TRADEOFF METHODS IN MULTIOBJECTIVE INSENSITIVE DESIGN
OF AIRPLANE CONTROL SYSTEMS

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This talk presents the latest results of an ongoing study of computer-aided design of airplane control systems, which is based on satisfying requirements on multiple objectives. Constrained minimization algorithms are used, with the design objectives in the constraint vector [1]. We briefly review the concept of Pareto optimality and show how an experienced designer can use it to find designs which are well-balanced in all objectives [2,3]. Then we will discuss the problem of finding designs which are insensitive to uncertainty in system parameters, introducing a probabilistic vector definition of sensitivity which is consistent with the deterministic Pareto optimal problem [4]. Insensitivity is important in any practical design, but it is particularly important in the design of feedback control systems, since it is considered to be the most important distinctive property of feedback control. Methods of tradeoff between deterministic and stochastic-insensitive (SI) design are described, and tradeoff design results are presented for the example of a Shuttle lateral stability augmentation system. This example is used because careful studies have been made of the uncertainty in Shuttle aerodynamics [5]. Finally, since accurate statistics of uncertain parameters are usually not available, the effects of crude statistical models on SI designs are examined.

OUTLINE

- REVIEW PARETO-OPTIMAL MULTIOBJECTIVE DETERMINISTIC AND STOCHASTIC-INSENSITIVE (SI) DESIGN.

- FORMULATE METHODS OF TRADEOFF BETWEEN DETERMINISTIC AND STOCHASTIC-INSENSITIVE DESIGN.

- DISCUSS TRADEOFF DESIGN RESULTS FOR SHUTTLE LATERAL STABILITY AUGMENTATION SYSTEM EXAMPLE.

- EXAMINE EFFECTS OF INACCURATE STATISTICAL MODELS ON STOCHASTIC-INSENSITIVE DESIGN.

MULTIOBJECTIVE DESIGN BY CONSTRAINED MINIMIZATION

The Pareto-optimal formulation of multiobjective design is not an optimization method in the usual sense, since it does not determine a unique solution. Pareto-optimal solutions comprise that portion of the boundary of the achievable domain which is noninferior to all others in the sense that every other solution must be worse in at least one objective. In the literature on multiobjective optimization it is generally assumed that some higher-level "decision maker's" logic exists which can lead to an optimal solution. We assume, to the contrary, that no optimal solution exists for practical multiobjective design problems. Our Pareto-optimal algorithm is a valuable tool for the designer, since it enables the computer to calculate example Pareto-optimal solutions using a constrained minimization algorithm. However, the quality of the design depends on critical decisions made by the designer, who must choose the objective functions and values of associated scaling parameters which lead to solutions which are well-balanced in the disparate objectives, control the tradeoff iterations, and choose the final design. Rather than seeking some undefinable optimization index for complex systems, the design process is based on whatever computable objectives the designer considers important, with consideration of computational cost subordinated to the designer's judgment.

- PARETO-OPTIMAL FORMULATION

- THERE IS NO OPTIMAL SOLUTION

- COMPUTER CALCULATES EXAMPLE PARETO-OPTIMAL SOLUTIONS USING CONSTRAINED MINIMIZATION ALGORITHM

- DESIGNER INTERACTION IS ESSENTIAL
  - CHOOSES AND SCALES OBJECTIVES
  - CONTROLS TRADEOFF ITERATIONS
  - CHOOSES FINAL DESIGN

In this figure we present the constrained minimization formulation which leads to the example Pareto-optimal designs. Let z be the vector of design variables, η a scalar dummy variable, and $f(z)$ the vector of objective functions, and let $g(z) \leq 0$ represent a vector of auxilliary constraints. Then for arbitrary vectors a and b (with $b_j > 0$), solution of the constrained minimization problem on the first line leads to a design on the boundary of the achievable domain which is at least locally Pareto optimal. It is well known that this minimization problem is equivalent to the min-max problem on the second line. The particular solution obtained depends on the choice of a and b. Suppose the designer chooses for a_j values of the objectives which he considers marginally acceptable, and another set of very desirable objectives, a_{D_j}. Then defining $b = a - a_D$ should yield a solution well balanced in the objectives, since a and a_D have been so chosen. This method, known as the "Goal Attainment Method" [6], is illustrated in the sketch. The cross-hatched curve indicates the boundary of the achievable domain in objective space, and the part between the cross-hatched bars is the Pareto domain. At any iteration the constraints on f are at $(a + \eta b)$. As η is minimized the constraints move toward the boundary, and the solution is forced to the deterministic optimal, f_D^*, corresponding to the minimum η_D^*. The line joining f_D^* and a plays an important role in the tradeoff formulations.
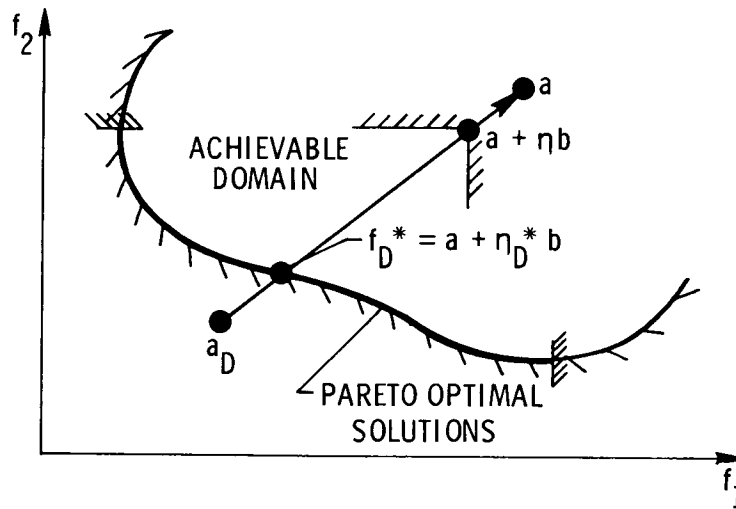
$$\text{MIN } \eta \text{ s.t. } f(z) \leq a + \eta b \text{ AND } g(z) \leq 0$$
$$z, \eta$$

$$\text{EQUIVALENT TO: } \text{MIN}_{z} \text{ MAX}_{j} \left[\frac{f_j(z) - a_j}{b_j} \right], \quad b_j > 0$$

$$\text{GOAL ATTAINMENT: } b = a - a_D$$

We now formulate the SI design algorithm and two tradeoff methods. The designer must specify a vector, y, of parameters with significant uncertainties and their probability distributions. Then the objective functions are f(z,y), and the stochastic sensitivity vector, s(z), is defined by the probabilities that specified requirements will be violated; i.e., that $f_j(z,y) > \hat{f}_j$, where $\hat{f}$ is a vector of requirement values.

Since this definition is only useful when $\hat{f}_j > f_{D_j}^*$, it is desirable to solve the deterministic problem first. Defining the Pareto-optimal SI design as that which minimizes the maximum sensitivity, the constrained minimization algorithm takes the form shown. Computational problems will be discussed later, but it is worth noting that insensitive design does not require accurate calculation of the probabilities.

Both tradeoff methods use a scalar parameter to vary a vector inequality along the line of varying constraints shown on the sketch for the deterministic design. For $\hat{f} \approx f_D^*$, the SI designs must be very like the deterministic. Introducing a scalar parameter, $\hat{\tau}$, and defining $\hat{f}(\hat{\tau})$ as in Method 1, $\hat{\tau} \approx 1$ gives deterministic-like solutions, and decreasing $\hat{\tau}$ provides a sort of tradeoff procedure, with increasing emphasis on insensitive design. Method 2 is a more precise tradeoff. Here $\hat{f}$ is fixed at a value giving insensitive design, and constraints on nominal objectives, $\bar{f}$, are varied in a similar manner giving a tradeoff between sensitivity and nominal values of objectives.

PARETO-OPTIMAL STOCHASTIC-INSENSITIVE DESIGN (SI)

<u>DEFINE</u>: $s(z) \triangleq \underset{y}{PROB} [f(z,y) > \hat{f}], \quad \hat{f} > f_d^* = a + \eta_d^* b$

$$MIN \; \eta \; S.T. \; s(z) \leq \eta, \; g(z) \leq 0$$
$$z, \eta$$

TRADEOFF METHODS IN SI DESIGN

1. VARY $\hat{f}$ WITH SCALAR $\hat{\tau}$.

$$\hat{f}(\hat{\tau}) = a + \hat{\tau} \; \eta_d^* b$$

2. VARY CONSTRAINTS ON NOMINAL f-VALUES

FIX $\hat{f}$ AND CONSTRAIN $f(z,\bar{y}) \triangleq \bar{f}(z) \leq a + \bar{\tau} \; \eta_d^* b$

The example case is design of a lateral stability augmentation system (SAS) for Shuttle entry at $M = 2.5$. The linearized lateral response equations are 4th order. System states are sideslip angle (β), yaw rate (r), roll rate (p) and bank angle (ϕ). Controls are aileron and rudder. The control law has 6 feedback gains (SAS design does not require bank angle feedback) and 2 feedforward gains from the pilot's stick input (δ_{ap}) to the controls. The general design objective is to obtain rapid, stable roll response to the stick input, with small sideslip. This example was chosen because statistical uncertainties in Shuttle aerodynamics have been carefully studied, and at $M = 2.5$ these uncertainties have been found to cause unacceptable variation in lateral response using aerodynamic controls [5]. Nevertheless, in the example we use only aerodynamic controls. The design parameter vector z is comprised of the 8 gains. The uncertain parameter vector y contains all 6 aerodynamic control effectiveness coefficients and the 3 sideslip coefficients. (The ϕ-equation is kinematic and contains no aerodynamic effects.) Uncertainty in control effectiveness will clearly have a strong effect on control system design, and lateral response is sensitive to the sideslip coefficients. In stability axes the standard deviations of the 3 types of coefficients are fairly consistent, and approximate values are shown for sideslip (A_{i1}), aileron (B_{i1}) and rudder (B_{i2}) coefficients. Rudder effectiveness is most uncertain. The y-statistics are considered gaussian and include correlation estimates.

$$\dot{x} = Ax + Bu, \quad x^T = (\beta, r, p, \varphi), \quad u^T = (\delta_a, \delta_r)$$

$$u = Kx + C\,\delta_{ap}, \quad K \triangleq \begin{bmatrix} K_{11} & K_{12} & K_{13} & 0 \\ K_{21} & K_{22} & K_{23} & 0 \end{bmatrix}, \quad C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$

z-VECTOR: 8 CONTROL SYSTEM GAINS

y-VECTOR: 6 CONTROL EFFECTIVENESS (B_{ij}) AND 3 SIDESLIP (β) DERIVATIVES (A_{i1})

σ_y-VALUES $(M = 2.5)$: $\sigma(A_{i1}) \approx 14\%$, $\sigma(B_{i1}) \approx 12\%$,

$\sigma(B_{i2}) \approx 20\%$

DESIGN OBJECTIVES FOR SHUTTLE LATERAL SAS

In this study, 11 deterministic design objectives are considered. Generally, these are based on military handling-qualities requirements for large transports. Stability is a basic requirement, and the 4 characteristic roots must be considered separately because the requirements in the various modes differ. The bank angle achieved in 6 seconds is the speed of response objective. Decoupling of the rolling motion from yaw-sideslip is achieved by keeping the peak sideslip small and $\left|\omega_\phi^2/\omega_d^2\right|$, a ratio of coefficients in the roll transfer function, near unity. For the Shuttle, small sideslip is also a heat-load requirement. It is always desirable to keep control effort small. Since the natural stability of the Shuttle is inadequate, it is clear that saturation in control deflection must be avoided. Rate saturation can lead to violent nonlinear instability. Therefore, the objectives of minimizing the peak control deflections and rates are included. Finally, the sensitivities of the 11 objectives, as previously defined, are also included as objectives. Although the functions $f(z,y)$ are nonlinear in y, the stochastic sensitivities were first calculated using a linear-gaussian assumption. These probabilities were checked using a Monte Carlo program, and all but the peak value probabilities were acceptably accurate. Acceptable accuracy was obtained by replacing the probability of violation for the maximum peak by the worst probability for any pair of peaks, using a bivariate gaussian routine. These approximate probabilities are used as the sensitivity functions in the tradeoff studies.

CATEGORY

OBJECTIVE

DETERMINISTIC, $\bar{f}_J(z)$:

CATEGORY	OBJECTIVE	
STABILITY	CHARACTERISTIC ROOTS	(4)
SPEED OF RESPONSE	BANK ANGLE IN 6 SECONDS	(1)
DECOUPLING	PEAK SIDESLIP AND $\left|\omega_\varphi^2/\omega_d^2\right|$	(2)
CONTROL EFFORT	PEAK MAGNITUDES AND RATES	(4)

STOCHASTIC SENSITIVITIES, $s_J(z)$:

|		
PROBABILITIES OF VIOLATING $\hat{f}$-REQUIREMENTS	$\underset{y}{PROB}\left[f_J(z,y) > \hat{f}_J\right]$	(11)

This figure shows how the sensitivity of the control system varies for the simpler tradeoff method, varying the value of $\hat{f}$. The 4 solid curves give Monte Carlo results for SI designs with $\hat{f}$ values at $\hat{\tau} = 0$, .2, .4 and .6. The Monte Carlo method uses the nonlinear objective functions, so that these probabilities are a more realistic estimate of the sensitivities obtainable using the linear-gaussian approximation in the SI program. Decreasing values of design τ give designs with more emphasis on insensitivity. The heavy dot on each curve shows the Monte Carlo sensitivity at the design value of $\hat{f}$. Since the probability of violation depends on $\hat{f}$, the curve shows the sensitivity variation with $\hat{f}$, to give a more complete picture of the sensitivity properties. Each curve can be thought of as a sort of vector cumulative distribution function, showing how the worst P_j increases from 0 to 1 as $\hat{\tau}$ increases. The curves are not smooth, because different P_j become worst as $\hat{\tau}$ varies. For comparison, the calculated optimal sensitivities and the Monte Carlo sensitivity values for the deterministic design are also shown. Although the Monte Carlo sensitivities for the 4 SI designs are much larger than the calculated values, comparison with the deterministic results shows that the SI designs are an order of magnitude less likely to have bad values of the objective functions. The usefulness of the probability approximation appears questionable for design $\hat{\tau} < 0.4$. For example, at $\hat{\tau} = 0$ the Monte Carlo sensitivity for the $\hat{\tau} = 0$ design is somewhat larger than those for the $\hat{\tau} = .2$ or .4 designs. Nevertheless, it is clear that the approximation is adequate to yield very significant decreases in sensitivity for SI designs compared to deterministic designs.
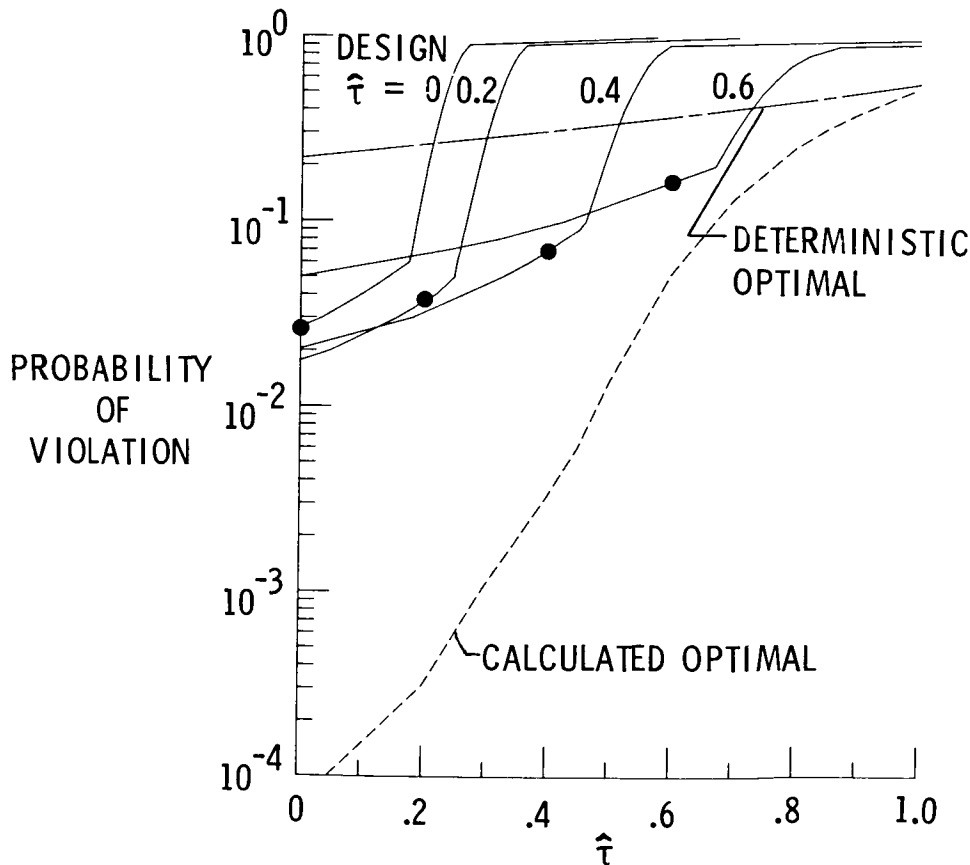
VARIATION OF SI DESIGNS WITH $\hat{f}(\hat{\tau})$

It is interesting to examine how important properties of the design
vary as $\hat{\tau}$ varies from near unity (deterministic-like designs) to lower
values, with increasing emphasis on insensitivity. The variation of
4 typical control system gains is shown on the left, and the nominal values
of 3 typical objectives and their standard deviations on the right. There
are clearly significant changes in design properties in the transition from
deterministic designs to those emphasizing low sensitivity. However, as
noted in the previous figure, there seems little significant change in gains
or other system properties in designs for $\hat{\tau} < 0.4$. As seen on the right,
the main tradeoff penalty in nominal objectives for decreased sensitivity
is loss of speed of response, as indicated by the bank angle at 6 seconds,
$\bar{\phi}(6)$. Typical of the other objectives are the oscillatory damping ratio,
$\bar{\zeta}$, which is relatively unchanged, and the damping in roll, $|\bar{\lambda}_R|$, which
increases. Note that 2 of the standard deviations decrease for the insen-
sitive designs, but σ_λ actually increases. This is permitted because
of the large increase in $|\bar{\lambda}_R|$. The computer finds gains to meet the
varying probability constraints, with freedom to use whatever combinations
of $\bar{f}$-values and σ-values are required.
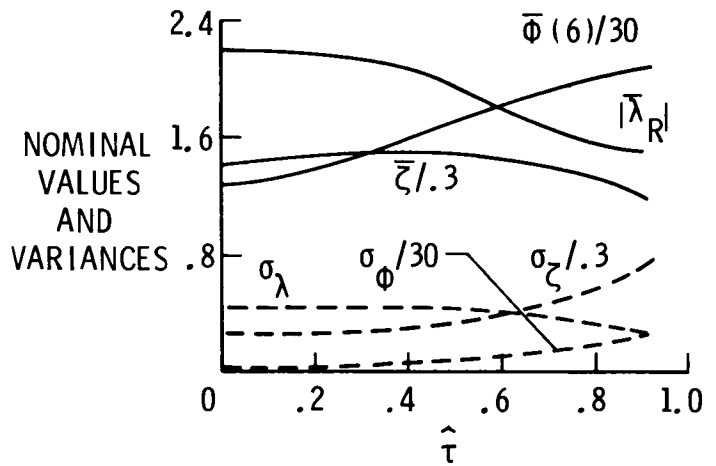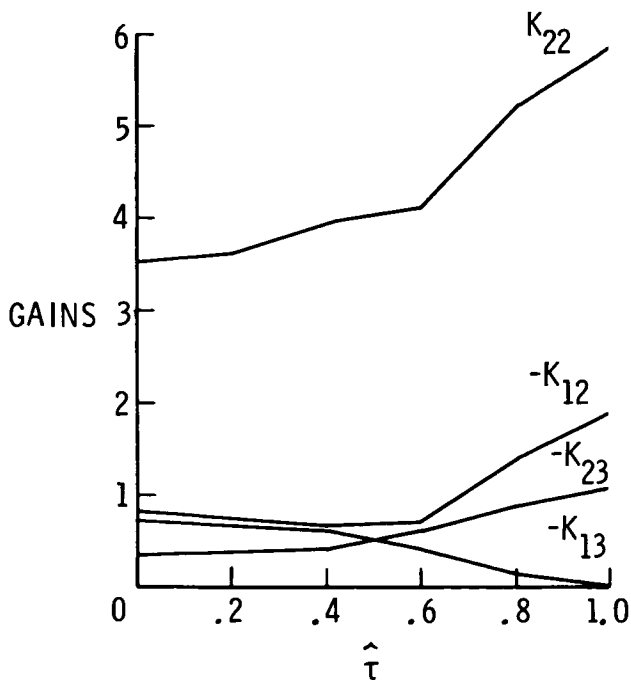
This figure presents Monte Carlo results of the more precise tradeoff between insensitivity and nominal values of objectives. Starting with the SI design at $\hat{\tau} = 0$ as the unconstrained design emphasizing insensitivity, increasingly stringent constraints are imposed on the nominal values by varying $\bar{\tau}$ in $\bar{f}(z) \leqq a + \bar{\tau} \eta_D * b$. The probability of violation for the constrained designs is shown in the solid curves. Although this method gives more precise control of the values of $\bar{f}_j$ obtained in each SI design, this set of solutions seems similar to the set obtained by simply varying $\hat{f}$. In the tradeoff varying $\hat{f}$, there was a significant increase in the probability of violations at low $\hat{\tau}$ between designs at $\hat{\tau} \leqq .4$ and $\hat{\tau} = .6$. Here the corresponding increase in sensitivity (i.e., the probability of bad objective values) occurs between the designs for $\bar{\tau} = .6$ and $\bar{\tau} = .8$. In problems where the probabilities can be calculated accurately (probabilistic design rather than insensitive design), this more precise method might be preferred, in spite of the added computational burden of adding the hard constraints. Also, there is a certain logical appeal to constraining the nominal objectives to good values while minimizing the probability that the objectives will be worse than marginally acceptable. For our applications, however, accurate statistics are not obtainable, and the simpler method seems preferable.
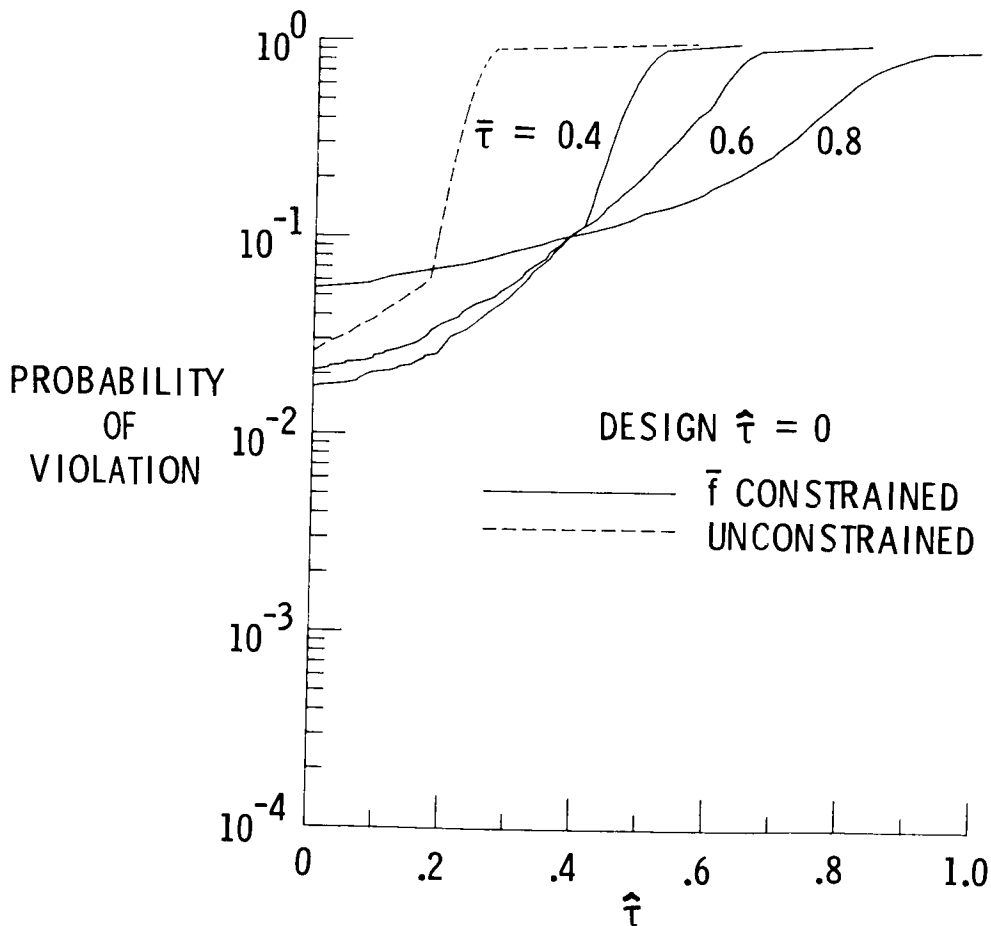
# EFFECTS OF CRUDE STATISTICAL MODEL ON SI DESIGN

In practice, inaccuracy in f-statistics resulting from the linear assumption is likely to be dominated by inaccuracy of the input values of the y-statistics. The statistics for the Shuttle example are more accurate and detailed than would usually be available for control system design. To investigate the effects of using a cruder estimate of the y-statistics on SI design, it was assumed that the sideslip, aileron and rudder coefficients had standard deviations equal to 15%, 15% and 20% of their nominal values, respectively, with no correlations. These crude statistics were used for SI design at $\hat{\tau} = 0.4$, and this design is compared with the original design at $\hat{\tau} = 0.4$ and the deterministic design. The figure shows Monte Carlo probabilities based on the Shuttle statistics. The curves are cumulative distribution functions for $\phi(6)$, the objective which always shows a large penalty in expected value in SI designs, and peak δ_r, which is always critical in the calculated probabilities for the SI design. The simplified input statistics give an SI design which has the same basic properties and approximately the same sensitivity as obtained with the more accurate statistics. Although the effectiveness of the SI design does not seem to require an accurate statistical model, accurate calculation of the probabilities does require accuracy of the statistical model. For both SI designs, the simplified statistics predict much larger probabilities of violation than the accurate statistics, and it was found that almost all the discrepancy was caused by neglecting the y-correlations.

DETERMINISTIC

STOCHASTIC, $\hat{\tau} = 0.4$ ROCKWELL STATS.

STOCHASTIC, $\hat{\tau} = 0.4$ SIMPLIFIED STATS.
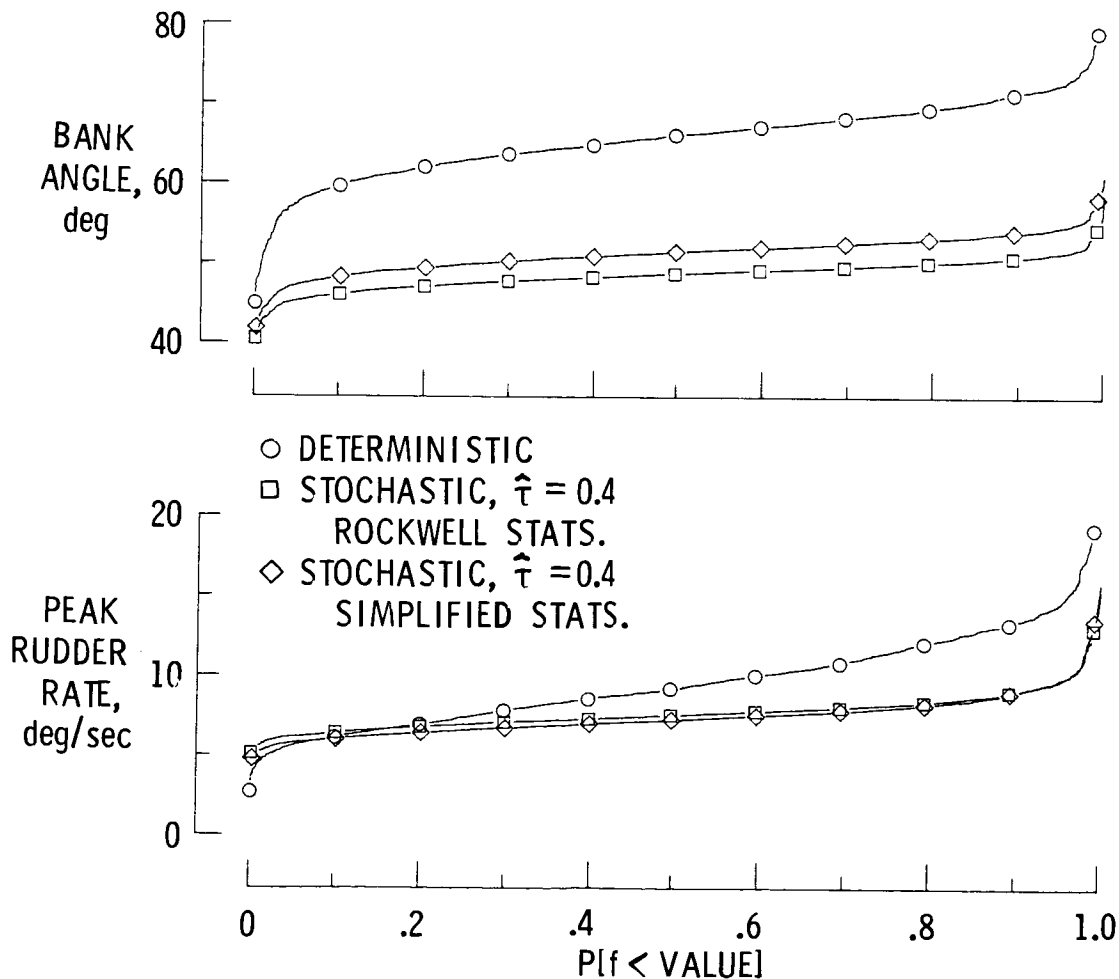
## NOMINAL AND OFF-NOMINAL RESPONSES FOR 3 SI DESIGNS

NO CONTROL LIMITING

Although statistical distribution curves are the best way to compare
designs for sensitivity to off-nominal parameters, simulated time histories
of off-nominal responses are also useful. The Monte Carlo random set of
responses for each control system was ranked using a weighted sum of viola-
tions of desired objective values, and time histories of nominal and 5
off-nominal responses at the 99th percentile for 3 SI designs are compared
in this figure. The solid curves show the responses of the nominal system
and the broken curves are the off-nominal responses. These cases are from
the set shown in the tradeoff varying $\hat{\tau}$, in which it was noted that there is
a significant increase in the probability of bad objective values for
design at $\hat{\tau} \geq 0.6$. This increased sensitivity is shown here by the
increase in scatter of the off-nominal responses for the design at
$\hat{\tau} = 0.6$. The tendency for decreased nominal speed of response for the less
sensitive designs is evident in the roll rate responses, $p(t)$, and the
tendency for large peak values of rudder and rudder rate in the off-nominal
responses is evident in the $\delta_r(t)$ responses. In fact, the $\hat{\tau} = 0.6$
off-nominal responses all violate the rudder rate limit of 12°/sec. The
next figure includes the control limits in the integration routine to show
the destabilizing effect of rate limiting.
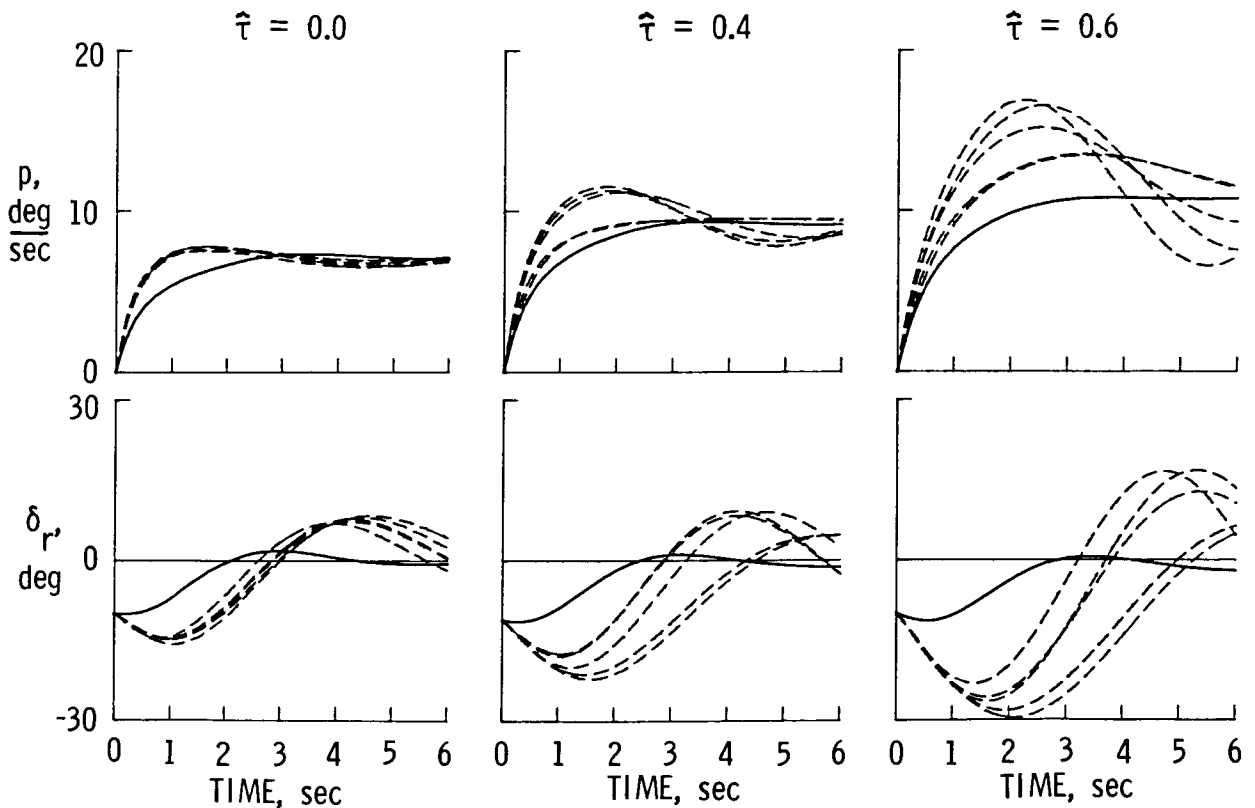
NOMINAL AND OFF-NOMINAL RESPONSES FOR 3 SI DESIGNS

INCLUDES CONTROL LIMITING

This figure shows the importance of using peaks in control deflections and rates as design objectives when it is likely that control limiting may occur. Deflection limiting is dangerous when the uncontrolled airplane is unstable, but the nonlinear delays introduced by rate limiting can cause violent instability in an inherently stable system, as shown in these responses for $\hat{\tau} = 0.6$. Although the SI design method calculates only the linear responses, the designer can control the probability that the peaks will violate the control limits, as shown in the results for $\hat{\tau} = 0$ and 0.4. In this case the a_j values for control peaks were chosen at the limiting values and the a_{D_j} values were 20% below the limits. The probabilities at $\hat{\tau} = 0$ are the probabilities that limiting will occur in the linear responses, and keeping these low implies that the probability of control-limiting instability will be low.
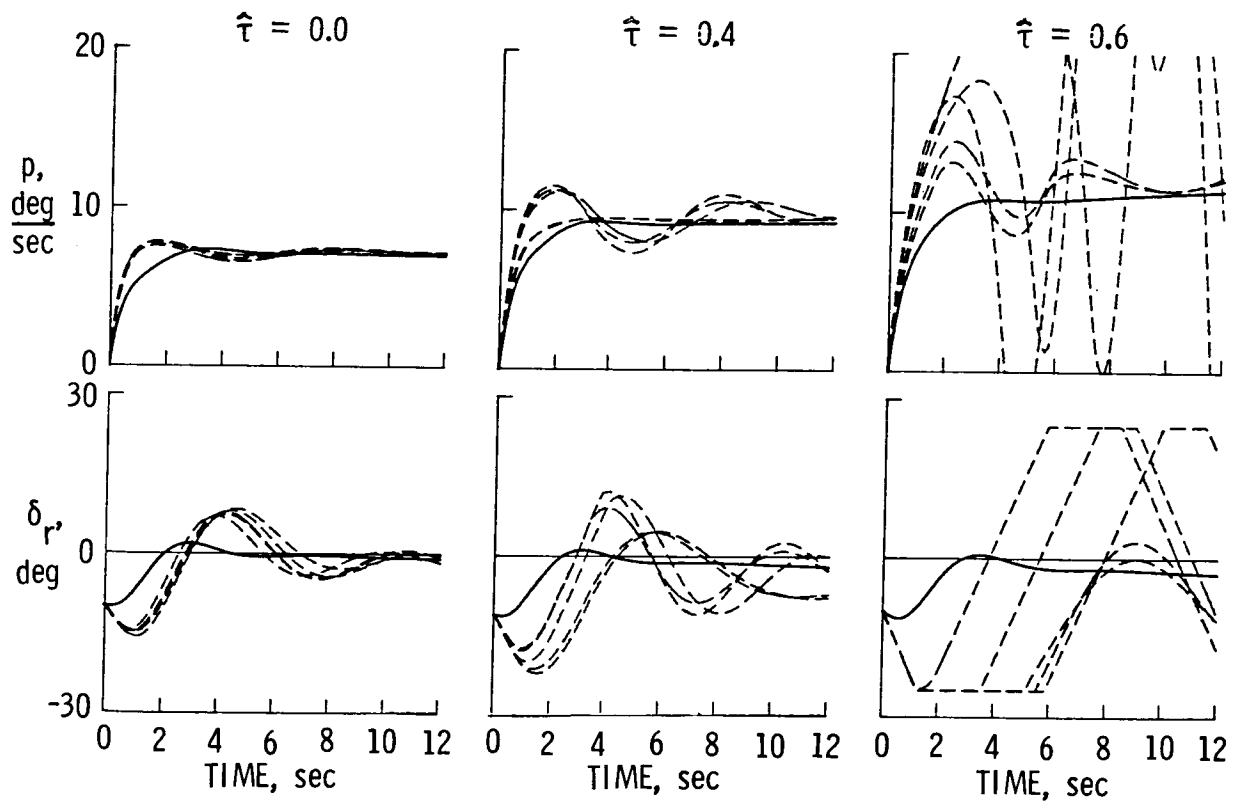
The Pareto-optimal stochastic insensitive design method defines a vector sensitivity which is related in a very natural way to a set of objectives chosen by an experienced designer. The designer must also make important decisions to formulate the constrained minimization algorithm for obtaining Pareto-optimal insensitive designs which are well balanced in the objectives and for trading off between insensitivity and nominal values. The designer, not the computer, makes the critical decisions which determine the quality of the design. The effectiveness of the method depends on the designer's judgment, but this makes it easy for him to interact with the program.

The main conclusions of this study are listed on the figure. The SI method yields control system designs with very significant decreases in sensitivity to parameter uncertainty. The effectiveness of the method does not depend on having an accurate statistical model. The tradeoff studies show that there are distinct differences between designs emphasizing insensitivity and deterministic designs. For example, there are large gain changes as emphasis on insensitivity increases. The two tradeoff methods are both effective in compromising between insensitivity and nominal values of objectives. Although the method utilizes only linear response calculations, it produces designs which are less likely to encounter nonlinear control-limiting instabilities. Finally, in the example case, the main penalty for achieving insensitivity was decreased nominal speed of response. It will be interesting to see if further study shows this to be a general property of insensitive control system designs.

STOCHASTIC-INSENSITIVE DESIGN GIVES A SIGNIFICANT DECREASE IN SENSITIVITY TO PARAMETER UNCERTAINTY IN SPITE OF INACCURACY OF CALCULATED PROBABILITIES.

TRADEOFF STUDIES SHOW THAT SI DESIGNS ARE DISTINCTLY DIFFERENT FROM DETERMINISTIC DESIGNS.

SEVERAL EFFECTIVE METHODS WERE DEVELOPED FOR OBTAINING DESIGNS WHICH COMPROMISE BETWEEN INSENSITIVITY AND NOMINAL OBJECTIVE VALUES.

INSENSITIVE DESIGN CAN BE ESPECIALLY EFFECTIVE WHEN CONTROL LIMITING IS A PROBLEM.

IN THE SHUTTLE LATERAL SAS EXAMPLE, THE MAIN PENALTY FOR ACHIEVING INSENSITIVE DESIGNS WAS REDUCED VALUE OF NOMINAL RESPONSE SPEED.

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