

ELEVATED TEMPERATURE CRACK GROWTH

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1. INTRODUCTION

Critical gas turbine engine hot section components such as blades, vanes, and combustor liners tend to develop minute cracks during the early stages of operation. These cracks may then grow under conditions of fatigue and creep to critical size. Current methods of predicting growth rates or critical crack sizes are inadequate, which leaves only two extreme courses of action. The first is to take an optimistic view with the attendant risk of an excessive number of service failures. The second is to take a pessimistic view and accept an excessive number of "rejections for cause" at considerable expense in parts and downtime. Clearly it is very desirable to develop reliable methods of predicting crack growth rates and critical crack sizes.

To develop such methods, it is necessary to relate the processes that control crack growth in the immediate vicinity of the crack tip to parameters that can be calculated from remote quantities, such as forces, stresses, or displacements. The most likely parameters appear to be certain path-independent (P-I) integrals, several of which have already been proposed for application to high temperature inelastic problems. A thorough analytical and experimental evaluation of these parameters needs to be made which would include elevated temperature isothermal and thermomechanical fatigue, both with and without thermal gradients.

In any investigation of fatigue crack growth, the problem of crack closure must be addressed in order to develop the appropriate crack growth model. Analytically, this requires the use of gap elements in a nonlinear finite element code to predict closure loads. Such predictions must be verified experimentally through detailed measurements; the best method for measuring crack closure has not been established in previous studies.

It is the purpose of this contract (NAS3-23940) to determine the ability of currently available P-I integrals to correlate fatigue crack propagation under conditions that simulate the turbojet engine combustor liner environment. The utility of advanced fracture mechanics measurements will also be evaluated and determined during the course of the program. These goals will be accomplished through a two year, nine task, combined experimental and analytical program. To date, an appropriate specimen design and a crack displacement measurement method

have been determined. Alloy 718 has been selected as the analog material based on its ability to simulate high temperature behavior at lower temperatures in order to facilitate experimental measurements. Available P-I integrals have been reviewed and the best approaches are being programmed into a finite element postprocessor for eventual comparison with experimental data. These experimental data will include cyclic crack growth tests under thermomechanical conditions and, additionally, thermal gradients.

2. A REVIEW OF P-I INTEGRALS

The utility of the J integral as a parameter for predicting crack growth in the elastic-plastic regime is rather limited. The theoretical basis of the J integral does not allow the extension of its usage to nonproportional loading and unloading in the plastic regime, nor can it be utilized in the presence of a temperature gradient and material inhomogeneity. A typical example where all these limiting factors are operative would be the hot section components of a gas turbine in mission cycles.

In recent years there has been a considerable effort to modify or reformulate the P-I integral. Consequently, a number of new P-I integrals have emerged in the literature. These include the J^* , \hat{J} , J_θ , ΔT_p , ΔT_p^* integrals and two thermoelastic integrals, J_w and J_G (see Appendix for definition of these). These P-I integrals have been critically reviewed in this program. The theoretical background has been examined with particular attention to whether or not the path-independence is maintained in the presence of nonproportional loading; unloading in the plastic regime; and a temperature gradient and material inhomogeneity. The relation among the P-I integrals, salient features and limitations were investigated. The physical meaning, the possibility of experimental measurement, and the computational ease were also examined. The summary of the review is presented in Table 1. In view of the requirements associated with performing the forthcoming tasks in this program the following conclusions were made:

- i) The J^* , \hat{J} , ΔT_p and ΔT_p^* integrals maintain the path-independence under the thermomechanical cycles which will be used in the tests in this program and will be simulated numerically in subsequent tasks. Although the physical meaning of these P-I integrals needs to be further pursued, they would be the logical choices for further evaluation in this program.
- ii) The J , J_w , J_G and J_θ integrals have limited capabilities. The J_w and J_G integrals are usable only for thermoelastic problems with homogeneous material properties. These integrals may be useful for prediction of crack growth in a rather small temperature gradient field and under small scale yielding conditions. The J_θ integral is a modified version of J to include the thermal strain. Therefore, it cannot be used with substantially

nonproportional loading and unloading in the plastic regime. It would be worthwhile, however, to investigate the utility of operationally defined J and possibly J_{θ} for the test cycles in this program.

3. NUMERICAL COMPUTATION

All selected P-I integrals have been implemented in a postprocessor to the General Electric nonlinear finite element program, CYANIDE. Numerical values of the integrals will be evaluated and examined for cracks subjected to various situations such as monotonic/cyclic loadings, uniform/non-uniform temperature distributions, stationary/propagating cracks, etc. Best formulations suitable for all situations will be selected and used to correlate with the test results. The relationship between the analytical CTOD (or CMOD) displacements and the values of P-I integrals will be established to identify the displacement which must be measured to determine operational P-I integral.

In computation, an integration path is first selected which should start from a node on one side of the crack surface, extend along edges of elements, and end at a node on the other side of the crack surface. The path can be selected by either providing all nodal numbers to be included in the path, or simply inputting only a few key nodes and letting the program search for a proper path. Once the path is defined, the program will identify whether an element is inside or outside the path, thus, computation of the area integral can be performed. Displacement, stress, and strain data required for computation are to be read directly from random CYANIDE output files. Each integral is to be set up in an individual calling subroutine thus addition of other new integrals can be easily done without affecting the entire program structure. The program is in the FORTRAN format and to be run in the timesharing mode. Options are provided for selecting types of integrals, an integration path, and load cases. The basic structure of the program has been established and successfully tested through the computation of the conventional J integral. The results for an elastic compact tension problem have shown very stable trend of path independence and are about four percent different from a handbook solution. For the plastic case, however, the results were shown to be increasingly deviating from the referenced one as the load increased. This may be due to the fact that crack tip blunting was not simulated in the analysis which used triangular elements, while the blunting effect was reportedly included in developing the handbook curves.

Implementation of other P-I integrals has been completed and further checkouts are in progress. Algorithms for the integral forms from Blackburn, Ainsworth, Kishimoto, and Atluri have been implemented. Example results for the same compact tension specimen problem are compared in Figure 2 along with the solution from the conventional J integral. It was found that for isothermal monotonic loading condition, several relationships exist between P-I integrals

$$J \text{ (Rice)} = J_{\theta} \text{ (Ainsworth)}$$

$$\Sigma \Delta T_p^*(Atluri) = J^* \text{ (Blackburn)}$$

$$\Sigma \Delta T_p(Atluri) = J \text{ (Rice)}$$

The first equation is theoretically verifiable, but the second and third equations are pertinent to the particular geometry and loading condition under consideration. Examining path independence for the integrals, it was revealed that most integrals showed a drastic drop in results evaluated in the crack tip region, and a slightly increasing trend as the integration path was further removed from the crack tip. Except for Kishimoto's integral, contribution from the area integral term are significantly smaller than the line integral. Both the ΔT_p and ΔT_p^* integrals were computed for a complete loading/unloading cycle. The trend for the results (Figure 3) computed from a remote path agrees well with Nagakaki's findings. However, it was also found that both ΔT_p and ΔT_p^* from the near field path were gradually losing the characteristics of path independence as the unloading proceeds further down. This may be due to the use of large unloading increments in the analysis. The capabilities of all integrals for problems with temperature gradients, varying material properties and propagating cracks are yet to be investigated.

In a parallel work, the proposed single edge crack specimen is being analyzed using the three-dimensional elastic-plastic finite element method. Stress and displacement distributions at some remote location in the gauge section for a cracked specimen subjected to displacement boundary conditions at the buttonhead were obtained. The purpose of this analysis is twofold: (1) to provide guidelines for experimental set-up and data measurement, and (2) to provide necessary boundary conditions for further analysis of local details.

In another effort, a mesh generation program has been developed to generate finite element mesh for crack problems. The program allows gradual transitioning of the mesh arrangement from a relatively coarse mesh in the remote field to very fine one near the crack area. Two basic types of mesh arrangement, namely, the fan type and square type, are options. The fan mesh is currently used in all the preliminary analyses, while, the square mesh will be extensively used for simulating crack growth behavior and crack closure phenomenon.

APPENDIX

The P-I integrals reviewed in this program are presented here. The index notation was used. The common variables are: σ_{ij} = stress tensor, ϵ_{ij} = strain tensor, u_i = displacement vector, t_j = traction vector, θ = relative temperature, α = thermal expansion coefficient, μ and λ = Lamé's constants. For integration paths and areas the reader is referred to Figure 1.

Rice's J-Integral [1]

$$J = \int_{\Gamma} (n_j W - t_j u_{i,j}) ds$$

$$\text{where } W = \int_0^{\epsilon_{ij}} \sigma_{ij} d \epsilon_{ij}$$

Wilson and Yu's Thermo-Elastic Integral [2]

$$J_W = \int_{\Gamma} (n_1 W - t_i u_{i,1}) ds - \alpha (3\lambda + 2\mu) \int_A [\frac{1}{2} (\Theta \epsilon_{ij})_{,1} - \epsilon_{ij} \Theta_{,1}] dA$$

where

$$W = \frac{1}{2} \sigma_{ij} \epsilon_{ij},$$

Gurtin's Thermo-Elastic Integral [3]

$$J_G = \int_{\Gamma} [n_1 W - t_k u_{k,1} - \frac{\alpha^2 (3\lambda + 2\mu)^2}{2(\lambda + \mu)} \Theta^2 n_1 + \frac{\alpha \mu (3\lambda + 2\mu)}{(\lambda + \mu)} (\Theta \frac{\partial u_1}{\partial n} - u_1 \frac{\partial \Theta}{\partial n})] ds$$

where

$$\frac{\partial}{\partial n} = n_j \frac{\partial}{\partial x_j}$$

and

$$W = \mu \epsilon_{ij} \epsilon_{ij} + \frac{\lambda}{2} (\epsilon_{kk})^2$$

The J_{Θ} -Integral by Ainsworth et. al [4]

$$J_{\Theta} = \int_{\Gamma} (n_1 W - t_i u_{i,1}) ds + \int_A \sigma_{ij} \epsilon_{ij,1} dA$$

where

$$W(\epsilon'_{ij}) = \int_0^{\epsilon'_{ij}} \sigma_{ij} d\epsilon'_{ij}, \text{ and } \epsilon'_{ij} = \epsilon_{ij} - \epsilon_{ij}^{\Theta}$$

The J^* -Integral by Blackburn [5]

$$J^* = \int_{\Gamma + \Gamma_C} (\frac{1}{2} \sigma_{ij} u_{i,j} dx_2 - t_i u_{i,1} ds) + \int_A (\frac{1}{2} \sigma_{ij} u_{i,j,1} - \frac{1}{2} \sigma_{ij,1} u_{ij}) dA$$

The \hat{J} -Integral by Kishimoto, Aoki and Sakata [6]

$$\hat{J} = - \int_{\Gamma + \Gamma_C} t_i u_{i,1} ds + \int_A \sigma_{ij} \epsilon_{ij,1} dA$$

The ΔT -Integrals by Atluri et. al [7]

$$\Delta T_p^* = \int_{\Gamma + \Gamma_C} [n_i \Delta W - (t_i + \Delta t_i) \Delta u_{i,1} - \Delta t_i u_{i,1}] ds + \int_A [\Delta \sigma_{ij} (\epsilon_{ij,1} + \frac{1}{2} \Delta \epsilon_{ij,1}) - \Delta \epsilon_{ij} (\sigma_{ij,1} + \frac{1}{2} \Delta \sigma_{ij,1})] dA$$

$$\Delta T_p = \int_{\Gamma + \Gamma_C} [n_1 \Delta W - (t_i + \Delta t_i) \Delta u_{i,1} - \Delta t_i u_{i,1}] ds + \int_{A_S - A_{\Gamma}} [(\sigma_{ij,1} + \frac{1}{2} \Delta \sigma_{ij,1}) \Delta \epsilon_{ij} - (\epsilon_{ij,1} + \frac{1}{2} \Delta \epsilon_{ij,1}) \Delta \sigma_{ij}] dA$$

where

$$\Delta W = (\sigma_{ij} + \frac{1}{2} \Delta \sigma_{ij}) \Delta u_{i,j},$$

and A_S is the total area and A_{Γ} is the area in Γ .

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Table 1: Summary of P-I Integrals

P-I Integral	Measure of Crack Tip Severity	Physical Meaning (5)		Capability to Handle			Material Inhomogeneity	Computation (Integrals involved)	Experimental Measurement (1)
		Elastic	Thermo-elastic	Plastic	Prop. loading (plastic regime)	Nonprop. & Loading Unloading (plastic regime)			
J	Yes	$-\frac{\partial P}{\partial a}$	-	$-\frac{\partial P}{\partial a}$	Yes	No	No	Line	Yes
J _w	Yes	$-\frac{\partial P}{\partial a}$	$-\frac{\partial \phi}{\partial a}$	-	No	No	No	Line + Area	Yes
J _G	Yes	$-\frac{\partial P}{\partial a}$	$-\frac{\partial \phi}{\partial a}$	-	No	No	No	Line	Yes
J _θ	Yes	$-\frac{\partial P}{\partial a}$	$-\frac{\partial \phi}{\partial a}$	$-\frac{\partial P(2)}{\partial a}$	Yes	No	No	Line + Area	Yes
J*	Yes	$-\frac{\partial P}{\partial a}$	$-\frac{\partial \phi}{\partial a}$	Unknown	Yes	Yes	Yes	Line + Area	No
\hat{J}	Yes	Rate of work done to crack tip by surrounding material (3)			Yes	Yes	Yes	Line + Area	No
ΔT_p^*	Yes	$-\frac{\Delta \Pi}{da}$ for prop. loading (4)			Yes	Yes	Yes	Line + Area	No
ΔT_p	No	$-\frac{\Delta \Pi}{da}$ for prop. loading (4)			Yes	Yes	Yes	Line + Area	Yes

Note: (1) Yes if it can be expressed as the rate of a potential or if it has only line integrals.

(2) $-\frac{\partial \phi}{\partial a}$ for thermoplastic proportional loading

(3) With the assumption of a rigid fracture process zone at the crack tip independent of the crack size

(4) Further study is needed for the case of thermomechanical loading

(5) P = Potential energy, ϕ = Global thermodynamic potential, $\Delta \Pi$ = Incremental potential

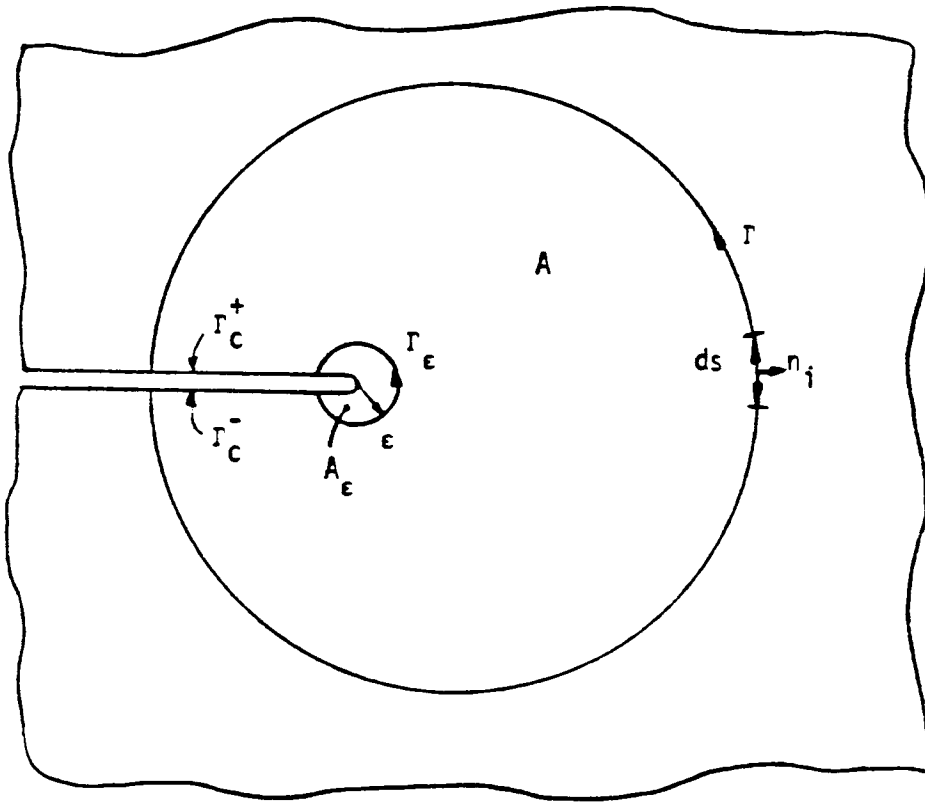


Figure 1: Integration Paths and Areas

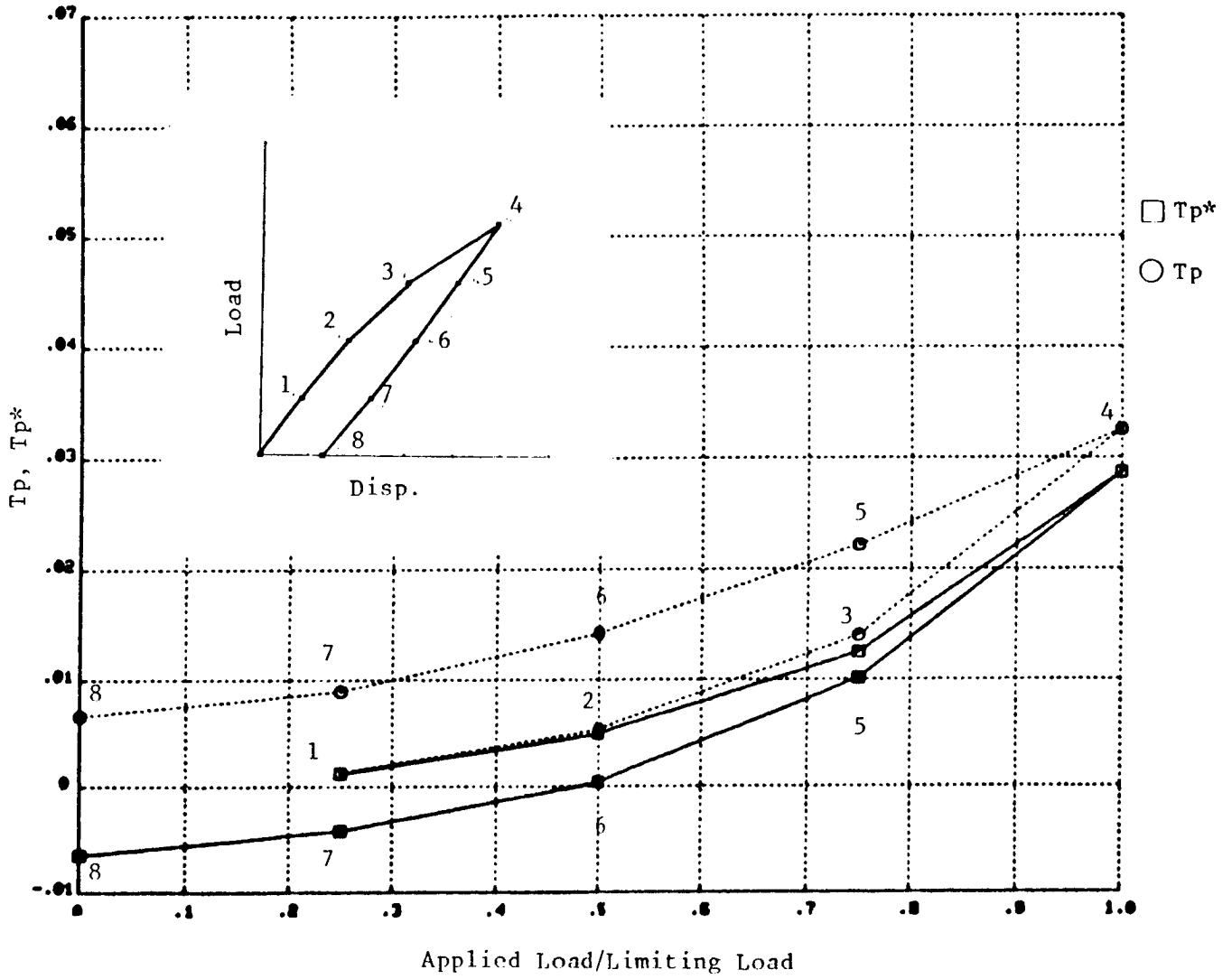


Figure 2. Comparison of Atluri's T_p and T_{p^*} Integrals for a Complete Load Cycle.

- Rice, Ainsworth
- Blackburn
- △ Kishimoto
- + Atluri (Tp*)
- × Atluri (Tp)

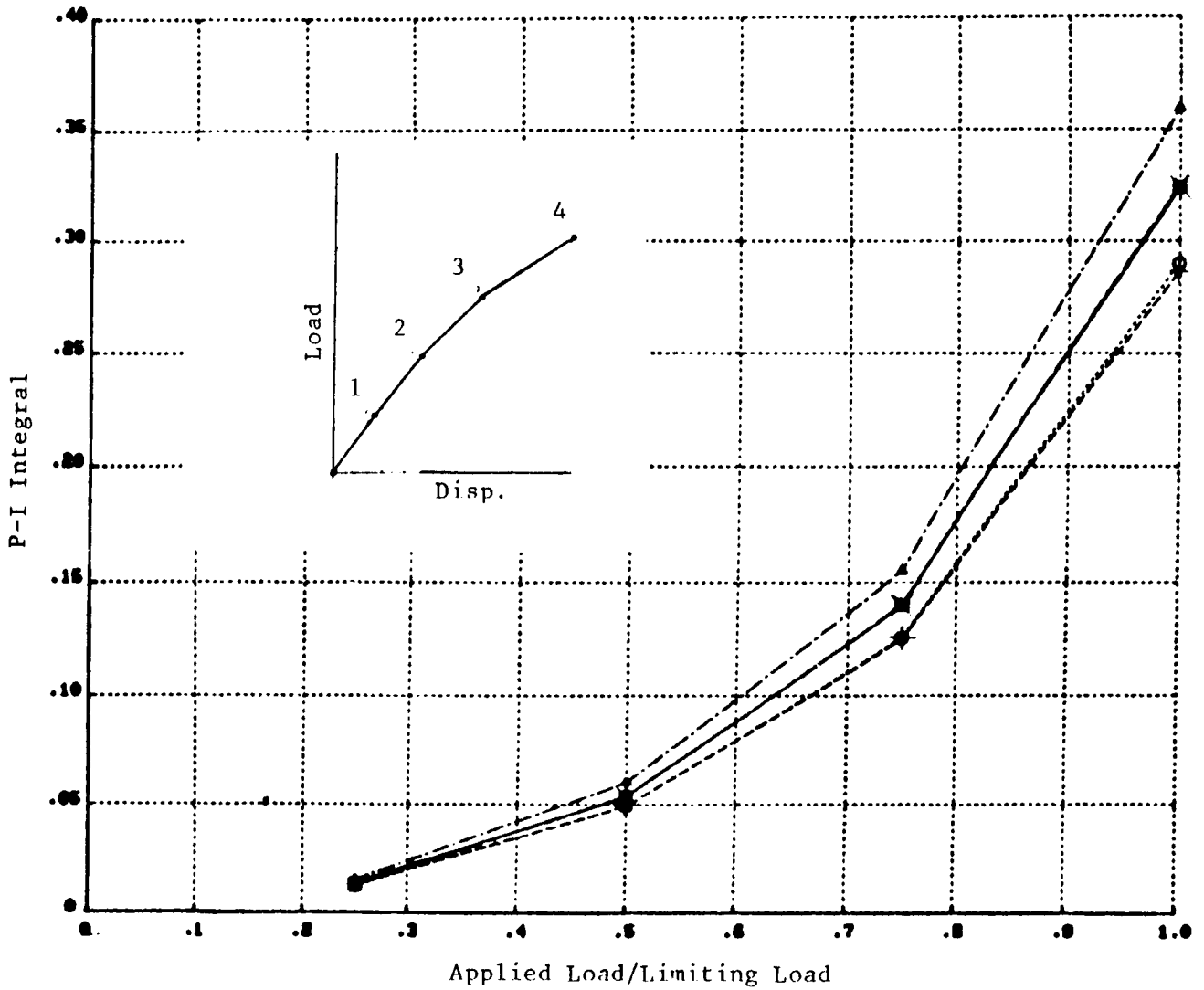


Figure 3. Comparison of Various P-I Integral results for a Compact Tension Specimen Problem.