CONSTITUTIVE MODELING FOR ISOTROPIC MATERIALS

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INTRODUCTION

Accurate analysis of stress-strain behavior is of critical importance in the evaluation of life capabilities of hot section turbine engine components such as turbine blades and vanes. The constitutive equations used in the finite element analysis of such components must be capable of modeling a variety of complex behavior exhibited at high temperatures by cast superalloys. The classical separation of plasticity and creep employed in most of the finite element codes in use today is known to be deficient in modeling elevated temperature time dependent phenomena. Rate dependent, unified constitutive theories can overcome many of these difficulties and may be more suitable for the analysis of the complex behavior of high temperature superalloys. However, many aspects of the unified theories have not been fully evaluated. There is an urgent need for a comprehensive evaluation and further refinement of the capabilities of unified constitutive models for analysis of high temperature superalloy behavior.

OBJECTIVE

It is the purpose of this contract (NAS3-23927) to thoroughly evaluate the unified constitutive theories for application to typical isotropic cast nickel base superalloys used for air-cooled turbine blades and vanes. The specific modeling aspects evaluated are: uniaxial, monotonic, cyclic, creep, relaxation, multiaxial, notch and thermomechanical behavior. Further development of the constitutive theories to model thermal history effects, refinement of the material test procedures, evaluation of coating effects and verification of the models in an alternate material will be accomplished in a follow-on for this base program.

APPROACH

The scope of the overall program covers several aspects of the development of constitutive models for material behavior. The objectives of the base program is being accomplished through a two year combined analytical and experimental program. This is divided into several tasks, each task focusing on a specific objective. First an extensive literature survey was made to identify possible constitutive models for detailed evaluation. Based on the detailed evaluation, two models have been selected for implementation into a finite element code. A comprehensive uniaxial smooth specimen material test program is defined so as to investigate the constitutive behavior patterns of Rene' 80, which is the base material. These experimental results are being used for both the determination of the material parameters and further evaluation of the predictive capabilities of the two models.

The models will be evaluated for multiaxial analysis capabilities, based on multiaxial test data. Two types of multiaxial tests are being performed - tension-tension type at a notch root using an extended ISG technique at Michigan State University (MSU) (Ref. 1) and tension-torsion type on hollow tubes at the General Electric Turbine Technology Laboratories. The notch root behavior prediction capability of the models will be evaluated based on several benchmark notch verification experiments. This part is similar to the work conducted by General Electric on the NASA sponsored Benchmark contract (Ref. 2).

The capability of the constitutive models to analyze the behavior of an actual engine component will be verified by performing a finite element analysis of a turbine blade tip, similar to that described in Reference 3.

PROGRESS

A. MODEL EVALUATION AND SELECTION

A comprehensive survey of various unified constitutive theories available in the literature has been completed. From the 13 models surveyed, 5 theories were selected for detailed evaluation. They are the models of (1) Bodner et. al., (2) Krieg, Swearengen, Rohde (3) Miller (4) Robinson and (5) Walker (References 4, 5, 6, 7, 8). Each of these models was programmed as subroutines in a computer program, which performs a simple numerical integration of the basic equations. All these models involve a number of material parameters. For the purpose of evaluation of the theories, constants available for different materials and temperature in the published literature were used. Each model was subjected to a variety of appropriate loading conditions, so as to evaluate their ability to model several basic aspects of high temperature superalloy behavior. These include: (1) strain rate sensitivity (2) creep (3) stress relaxation (4) history dependence (5) cyclic hardening/softening (6) anelasticity. In addition, the models were evaluated in terms of their complexities in numerical implementation and material parameter evaluation.

During the course of this detailed evaluation, several generic features of the models have become more evident, such as the roles played by the backstress and drag stress. The numerical difficulties special to each of the models have also become apparent. Based on this evaluation process, two models were selected for further detailed investigation. These were (1) the Bodner Model and (2) a generic backstress/drag stress model. In the generic backstress/drag stress model, the specific functional forms are being chosen based on the behavior observed in the Rene' 80 test program.

B. EXPERIMENTAL PROGRAM

All specimens used in this program are cast as solid specimens and machined to the desired configuration. Because of the desire to determine thin wall constitutive relationships applicable to airfoils, the specimens are tubular, with approximately .030 in. wall thickness. Tables 1a and 1b show all the monotonic tests and creep tests completed to date. Table 1c shows all the isothermal cyclic tests that have been performed. The tests cover a range of strain rates from .002 in/in/min to .2 in/in/min and the temperature range from 538C (1000F) to (982C) 1800F.

The tests are specially designed to meet the needs of constitutive model development. At the same time, efforts were taken to maximize the types of data obtained. For example, at the end of the strain rate controlled monotonic tests, a stress relaxation test is performed. The cyclic tests have an automatic data acquisition system, which is capable of getting up to 200 data points for each hysteresis loop. Some examples of this can be seen in Fig. 4.

The tension-tension multiaxial tests are being done by Prof. J.F. Martin at MSU. These tests utilize an axisymmetric notched round bar with three indentations at the notch root. Both the hoop and axial strains will be measured using the interferometric strain gage, similar to that used in the benchmark test program (Ref. 2).

Data reduction procedures also reflect the special needs of constitutive model development. For each test, the elastic modulus is first determined, based on the initial stress-strain readings. Then the inelastic strain is calculated. Since time is recorded at each data point, the time derivatives of all measured quantities is calculated. Thus stress rate and inelastic strain rate is calculated at each point using a 7 point sliding polynomial technique. All the results are stored in a computer file which can be directly used as input in material parameter evaluation.

C. EVALUATION OF MATERIAL PARAMETERS

It has been widely recognized that one of the major sources of difficulty in the use of unified constitutive theories is the determination of the material parameters. No generalized procedures of determining these material parameters are currently available. Considerable effort has been made to develop such a method in the present contract.

The approach that is adopted is to develop a computer program which directly uses the various test results as input and generates the various material parameters as output. The computer program developed is kept as flexible as possible, so that different functional forms can be used. Such an approach also assures consistency in the treatment of the various test data. However, it should also be noted that, while conceptually simple, such an approach can be very challenging, mainly due to the non-linear equations involved. Such a computer program has been developed for a generic backstress-drag stress model.

The generic backstress-drag stress model is described by the following set of equations for the uniaxial case:

$$\dot{\varepsilon}^{I} = \left(\frac{\sigma-\Omega}{Z}\right)^{N} \operatorname{sgn}(\sigma-\Omega) \qquad 1$$

$$\dot{\Omega} = f_1\,\dot{\varepsilon}^{I} - f_2|\dot{\varepsilon}^{I}|\Omega - R_1 \qquad 2$$

$$\dot{Z} = g_1\,\dot{\varepsilon}^{I} - g_2|\dot{\varepsilon}^{I}|Z - R_2 \qquad 3$$

In the above,

$\dot{\varepsilon}^{I}$ = Inelastic strain rate
Ω = Backstress
Z = Drag stress

R_1 and R_2 are static thermal recovery functions.

f_1 and g_1 are the hardening functions.

f_2 and g_2 are the dynamic recovery functions.

Equations 1, 2 and 3 are a set of coupled non-linear differential equations. The specific forms for the various hardening and recovery functions are significantly different for the various models that have been published. The approach taken in this project is to choose those forms that appear most appropriate for Rene' 80 behavior. To determine the various material parameters involved, an iterative approach is used. In this, a set of starting assumptions are made which are subsequently relaxed. Then successive non-linear optimizations are performed in equations 1, 2 and 3 using the experimentally measured quantities as the basis.

Rene' 80 test data at 982C (1800F) has been analyzed in detail using a computer program incorporating the procedure described above. Some of the notable results are as follows:

(1) f_1 = constant, f_2 = constant appears to work reasonably well for this case. The constants in R_1 have been found using slow strain rate monotonic and creep tests. However, the overall contribution of the above term seems extremely small, as compared to the hardening and dynamic recovery terms.

(2) Fig. 1a shows the results of the iteration procedure after 5 iterations, using only the high $\dot{\varepsilon}$ monotonic test (.2 in/in/min). It appears that parameters determined using the computer program can reproduce the stress strain behavior reasonably well. Fig. 1b shows the same result, but using only the small strain rate monotonic test (.002 in/in/min). The constants for these two strain rates are significantly different.

(3) The monotonic based constants are not able to predict the cyclic behavior. Fig. 2a and 2b show the cyclic loop predictions using monotonic based constants. Fig. 3a indicates that softening is continuing. In the model, the drag stress equation constants control cyclic softening.

(4) Fig. 3 shows the results of using constants based on all monotonic tests. It is seen that these parameters overpredict the high $\dot{\varepsilon}$ tests and underpredict the low $\dot{\varepsilon}$ tests. Thus, although the model appears good for a specific strain rate, it does not seem capable of representing the entire strain rate spectrum used here (0.002 in/in/min to 0.2 in/in/min).

(5) Fig. 4a shows the comparison of the test data and model prediction for a cyclic test at .2 in/in/min. The result shown is for the 96th cycle. The initial hardening shown in the plot is to be disregarded, because the prediction was made for only 2 cycles and not the entire 96 cycles. Fig. 4b shows similar results as above for the 0.002 in/in/min cyclic test. Both Fig. 4a and 4b indicate that the procedure works well for each strain rate. However, the material parameters are significantly different for the two cases. Here again, the difference is believed to be caused by the drag stress equation parameters, as in the monotonic case. This points to the limitations of the particular model in representing a wide range of strain rate behavior.

Current work is evaluating the Bodner model, and extending the analysis to lower temperatures where less strain rate sensitivity is anticipated.

(D) FINITE ELEMENT CODE IMPLEMENTATION

The 2-D finite element code containing Bodner's constitutive model has been completed and tested. The 2-D finite element code utilizes two dimensional constant strain triangles and an incremental initial strain iteration technique. To facilitate the simulation of arbitrary load histories, the load history is partitioned into piecewise linear segments with steady state thermal conditions during each segment. In order to simplify input, reduce convergence problems and minimize cost, a dynamic time stepping procedure is incorporated. The 3-D finite element code using 20 noded isoparametric bricks is currently being developed.

In order to verify the 2-D finite element code with Bodner's model a number of uniaxial test cases were run and compared with published results. (References 9-11). In addition, a large two dimensional model (Fig. 5a) of the benchmark notch specimen (Reference 2) was constructed and run with three different loading histories and compared with published experimental results. An example of these comparisons can be seen in Figure 5b. The overall performance of the finite element code with Bodner's model was quite good. The cost of running the code is comparable to one using a conventional uncoupled plasticity and creep constitutive model.

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TABLE I

ORIGINAL PAGE IS OF POOR QUALITY

(a) TENSILE SPECIMEN TEST MATRIX

Test Temperature C (F)	Strain Rate 0.002 Min^{-1}	0.02 Min^{-1}	.06 Min^{-1}	0.2 Min^{-1}
538 (1000)	T	T + SR		T + SR
649 (1200)		T + SR		
760 (1400)		T + SR		
871 (1600)		T + SR		
982 (1800)	T + SR	T + SR	T	T + SR

T indicates a constant strain rate tension test terminated at a strain of 0.03.

SR is a stress relaxation test to be performed at a constant strain of 0.03.

(b) CREEP SPECIMEN TEST MATRIX

Test Temperature C (F)	Initial Applied Stress Levels mPa (ksi)
982 (1800)	110 (16.0)
982 (1800)	217 (31.5)
982 (1800)	303 (44.0)
871 (1600)	493 (71.5)
871 (1600)	414 (60.0)
871 (1600)	312 (45.3)
760 (1400)	554 (80.3)
760 (1400)	685 (99.3)
760 (1400)	634 (92.0)
1093 (2000)	114 (16.6)

(c) UNI-AXIAL FATIGUE SPECIMEN TEST MATRIX

($|\epsilon_{max}|$ or $|\epsilon_{min}|$ = 0.0015, 0.0030, 0.0045)

Continuously Cycled Tests (Strain Controlled)

Test No.	Temperature - C (F)	A_ϵ	$\dot{\epsilon}$ (min^{-1})
1	538 (1000)	∞	0.2
2	871 (1600)	∞	0.2
3	982 (1800)	∞	0.2
4	538 (1000)	∞	0.002
5	871 (1600)	∞	0.002
6	982 (1800)	∞	0.002
7	538 (1000)	+1	0.2
8	538 (1000)	+1	0.002
9	871 (1600)	+1	0.2
10	871 (1600)	-1	0.2
11	982 (1800)	-1	0.2
12	982 (1800)	-1	0.002

Hold Time Tests ($A_\epsilon = \infty$, $\dot{\epsilon} = 0.2$ min^{-1}, strain controlled)

Test No.	Temperature - C (F)	Maximum or Minimum Strain Hold	Hold Time (Sec)
13	538 (1000)	Maximum	12
14	538 (1000)	Maximum	120
15	871 (1600)	Maximum	12
16	871 (1600)	Maximum	120
17	871 (1600)	Minimum	12
18	871 (1600)	Minimum	120
19	982 (1800)	Minimum	12
20	982 (1800)	Minimum	120

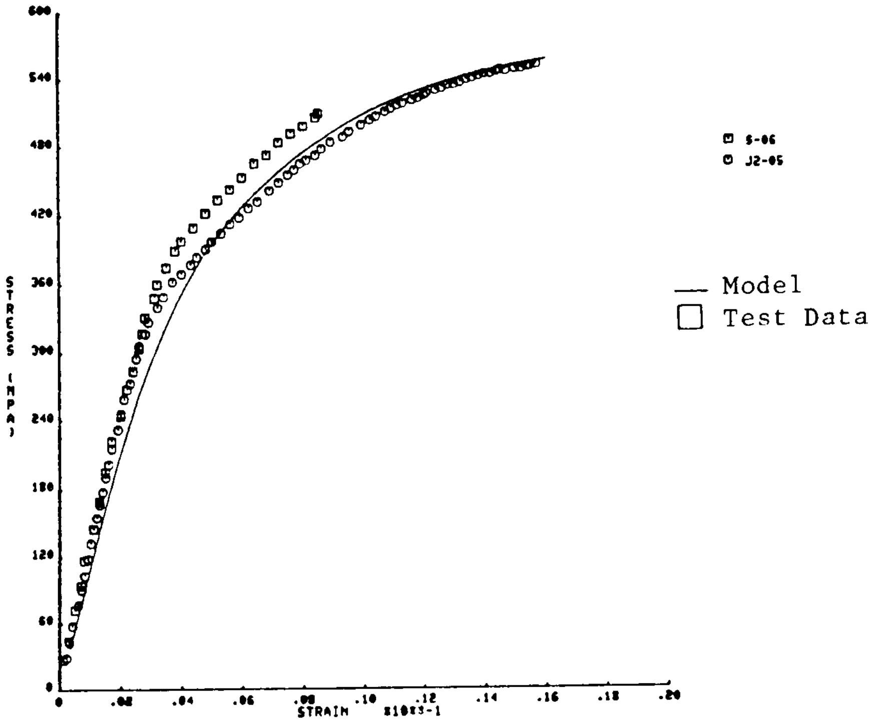


Figure 1a. Rene 80 982 C (1800 F) Monotonic Test at 0.2 in/in/min.

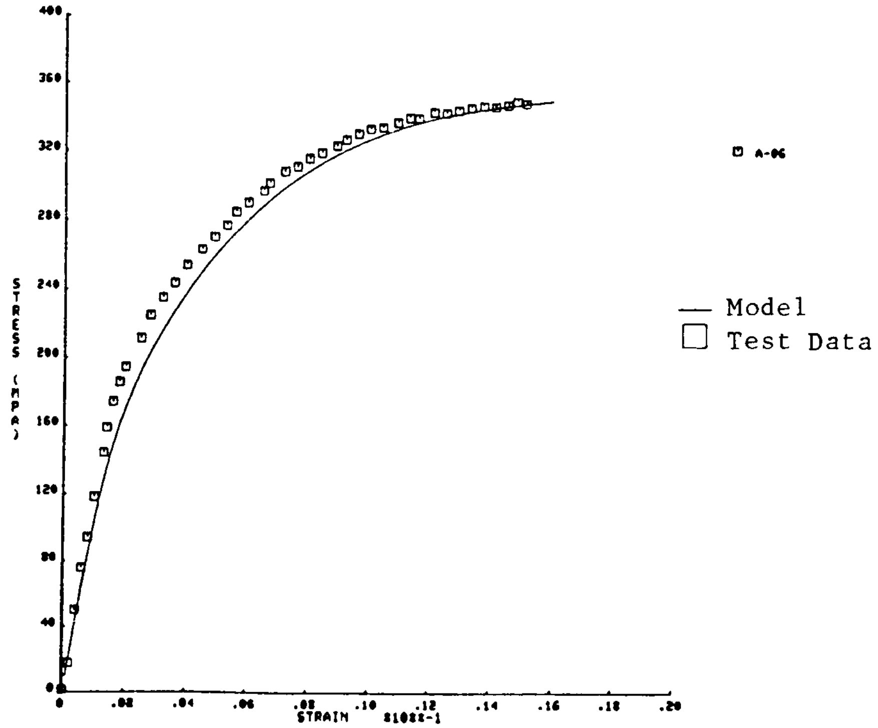


Figure 1b. Rene 80 982 C (1800 F) Monotonic Test at 0.002 in/in/min.

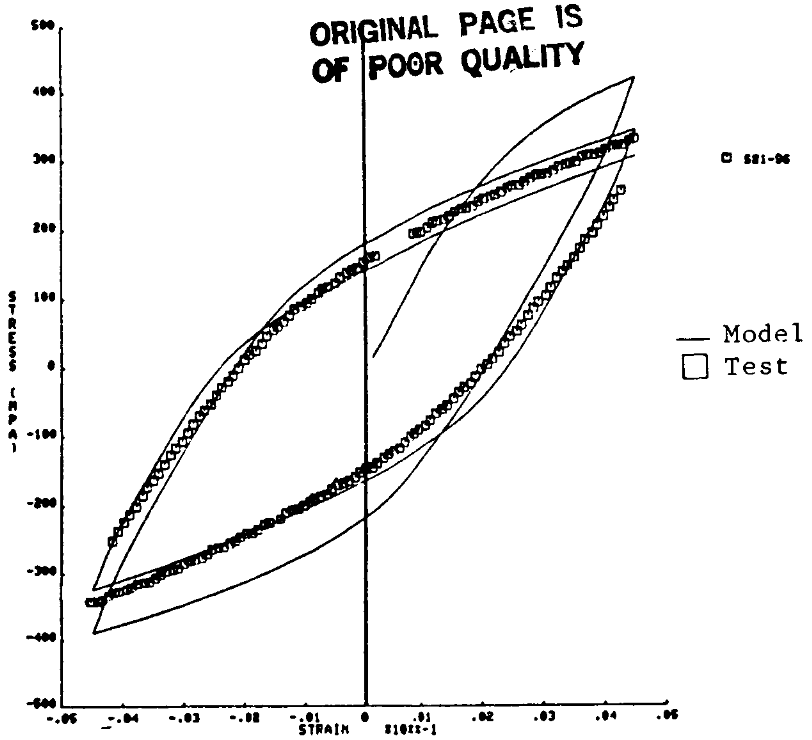


Figure 2a. Rene 80 982 C (1800 F) 0.2 in/in/min Cyclic Test.
Model Prediction Using Monotonic Based Material Parameters.

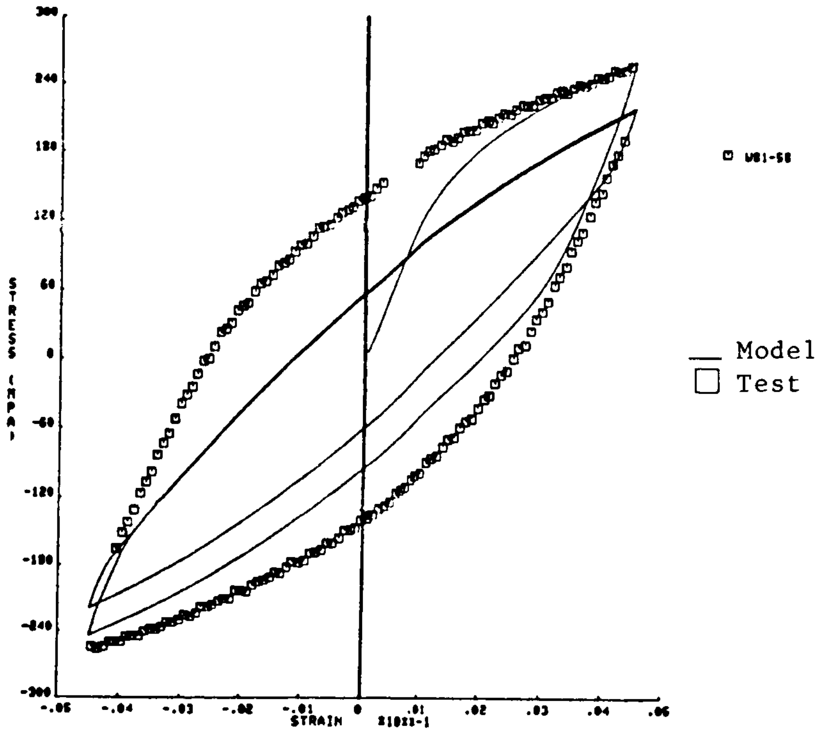
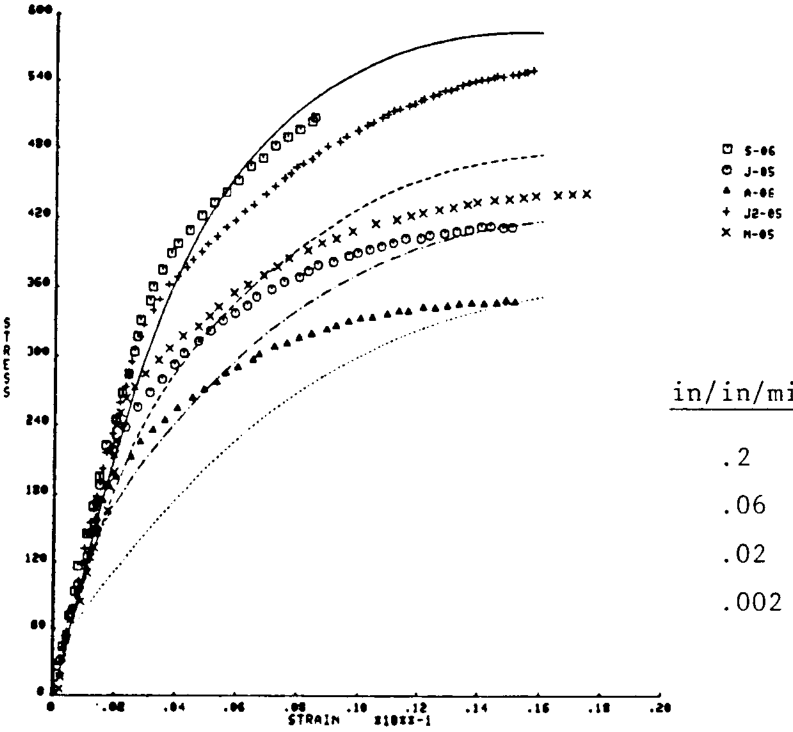


Figure 2b. Rene 80 982 C (1800 F) 0.002 in/in/min Cyclic Test.
Model Prediction Using Monotonic Based Material Parameters.



in/in/min	Test	Model
.2	□ +	——
.06	×	— —
.02	○	— · —
.002	△	......

Figure 3. Rene 80 982 C (1800 F) Monotonic Tests.

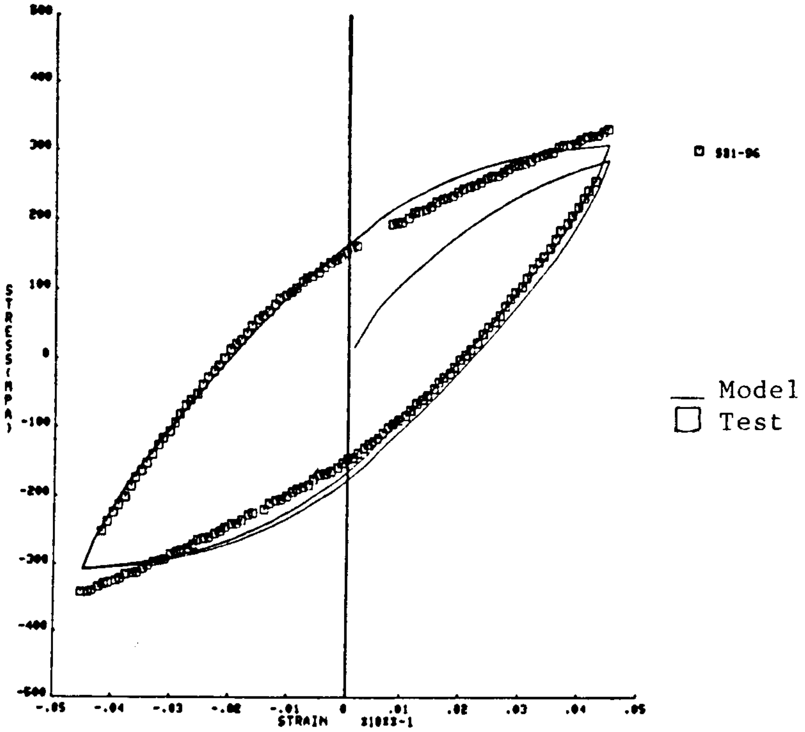


Figure 4a. Rene 80 982 C (1800 F) Cyclic Test 0.2 in/in/min, 96th Cycle.

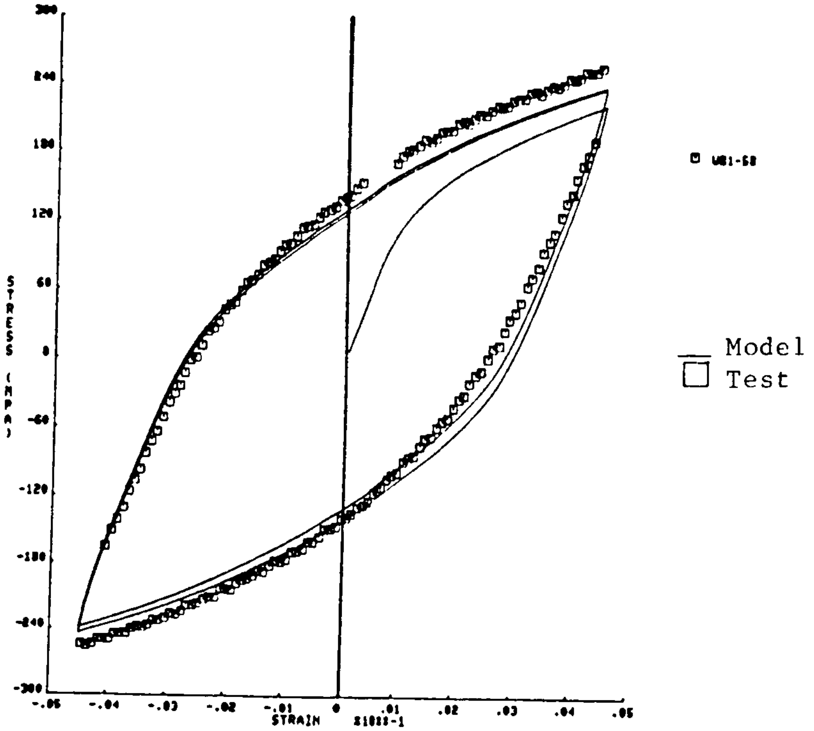


Figure 4b. Rene 80 982 C (1800 F) Cyclic Test 0.002 in/in/min, 58th Cycle.

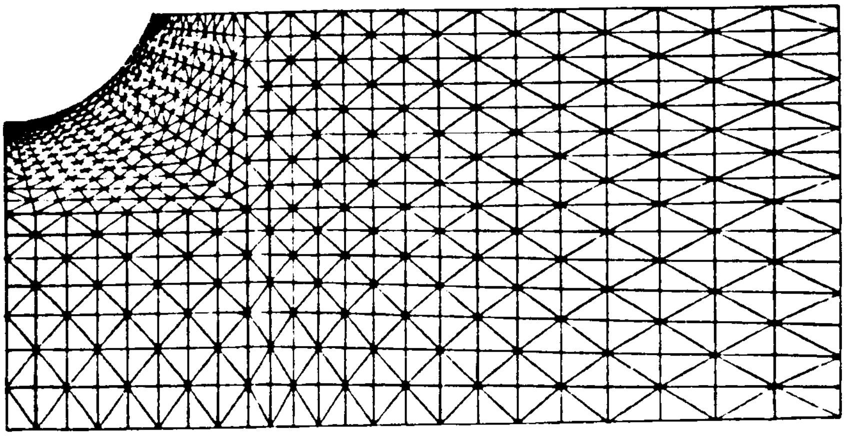


Figure 5a. Finite Element Mesh for Benchmark Notch Specimen.

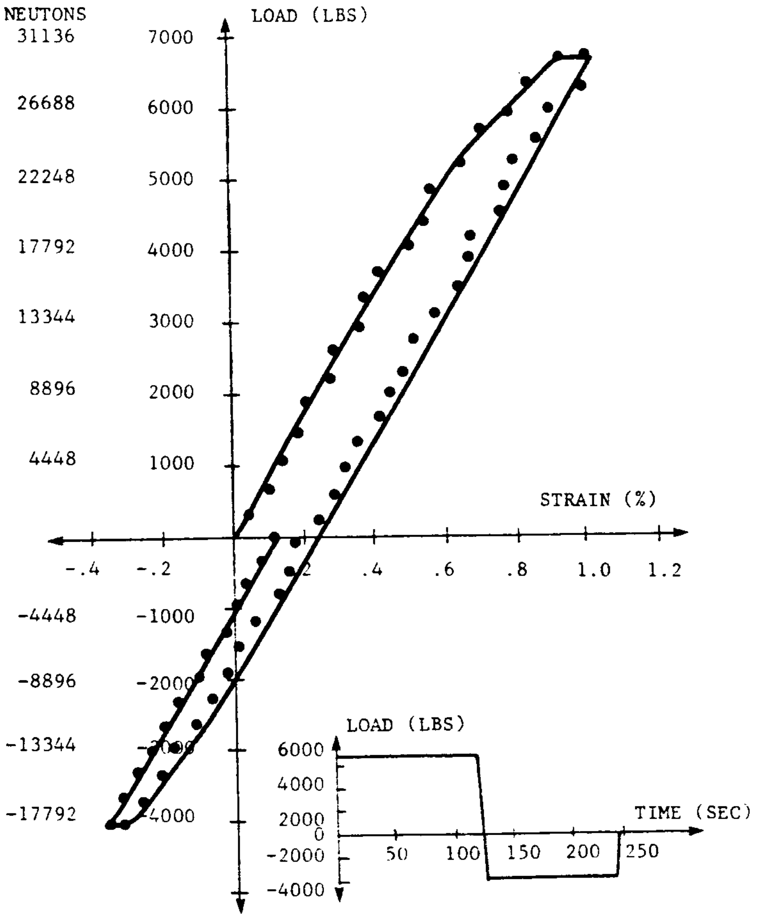


Figure 5b. Bodner Model Predictions for Benchmark Notch Specimen.