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FINITE ELEMENT PREDICTION OF LOSS FACTORS FOR STRUCTURES WITH FREQUENCY-DEPENDENT DAMPING TREATMENTS

Gordon C. Everstine and Melvyn S. Marcus

Numerical Mechanics Division
David Taylor Naval Ship R&D Center
Bethesda, Maryland 20084

ABSTRACT

A finite element procedure is described for calculating the loss factors for elastic structures to which frequency-dependent viscoelastic damping treatments have been applied. The frequency dependence of the viscoelastic damping material is treated by approximating its shear modulus with a second-order polynomial so that the stiffnesses associated with the constant, linear, and quadratic terms can be combined, respectively, with the stiffness, damping, and mass matrices assembled for the rest of the structure. A single complex eigenvalue analysis is then performed in which the eigenvalues are purely imaginary. The loss factor is computed by the modal strain energy (MSE) approach first formulated in 1962 by Ungar and Kerwin and recently recast in the finite element context by Johnson, Kienholz, and Rogers. In the MSE approach, the loss factor of a composite structure vibrating in one of its natural modes may be visualized as a weighted average of the loss factors of the component parts, with the relative stored energies as weighting constants. The finite element procedure, which can treat very general geometries, is illustrated for the case of a vibrating constrained-layer damped plate.

INTRODUCTION

Damping treatments are frequently applied to engineering structures to reduce both vibration and noise. For example, laminated plates composed of alternate layers of elastic and viscoelastic materials have been used as structural members which can dissipate vibratory energy as well as maintain the required structural integrity.

The finite element prediction of the dynamic response of such structures is complicated considerably by the frequency-dependence of the viscoelastic material properties. Because the structure's stiffness matrix is frequency-dependent, the finite element equations are nonlinear rather than linear, and conventional analysis cannot be used. For example, the calculation of time-harmonic forced response by a direct approach (in the physical, rather than modal, coordinates of the structure) would require the complete re-assembly and solution of the system equations for each drive frequency. Also, with most finite element codes, the overall damping characteristics of the structure cannot be determined by solving the complex eigenvalue problem since the frequency-dependence of the coefficient matrices results in a nonlinear eigenvalue problem.

This paper describes how constant coefficient matrices in finite element analysis can be restored so that the difficulties associated with frequency-dependent material properties can be reduced. In essence, a frequency-dependent material property is approximated by a polynomial quadratic in the circular frequency ω, and the coefficients of the first and second order terms of the polynomial are combined with the system damping and mass matrices, respectively. The main requirements for this approach are (1) that the frequency-dependence can be adequately represented by a quadratic, and (2) that the finite element code can handle complex stiffness, damping, and mass matrices. NASTRAN, for example, allows these matrices to be complex.

This paper discusses two problems for structures to which frequency-dependent viscoelastic damping treatments have been applied:

1. forced time-harmonic response analysis using physical, rather than modal, coordinates (called "direct frequency response" in NASTRAN), and
2. prediction of the overall system loss factor (structural damping coefficient).

Convenient solution of the first problem requires only the quadratic polynomial approximation to the frequency dependence of the viscoelastic material properties. For the second problem, the approach taken here is to use the quadratic fit only for the real part of the viscoelastic modulus, and then to solve an undamped (but mathematically complex) eigenvalue problem and apply the modal strain energy (MSE) approach to extract the system loss factor.

APPROXIMATION FOR FREQUENCY-DEPENDENT MATERIALS

The formulation of a structural dynamics problem for finite element solution results, in the time domain, in the matrix equation

$$M\ddot{u} + B\dot{u} + Ku = F(t) \tag{1}$$

where M, B, and K are the mass, viscous damping, and stiffness matrices, respectively, u is the vector of unknown displacement components at the grid points, and F is the vector of applied forces at the grid points.

For time-harmonic loading,

$$F = F_o\, e^{i\omega t} \tag{2}$$

$$u = u_o\, e^{i\omega t} \tag{3}$$

where ω and F_o are, respectively, the circular frequency and complex amplitude of the applied force, and u_o is the complex amplitude of the displacement response. In that case, the time-harmonic form of Equation (1) is

$$(-\omega^2 M + i\omega B + K)\, u_o = F_o \tag{4}$$

Consider a structure made of one or more materials, only one of which is damped. Assume that the damped material (which may be anisotropic) is modeled mathematically by the complex modulus approach, in which the modulus of elasticity is the complex number $E(1+i\eta)$, where η is the loss factor for the material. Assume also that all moduli for that material have the same loss factor η and, hence, that the Poisson's ratios are real and frequency-independent. Since every term of the finite element material matrix D (which converts the strain vector to the stress vector for an element) is proportional to one of the elastic moduli, the frequency dependence can be factored out as a common dimensionless scalar factor:

$$D(\omega) = \alpha(\omega)\, D_o\, (1 + i\eta(\omega)) \tag{5}$$

where D_o is a real, frequency-independent reference material matrix, $\alpha(\omega)$ is a dimensionless factor which expresses the frequency dependence of the real part of the elastic moduli, and $\eta(\omega)$ is the frequency-dependent loss factor for the material. In effect, D_o is the material matrix for some arbitrary frequency ω_o, and αD_o is the real part of the material matrix for some other frequency ω; thus, $\alpha(\omega_o) = 1$. It therefore follows that the stiffness matrix K_d for the portion of the structure made of the damped material is of the form

$$K_d = \alpha K_o(1 + i\eta) \tag{6}$$

where K_o is the real part of K_d at the reference frequency ω_o. The stiffness matrix for the remainder of the structure (the undamped portion) will be denoted K_e.

With damping modeled using the complex modulus approach, M in Equation (4) is real, B = 0 (unless viscous damping is also present), and K is complex. The real part of K includes the contributions from all components and materials in the structure. However, if only one material has complex moduli, the imaginary part of the system stiffness matrix K arises only from the damped material, thus allowing the stiffness matrix for the damping material to be extracted from the overall system stiffness matrix. If the loss factor (structural damping coefficient) η for the damping material is specified as unity on the finite element material properties data card,

$$\alpha K_o = \mathrm{Im}(K_d)|_{\eta=1} = \mathrm{Im}(K)|_{\eta=1} \tag{7}$$

The stiffness matrix K_e for the remainder of the structure (the undamped portion) is then obtained by subtracting from the system matrix K the contribution from the damping material:

$$K_e = \mathrm{Re}(K) - \mathrm{Im}(K)|_{\eta=1} \tag{8}$$

where K is the (complex) stiffness matrix for the entire structure assuming that $\eta = 1$ is specified for the damped portion.

It will be convenient to write Equation (6) in the form

$$K_d = K_o(\alpha + i\beta) \tag{9}$$

where

$$\beta(\omega) = \alpha(\omega)\eta(\omega) \tag{10}$$

If the dependence of α and β on ω were quadratic, it is clear from Equation (4) that the frequency-dependent terms could be absorbed into the system's mass and viscous damping matrices, thereby transforming the problem into standard form (although with complex coefficient matrices).

Therefore, we attempt to approximate $\alpha(\omega)$ and $\beta(\omega)$ with the quadratic polynomials

$$\alpha(\omega) \cong \alpha_o + \alpha_1\omega + \alpha_2\omega^2 \tag{11}$$

$$\beta(\omega) \cong \beta_o + \beta_1\omega + \beta_2\omega^2 \tag{12}$$

The six unknown coefficients in Equations (11) and (12) can be determined by standard least-squares procedures (which are summarized in the next section).

With quadratic frequency-dependence of the damping material, the coefficient matrices M, B, and K in Equation (4) can be replaced by new complex matrices M*, B*, and K* defined by the equality

$$-\omega^2 M + i\omega B + K_e + K_o[\alpha_o + \alpha_1\omega + \alpha_2\omega^2 + i(\beta_o + \beta_1\omega + \beta_2\omega^2)] = -\omega^2 M^* + i\omega B^* + K^* \quad (13)$$

where K_e is the system stiffness matrix for the undamped part of the structure, B is the damping matrix for the viscous dampers, if any, and M is the original mass matrix for the entire structure. Thus, by equating coefficients of like powers of ω in Equation (13), we obtain

$$K^* = K_e + (\alpha_o + i\beta_o)K_o \quad (14)$$

$$B^* = B + (\beta_1 - i\alpha_1)K_o \quad (15)$$

$$M^* = M + (-\alpha_2 - i\beta_2)K_o \quad (16)$$

where K_e is determined from Equation (8). In this form, the new coefficient matrices M*, B*, and K* are complex and independent of frequency.

To summarize, the frequency dependence of the real and imaginary parts of the moduli of the viscoelastic material is replaced by a quadratic so that such dependence can be absorbed into the system mass and damping matrices. The principal assumptions made are (1) that the frequency dependence of the material can be described by a single scalar function, (2) that this function can be adequately represented by a quadratic polynomial, and (3) that the finite element code is general enough to allow complex coefficient matrices.

CURVE-FITTING

Let y(x) represent one of the two frequency-dependent material property functions $\alpha(\omega)$ and $\beta(\omega)$ defined in Equation (9). Assume that y(x) is known at n points (x_i, y_i), i=1,2, ... ,n. We wish to approximate y(x) by the quadratic polynomial

$$p(x) = a_o + a_1 x + a_2 x^2 \quad (17)$$

in such a way that the residual

$$R = \sum_{i=1}^{n} w_i[y_i - (a_o + a_1 x_i + a_2 x_i^2)]^2 \quad (18)$$

is minimized, where w_i is the weighting factor for point i. This is the classical least squares problem. To minimize the squares of the absolute errors, $w_i = 1$. To minimize the squares of the relative errors, $w_i = 1/y_i^2$.

The polynomial coefficients which minimize the residual R are the solutions of the symmetric system [1]

$$\begin{bmatrix} \sum w_i & \sum w_i x_i & \sum w_i x_i^2 \\ \sum w_i x_i & \sum w_i x_i^2 & \sum w_i x_i^3 \\ \sum w_i x_i^2 & \sum w_i x_i^3 & \sum w_i x_i^4 \end{bmatrix} \begin{Bmatrix} a_o \\ a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} \sum w_i y_i \\ \sum w_i x_i y_i \\ \sum w_i x_i^2 y_i \end{Bmatrix} \qquad (19)$$

where all summations are from $i=1$ to n.

MODAL STRAIN ENERGY APPROACH TO DAMPING

The relationship between damping and energy concepts for nonhomogeneous structures was apparently first formulated by Ungar and Kerwin in 1962 [2]. In general terms, they showed that the loss factor of a composite structure may be computed as a weighted average of the loss factors of the component parts, with the relative stored energies as weighting factors. The Ungar-Kerwin ideas were recently recast in the finite element context by Johnson, Kienholz, and Rogers [3-5]. For completeness, we summarize (in a slightly different form) the aspects of this work needed here.

The complex Rayleigh quotient for a damped structure may be written in the form

$$\omega^2(1 + i\eta) = \phi_d^T(K_T + i\eta_d K_d)\phi_d/(\phi_d^T M\phi_d) \qquad (20)$$

where K_T = stiffness matrix (real) for the entire structure
K_d = stiffness matrix (real) for the part of the structure which is damped (i.e., the viscoelastic material)
η_d = loss factor (real) for the viscoelastic material
M = mass matrix (real) for the entire structure
ϕ_d = complex eigenvector (mode shape) for a damped mode of the structure
ω = circular natural frequency (real) for the mode
η = composite loss factor (real) for the entire structure

For lightly damped structures, the damped mode shape ϕ_d may be approximated by the real eigenvector ϕ obtained by solving the real, undamped eigenvalue problem.

Using this approximation ($\phi_d = \phi$) and equating the imaginary parts of Equation (20) to each other yields

$$\eta = \eta_d(\phi^T K_d\phi)/(\omega^2\phi^T M\phi) \qquad (21)$$

Since the real eigenvector ϕ satisfies

$$\omega^2\phi^T M\phi = \phi^T K_T\phi \qquad (22)$$

we obtain

$$\eta = \eta_d(\phi^T K_d\phi)/(\phi^T K_T\phi) \qquad (23)$$

or

$$\eta = \eta_d(k_d/k_T) \qquad (24)$$

where k_T is the generalized stiffness of the mode, and k_d is the contribution of the viscoelastic material to the generalized stiffness. Since the generalized stiffness for a mode is equal to twice the elastic strain energy in the mode, Equation (24) may be interpreted as stating that the ratio of the composite loss factor η to the loss factor of the viscoelastic material is equal to the fraction of the total strain energy contained in the damping material.

More generally, for structures containing several damping materials with individual loss factors η_1, η_2, η_3, ..., the composite loss factor for the structure is

$$\eta = \eta_1(k_1/k_T) + \eta_2(k_2/k_T) + \ldots \tag{25}$$

where k_i is the contribution of the ith damping material to the total generalized stiffness k_T.

Equation (24) is implemented within the finite element procedure by performing an undamped eigenvalue analysis and extracting for each mode the total strain energy $2k_T$ and the strain energy $2k_d$ contained in the viscoelastic material. In the undamped analysis, the damping material is modeled and assigned a zero loss factor.

NASTRAN IMPLEMENTATION

With NASTRAN analysis, the specification of quadratic frequency dependence for part of a structure requires that the stiffness matrix for that part be available and that various scalar multiples of that matrix be combined with the original stiffness, mass, and damping matrices for the entire system.

A convenient way to obtain the stiffness matrix for a subset of elements (the viscoelastic material) in a structure is to specify a unit material damping constant (g = 1) on the material card for that substructure. In that case, the reference elastic stiffness matrix K_o for the damping material is merely the NASTRAN data block K4GG. Thus, from Equation (8), the elastic stiffness matrix K_e for the rest of the structure (all material except the damping material) is

$$K_e = KGGX - K4GG \tag{26}$$

where KGGX is the NASTRAN data block containing the real part of the complete system stiffness matrix. With the use of NASTRAN data block terminology, Equations (14) - (16) then become

$$K^* = KGGX + (\alpha_0 - 1 + i\beta_o)\ K4GG \tag{27}$$

$$B^* = BGG + (\beta_1 - i\alpha_1)\ K4GG \tag{28}$$

$$M^* = MGG + (-\alpha_2 - i\beta_2)\ K4GG \tag{29}$$

where the new complex matrices K*, B*, and M* replace the original matrices KGGX, BGG, and MGG, respectively. This replacement is effected with an ALTER to the rigid format. The scalar multipliers in Equations (27) - (29) are defined in NASTRAN using three complex parameters:

$$\left.\begin{aligned} PARMK &= (\alpha_o - 1,\ \beta_o) \\ PARMB &= (\beta_1,\ -\alpha_1) \\ PARMM &= (-\alpha_2,\ -\beta_2) \end{aligned}\right\} \tag{30}$$

where the α's and β's are the coefficients of the quadratic curve fits in Equations (11) and (12).

Forced Response

For the calculation of forced time-harmonic response (NASTRAN's Rigid Format 8), an ALTER to effect Equations (27) - (29) would be sufficient to enforce quadratic frequency dependence of the properties of one material. The DMAP ALTER which implements these changes is shown in Figure 1. The first section (ALTER 41) replaces the original coefficient matrices with the new matrices K*, M*, and B* defined in Equations (27) - (29). The other three sections replace NASTRAN's functional modules SMP1 and SMP2, which perform the static condensation (Guyan reduction) on the stiffness, mass, and damping matrices. These replacements are needed because SMP1 and SMP2 do not allow complex input.

```
ALTER 41 $ APR 84, R.F. 8, FREQ-DEP MATL
  PARAM //*MPY*/NOBGG/1/1 $ YES BGG
  PARAM //*MPY*/NOK4GG/1/-1 $ NO K4GG
  ADD KGGX,K4GG/KBAR//C,Y,PARMK $ NEW K
  ADD MGG ,K4GG/MBAR//C,Y,PARMM $ NEW M
  ADD BGG ,K4GG/BBAR//C,Y,PARMB $ NEW B
  EQUIV KBAR,KGGX//MBAR,MGG//BBAR,BGG $ NEW K,B,M
  PURGE K4GG $
ALTER 78,78 $ REPLACE SMP1 FOR COMPLEX K
  UPARTN USET,KFF/KAAB,KOA,,KOO/*F*/*A*/*O* $
  SOLVE KOO,KOA/GO/1/-1 $
  MPYAD KOA,GO,KAAB/KAA/1 $
  DIAGONAL KAA/AVEC/*COLUMN*/0. $ VECTOR OF ONES
  ADD AVEC,/PVEC/(0.0,0.0) $ VECTOR OF ZEROS (P-VEC)
  MERGE KAA,,,,PVEC,/KAASYM/-1//6 $ DUMMY MERGE FOR K
  EQUIV KAASYM,KAA $ KAA TRAILER NOW SYMMETRIC
ALTER 80,80 $ REPLACE SMP2 FOR COMPLEX M
  UPARTN USET,MFF/MAAB,MOA,,MOO/*F*/*A*/*O* $
  MPYAD MOO,GO,MOA/TEMP1/1 $
  MPYAD GO,TEMP1,MAAB/TEMP2/1 $
  MPYAD MOA,GO,TEMP2/MAA/1 $
  MERGE MAA,,,,PVEC,/MAASYM/-1//6 $ DUMMY MERGE FOR M
  EQUIV MAASYM,MAA $ MAA TRAILER NOW SYMMETRIC
ALTER 83,83 $ REPLACE SMP2 FOR COMPLEX B
  UPARTN USET,BFF/BAAB,BOA,,BOO/*F*/*A*/*O* $
  MPYAD BOO,GO,BOA/TEMP3/1 $
  MPYAD GO,TEMP3,BAAB/TEMP4/1 $
  MPYAD BOA,GO,TEMP4/BAA/1 $
  MERGE BAA,,,,PVEC,/BAASYM/-1//6 $ DUMMY MERGE FOR B
  EQUIV BAASYM,BAA $ BAA TRAILER NOW SYMMETRIC
ENDALTER $
```

Figure 1 - DMAP ALTER for Rigid Format 8 for Time-Harmonic Response of Structures with Frequency-Dependent Material Properties

To summarize, the NASTRAN procedure for calculating the time-harmonic response of a structure with one frequency-dependent damping material is as follows:

1. Perform a least squares quadratic fit to one of the elastic moduli to determine the α's and β's in Equations (11) and (12). For viscoelastic materials (e.g., rubber), the shear modulus G is usually used, in which case

$$G(\omega) = G_0\,\alpha(\omega) \tag{31}$$

   and $\beta(\omega)$ is defined in Equation (10). G_0 is the reference material property and may be taken to be 1 psi. In Equation (10), $\eta(\omega)$ is the loss factor for the viscoelastic material.

2. Specify the material damping constant $g = 1$ and reference elastic properties (e.g., shear modulus $G = G_0$) on the material card for the viscoelastic material; specify the mass density and Poisson's ratios correctly. Specify $g = 0$ on the material card for the undamped material.

3. Perform frequency response analysis (Rigid Format 8) with the DMAP ALTER of Figure 1 and the parameters defined in Equation (30).

Loss Factors

Forced response predictions for structures with one damping material require only the replacement of the original K, M, and B matrices with their complex counterparts K*, M*, and B* defined in Equations (27) - (29). However, the same replacement cannot be used to solve the complex eigenvalue problem (Rigid Format 7) to obtain damping, because the assumed time dependence is different between Rigid Formats 7 and 8. Specifically, for forced response calculations, NASTRAN's time dependence is of the form $e^{i\omega t}$, whereas in the complex eigenvalue problem, the time dependence is of the form e^{pt} for complex p. The modal strain energy (MSE) approach is a means of avoiding this difficulty, since only undamped natural frequencies are computed, in which case

$$\beta_0 = \beta_1 = \beta_2 = 0 \tag{32}$$

in Equations (27) - (30). A complex eigenvalue analysis is still required, however, since the damping matrix B* is purely imaginary; K* and M* are both real.

To complete the calculation of the composite loss factor η in Equation (24), we first note that Equation (24) is equivalent to

$$\eta = \eta_d(\alpha k_o)/(k_e + \alpha k_o) \tag{33}$$

where k_o is the generalized stiffness for the damping material with some reference level of material properties chosen (e.g., shear modulus G = 1 psi), and k_e is the generalized stiffness for the remaining material. Since α is frequency-dependent, it is convenient to let NASTRAN compute k_e and k_o for each mode and to use a post processor to compute η for each mode.

The DMAP ALTER which implements all these changes in NASTRAN's complex eigenvalue analysis (Rigid Format 7) is shown in Figure 2. The first section (ALTER 41) replaces the original coefficient matrices with the new matrices K*, M*, and B* defined in Equations (27) - (29). The second section (ALTER 80,80) is the replacement for NASTRAN's static condensation module SMP2. The third section (ALTER 127) computes the generalized stiffnesses k_o and k_e for each mode. These generalized stiffnesses as well as the list of eigenvalues (contained in data block CLAMA) are written on an OUTPUT2 file for postprocessing to evaluate η for each mode according to Equation (33).

To summarize, the NASTRAN procedure for calculating loss factors of structures with one frequency-dependent damping material is as follows:

1. Perform a least squares quadratic fit to one of the elastic moduli to determine the α's in Equation (11). For damping materials (such as rubber), the shear modulus G is usually used; see Equation (31).

2. Specify the material damping constant g = 1 and reference elastic properties (e.g., shear modulus $G = G_0$) on the material card for the viscoelastic material; specify the mass density and Poisson's ratios correctly. Specify g = 0 on the material card for the undamped material.

3. Perform complex eigenvalue analysis (Rigid Format 7) with the DMAP ALTER of Figure 2. Use the parameters defined in Equation (30) with $\beta_0 = \beta_1 = \beta_2 = 0$. Retain the OUTPUT2 file (UT1).

```
ALTER 41 $ APR 84, R.F. 7, FREQ-DEP MATL
  PARAM //*MPY*/NOBGG/1/1 $ YES BGG
  PARAM //*MPY*/NOK4GG/1/-1 $ NO K4GG
  DIAGONAL KGGX/IDENTG/*SQUARE*/0. $ G SET IDENTITY MATRIX
  MPYAD IDENTG,KGGX,K4GG/KMAT2/1/1/-1 $
  ADD,KGGX,K4GG/KBAR//C,Y,PARMK $ NEW K
  ADD MGG ,K4GG/MBAR//C,Y,PARMM $ NEW M
  ADD BGG ,K4GG/BBAR//C,Y,PARMB $ NEW B
  EQUIV KBAR,KGGX//MBAR,MGG//BBAR,BGG $ NEW K,B,M
ALTER 80,80 $ REPLACE SMP2 FOR COMPLEX B
  DIAGONAL KAA/AVEC/*COLUMN*/0. $ VECTOR OF ONES
  ADD AVEC,/PVEC/(0.0,0.0) $ VECTOR OF ZEROS (P-VEC)
  UPARTN USET,BFF/BAAB,BOA,,BOO/*F*/*A*/*O* $
  MPYAD BOO,GO,BOA/TEMP3/1 $
  MPYAD GO,TEMP3,BAAB/TEMP4/1 $
  MPYAD BOA,GO,TEMP4/BAA/1 $
  MERGE BAA,,,,PVEC,/BAASYM/-1//6 $ DUMMY MERGE FOR B
  EQUIV BAASYM,BAA $ BAA TRAILER NOW SYMMETRIC
ALTER 127 $ GENERALIZED STIFFNESSES
  SMPYAD CPHIP,K4GG ,CPHIP,,,/GKMAT1/3////1/1 $
  SMPYAD CPHIP,KMAT2,CPHIP,,,/GKMAT2/3////1/1 $
  DIAGONAL GKMAT1/K1 $
  DIAGONAL GKMAT2/K2 $
  MATPRN K1,K2,CLAMA,,// $
  OUTPUT2 K1,K2,CLAMA,,// $
ENDALTER $
```

Figure 2 - DMAP ALTER for Rigid Format 7 for Loss Factors of Structures with Frequency-Dependent Material Properties

DISCUSSION

A finite element procedure has been described for predicting the loss factors of structures to which frequency-dependent viscoelastic damping treatments have been applied. This procedure, used on the simple test problem of a three-layer plate, yielded predictions similar to those of a generally-accepted analytical procedure for infinite plates.

The power of the finite element procedure, however, is that it is not restricted to such simple geometry but can readily handle such complications as different boundary conditions, variable plate thickness, and localized treatments. None of these effects can be treated by the infinite plate theory.

REFERENCES

1. Vandergraft, J.S., Introduction to Numerical Computations, second edition, Academic Press, New York, 1983.

2. Ungar, E.E., and E.M. Kerwin, Jr., "Loss Factors of Viscoelastic Systems in Terms of Energy Concepts," J. Acoust. Soc. of Amer., vol. 34, no. 7, July 1962, pp. 954-957.

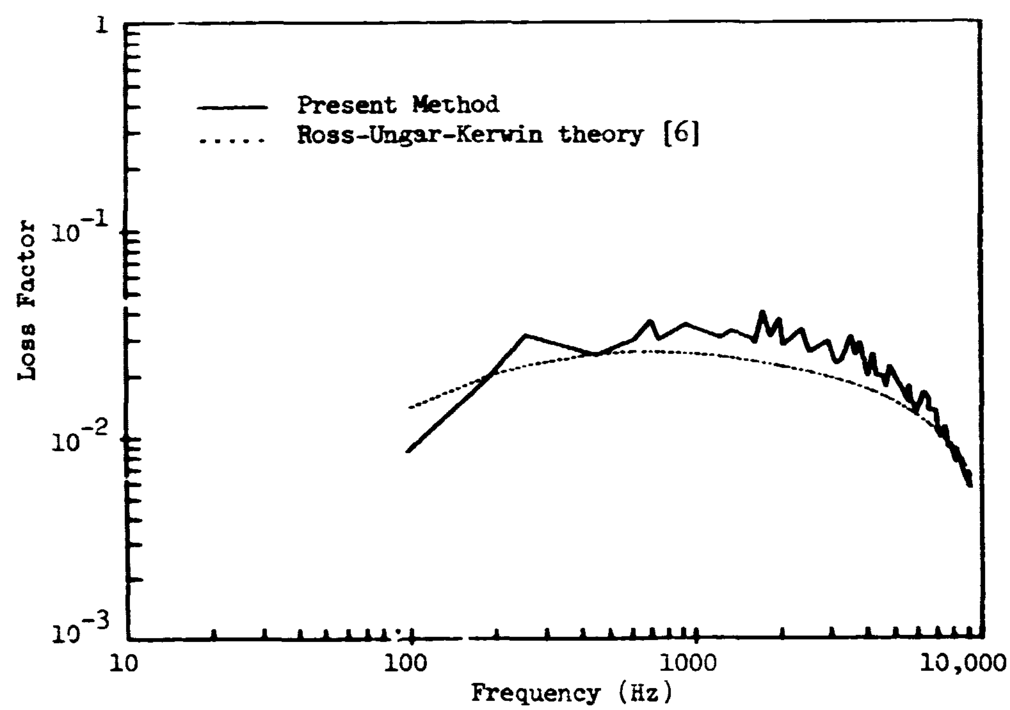

Figure 4 - Comparison of Loss Factors for Sandwich Plate as Predicted by Present Finite Element Method and by Ross-Ungar-Kerwin Theory [6]

4. With a postprocessor, read the UT1 file and compute for each mode the loss factor η according to Equation (33).

EXAMPLE

The procedure for calculating composite loss factors will be illustrated for the three-layer sandwich plate shown in Figure 3. We assume that the shear modulus G and loss factor η for the middle layer are given by

$$G(f) = 10^4 + 9f \quad \text{(psi)} \tag{34}$$

$$\eta(f) = 0.5 - (4\times10^{-5})f \tag{35}$$

where f is frequency in Hz. These relations imply considerable frequency dependence, since between zero and 10,000 Hz, G varies by a factor of ten, and η varies by a factor of five. The top and bottom layers are made of steel.

The plate was modeled with three layers of 20-node isoparametric ("brick") finite elements. A 13x5x3 mesh of elements was used. These elements were used rather than plate elements because of interest in fairly thick plates of variable thickness, in which case the engineering plate theory no longer applies. Guyan reduction was applied to this model in such a way that the only degrees of freedom retained were the normal translations at the corner nodes of each element on the top and bottom faces.

The composite loss factor computed using the finite element procedure described in the preceding sections is compared in Figure 4 with a calculation based on classical infinite plate theory presented by Ross, Ungar, and Kerwin [6]. That figure indicates a good agreement between the two predictions. The infinite plate theory is generally thought to provide a reasonably accurate prediction of the composite loss factor for plates with this simple geometry, and hence serves as a good check on the finite element result.

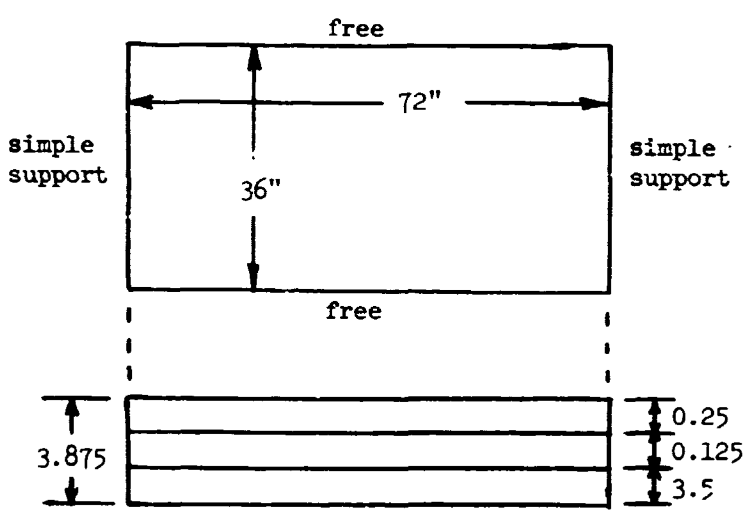

Figure 3 - Three-Layer Sandwich Plate Used for Calculating Loss Factors

3. Johnson, C.D., D.A. Kienholz, and L.C. Rogers, "Finite Element Prediction of Damping in Beams with Constrained Viscoelastic Layers," The Shock and Vibration Bulletin, bull. 51, part 1, May 1981, pp. 71-81.

4. Johnson, C.D., and D.A. Kienholz, "Finite Element Prediction of Damping in Structures with Constrained Viscoelastic Layers," AIAA J., vol. 20, no. 9, Sept. 1982, pp. 1284-1290.

5. Johnson, C.D., and D.A. Kienholz, "Prediction of Damping in Structures with Viscoelastic Materials," Proc. 1983 MSC/NASTRAN User's Conference, Los Angeles, California, March 1983.

6. Ross, D., E.E. Ungar, and E.M. Kerwin, Jr., "Damping of Plate Flexural Vibrations by Means of Viscoelastic Laminae," in Structural Damping, edited by J.E. Ruzicka, The American Society of Mechanical Engineers, New York, 1959, pp. 49-87.