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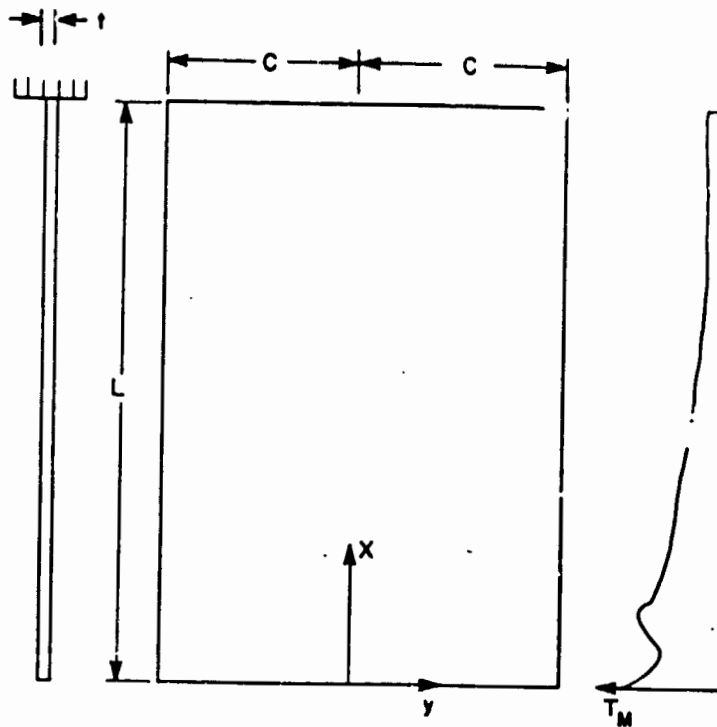
STRESS AND BUCKLING ANALYSIS

UNIVERSITY OF KENTUCKY

O. Dillon

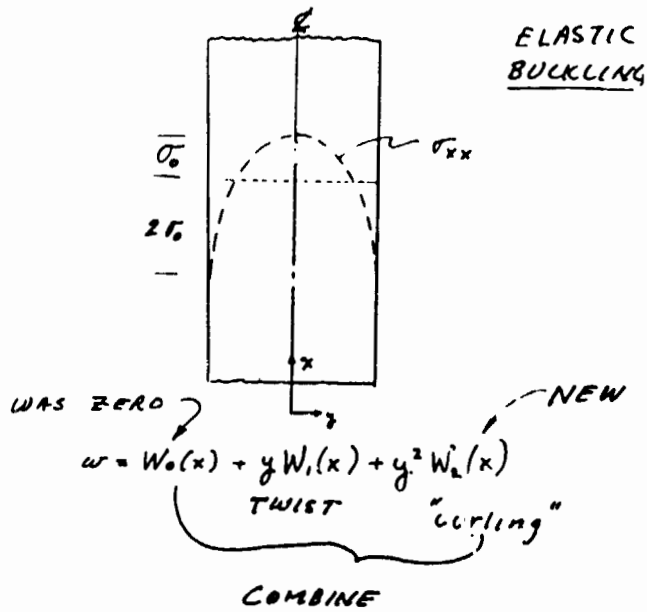
<p>TECHNOLOGY STRESS AND BUCKLING ANALYSIS</p>	<p>REPORT DATE 10/3/84</p>
<p>APPROACH MODEL MATERIAL BEHAVIOR MODEL BUCKLING DUE TO THERMAL STRESSES</p> <p>CONTRACTOR UNIVERSITY OF KENTUCKY</p>	<p>STATUS</p> <ul style="list-style-type: none"> • ELASTIC STRESS AND BUCKLING ANALYSIS COMPLETED FOR CONSTANT MATERIAL PROPERTIES. • CRITICAL SHEET THICKNESS VS. SHEET WIDE COMPARED FOR FOUR THERMAL PROFILES. • RESULTS ARE REASONABLY CONSISTENT WITH EXPERIMENT. <p>SURVEY OF THE MECHANICAL PROPERTIES OF SILICON</p> <p>PRE-BUCKLED STRESSES IN PLASTIC RANGE</p>
<p>GOALS</p> <ul style="list-style-type: none"> • PROVIDE GUIDANCE BASED ON ANALYSIS FOR THERMAL PROFILES FOR REDUCING STRESSES AND IMPROVING FLATNESS IN WIDE RIBBON. • HAVE RESULTS BE APPLICABLE TO ALL SHEET GROWTH SYSTEMS. 	

SILICON SHEET

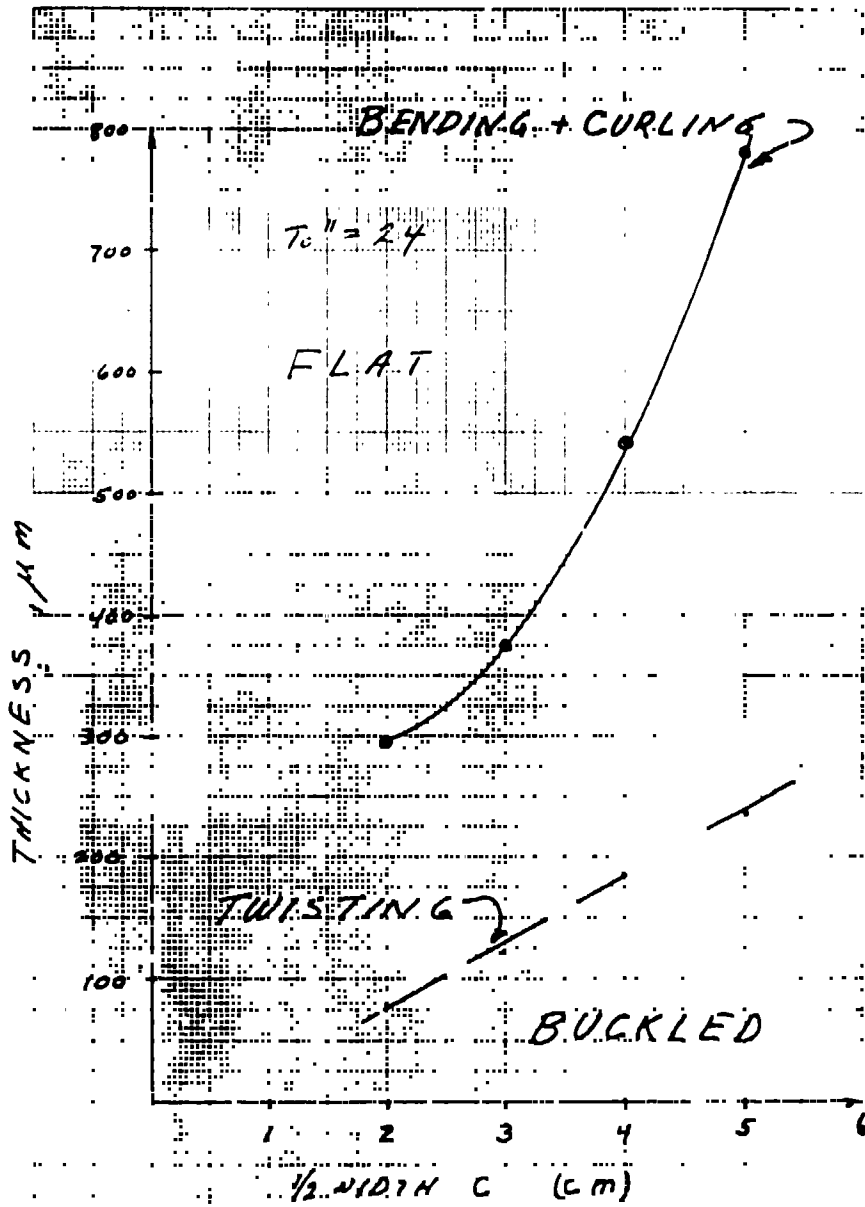


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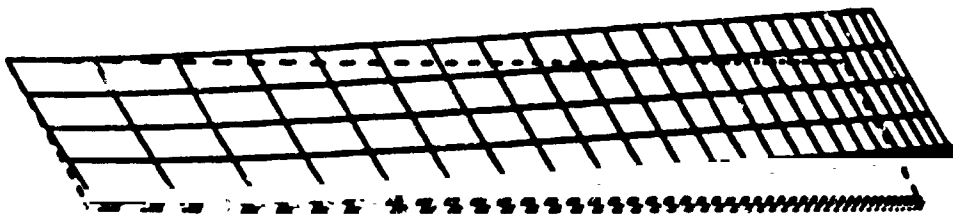
ASSUME: $F = (C^2 - y^2) f(x)$ (pre buckled)



USE ANALYTICAL METHODS
TO OBTAIN CRITICAL
THICKNESS



Calculated Buckled Web Shape



Sumino

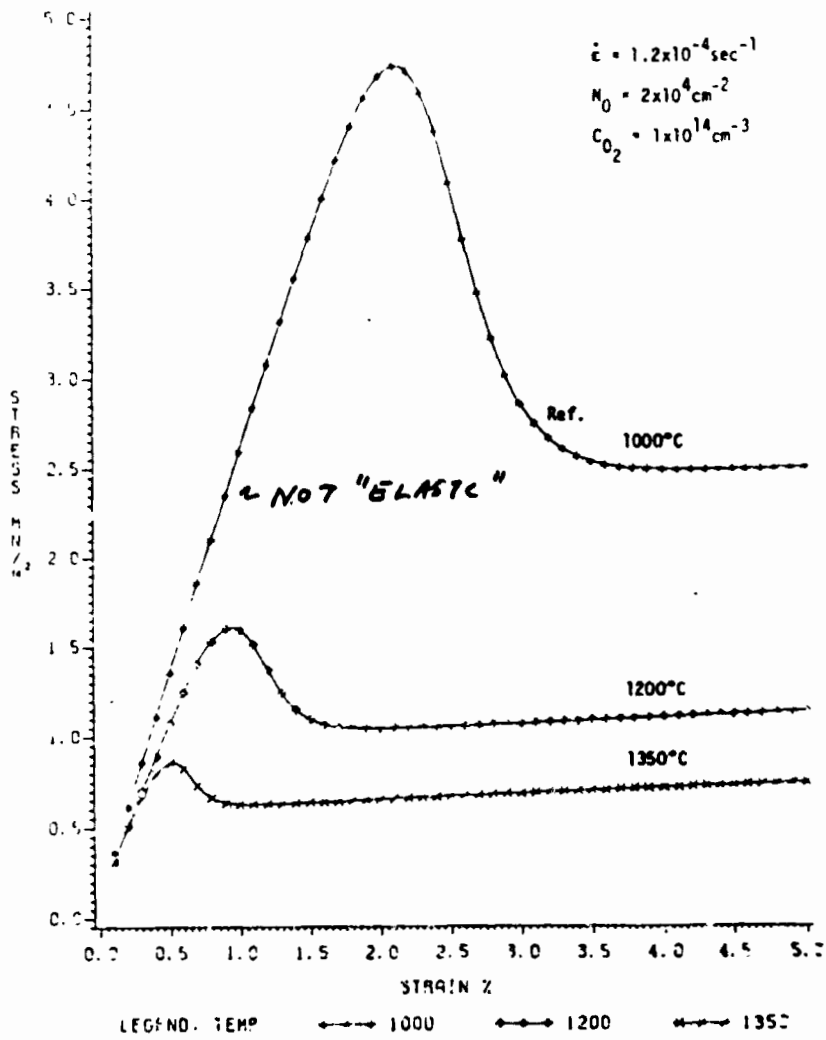
$$\dot{\epsilon}^{PL} = \frac{bB}{T_0^m} N_m (\tau - D/R_m)^m e^{-Q/RT}$$

MOBILE DISLOCATION DENSITY BACK STRESS

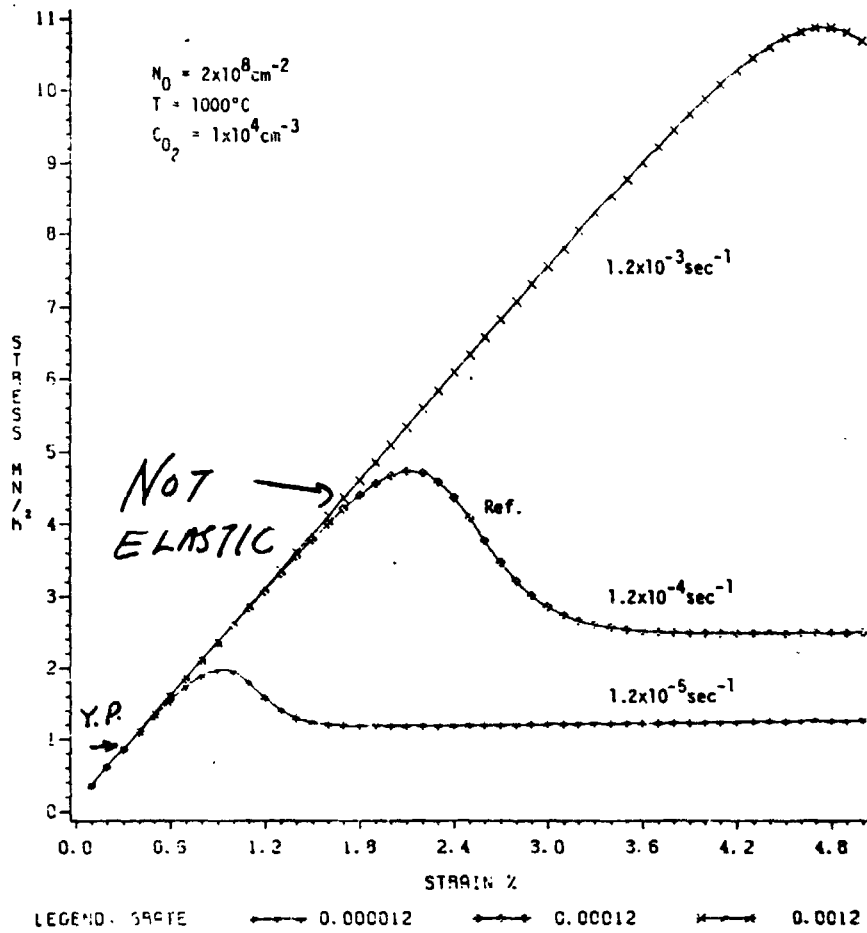
$$\dot{N}_m = k_1 N_m (1 - D/R_m)^{m+1} e^{-1/RT}$$

DENSITY CHANGES

Stress vs Strain for Si (Temperature in °C)

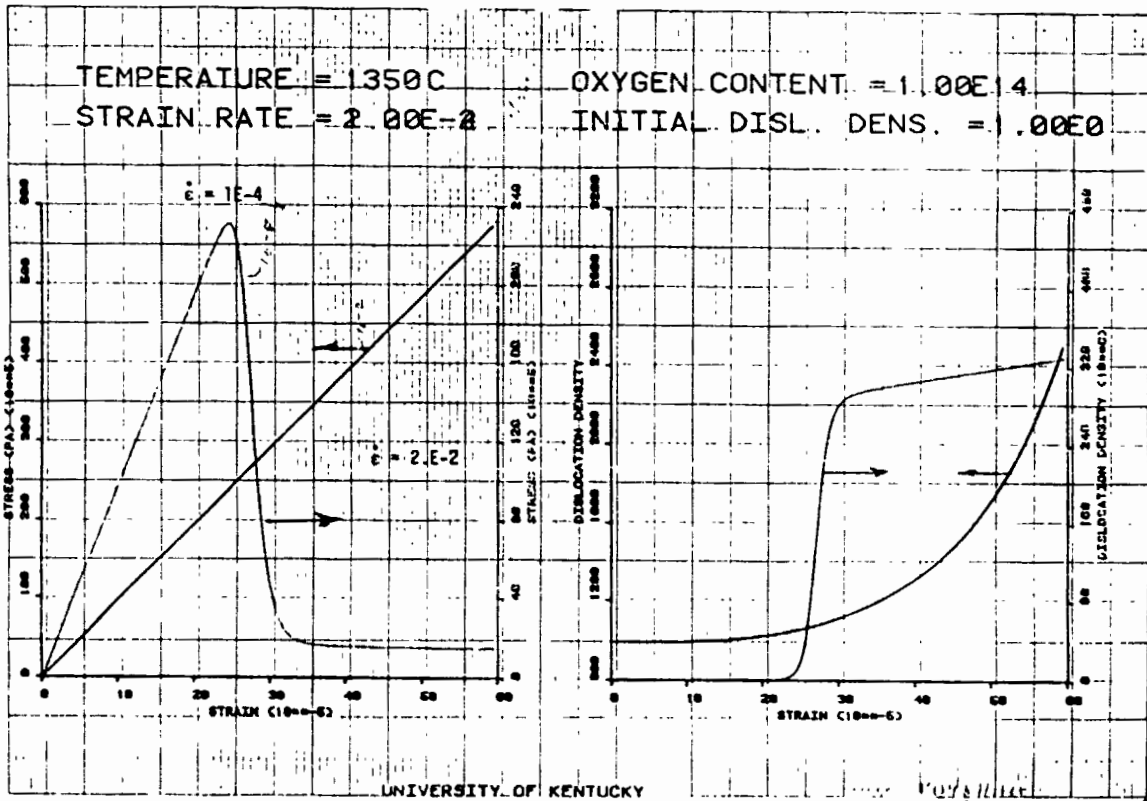


Stress vs Strain for Si (Strain Rate 1/sec)

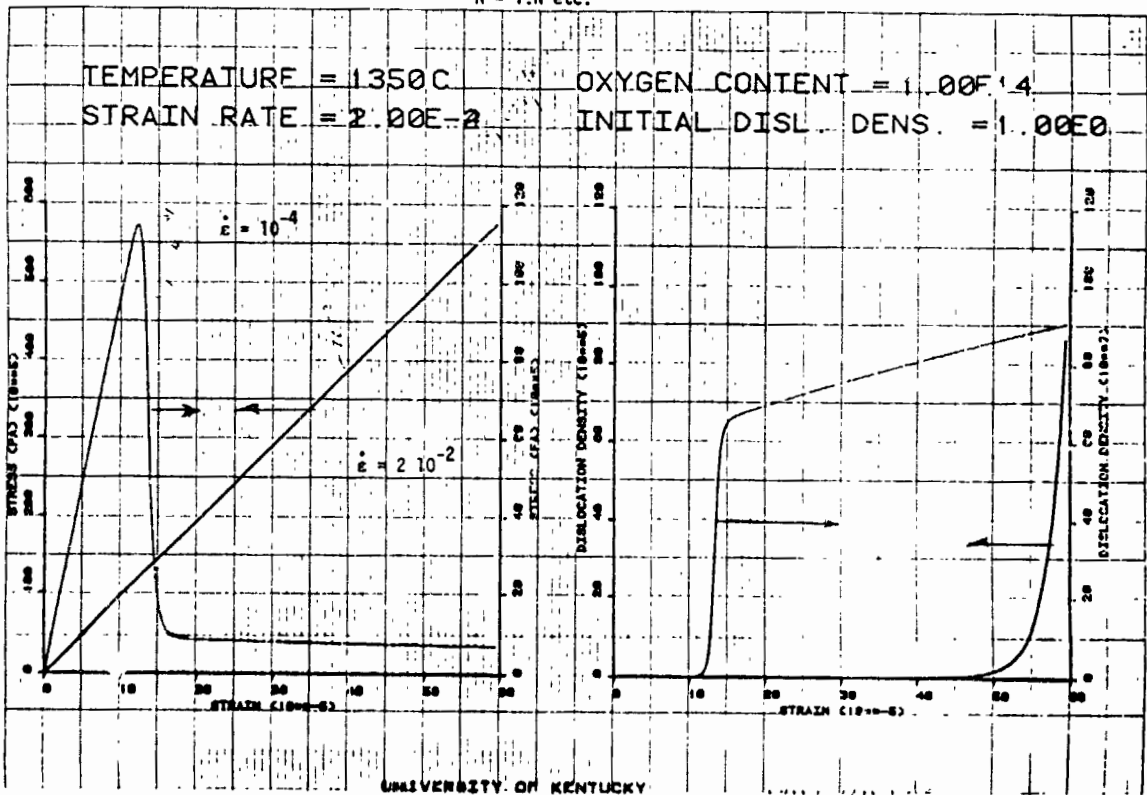


TEMPERATURE=1000 C

$\dot{\epsilon} = 1.N \text{ etc.}$



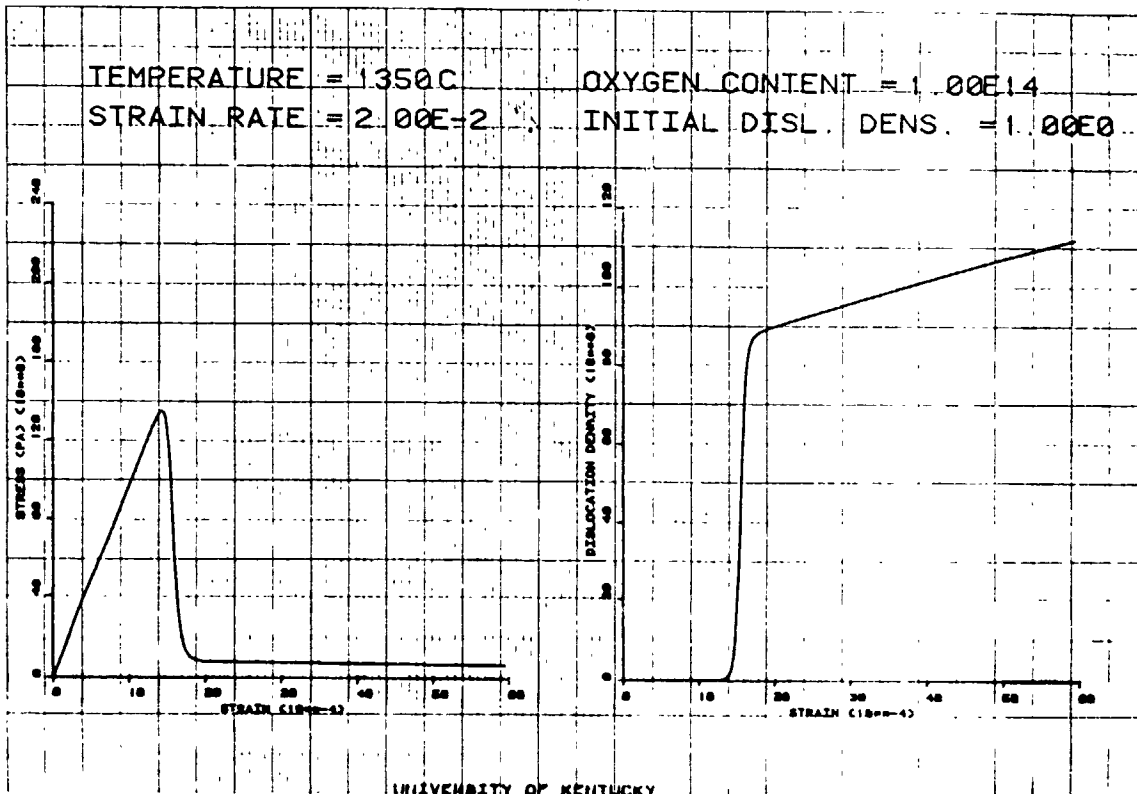
$\dot{\epsilon} = 1.N \text{ etc.}$



ORIGINAL PACES
OF POOR QUALITY

SILICON SHEET

$\dot{N} = .1N$ etc.



Pre-Buckled Stresses

ElasticAPPENDIX A

The equations governing the stress in a ribbon consist of those which reflect that the material is in equilibrium, that the deformations are compatible and that there is a material constitutive relation. Assuming that (a) one can neglect the three stresses τ_{12} , (b) that the material is elastic and that Young's modulus and Poisson's ratio are constant, the governing equations are

Equilibrium

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \quad (A-1)$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0$$

Compatibility

$$\frac{\partial^2 \epsilon_{xx}}{\partial y^2} + \frac{\partial^2 \epsilon_{yy}}{\partial x^2} = 2 \frac{\partial^2 \epsilon_{xy}}{\partial x \partial y} \quad (A-2)$$

Constitutive relations

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu}{E} \sigma_{yy} + \alpha T \quad (A-3)$$

$$\epsilon_{yy} = \frac{\sigma_{yy}}{E} - \frac{\nu}{E} \sigma_{xx} + \alpha T \quad (A-4)$$

$$\epsilon_{xy} = \frac{(1+\nu) \sigma_{xy}}{E}$$

Equations (A-1) combine to yield

$$\frac{\partial^2 \sigma_{xx}}{\partial x^2} = \frac{\partial^2 \sigma_{yy}}{\partial y^2} \quad (A-5)$$

While Eqs. (A-2) through (A-4) yield

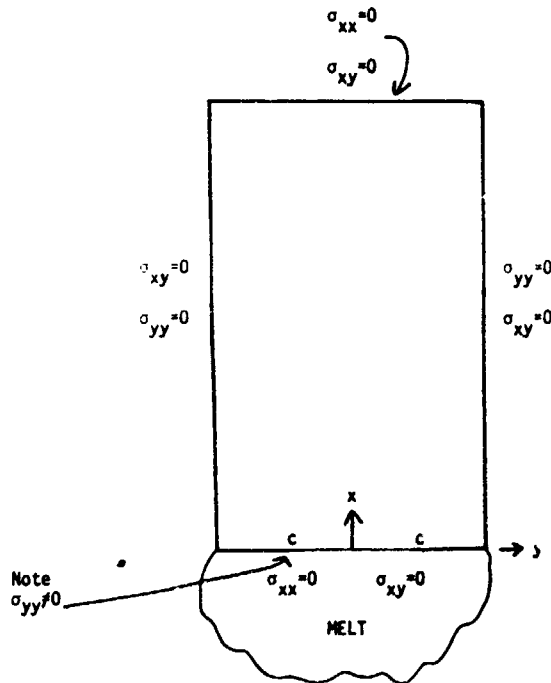
$$\frac{\partial^2 \sigma_{yy}}{\partial x^2} = - \frac{\partial^2 \sigma_{xx}}{\partial x^2} - \frac{\partial^2 \sigma_{yy}}{\partial y^2} - \frac{\partial^2 \sigma_{xx}}{\partial y^2} - \frac{\partial \epsilon \alpha^2 T}{\partial x^2} \quad (A-6)$$

Second order central differences equivalents of these equations are:

$$\begin{aligned} \sigma_{xx}(I+1,J) &= 2\sigma_{xx}(I,J) - \sigma_{xx}(I-1,J) \\ &+ [\sigma_{yy}(I,J+1) - 2\sigma_{yy}(I,J) \\ &+ \sigma_{yy}(I,J-1)] \cdot x^2/y^2 \end{aligned} \quad (A-5a)$$

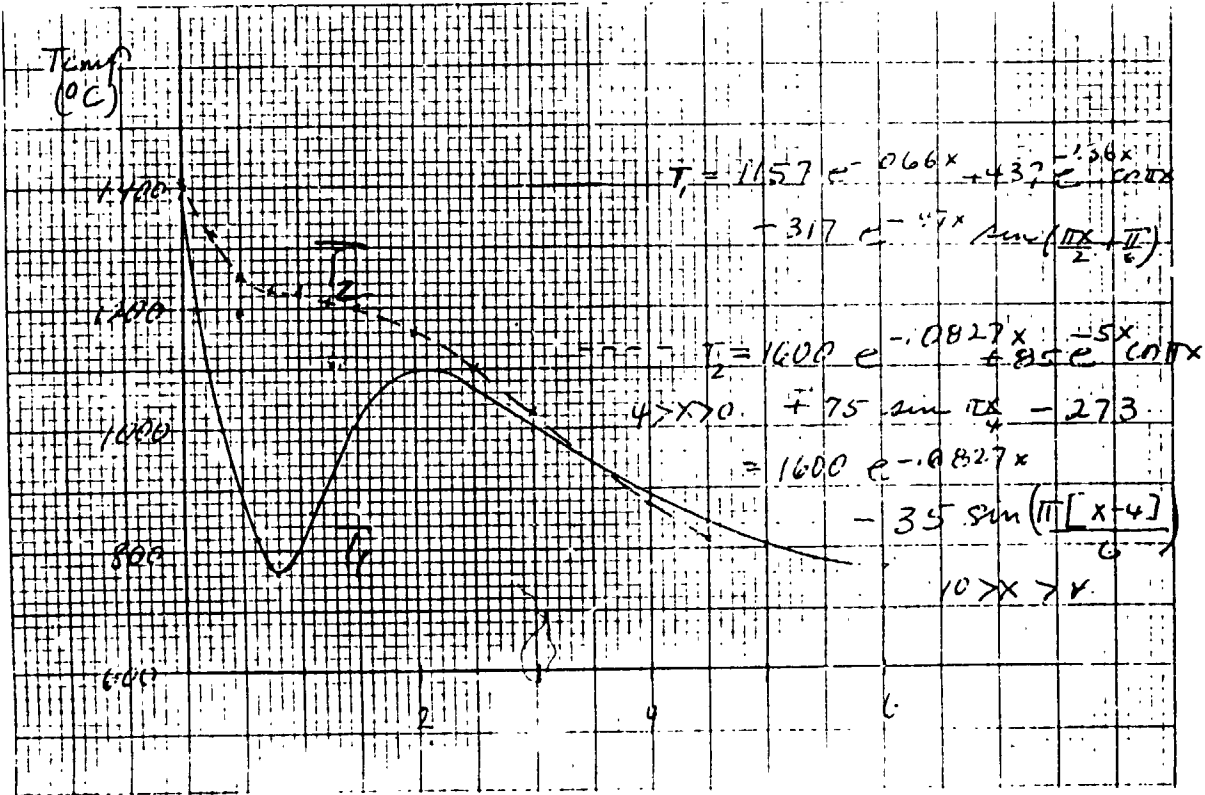
and

$$\begin{aligned} \sigma_{yy}(I+1,J) &= 2\sigma_{xx}(I,J) - \sigma_{yy}(I-1,J) \\ &- \sigma_{xx}(I+1,J) + 2\sigma_{xx}(I,J) \\ &- \sigma_{yx}(I-1,J) - \frac{2E\alpha^2 T}{\partial x^2} \Delta x^2 \\ &- \left(\frac{\Delta x}{\Delta y}\right)^2 [\sigma_{xx}(I,J+1) - 2\sigma_{xx}(I,J) \\ &+ \sigma_{xx}(I,J-1) + \sigma_{yy}(I,J+1) - 2\sigma_{yy}(I,J) \\ &+ \sigma_{yy}(I,J-1)] \end{aligned} \quad (A-6a)$$



Assumes $t > t_p$

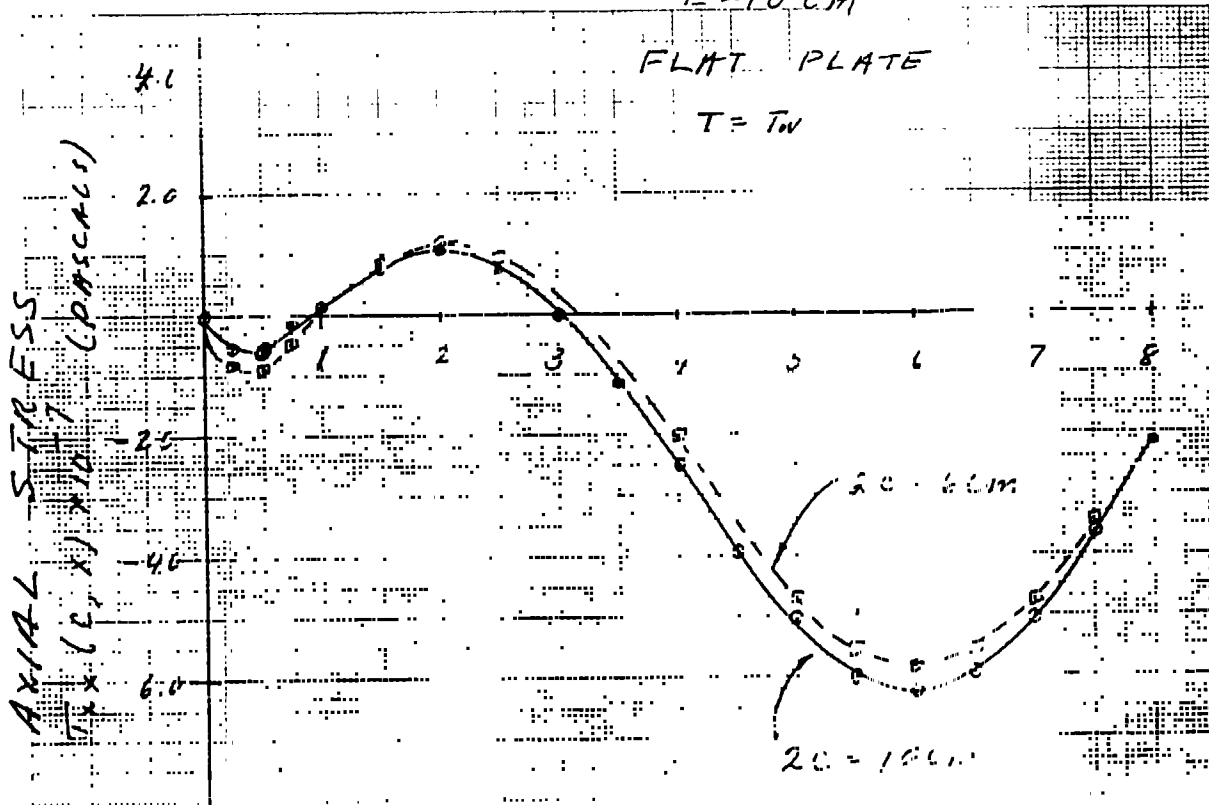
(NO BUCKLING)



□ Run V-0807 12 Sept 84

○ Run V-0100 15 Sept

$l = 10.0 \text{ cm}$



We assume the Sumino model of viscoplastic behavior, i.e.

$$\dot{\epsilon}_{ij}^{PL} = \frac{bB}{\tau_0^m} \frac{N_m (\sqrt{J_2} - D/N_m)^m e^{-Q/kT} (\sigma_{ij} - \sigma_{KK} \delta_{ij}/3)}{\sqrt{J_2}}$$

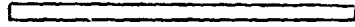
$$\sqrt{J_2} > D/N_m$$

DISLOCATION DENSITY CHANGES!!

$$\dot{N}_m = K_1 N_m (\tau - D/N_m)^{\lambda+m} e^{-Q/k_1}$$

MOBILE

NEW: SOME BASIS FOR A "SHAPE" FACTOR



VS.



$$K_1 \rightarrow \frac{K_1}{T_0}$$

$\left. \begin{array}{l} \lambda \\ b \\ B \\ \tau_0 \\ m \\ D \\ K_1 \\ Q \end{array} \right\} \text{MATERIAL "CONSTANTS"}$

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$$\frac{\partial^2 \sigma_{xx}}{\partial x^2} = \frac{\partial^2 \sigma_{yy}}{\partial y^2} \quad (1)$$

The compatibility equation is

$$\frac{\partial^2 \epsilon_{xx}}{\partial y^2} + \frac{\partial^2 \epsilon_{yy}}{\partial x^2} = 2 \frac{\partial^2 \epsilon_{xy}}{\partial x \partial y} \quad (2)$$

$$\dot{\epsilon}_{ij} = \frac{1+\nu}{E} \dot{\sigma}_{ij} - \frac{\nu}{E} \dot{\sigma}_{kk} \delta_{ij} + \alpha \dot{t} \delta_{ij} + \dot{\epsilon}_{ij}^{PL} \quad (3)$$

Eqs (2) and (3), to yield

$$\begin{aligned} & \frac{\partial^2 (\dot{\sigma}_{xx} + \dot{\sigma}_{yy})}{\partial x^2} + \frac{\partial^2 (\dot{\sigma}_{xx} + \dot{\sigma}_{yy})}{\partial y^2} \quad (4) \\ & = -E\alpha \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \dot{t} \\ & + \left(\frac{\partial^2 \epsilon_{xx}^{PL}}{\partial y^2} + \frac{\partial^2 \epsilon_{yy}^{PL}}{\partial x^2} - 2 \frac{\partial^2 \epsilon_{xy}^{PL}}{\partial x \partial y} \right) E \end{aligned}$$

PLASTIC

$$\frac{\partial^2(\sigma_{xx} + \sigma_{yy})}{\partial x^2} + \frac{\partial^2(\sigma_{xx} + \sigma_{yy})}{\partial y^2} = -\alpha E \frac{\partial^2 T}{\partial x^2}$$

$$+ \int_0^x \frac{E}{v} \left(\frac{\partial^2 \epsilon_{xx}^{PL}}{\partial y^2} + \frac{\partial^2 \epsilon_{yy}^{PL}}{\partial x^2} - 2 \frac{\partial^2 \epsilon_{xy}^{PL}}{\partial x \partial y} \right) dx$$

PULL RATE

$$\frac{\partial^2 \sigma_{xx}}{\partial x^2} = \frac{\partial^2 \sigma_{yy}}{\partial y^2}$$

ϵ_{ij}^{PL} = function of stresses and N_m

$$\dot{N}_m = K_1 N_m (\tau - D N_m)^{1+m} e^{-Q/RT}$$

** SOLVE VIA INTERATION!!!!

"0" = ELASTIC

"20"

OUTPUT STRESSES (x,y)

STRAIN RATES (x,y)

VERY NEW → DISLOCATION DENSITY (x,y)

SILICON SHEET

$$\begin{aligned}L &= 1 \text{ cm} & \dot{\epsilon} &\approx 10^{-2} \text{ sec}^{-1} \\C &= 2 \text{ cm} \\T &= T_w & \dot{\epsilon}^{PL} &\approx 10^{-7} \text{ sec}^{-1}\end{aligned}$$

$$\left. \begin{aligned}\sigma_{xx \max}^{el} &= .1147 \times 10^8 \text{ pascals} \\ \sigma_{xx \max}^{PL} &= .1086 \times 10^8 \text{ pascals}\end{aligned} \right\} \text{Typical}$$

$$N_{init} = 1.0/\text{cm}^2$$

$$N_{final} = 12/\text{cm}^2 \text{ at outer edge} \\ 6/\text{cm}^2 \text{ at center}$$

EVERY PROBLEM DIFFERENT

$$\dot{\epsilon} \approx 10^{-3} \rightarrow 10^{-2} \text{ sec}^{-1}$$

$$\dot{\epsilon}^{PL} \approx 10^{-9} \rightarrow 10^{-4} \text{ sec}^{-1}$$

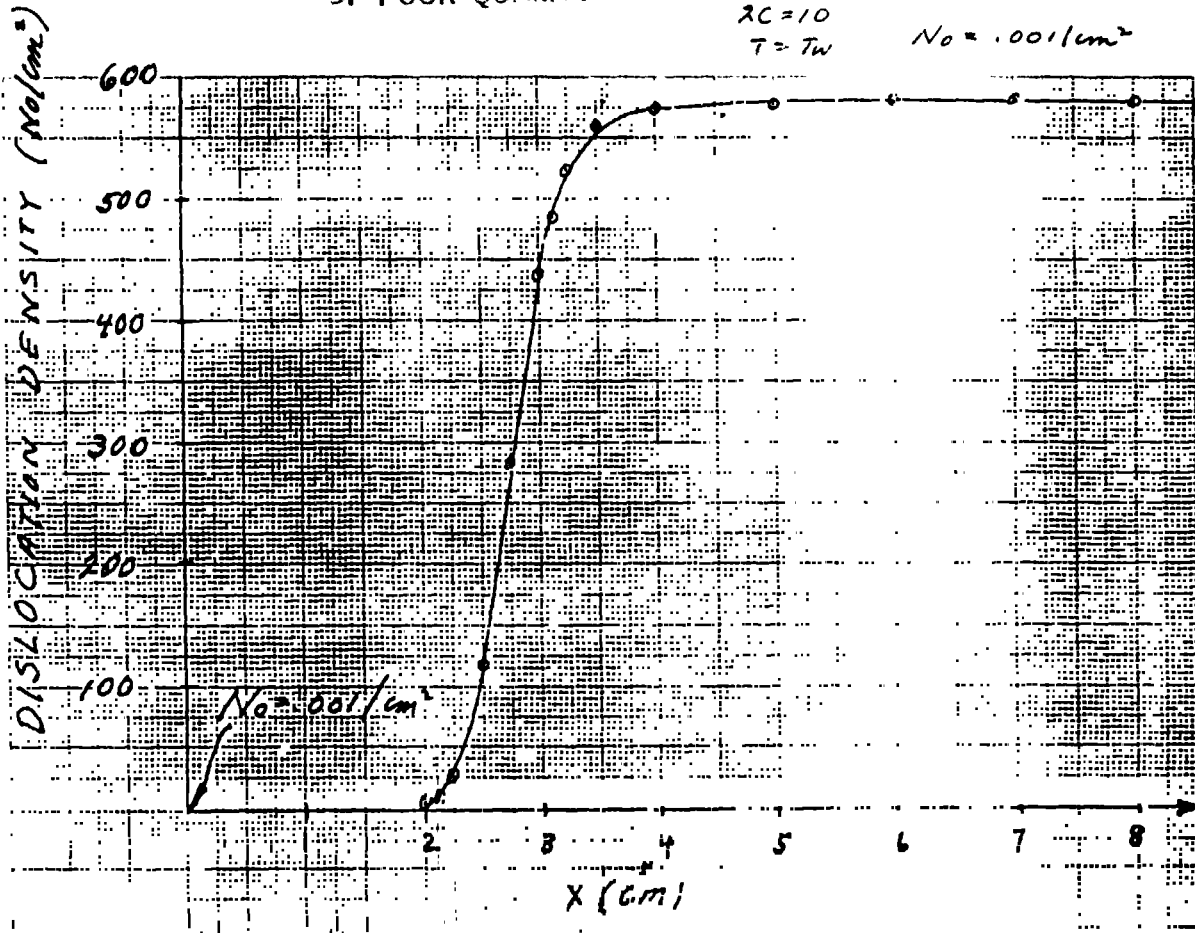
$$\rightarrow 10^{-3} \text{ sec}^{-1} \text{ once in awhile}$$

SILICON SHEET

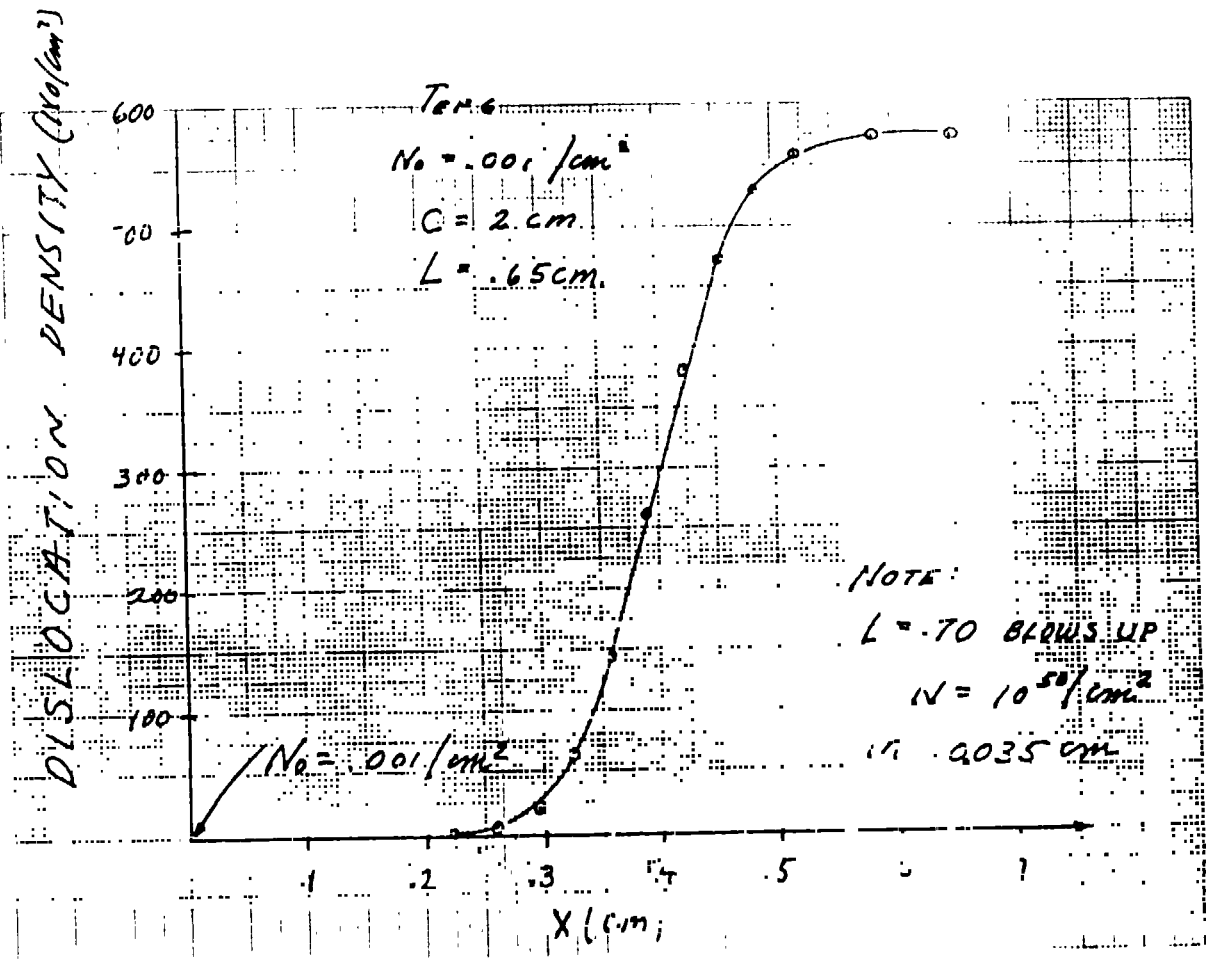
ORIGINAL PART OF
OF POOR QUALITY

$L=10$
 $\lambda C=10$
 $T=T_w$

$N_0 = 1.001/cm^2$

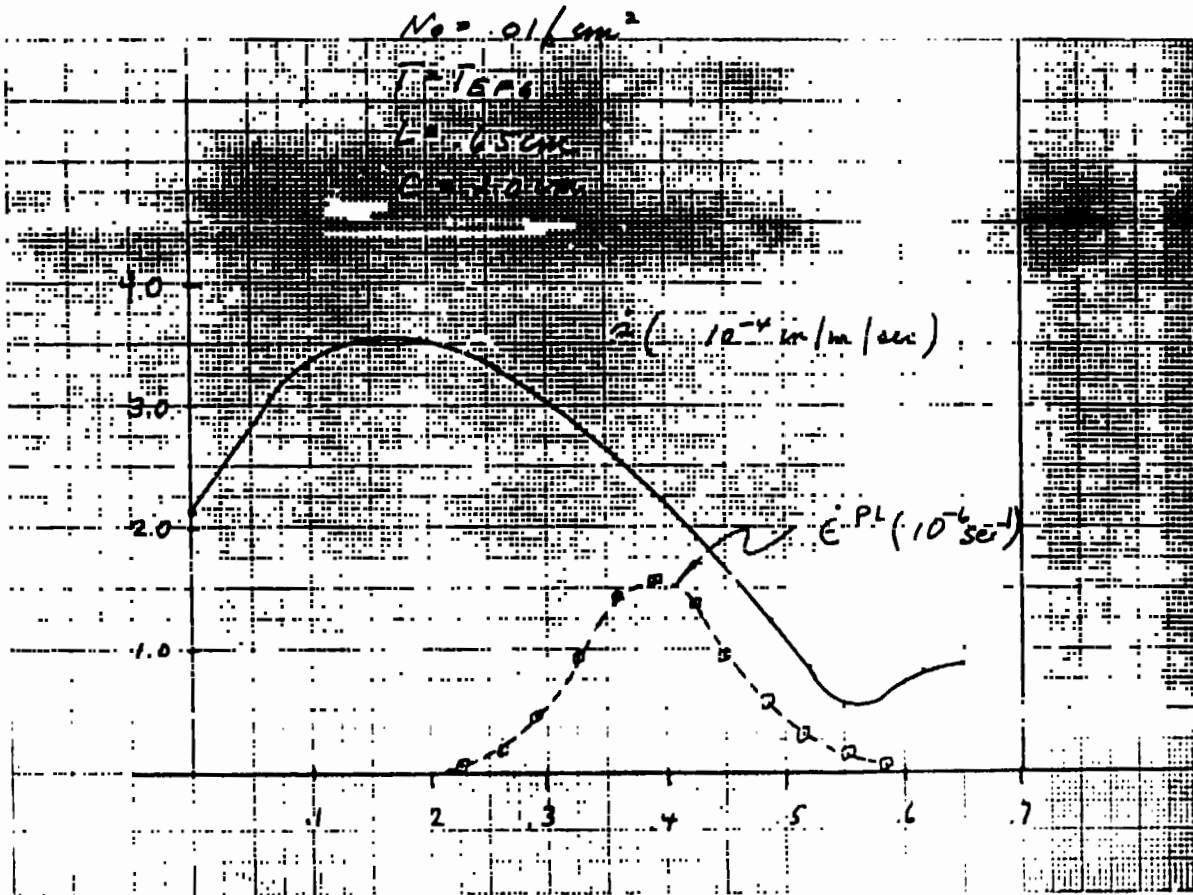


SILICON SHEET



ORIGINAL PAGES
OF POOR QUALITY

Run V-0895 21 Sept



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NUMERICAL PROCEDURE { CONVERGES RAPIDLY
change in σ is $\approx 10^{-4}\sigma$
DOES NOT CONVERGE

NOTE 1: ELASTIC REGION NO PROBLEM

NOTE 2: DOES NOT CONVERGE

MEANS $\dot{\epsilon}^{PL} = 10^3 \text{ sec}^{-1}$

$N_m = 10^{50}/\text{cm}^2$

NOTE 3:

MAJOR OBSERVATION { L = .65 cm CONVERGES
L = .70 cm DOES NOT

L = 0.5 - 10. CONVERGES WITH TEMP PROFILE

FOR STRESSES USE ELASTIC

FOR RESIDUAL STRESSES

PLASTICITY

$\dot{\epsilon}_{ij}^{PL}(x,y)$

THIS IS PROBLEM.

PLASTICITY IS SMALL BUT IS JUST AS BAD AS A LOT.

$\dot{\epsilon}$ IS HIGH σ_{ij} ARE HIGH

T = 750° x .75 cm

MODEL INADEQUATE

Re: DEFORMATION MODE CHANGES (twins)

Problems and Concerns

1. N_D at MELT INTERFACE ($x=0$)
2. VALIDITY OF CONSTRAINTS FOR SUMINO MODEL FOR RIBBON
3. SHOULD WE THINK "TWINS"?
4. SHOULD WE WORK OTHER PROFILES?
5. ROLE OF CHANGING "L"