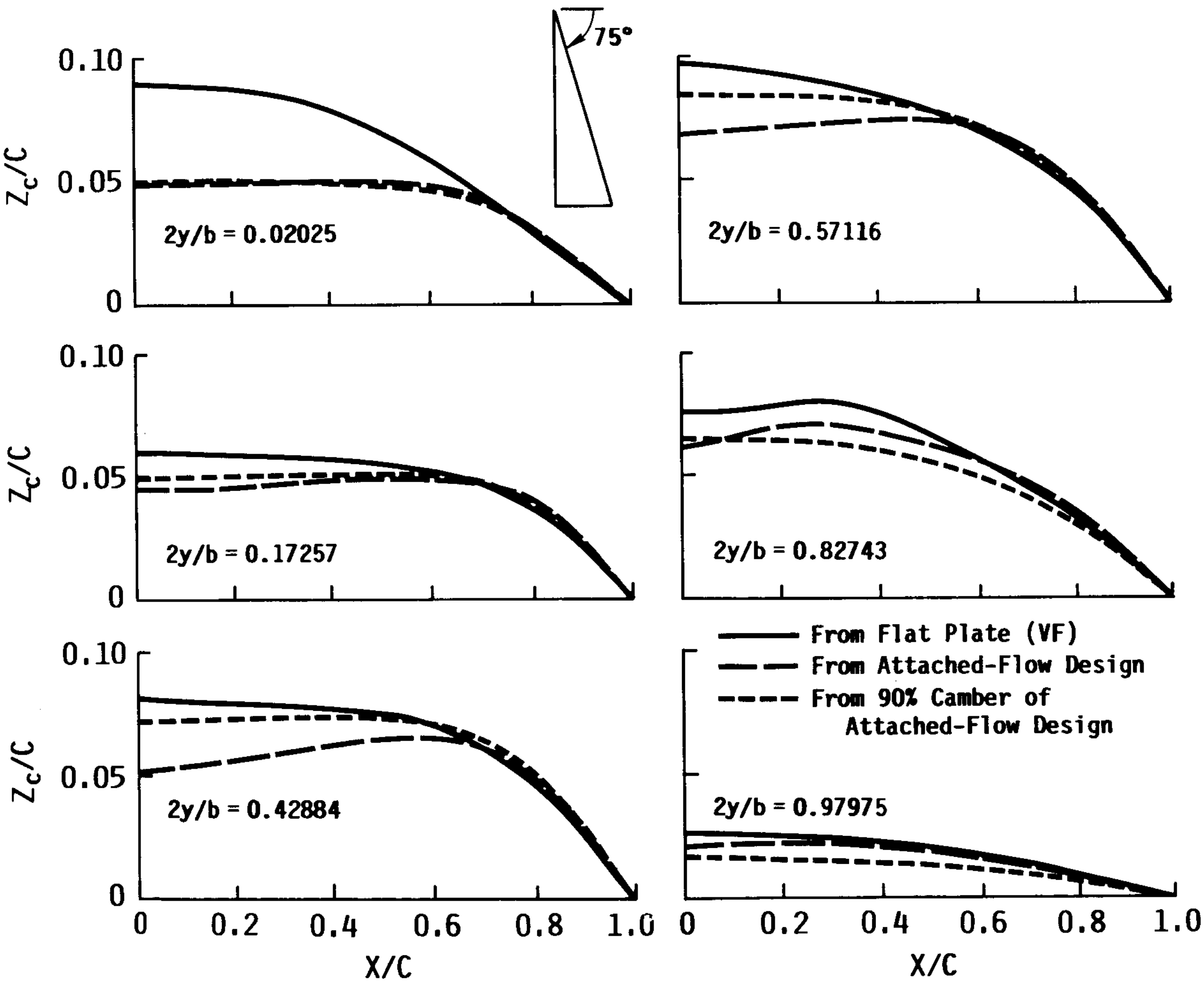


Figure 5

DESIGN FROM A FLAT SURFACE FOR A DESIGN LIFT COEFFICIENT OF 0.6, M = 0, and TWIST ≤ 8 DEG

By increasing the design lift coefficient to 0.6, the attached-flow concept did not produce a feasible design under the specified twist constraint. However, a converged feasible design was easily obtained under the vortex flow concept. It is seen from figure 6 that at high lift forward camber is needed to provide more forward-facing surface to produce a thrust force.

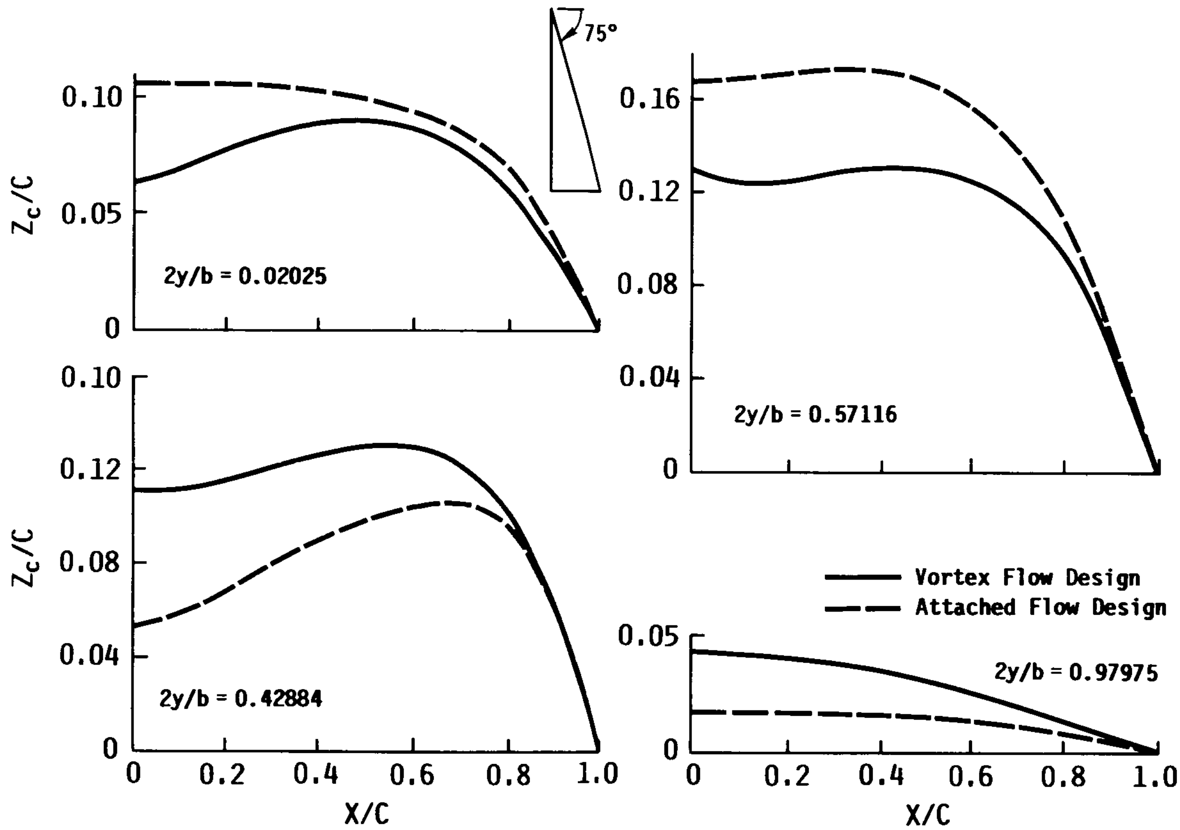


Figure 6

PARTIAL VORTEX FLOW DESIGN

$C_{L_{des}} = 0.3$, TWIST ≤ 8 DEG

In applications, there are always some other practical constraints, such as some inboard portion being specified. The present method allows a certain portion of the planform to be unchanged during the optimization process. For example, in figure 7 three types of vortex flow design are presented, one started from a flat surface, the other from the attached-flow design, and the last one (called the VPA design) also from the attached-flow design, but with the inboard one-third fixed. The results show that in the VPA design more forward-facing surface exists in the outboard portion to take advantage of the vortex-induced thrust.

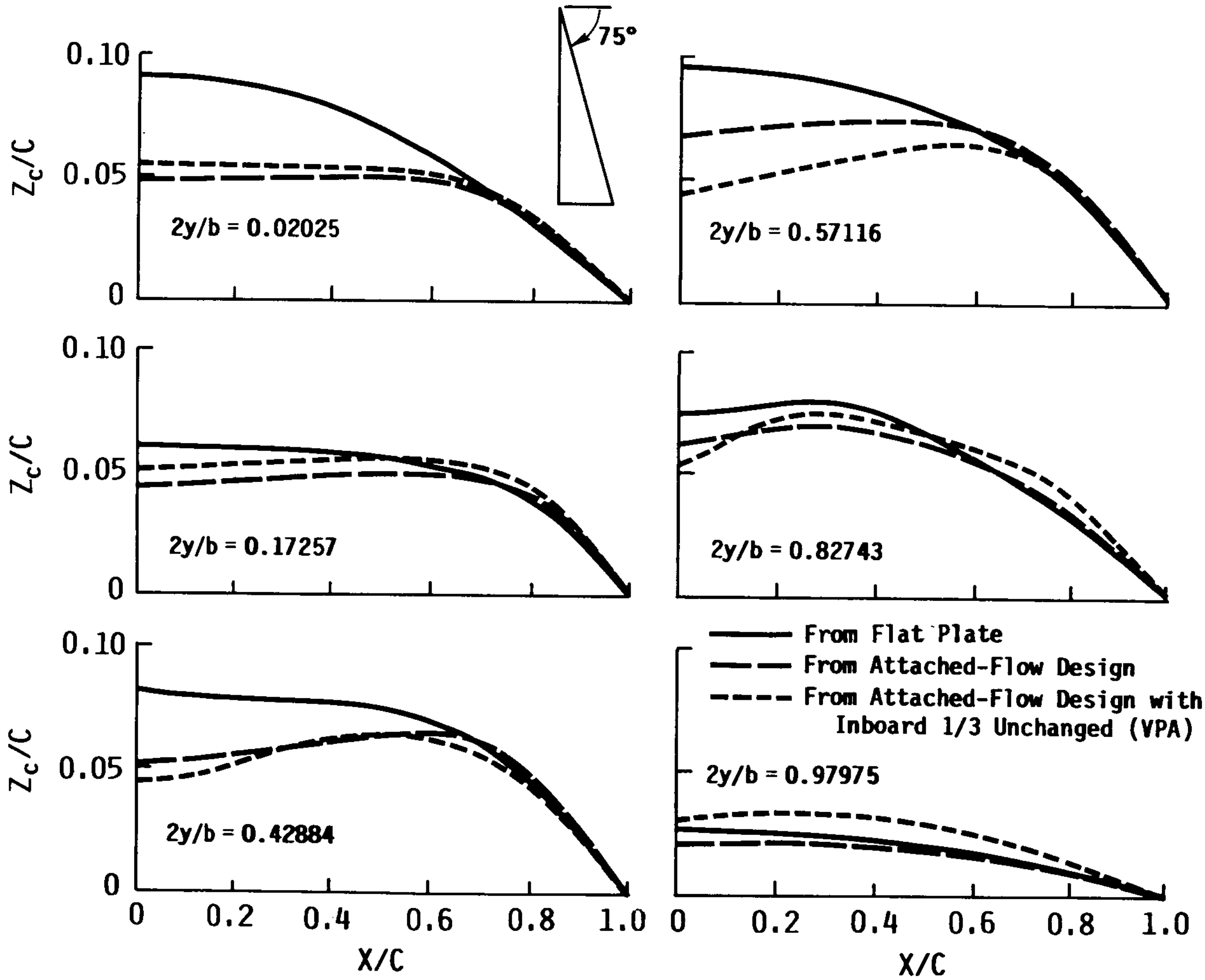


Figure 7

PERFORMANCE COMPARISON OF DIFFERENT DESIGN CONCEPTS

$C_{L_{des}} = 0.3$, $M = 0$, TWIST $\leq$ 8 DEG

At the design lift coefficient of 0.3, all three design configurations have about the same $C_{D_i}$, reaching the planar minimum value of $C_L^2/\pi A$. At off-design conditions, the attached-flow design (AF) would be superior if the flow could remain attached. When the flow is separated, the AF design (not shown) will have approximately the same performance as the VF design. On the other hand, the partial vortex flow design (VPA) seems to be superior throughout the off-design $C_L$. (See fig. 8.)

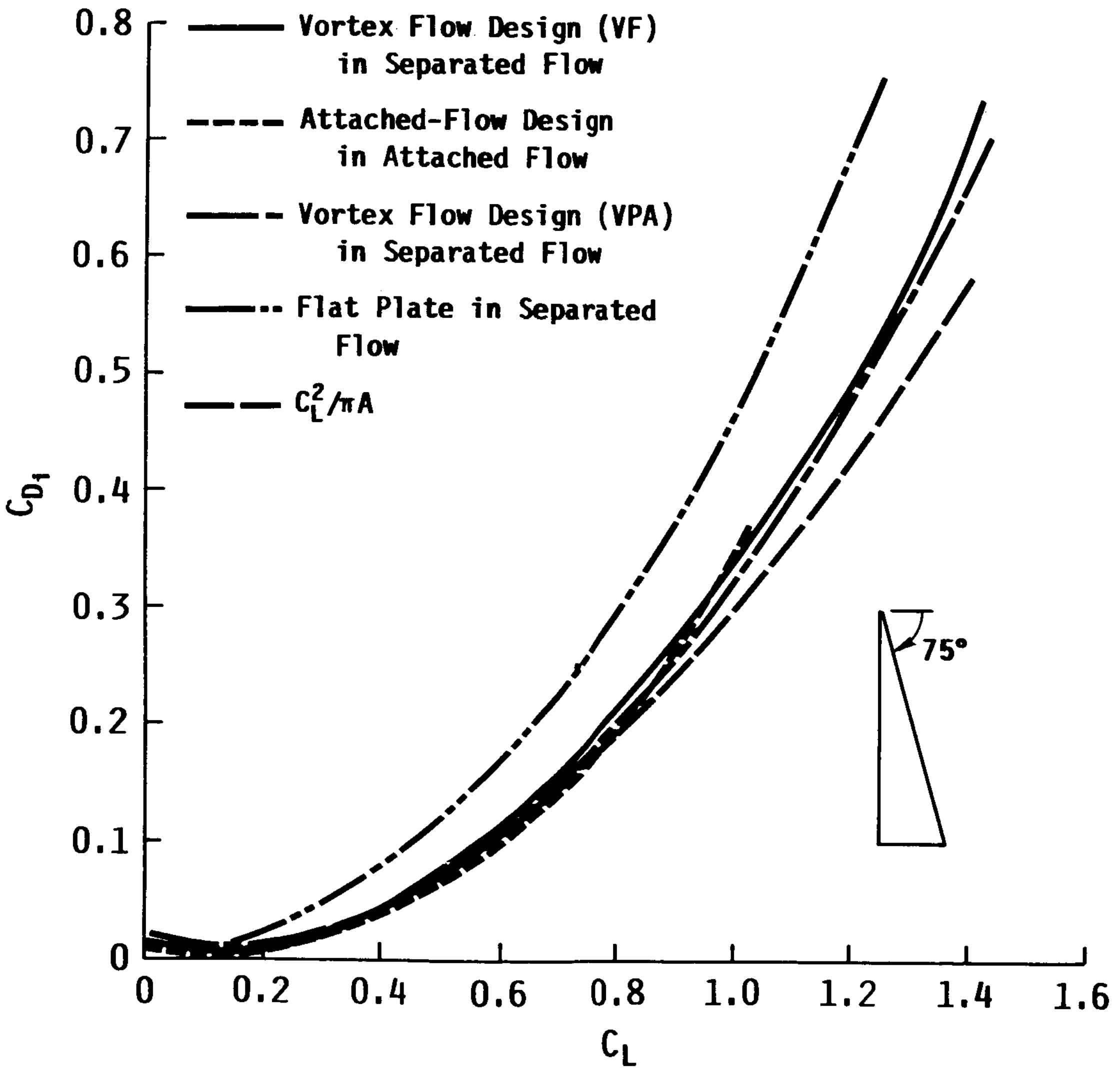


Figure 8

VORTEX FLAP DESIGN

$C_{L_{des}} = 0.3$, $M = 0$

A vortex flap is a leading-edge flap which takes advantage of vortex-induced thrust to reduce the drag. The present method is capable of determining the size and deflection angle of the flap in such a way that the drag is minimized, subject to the lift constraint. As presented in figure 9, by assuming an initial $\alpha$ of 5 deg. and $\delta_n = 0$ deg., an optimum size of the flap is determined with a final $\alpha$ of 7.7 deg. and $\delta_n = -15.1$ deg.

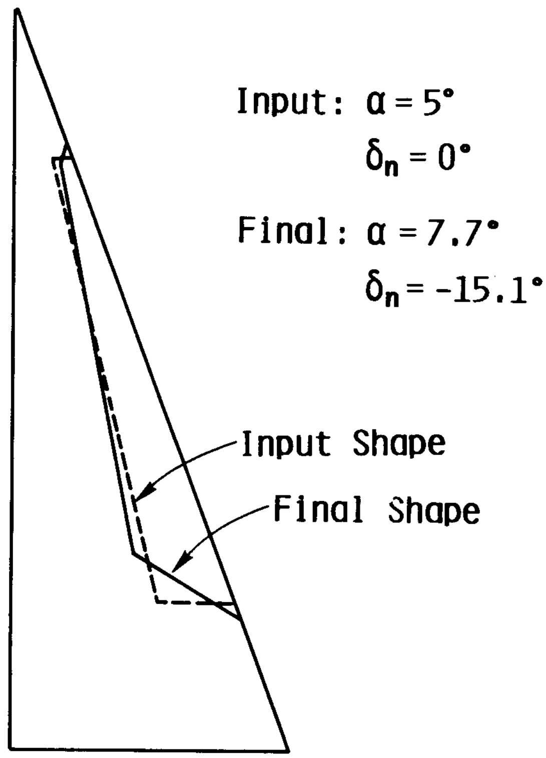


Figure 9

## SYMBOLS AND ABBREVIATIONS

| | |
|---|---|
| $A$ | aspect ratio |
| $[A]$ | aerodynamic influence coefficient matrix |
| AF | attached-flow design with a flat surface as the starting solution |
| $a_j$ | Fourier coefficients for the camber slope |
| $b$ | wing span |
| $c$ | local chord |
| $C_{D_i}$ | induced drag coefficient |
| $C_{D_o}$ | minimum drag coefficient |
| $C_L$ | lift coefficient |
| $C_{L_{des}}$ | design lift coefficient |
| $\Delta C_p$ | lifting pressure coefficient |
| $c_s$ | sectional suction coefficient |
| $c_t$ | sectional thrust coefficient |
| $F$ | objective function |
| $G$ | constraint function |
| $M$ | Mach number |
| VF | vortex flow design with a flat surface as the starting solution |
| VPA | vortex flow design with the attached-flow design as the starting solution and inboard one-third portion remaining unchanged |
| $x,y,z$ | a rectangular coordinate system |
| $z_c$ | camber ordinate |
| $z_\ell$ | camber ordinate along the leading edge |
| $\alpha$ | angle of attack |
| $\delta_n$ | leading-edge flap angle measured normal to hinge line, negative downward |
| $\gamma_x$ | streamwise vortex density |
| $\Lambda$ | leading-edge sweep angle |
| $\phi$ | perturbation velocity potential |

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