

### 3.7.8 EVIDENCE OF A SATURATED GRAVITY-WAVE SPECTRUM THROUGHOUT THE ATMOSPHERE

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#### 1. INTRODUCTION

A number of studies have addressed the frequency and wave number spectra of atmospheric motions over the past two decades and have revealed a surprising degree of consistency among different measurement techniques. This consistency in spectral shape and power has motivated two competing theories concerning the nature of atmospheric motions responsible for the spectral characteristics. One theory attributes atmospheric motions primarily to two-dimensional (essentially horizontal) turbulence (GAGE, 1979); the other identifies such motions with internal gravity waves (VANZANDT, 1982), analogous to the motion spectrum believed to exist in the oceans (GARRETT and MUNK, 1975).

In this paper, we adopt the view that the dominant mesoscale motions are due to internal gravity waves and show that previous and new vertical wave number spectra of horizontal winds are consistent with the notion of a saturation limit on wave amplitudes. We also propose that, at any height, only those vertical wave numbers  $m > m_*$  are at saturation amplitudes, where  $m_*$  is the vertical wave number of the dominant energy-containing scale. Wave numbers  $m < m_*$  are unsaturated, but experience growth with height due to the decrease of atmospheric density. The result is a saturated spectrum of gravity waves with both  $m_*$  decreasing and wave energy increasing with height. This saturation theory is consistent with a variety of atmospheric spectral observations and provides a basis for the notion of a "universal" spectrum of atmospheric gravity waves. It should be noted that the saturation spectrum argument has been advanced independently by DEWAN and GOOD (1985).

#### 2. A SATURATED VERTICAL WAVE NUMBER SPECTRUM

Vertical wave number spectra of horizontal wind fluctuations from several sources that span the height range from the ground to 130 km are plotted in Figure 1. The curve labelled "troposphere" is smoothed data obtained with the use of Jimspheres by ENDLICH et al. (1969). This curve is an average of the spectra from six wind profiles, acquired at roughly 2-hr intervals, from which the average wind profile was subtracted, and has a slope at high wave numbers of  $\sim -2.5$ . The best fit curve of DEWAN et al. (1984) to their stratospheric spectra is displayed as the curve labelled "stratosphere". This curve follows an  $m^{-2.7}$  power law (where  $m$  is vertical wave number) from a wave number of  $10^{-3}$  to  $5 \times 10^{-2}$  cycles/m (corresponding to vertical wavelengths of 20 m to 1 km). The curve labelled "mesosphere" is the inferred horizontal velocity spectrum from the average of summer-time spectra obtained near the mesopause with the Poker Flat MST radar as discussed by SMITH et al. (1985). The slope of this velocity spectrum is  $-2.5$ . Finally, the thermospheric spectrum was obtained from winds measured between 85 and 130 km with a three-axis accelerometer sphere flown as part of the STATE campaign. This spectrum is of

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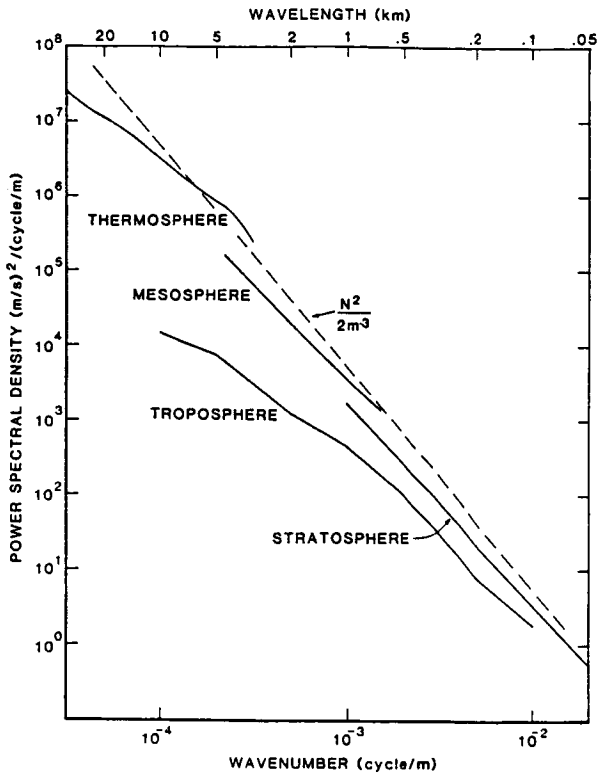


Figure 1. Vertical wave number spectra obtained in four regions of the atmosphere using a variety of different techniques. See text for details.

the scalar wind speed measured by the falling sphere and was smoothed by two passes of a three-point running average to increase the confidence level. It was not possible to remove the mean wind from this velocity data, however.

Figure 1 shows clearly that the spectral shape and the spectral power are very similar from the stratosphere to the lower thermosphere. This spectral similarity can be extended to the troposphere as well by scaling the spectra to a common value of the Brunt-Vaisala frequency,  $N$ . The data of DEWAN et al. (1984) and the radar and falling sphere data were obtained from regions with  $N \sim 0.02$  rad/s (a Brunt-Vaisala period near 5 min). However, a typical value of  $N$  in the troposphere is half that in the stratosphere. The linear saturation theory reviewed in the next section implies that spectral power should scale as  $N^2$ . Thus, an increase in the tropospheric data of ENDLICH et al. (1969) of  $\sim 4$  is required to permit its comparison with data obtained at greater heights.

Assuming that most of the spectral power is associated with vertically propagating gravity waves, the consistency of the scaled vertical wave number spectra with height clearly suggests that some process is acting to limit wave amplitudes, particularly at high vertical wave numbers, throughout the atmosphere. This is consistent with the suggestion, due to WEINSTOCK (1982), that gravity waves are saturated throughout much of the atmosphere.

### 3. LINEAR SATURATION THEORY

Linear saturation theory assumes that a monochromatic gravity wave will be limited to that amplitude at which the wave just reaches the point of convective instability, i.e., where

$$\theta_z = 0, \quad (1)$$

where  $\theta$  is the potential temperature. This is equivalent to the condition (see FRITTS, 1984)

$$u' = c - \bar{u}, \quad (2)$$

where  $u'$  and  $u$  are the horizontal perturbation velocity and the mean flow in the direction of wave motion. This may be written, using the dispersion relation for gravity waves with intrinsic frequencies  $\omega$  such that  $f^2 \ll \omega^2 \ll N^2$ , as

$$u' = N/m. \quad (3)$$

Thus, the power spectral density for horizontal velocity fluctuations inferred by assuming that each component of the gravity-wave spectrum is individually saturated is

$$E(m) = N^2/2m^3. \quad (4)$$

The third power of  $m$  is due to the wave number bandwidth corresponding to each component of the gravity-wave spectrum. This spectral power is shown for  $N = 0.02$  rad/s with a dashed line in Figure 1.

The theoretical curve exceeds the power spectral densities observed throughout the atmosphere by about 2. We believe this difference may be attributed to the superposition of gravity waves in the atmosphere, which seems to restrict amplitudes to less than monochromatic saturation values (SMITH and FRITTS, 1983; MEEK et al., 1985), in good agreement with the observed power spectral densities. Recent numerical results by FRITTS (1985) suggest a similar reduction in the wave amplitudes required for saturation due to superposition.

### 4. VARIATION OF GRAVITY-WAVE SPECTRUM WITH HEIGHT

Both the data presented in Figure 1 and the form of the analytic gravity-wave spectrum used to fit successfully previous oceanic and atmospheric gravity-wave spectra (GARRETT and MUNK, 1975; VANZANDT, 1982) suggest that the gravity-wave spectrum at any height departs from a saturated spectrum at sufficiently small vertical wave numbers. Observational evidence of this departure is most evident in the tropospheric curve in Figure 1, but may be present in the thermospheric data as well. The departure in the analytic spectral description consistent with a saturated high wave number spectrum enters as a spectral break at a wave number  $m_*$  via a term of the form

$$(1 + m/m_*)^{-3}, \quad (5)$$

providing limiting slopes of -3 and 0 for high and low wave numbers, respectively.

The departure from a saturated spectrum at small vertical wave numbers in the lower atmosphere suggests that these motions are not excited at large amplitudes. As these motions propagate upward, however, they are expected to

grow with height due to the decrease in atmospheric density. The results of this process are a gravity-wave energy and a dominant vertical wavelength that increase with height, as noted in a number of previous studies.

A model of the proposed vertical wave number spectrum and its variation with height is presented in Figure 2. This spectrum assumes a vertical wave number dependence given by (5). At vertical wave numbers  $m > m_*$ , for which the spectrum is assumed saturated, the energy levels are the same at all heights. At smaller (unsaturated) wave numbers, however, the growth of wave amplitudes with height causes the dominant vertical scale ( $m_*^{-1}$ ) to increase accordingly. This is shown with values of the dominant vertical wavelength ( $\lambda_z = m_*^{-1}$ ) of 1, 5, and 20 km in the troposphere, stratosphere, and mesosphere, respectively.

Evidence of this behavior is provided by a variety of observations, including gravity-wave energies that increase with height (BALSLEY and CARTER, 1982; BALSLEY and GARELLO, 1985) and detailed gravity-wave studies in the stratosphere, mesosphere, and lower thermosphere showing gravity-wave scales that increase with height (SATO and WOODMAN, 1982; BARAT, 1983; SMITH and FRITTS, 1983; MEEK et al., 1985; FRITTS et al., 1985). We should expect, however, that the energy at low wave numbers, and hence  $m_*$ , will be considerably more variable than the energy at  $m > m_*$ , because the gravity-wave energy at  $m < m_*$  is determined by the strengths of various gravity-wave sources at lower levels, whereas that at  $m > m_*$  is limited by saturation processes.

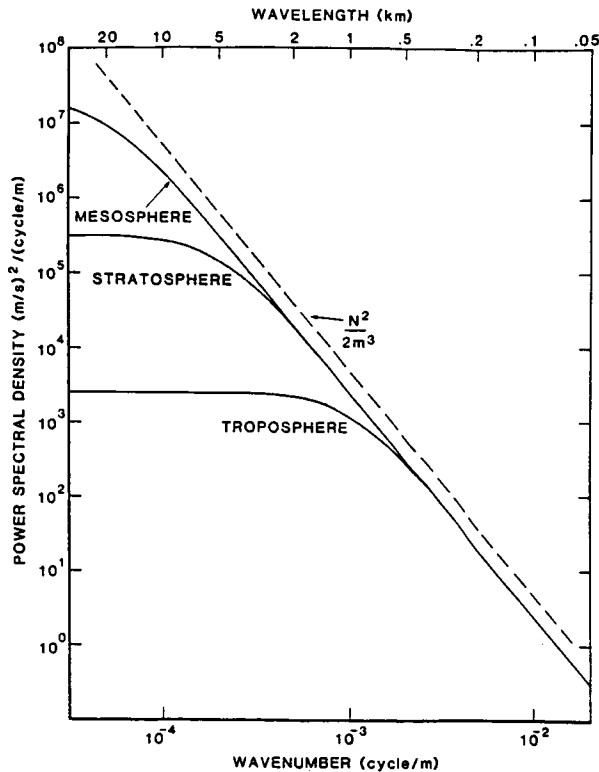


Figure 2. Model saturated gravity-wave spectrum scaled to a common  $N^2$  at three different heights. Note the constant (saturated) energies for large wave numbers and the increase in the dominant vertical scale with height.

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