A Simple Method for Designing or Analyzing an Optical Communication Link

J. R. Lesh, W. K. Marshall, and J. Katz
Communications Systems Research Section

A simple method is described for determining the performance of a free space optical communication link. The method can be used either in the system design (synthesis) mode or in the performance evaluation (analysis) mode. Although restricted to photo counting (e.g., photomultiplier tube or equivalent) based detection of pulse position modulated signals, the method is still sufficiently general to accommodate space-based, as well as ground-based, reception.

I. Introduction

Experience over the years with the Deep Space Network has produced a high degree of familiarity and intuitive understanding of RF communications. Not only engineers but managers and scientific users as well generally appreciate that doubling signal power can double the data rates and that a bit SNR of 10 dB results in quite good, but most likely not overdesigned, performance. This intuition is facilitated by the fact that an RF communications link is usually limited by the thermal noise of the communication receiver and the performance of the link is determined once the received signal-to-noise power ratio is specified.

For a deep space optical communications link, the dominant performance limitation is usually not related to noise *in* the receiver but due to received background noise (light). The performance of a link depends on the individual values of the signal and noise powers, not just on their ratio. Furthermore, there are meaningful examples for which a high "SNR" (defined here as a ratio of signal and noise powers or counts) can result in poor performance whereas there are likewise examples where a poor "SNR" can result in essentially error-free performance. This latter point deserves explanation.

Consider a very benign optical communication channel where the background (noise) count rate is 0.001 over some characteristic decision time whereas the signal count rate over that time is one count. Such an example is not too unusual for a high data rate free-space optical channel. Despite the fact that the "SNR" is 1000 (30 dB), the system performance is dominated by the weak signal pulse erasure rate which is e^{-1} or 37%. Now consider a much noisier channel where both the signal and background count rates are 1000. Here, the "SNR" is 1 (or 0 dB). The detector's job is to distinguish the absence of a signal pulse (which is characterized by a mean noise count rate of 1000) from a signal pulse present condition where the mean count rate (signal + noise) is 2000. However, the Poisson statistics that govern the count processes produce RMS variations around these mean values, which equal the square roots of those mean values. The RMS spread around the 1000-count mean is 33 and that of the 2000-count mean is only 45. These two distributions are clearly distinguishable with very low probability of error.

The above results notwithstanding, it is very desirable that people affected by optical communication technology comfortably build intuition about its performance. The purpose of this article is to provide a simple means by which that intuition can be acquired. It should be emphasized that this is a first-order design tool useful for scoping an optical link and illustrating trends therefrom, and does *not* replace the more rigorous (and accurate) analysis techniques that have been developed elsewhere. The information provided should permit a relatively rich set of cases to be addressed.

In the next section we will describe the procedure and illustrate its use by analyzing an example. The example will involve ground-based reception of a spacecraft signal from Mars with that planet in the background. Following this, the rules for scaling the results to other parameter values will be presented. These scaling rules permit a rather wide set of parameter values and conditions to be accommodated. To illustrate this we will then use the rules to determine the required aperture size for a space-based receiver for the above example. It should be pointed out, however, that these results only apply to direct detection (not heterodyne) systems which utilize photomultiplier tube (or equivalent) based detectors.

II. The Simplified Design and Analysis Procedure

There are basically three steps to evaluating the performance of an optical link:

(1) Determine the number of *detected* signal photons per pulse at the detector.

(2) Determine the number of *detected* background or noise generated photons per PPM slot at that same detector.

(3) Compare the number of detected signal photons per pulse with the number of detected noise photons per slot.[1]

The operations may be done in any order and are routinely done so depending on whether the job is to analyze a given link or determine requirements on parameters to meet a specific level of performance. The following example uses the procedure in the specified (analysis mode) order.

[1]From conventional PPM modulation, the slot width, τ_s (in seconds), is related to the data rate, DR (in bits/s), and the PPM word size M by $\tau_s = \log_2 M/DR*M$. However, for greater noise immunity, one can decrease the slot time by introducing dead time. Thus, τ_s is a free parameter as long as it is smaller than $\log_2 M/DR*M$.

A. Performance Analysis Example

In this example, we have assumed a 10-meter, ***ground-based*** receiver aperture with a rather broad (5 μrad) field of view to account for atmospheric turbulence broadening. This field of view admits about 100 times more background light into the detector than a spaceborne 10-meter aperture (assuming both apertures are non-diffraction limited at 10 times the diffraction limit), and it admits nearly 10^4 times more noise than a spaceborne diffraction limited aperture of the same size. The example also assumes a 400-milliwatt laser operating at a wavelength of 0.532 μm through a 10-cm transmitter telescope from Mars with the sunlit planet in the background. We assume a data rate of 30 kbps and 256-ary PPM modulation. All of the values used in the example are quite conservative, and will ***realistically*** permit variations on both sides of the parameter values.

1. Determine number of detected signal photons/pulse. A diffraction limited (transmit) telescope of diameter D meters produces a useable beamwidth (diameter) θ_t of approximately 1.5 λ/D radians, where λ is the optical wavelength in meters.[2] For $D = 10$ cm and $\lambda = 0.532 \times 10^{-6}$ meters, $\theta_t \sim 8.0$ μrad. At an Earth–Mars distance, R, of 2.3×10^8 km, the footprint diameter at the Earth is 1835 km. If light is collected by a 10-meter-diameter aperture, then the geometric signal level reduction factor is

$$\left(\frac{10 \text{ m}}{1.84 \times 10^6 \text{ m}}\right)^2 = 3 \times 10^{-11}$$

If 400 milliwatts is transmitted, then (assuming for the moment no losses other than beam spread), the power at the receiver is 1.2×10^{11} watts. At 30 kbps and PPM word size of 256 (8 bits/pulse), the pulse rate is 3750 pulses/second. Thus, each signal pulse has 3.2×10^{-15} joules of energy. If the energy per photon is $h\nu = hc/\lambda = 3.7 \times 10^{-19}$ joules, then each pulse contains 8.6×10^3 photons. Additionally, one must consider other (non-space) losses. Let us assume the following efficiencies:

Transmit optics	50%
Receive optic	50%
Atmospheric attenuation	50%
Detector quantum efficiency	30%
Total *detected* photon efficiency	3.75%

Then, the number of *detected* photons per pulse is 321.

[2]The factor of 1.5 takes into account the effects of Gaussian beam illumination of the aperture and 20% subreflector blockage. For details, see the appendix.

2. **Determine number of detected noise photons/slot.** The next task is to determine the number of background-generated noise photons which the detector senses (on the average) per PPM slot time. Table 1 shows the approximate count rates for a variety of extended sources (i.e., bigger than the detector field of view) as well as a number of point sources (those smaller than the detector field of view). The parameters assumed in the calculations are shown in the lower portion of the table.

By referring to this table we see that unless the Sun, or a very bright star (of which there are very few), is in the field of view, the background count rate, even with daytime viewing, will be only about 0.2 counts/slot (0.1 from Mars and 0.1 from daytime sky).

3. **Compare detected signal photons per pulse with detected noise photons per slot.** The final step involves assessing the performance of the link given the appropriate signal and noise count rates. Figure 1 shows a curve of the number of detected signal photons per pulse required to achieve an uncoded error rate of 10^{-3} as a function of the number of detected noise photons per slot. The curve was calculated for a PPM word size, M, of 256, but is "first-order accurate" for word sizes as small as $M = 2$ or as large as many thousands. Furthermore, the error rate can be reduced very substantially by even rudimentary coding. (For example, an 8-bit Reed–Solomon code used at this uncoded bit error rate would produce a coded error rate of $\sim 10^{-22}$.) From this curve we see that for a detected noise count rate of 0.2 counts/slot, the required number of signal photons per pulse is approximately 11. Comparing this with the available number of detected signal photons from above, we see that this example link provides a margin of 14.6 dB!

B. Design Procedure Example

The above example was calculated based on the number of geometrically intercepted signal photons at the receiving aperture (reduced by the appropriate set of inefficiencies) and the corresponding background count rates from Table 1. For different situations, the geometry and efficiencies still determine the signal counts per transmitted pulse. However, the noise count rates must be scaled to the new set of conditions. Table 2 gives the scaling rules for this operation. To see how these rules are used, we will now calculate the aperture size required for spaceborne reception in the above example which will produce approximately a 3-dB link margin.

We note that moving the receiver outside the Earth's atmosphere will reduce the background count rate by a factor of two due to deletion of the daylit sky. However (see Table 2), the count rate from Mars will double due to the loss of the Earth's atmospheric attenuation. Thus, a 10-meter aperture in space will have the same background count rate as one on the ground (in this particular example and assuming the same detector fields of view). Recall that in the previous example the link was more than 10 dB overdesigned and the detector field of view (5 μrad) was limited by atmospheric seeing. Let us assume that for spaceborne reception the field of view is reduced to 2 μrad (which is still substantially larger, i.e., worse, than diffraction limit). Note from Table 2 that reducing the field of view to 40% of its original value reduces the background noise effects by a factor of 0.16, and reducing the aperture diameter (anticipated due to the overdesign of the previous example) further reduces noise quadratically. However, from Fig. 1 we see that reducing the background count by several orders of magnitude still leaves the required count rate relatively unchanged at around 8 counts per pulse. Thus, we can use this count rate to determine the required aperture size.

Recall that atmospheric attenuation affects both signal as well as background. Thus, the 321 counts/pulse of the previous example, which corresponds to 642 counts/pulse outside the atmosphere, can be reduced to 16 (3-dB margin over the 8) by reduction of the aperture. This corresponds to a receiver aperture diameter of 1.6 meters. Using this diameter to further refine the background count rate estimates we have that

$$N_b = \underbrace{(0.2 \text{ cts/slot})}_{\substack{\text{Mars count rate}\\ \text{at 10-m and}\\ 5\text{-}\mu\text{rad FOV}}} \underbrace{(2/5)^2}_{\substack{\text{FOV}\\ \text{reduction}}} \underbrace{(1.6/10)^2}_{\substack{\text{Aperture}\\ \text{reduction}}} = 8 \times 10^{-4} \text{ cts/slot}$$

which, from Fig. 1, implies that only 7 detected photons per pulse are actually required. Note also from Fig. 1 that the detector field of view could be increased substantially (and thereby greatly reducing the cost of the receiver aperture) without requiring more than 8 detected photons per pulse (16 with a 3-dB margin).

III. Concluding Remarks

We have shown a simplified procedure for analyzing or designing a direct detection, photon counting optical link with background noise. The procedure consists of a simple geometric calculation of the received signal pulse intensity, a table look-up method for background noise, and a single curve against which to compare the two. Additionally, scaling rules for calculating other situations were given and their use illustrated through an example. This method will aid those who prefer not to dig more deeply into the theory of optical communications to easily build an intuitive understanding of the field.

Reference

1. Klein, B. J., and Degnan, J. J., "Optical Antenna Gain. 1: Transmitting Antennas," *Appl. Opt.*, Vol. 13, No. 9, pp. 2134–2141, September 1974.

Table 1. Background noise counts in ground-based optical communications

Extended sources	Number of noise counts/slot
Sun	10^5
Mercury	0.8
Venus	3.0
Earth (typical)	0.6
Mars	0.1
Jupiter	0.03
Saturn	0.01
Uranus	0.003
Neptune	0.001
Moon	0.2
Clear sky:	
Day	0.1
Moonlit night	10^{-7}
Moonless night	10^{-8}
Background noise outside the atmosphere	10^{-8}

Point Sources	Number of noise counts/slot
Zero magnitude star	6
6th magnitude star	0.02
Pluto	3×10^{-5}

Parameter values assumed for the calculations:

1. Wavelength: 0.5 μm
2. Slot time: 10 ns
3. Receiver diameter: 10 m
4. Detector field of view: 5 μrad
5. Receiver optics efficiency: 0.5
6. Detector quantum efficiency: 0.3
7. Optical filter bandwidth: 10 Å
8. Atmospheric transmission: 0.5

Table 2. Scaling rules for Table 1

The number of noise counts varies according to the following rules:

1. Linearly with the optical filter bandwidth, slot time, detector quantum efficiency, atmospheric transmission, and receiver optics efficiency.
2. Quadratically with the receiver diameter.
3. Quadratically with the detector field of view (only for noise generated by extended background sources). Note, however, that for space-based reception, and assuming constant surface figure requirements, the field of view scales inversely with receiver diameter.
4. According to Blackbody radiation law (illumination by Sun at 5900 K) with the wavelength. Typically, noise counts in the 0.8–1-μm region will be 2 to 3 times smaller than in 0.5 μm.

Notes:

1. Planets appear as extended sources only for fields of view smaller than their own angular extent. Thus, the field of view scaling should be checked for values above 10 μrad and below 2 μrad.
2. For space-based reception extraterrestrial background sources (as well as the desired signal source) should be increased by a factor of two.
3. Noise contributions from planets vary substantially with wavelength, phase angle, and other factors (e.g., contributions from Saturn's rings). Values used are believed to represent the worst case situation.

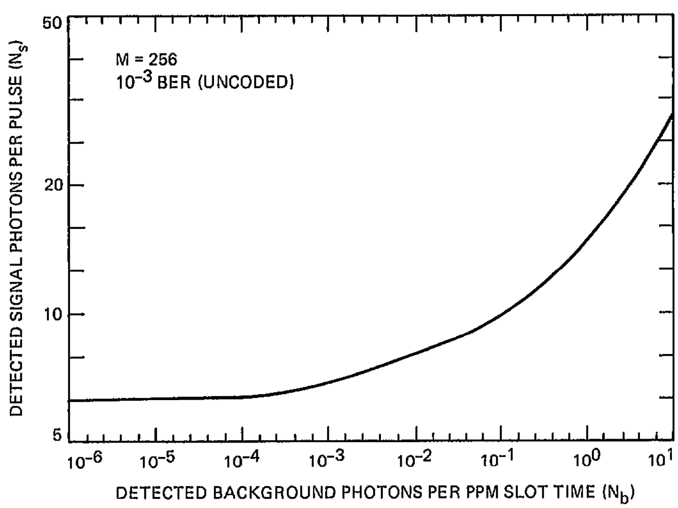

Fig. 1. Required signal pulse intensity at detector vs detected background count rate

Appendix

Equivalent "Uniform Cone" Beamwidth

In the first-order analyses of communications links, it is sometimes desirable to calculate the signal power density at the receiver by means of a "cone of uniform intensity" approximation for the transmitted beam. To use this approach, however, it is necessary to choose the proper beamwidth for a given transmitter aperture size. The analysis below indicates that the beam diameter should be assumed to be 1.5 λ/D (radians). Results obtained using this approach should be accurate to within 1 dB in most cases.

Consider an antenna/telescope which transmits power over a uniform cone of half-angle θ. The on-axis gain of such an antenna would be

$$g = \frac{4\pi L^2}{\text{spot area at distance } L}$$

$$= \frac{4\pi L^2}{\pi(\theta L)^2}$$

$$= \frac{4}{\theta^2} \qquad \text{(A-1)}$$

Consider now an "ideal" circular antenna of diameter D. This antenna would have an on-axis gain of

$$g = \frac{4\pi(\pi D^2/4)}{\lambda^2}$$

$$= \frac{\pi^2 D^2}{\lambda^2} \qquad \text{(A-2)}$$

Hence, an *ideal* transmitting antenna (with no pointing losses) can be modeled *exactly* via the "uniform cone" method by choosing the beam *half-angle* to be

$$\theta = \frac{2\lambda}{\pi D} = 0.637\frac{\lambda}{D} \qquad \text{(A-3)}$$

so that the on-axis gains given by Eqs. (A-1) and (A-2) are equal.

Now note that a real antenna of diamter D has a gain lower than that given by Eq. (A-2), due to several effects:

(1) Transmission/reflection losses.

(2) Pointing losses.

(3) Beam truncation and aperture obscuration losses.

Losses due to (1) are usually handled explicitly in calculating the received signal. Losses due to (2) are usually assumed to be small, for example, less than 1 dB. Losses due to (3) can be handled easily within the "uniform cone" method by using an effective aperture size in Eq. (A-3) rather than the actual diameter. For realistic telescopes with Gaussian beams,

	Factor	
Truncation loss	0.81	(0.9 dB)
Obscuration loss	0.85	(0.7 dB)
Total	0.69	(1.6 dB)

Hence, one typically has $D_{\text{eff}} = \sqrt{0.69}\,D = 0.83\,D$

Equation (A-3) becomes

$$\theta = 0.637\frac{\lambda}{D_{\text{eff}}}$$

$$= 0.637\frac{\lambda}{0.83\,D}$$

$$= 0.77\frac{\lambda}{D}$$

Therefore, when using the "uniform cone" method, the diametrical beamwidth should be $2 \times 0.77\ \lambda/D \sim 1.5\ \lambda/D$.

As a final note, it is important that the telescope be pointed accurately compared to the width of the central far-field lobe. This width is typically about 2.4 λ/D (diameter), so the requirement is

$$\text{pointing error} \ll 2.4\ \lambda/D$$

For more information see Ref. 1.

Maximum Likelihood Estimation of Signal-to-Noise Ratio and Combiner Weight

S. Kalson and S. J. Dolinar
Communications Systems Research Section

An algorithm for estimating signal-to-noise ratio and combiner weight parameters for a discrete time series is presented. The algorithm is based upon the joint maximum likelihood estimate of the signal and noise power. The discrete-time series are the sufficient statistics obtained after matched filtering of a biphase modulated signal in additive white gaussian noise, before maximum likelihood decoding is performed.

I. Introduction and Problem Model

This article investigates maximum likelihood estimation of signal-to-noise ratio and combiner weight parameters for a discrete time series. The discrete time series are the sufficient statistics obtained after matched filtering of a biphase modulated signal (Ref. 1). In order to show the underlying assumptions and limitations of the estimation problem, we first examine the communication system that gives rise to the discrete time series.

We take as our model that given in Fig. 1. The channel encoder maps the binary digital source encoder output $\{I_k\}$ into the binary channel symbols $\{C_k\}$, where the channel symbols are produced with rate $1/T$. The modulation is biphase. That is, the modulator produces the baseband signal

$$s(t) = \sum_k A_k q_k(t) \tag{1}$$

where the $\{A_k\}$ are chosen according to

$$A_k = \begin{cases} -\sqrt{E_s}, & C_k = \text{"0"} \\ +\sqrt{E_s}, & C_k = \text{"1"} \end{cases} \tag{2}$$

Here, E_s is the channel symbol energy, and the $\{q_k(t)\}$ are orthonormal basis functions. We assume that the $\{q_k(t)\}$ are time-displaced replicas of a single function of duration T, namely,

$$q_k(t) = q(t-(k-1)T) \tag{3}$$

where

$$q(t) = 0, \ t<0 \text{ or } t>T \tag{4}$$

$$\int_0^T q(t)^2\, dt = 1 \tag{5}$$

The baseband signal $s(t)$ is transmitted over an additive white gaussian noise channel with one-sided noise spectral