Boundary to Constructive Solid Geometry Mappings: A Focus on 2-D Issues

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Most solid modelers (SMs) use either a boundary representation (B-rep) or a constructive solid geometry representation (CSG-rep) as their primary, internal representation. Each method has its advocates; each has its detractors; each has its strong points; each has its drawbacks. The need for both representations is becoming increasingly evident, and it is becoming necessary to be able to convert from one to the other.

The need to go from a CSG-rep to a B-rep (or a simple wireframe rep) has long been apparent because an object that is defined only by a CSG-rep is difficult, if not impossible, for a human to interpret. Before fast, shaded display capabilities became available, the wireframe and the B-rep were the only practicable techniques for psuedo-interactive display. Also, the implementation of the Boolean operations between two solid objects of set union, intersection and difference, which was quickly appreciated as an important capability expected of SMs, utilized techniques of a nature similar to those required in converting from a CSG-rep to a B-rep. As a consequence, the problem of generating a B-rep from a CSG-rep has been extensively studied and is now relatively well understood.

In contrast, until recently, there has been little reason to generate a CSG-rep from a B-rep. However, user experience has now strongly established the convenience of using swept contour (profile) curves in the process of defining objects. Thus, SM vendors have an incentive to provide this input mechanism for their users. But, while the user input data to define a swept object is often trivially related to a B-rep of the object, it has proven more difficult to use these data to construct a CSG-rep of it. Thus, much of the current impetus for looking at B-rep to CSG-rep mappings comes from the developers of CSG-rep SMs who need to provide congenial user interaction. They have pursued various approaches in attacking the problem, but the emphasis has been on finding heuristically justified, algorithmic solutions.

Other applications, e.g., N/C verification, also reinforce the importance of this, and similar, representational mappings.

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The importance of the problem warrants a better mathematical understanding of it. This talk will primarily address that aspect of it, although some algorithmic considerations will be briefly mentioned.

Although SMs deal with 3-D objects, it turns out that "most" of the geometry for swept objects of current interest is 2-D. Thus, the crux of the matter for generating B-rep to CSG-rep mappings for these cases devolves to solving a 2-D problem. Also, it is more insightful to cast the problem in terms of using "halfspaces" rather than CSG-rep "primitives." This is partly because a solution to the 2-D problem, with 2-D halfspaces, immediately provides a solution to the 3-D problem by simply replacing, in an obvious way, 2-D halfspaces by 3-D halfspaces. As primitives themselves are expressed by (often implicit) binary trees whose leaves are halfspaces, it is true, of course, that a solution with them provides a solution with halfspaces. However, this approach, in general, tends to be profligate in the total number of halfspaces that appears in the CSG-rep of a swept object. Furthermore, as will be shown, the binary tree found for a halfspace solution may be used to determine an excellent binary tree for a solution with primitives.

The problem will be discussed in two phases. The first phase entails finding a CSG-rep that defines the region bounded by a polygonal profile curve. The second phase utilizes the results of the first phase to find a CSG-rep for many non-polygonal profile curves.

A mathematically concise representation of a region bounded by a polygonal will be presented. Namely, any polygonal region bounded by an n sided polygon may be represented by a binary tree which has at most n planar halfspaces as leaves. A structure for this representation and an algorithm for calculating it will be discussed.

If the profile curve is non-polygonal, i.e., has non-straight edges, then a general method for the determination of a concise CSG-rep that defines the region it bounds is not yet known. Nonetheless, while a totally satisfactory solution is lacking, a concise CSG-rep for many regions of interest in CAD applications does exist. This is true, for example, for most regions for which it is possible to determine "an underlying polygonal region" and a set of "edge regions," each associated with a non-straight edge. Most of these regions may be represented by a binary tree that has the polygonal region as one leaf and the edge regions as the other leaves. For such a representation, and for most edges of interest in the present CAD environment, this implies that the region determined by a profile curve with n edges, k of which are non-straight, has a binary tree representation with no more than n+2k halfspace leaves, of which at most n+k halfspaces are distinct. Conditions which assure that this representation exists, and considerations for an algorithm to generate it, will be discussed.