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## 3.1.6 ATMOSPHERIC WAVES AND THE NATURE OF BUOYANCY TURBULENCE IN THE CONTEXT OF THE WAVES VS 2D-TURBULENCE DEBATE

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## INTRODUCTION

An interesting question is "How does one empirically distinguish between velocity fluctuations due to turbulence and those due to waves?" The subject is more interesting at present because there is a controversy between those who interpret such velocity fluctuations as being due to 2-D turbulence (GAGE, 1979; LILLY, 1983) vs those who attribute them to waves (VANZANDT, 1982; DEWAN, 1979). Is there a way to determine by means of experiment which view is correct, or when one or the other is more appropriate? Unfortunately, the power spectral density (PSD) does not help very much with this problem.

The goal of this and the companion paper to follow is to address this problem. It will, however, be necessary to first discuss the physical differences between waves and turbulence. One of the main purposes of this paper is to display certain new theoretical ideas on the subject of buoyancy range turbulence in this context. The companion paper presents a proposed empirical test to distinguish between 2-D turbulence and gravity waves.

## WAVES VS TURBULENCE

Inertial range turbulence (IRT) involves an energy cascade or, to put it another way, strong mode interactions. The PSD has a $k^{-5/3}$ dependence where k is the wave number. IRT also involves strong mixing and it is isotropic. The cascade was vividly described by TENNEKES and LUMLEY (1972) in terms of a 3-D vortex stretching interaction between scales. RICHARDSON (1972) used poetry for the same purpose.

In contrast, buoyancy range turbulence is not isotropic but is strongly affected by buoyancy. It was described by Bolgiano (1959) who gave a $k^{-11/5}$ dependence for the spectrum, and by LUMLEY (1964) who gave $k^{-3}$ for the dependence. There were other prominent contributors as well, but, the main point is that this work led to a particular length scale known as the buoyancy length $l_B = (\varepsilon/N_B^3)^{1/2}$ where $\varepsilon$ is the dissipation rate and $N_B$ the buoyancy frequency. This scale separates IRT from BRT according to these early authors. As we shall see, this is indeed correct, but a slight extension of this concept leads to a scale which separates waves from BRT.

Table 1 summarizes some basic differences between waves and turbulence. A particularly useful distinction involves the interaction time between modes. BRT can be best regarded as a field of wave modes which interact so strongly that a given mode dies within one period or so of oscillation. In contrast, the fluctuations which can properly be called waves oscillate for very many periods and do so in a linear fashion i.e., without significant mode interaction.

## INTERACTION TIME AND THE BRT/WAVE SEPARATION SCALE

While there is very little interaction between waves, Phillips and others have shown that under certain resonant conditions (PHILLIPS, 1977) there is indeed some interaction. He has shown (PHILLIPS, 1960) that the interaction time, $T_i$, is to a certain approximation,

$$T_i \cong (k_1 v_1 k_2 v_2)^{-1/2} \tag{1}$$

where $k_i$ and $v_i$ refer to the wave numbers and particle velocities of components of a resonant triad of interacting waves. If one ignores constants of order unity and if we let k and v refer to the primary wave, then we can, for our purposes, use the approximation

$$T_i \cong (kv)^{-1} \tag{2}$$

The dispersion relation for an incompressible buoyancy wave is simply

$$T_w = (N_B \cos \theta)^{-1} \tag{3}$$

where $T_w$ is the wave period, $\theta$ is the angle between the wave vector k, and the horizontal. In this paper, we shall ignore factors of 2.

In view of the above discussion, we shall characterize waves by

$$T_i << T_w \tag{4}$$

and BRT by the reverse of this inequality. It follows that the boundary between the two regimes is given by $T_i = T_w$. From this it follows that at the boundary

$$(N_B \cos \theta) = (kv) \tag{5}$$

from (2) and (3).

It is useful to eliminate v from (5). For this purpose we consider the case where $\theta = 0$. As will be shown below, this leads to the transitional scale that separates horizontally propagating waves and IRT. We next assume that at this scale all the energy of this borderline wave with velocity v is fed into the IRT cascade and that the energy emerges from the small scale end of that cascade in the form of $\varepsilon$, i.e., dissipation. With this in mind, and using the definition that a borderline wave dumps all of its energy in one period, we obtain

$$\varepsilon = \frac{v^2}{T_w} \tag{6}$$

This is used in Equation (5) to eliminate v and hence

$$N_B = k_B \left(\frac{\varepsilon}{N_B}\right)^{1/2} \tag{7}$$

or

$$k_B = (N^3/\varepsilon)^{1/3} \tag{8}$$

an equation which has what may be a surprisingly familiar look to it. It is, of course, the inverse of the well-known buoyancy length, but it appears in a novel context. At first it seems to contradict the assertion that this length separates IRT and BRT; however, the seeming contradiction will soon be resolved below.

To address the above paradox, we now turn to the general case where $\theta$ is allowed to be arbitrary. In this case, the borderline condition which is given by $T_w = T_i$ leads to

$$k_B = (N_B{}^3 \cos^3 \theta / \varepsilon')^{1/2} \tag{9}$$

where $\varepsilon$ has been replaced by $\varepsilon'$ in anticipation of the discussion to follow. For BRT we claim that the clearest physical description is that it consists of a cascade of strongly interacting waves of large amplitude. This cascade resembles IRT in the sense that, for the most part, the flow of energy is down the scale. (The subharmonic instability is a good candidate for the mechanism of interaction). Now an interesting observation is that, as $\theta$ is increased, $T_w$ is also increased (in accordance with the dispersion relation). This means that for any $T_i$ one can find a $T_w$ such that $T_i = T_w$. In principle, as $\theta$ goes to 90°, $T_w$ goes to $\infty$. It is these waves where $\theta > 0$ but where $T_w < T_i$ which are not proper waves in the usual sense but are, rather, the strongly interacting buoyant modes which constitute what is designated at BRT.

As promised, we now discuss $\varepsilon'$. In BRT, there are two additional ways for energy to be dissipated in contrast to IRT which has only $\varepsilon$. These are, namely, $\varepsilon_{PE}$ which is the dissipation rate of potential energy brought about by mixing in the stratified fluid, and $\varepsilon_{RAD}$ which is due to the radiation of energy in the form of gravity waves that are generated by a certain amount of "up scale" energy flow caused by mode interactions. Thus,

$$\varepsilon' = \varepsilon_{PE} + \varepsilon_{RAD} + \varepsilon \tag{10}$$

Unfortunately, the numerical values of $\varepsilon_{RAD}$ and $\varepsilon_{PE}$ are not known. As can be seen from Equation (9), BRT can exist for wavelengths ranging from the usual "buoyancy length border" or, the outer scale of IRT, to scales that are boundless for $\theta$ close to 90°. The very large wavelengths and associated long periods correspond to nearly horizontal particle motion as can be seen from the incompressibility condition $\vec{k} \cdot \vec{v} = 0$. In other words, as $\theta$ is increased, $\vec{k}$ becomes more vertical, the period lengthens, and particle motion becomes horizontal. Thus, the question arises, "Does this type of BRT represent what is usually called 2D-turbulence?" The answer seems to be "no" for the following reason. The 2D-turbulence in the literature involves a cascade in the direction of small to large scale, which is to say, a reverse cascade. BRT does not seem to fit this description.

We leave as an unanswered question "Where does 2-D turbulence fit as strongly interacting buoyancy-affected modes such that

$$k < (N_B^3 \cos^3\theta/\varepsilon')^{1/2}$$

Equation (11), in principle, could lead to an empirical test between BRT and waves. Further discussion will be given in the companion paper.

Table I

Physical distinctions between waves and turbulence

| Waves | Turbulence |
|---|---|
| 1. Linear Superposition | 1. NonLinear "Promiscuous" Mode-Interaction ("Cascade" in k-space) |
| 2. No Fluid Mixing | 2. Fluid Mixing (Dispersion) |
| 3. Wave Pattern is Global (In Space & Time Propagation Lasts many periods Coherence | 3. Eddies are Local (In Space & Time) No Propagation Decays in about one period Incoherence |
| 4. Obeys Dispersion Relation | 4. No Dispersion Relation |

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