# 7.1.3 DETERMINATION OF U, V, AND W FROM SINGLE STATION DOPPLER RADAR RADIAL VELOCITIES

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## INTRODUCTION

The ST/MST clear-air Doppler radar, or wind profiler, is becoming an important tool in observational meteorology because of its capability to remotely observe dynamic parameters of the atmosphere. However, there are difficulties, which have long been recognized in work with precipitation sensitive Doppler radars, in transforming the observed radial velocities into meteorological wind components. As WALDTEUFEL and CORBIN (1979) put it "... the problem lies in the fact that one would like to know as much as possible about the wind vector field, whereas a single-Doppler radar yields only one wind component, the radial. Every attempt to gain some knowledge of the vector field, therefore, must compound the data field with additional hypotheses or simplifications." In this paper, we review how this problem has been treated in the past, and recast some of the analysis to a form more suited to the high elevation angle, fixed beam ST radar profiling techniques. We then examine the diagnostic abilities of a number of fixed beam configurations with reference to a linear wind field. The results, in conjunction with other work which treats problems such as the effects of finite sample volumes in the presence of nonhomogeneous atmospheric reflectivity (e.g., KOSCIELNY et al., 1984), have implications important to the design of both individual MST/ST radars and MST/ST radar networks.

## BACKGROUND

The use of pulse Doppler radar to measure horizontal winds seems to have been first suggested in the literature by PROBERT-JONES (1960) in connection with a 3-cm precipitation sensitive system. LHERMITTE and ATLAS (1961) suggested the VAD (velocity azimuth display) method of retrieving the mean horizontal velocity from radial velocity data taken in horizontal circles centered along the vertical of the radar site. CATON (1963) and BROWNING and WEXLER (1968) confined the analysis to a horizontal plane to deduce mean convergence, stretching and shearing deformation as well as the mean horizontal velocities from VAD observations. All of these authors treated only the stratiform situation, where $\partial w/\partial x = \partial w/\partial y = 0.0$.

EASTERBROOK (1975) examined the processing of data in a conical sector, wherein values are estimated at a point not centered on the radar. In this case, although five parameters of the wind field can be extracted, the mean horizontal velocity is contaminated by vorticity and cannot be independently determined.

The papers by WOODMAN and GUILLEN (1974) describing results using the Jicamarca incoherent-scatter radar, and GREEN et al. (1975) describing the results from the prototype VHF ST wind profiling radar at Sunset, Colorado, mark the arrival in the literature of VHF clear-air Doppler radars as important wind profile measurement tools. These radars are generally fixed-multibeam systems, utilizing large elevation angles consistent with wind height-profiling through the troposphere and into the lower stratosphere, as opposed to the conventional meteorological radars, which usually have a single rotating beam, and utilize low zenith angles consistent with measuring winds at low heights.

Because precipitation sensitive Doppler radars work under conditions of relatively high signal to noise, a practical radar may utilize a rotatable dish antenna. Then it is natural to apply data-redundant least squares techniques taking advantage of the easily produced circular symmetry, such as the VAD technique (BROWNING and WEXLER, 1969). Although clear-air Doppler radars must work with the much poorer signal-to-noise ratio provided by echoes from irregularities in the refractive index of the air itself, special high performance clear-air radars are able to create sufficient power-aperture by using very high power to allow use of a rotatable dish antenna. Under these conditions, VAD techniques can be used in clear air, as demonstrated by PETERSON and BALSLEY (1979). They used the Chatanika 23 cm Doppler radar to compare the accuracy of VAD, VED (velocity elevation display), and direct vertical probing in measuring w, the vertical wind component. They found direct measurement with the vertical beam superior to the other two methods.

With typical ST radars, however, the power-aperture requirements couple with economic considerations to dictate large, immobile antennas for economical systems. It is not practical to steer these antennas by physically moving them, and though electronic beam steering methods are available (GREEN et al., 1984, CLARK and GREEN, 1984, FUKAO et al., 1985), they have not yet been generally applied. Thus, each beam position is expensive to implement, so that economical ST systems are designed to obtain the minimum amount of data to measure the wind components with as little bias as possible.

CLARK et al. (1985) show the significant bias reduction obtained in u, v estimates over mountainous terrain obtained by adding a vertical beam to measure w. It will be apparent from the next section that the addition of additional beams can reduce the bias even further when stratiform conditions prevail. However, as discovered in the previous work with precipitation sensitive radars, it is not possible to eliminate bias completely with single-Doppler radar techniques. The following discussion will try to clarify the nature of this bias, and serve as an aid in design of economical ST radar systems.

## THE GENERAL SCALING EQUATION FOR A LINEAR WIND FIELD

We adopt the usual Cartesian coordinate system x, y, z, representing distances to the east, north and zenith, respectively, with origin at the center of the radar antenna (Figure 1). The primary assumption applied to the wind field is linearity in the region about $(x_o, y_o, z_o)$, the point in space at which the flow parameters are to be determined. The vector distance to a measuring volume from the antenna is defined as

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

where $\hat{i}$, $\hat{j}$, and $\hat{k}$ are the usual unit vectors. The wind vector at $\vec{r}$ is

$$\vec{v} = u\hat{i} + v\hat{j} + w\hat{k}$$

where it is assumed

$$\begin{aligned} u &= u_o + (x-x_o)u_x + (y-y_o)u_y + (z-z_o)u_z \\ v &= v_o + (x-x_o)v_x + (y-y_o)v_y + (z-z_o)v_z \qquad (1) \\ w &= w_o + (x-x_o)w_x + (y-y_o)w_y + (z-z_o)w_z \end{aligned}$$

The x, y, z subscripts denote partial differentiation (e.g., $u_x = \partial u/\partial x$); $u_o$, $v_o$, and $w_o$ are the primary parameters we wish to measure, being the values of u, v, and w at the point $(x_o, y_o, z_o)$.

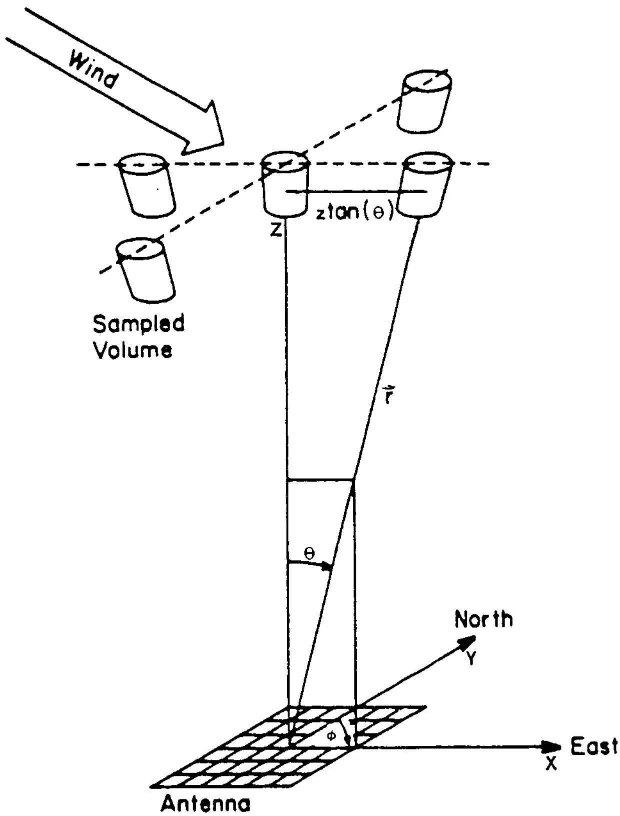


Figure 1.

Often at this point in the analysis, a transformation to polar coordinates convenient to the operation of a radar is made. Instead, we will stay in Cartesian coordinates to facilitate the analysis. If we intend to make measurements of wind velocity at points $(x_o, y_o, z_o)$ for large ranges not centered on the radar antenna, it would be necessary to consider the curvature of the earth (DOVIAK and ZRNIC, 1984, p 261). Here we confine analysis to profile measurement directly above the antenna, where z is identical to height.

The mean radial velocity $v_r$ observed within a sample volume at a given $\vec{r}$ is related to the wind vector $\vec{v}$ there by the dot product

$$rv_r = \vec{v}\cdot\vec{r} = ux + vy + wz \tag{2}$$

Inserting the linear flow relations of equation (1) into equation (2), we get the general relation relating the parameters of the linear wind field, which we would like to measure, to the observed $v_r$'s:

$$\begin{aligned} rv_r = {} & (u_o + u_x(x-x_o) + u_y(y-y_o) + u_z(z-z_o))x \\ & + (v_o + v_x(x-x_o) + v_y(y-y_o) + v_z(z-z_o))y \\ & + (w_o + w_x(x-x_o) + w_y(y-y_o) + w_z(z-z_o))z \end{aligned} \tag{3}$$

Equation (3) is an arrangement of the general scaling equation convenient for developing sample location strategies. Much of the rest of the paper is concerned with describing some of these strategies. First, however, we will examine a second arrangement of the scaling equation in which general characteristics of solution are more apparent:

$$
\begin{aligned}
rv_r = {} & x[u_o - u_x x_o - u_y y_o - u_z z_o] \\
& + y[v_o - v_x x_o - v_y y_o - v_z z_o] \\
& + z[w_o - w_x x_o - w_y y_o - w_z z_o] \\
& + x^2[u_x] \\
& + y^2[v_y] \\
& + z^2[w_z] \\
& + xy[u_y + v_x] \\
& + xz[u_z + w_x] \\
& + yz[v_z + w_y]
\end{aligned}
\tag{4}
$$

This is a linear equation with nine parameters (the quantities in brackets), which may be solved from simultaneous observation of $v_r$ at nine spatially independent locations. Of these nine parameters, only three correspond directly to the twelve parameters that specify the linear wind field. The other six are couples of these parameters, so that, in general, only linear combinations may be found. Specifically, solution of equation (4) for any given position ($x_o$, $y_o$, $z_o$) provides: 1) the parameters necessary to describe the divergence of the flow $\nabla \cdot \vec{v} = [u_x] + [v_y] + [w_z]$; 2) the additional three parameters $[u_y + v_x]$, $[u_z + w_x]$, and $[v_z + w_y]$ necessary to specify the deformation of the flow; and 3) parameters containing $u_o$, $v_o$, and $w_o$ coupled inextricably with spatial derivatives.

## PROFILING: MEASUREMENT OF THE WIND FIELD ALONG THE VERTICAL

In volumes directly above the radar we have that $x_o = y_o = 0.0$, so equation (3) becomes

$$
\begin{aligned}
rv_r = {} & x[u_o + xu_x + yu_y + (z-z_o)u_z] \\
& + y[v_o + xv_x + yv_y + (z-z_o)v_z] \\
& + z[w_o + xw_x + yw_y + (z-z_o)w_z]
\end{aligned}
\tag{5}
$$

Once again, this equation may be rearranged to give

$$
\begin{aligned}
rv_r = {} & x[u_o - z_o u_z] \\
& + y[v_o - z_o v_z] \\
& + z[w_o - z_o w_z] \\
& + x^2[u_x] \\
& + y^2[v_y]
\end{aligned}
\tag{6}
$$

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$$+ z^2[w_z]$$
$$+ xy[u_y + v_x]$$
$$+ xz[u_z + w_x]$$
$$+ yz[v_z + w_y]$$

where the bracketed terms represent the nine parameters that may be evaluated. The difference between this equation and equation (4) is the disappearance of all but the vertical shear terms from the first three parameters, leaving $u_o$, $v_o$, and $w_o$ coupled only to $u_z$, $v_z$, and $w_z$, respectively. From the last two terms we see that when the stratiform condition is present (i.e., $w_x$ and $w_y$ may be neglected), $u_z$ and $v_z$ may be evaluated, thus allowing unbiased solution for $u_o$ and $v_o$.

VERTICAL BEAM MEASUREMENT OF $w_o$ and $w_z$

Measurements made with a vertically pointing beam are particularly simple to analyze, since they only contain information on two of the linear flow parameters: $w_o$ and $w_z$. Inspection of the scaling equation with $x = y = 0$ shows that the observed $v_r$ is identically $w_o$, the vertical velocity component at height $z_o$. Furthermore, the values of $v_r$ observed at two closely spaced heights, $z_o$ and $z_1$, yield $w_z$, so that

$$w_o = v_{ro}$$
$$w_1 = v_{r1} \qquad (7)$$

and

$$w_z = \frac{v_{ro} - v_{r1}}{z_o - z_1}$$

Thus a vertically directed beam allows unbiased determination of $w_o$ and $w_z$ in the presence of linear flow.

OBLIQUE BEAMS

Assuming that $w_o$ and $w_z$ have been determined using a vertically pointing beam, we may treat these values as knowns and move them to the left side of the equation. It is then convenient to define the quantity $V_r$ as

$$V_r = rv_r - z[w_o - z_o w_z] - z^2 w_z$$

which contains only known quantities. Then we can write equation (6) as

$$V_r = x[u_o - z_o u_z]$$
$$+ y[v_o - z_o v_z]$$
$$+ x^2[u_x]$$
$$+ y^2[v_y] \qquad (8)$$
$$+ xy[u_y + v_x]$$

$$+ xz[u_z + w_x]$$

$$+ yz[v_z + w_y]$$

This is the general overhead scaling equation when the vertical velocity and vertical velocity shear are known or negligible. It has seven parameters, requiring seven spatially independent observations for solution.

ORTHOGONAL VERTICAL PLANES

If measurement of the deformation of the flow is not required we can require all beams to be either in the xz or yz vertical plane. Then either y = 0 or x = 0 for all beams, so that the fifth term of equation (8) disappears. Under these conditions, equation (8) divides into two independent equations. For y = 0

$$\frac{V_r}{x} = [u_o + z_o u_z] + x[u_x] + z[u_z + w_x] \tag{9}$$

and for x = 0

$$\frac{V_r}{y} = [v_o + z_o w_y] + y[v_y] + z[v_z + w_y]$$

In each plane, we now have three unknown parameters, requiring three spatially independent observations for solution. Thus, considering both planes and the two vertical beam observations used to evaluate $V_r$, eight independent observations are needed altogether when using orthogonal vertical planes. This does not mean we need eight different beams, since observations may be independent if they are separated in range along a beam. This technique will be considered in the next section.

SINGLE BEAM ANALYSIS

It would be very efficient if equation (9) could be solved by utilizing observations in three sample volumes along a single beam. However, such a scheme cannot lead to complete solution since z and x are not independent, being related by

$$z = x \operatorname{ctn}\theta$$

where $\theta$ is the zenith angle of the beam. For example, in the xz plane substituting for z using this relation yields

$$\begin{aligned} V_r &= x[u_o - z_o u_z] \\ &\quad + x^2[u_x + \operatorname{ctn}\theta\,(u_z + w_x)] \end{aligned} \tag{10}$$

The number of parameters is thus reduced to two. While $u_o$ is still biased by $z_o u_z$, the stratiform approximation no longer allows solution for $u_z$, which is now coupled to both $u_o$ and $u_x$. Thus, neither $u_o$ nor $\vec{\nabla}\cdot\vec{v}$ may be measured in unbiased form in the presence of $u_z$.

CONSTANT HEIGHT ANALYSIS

Starting again with equation (5), the vertical shear terms may be eliminated if we require that all observations be for $z = z_o$ (i.e., constant height analysis), giving

$$rv_r = x[u_o + xu_x + yu_y] \\ + y[v_o + xv_x + yv_y] \\ + z_o[w_o + xw_x + yw_y] \tag{11}$$

Gathering like terms

$$rv_r = z_o[w_o] \\ + x[u_o + z_o w_x] \\ + x^2[u_x] \\ + xy[u_y + v_x] \\ + y[v_o + z_o w_y] \\ + y^2[v_y] \tag{12}$$

This is a linear equation with six unknowns. The bias to $u_o$ and $v_o$ is now the $z_o w_x$ and $z_o w_y$ terms, respectively, so that once again the straiform approximation allows solution for $u_o$ and $v_o$.

USE OF THE VERTICAL BEAM

It is easy to see that for a vertically pointing beam (i.e., $x = y = o$),

$$v_r = w_o \tag{13}$$

Thus, $w_o$ is measured directly, with no bias.

USE OF ORTHOGONAL VERTICAL PLANES

Restricting beam positions such that either $x = 0$ or $y = 0$ splits equation (12) into two independent equations.

$$rv_r - z_o w_o = x[u_o + z_o w_x] + x^2[u_x] \tag{14}$$

and

$$rv_r - z_o w_o = y[v_o + z_o w_y] + y^2[v_y]$$

Thus, two beams in the xz plane plus two beams in the yz plane plus a vertical beam yield $[w_o]$, $[u_o + z_o w_x]$, $[v_o + z_o w_y]$, $[u_x]$, and $[v_y]$.

DISCUSSION AND CONCLUSIONS

The ability of ST radars to measure vertical velocity is unmatched at the present time by any other technique. It is straightforward and without bias, even if the linear wind field assumption is poor. Only under fiscal restraints of the most severe kind, or for very special purposes, should this beam be left out of ST radar systems, since this parameter is not only of general interest, but is necessary to remove bias from u and v measurements (CLARK et al., 1985).

The key parameters to uncoupling terms in the scaling equations are $w_x$ and $w_y$. Whenever the stratiform condition, which states that these two parameters are negligible, is satisfied, a five-beam ST radar may determine unbiased (with reference to a linear wind field) values of u, v, and w for sample volumes directly above the radar. Furthermore, the divergence and partial deformation of the flow may be determined.

Three-beam systems can determine w and $w_z$, but are unable to obtain u and v wind components uncontaminated by vertical sheer terms, even when the stratiform condition is satisfied.

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