N85-25887

EXPERIENCE WITH FREE BODIES

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BUTLER ANALYSES

It is encouraging to see that analysis and test activities are moving away from competitive postures and toward mutually beneficial cooperation. When some structure is examined for its vibration characteristics by both analysis and test, and the results are to be compared; the matching of boundary conditions becomes a problem. Free bodies can be simulated easily by both disciplines, so it is often the condition to be prescribed by a project. In order for a free body analysis to be valid it must exhibit six clean rigid body (zero frequency) modes. Free body analyses are some of the easiest kinds of analysis to perform if one does everything right. They can be real headaches if one goofs too much. This paper deals with some of the problems that confront an analyst in modeling, if he is to satisfy rigid body conditions; and with some remedies for these problems. It also deals with the problems of detecting these culprits at various levels within the analysis. The paper concludes with the publication of a new method within NASTRAN for checking the model for defects very early in the analysis without requiring the analyst to bear the expense of an eigenvalue analysis before discovering these defects.

MECHANISMS-SPC

Single point constraints (SPC's), multipoint constraints (MPC's), and springs to ground are candidates for interfering with free body motion. The ways that they interfere are obvious in some cases, but for others are fairly subtle. These topics will be elaborated in order; first SPC's. There is a need to constrain the rotational degree of freedom, of grid points to which only plates connect, about the normal to the plate. NASTRAN reports a singularity for any failure to apply such constraints. These constraints are usually invoked on the GRID card and, after initial debugging, usually end up with the necessary --and only-- the necessary rotational constraints applied. Overconstraints inevitably arise when changes to a design occur and modifications are made to its analytical model. Any new constraints always get added, but many of the old constraints, which now need to be purged, are often overlooked, resulting in overconstraints. Thus when the analysis for rigid body eigenvectors is run, some of the six rigid body modes don't appear and elastic modes erupt in their place. The only remedy for points overconstrained by SPC's is to ferret them out. Various ways to detect them will be explored later.

Another more subtle case arises when the normal to a plate is <u>not</u> aligned with one of the displacement coordinate axes. NASTRAN's "GPSP" module is satisfied if a rotational freedom about any one of the coordinate directions is constrained, just so the constrained coordinate provides a component in the direction of the normal to the plate. Satisfying GPSP is not enough for purposes of free motion. There remains a

connection to ground in the constrained rotational degree of freedom, such that a component of elastic action still feeds to ground, so it sabotages the rigid body mode. See figure 1.

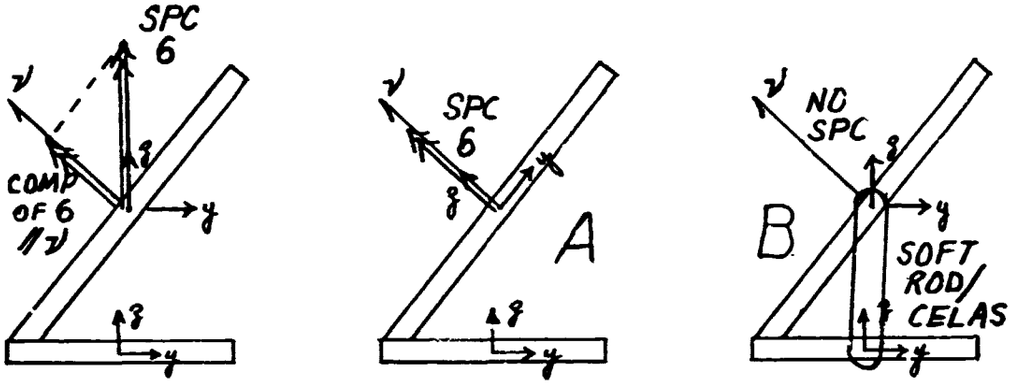

Figure 1

There are at least 3 ways to remedy this intolerable situation, and still satisfy GPSP.

A. Change the displacement coordinate system at the offending grid point so that one coordinate axis is aligned with the normal; then constrain that rotational dof.

B. Opt not to constrain the normal rotation, but provide instead an alternate elastic path many orders of magnitude softer than the neighboring elements. Two simple devices can be employed:
   B1. Use a CONROD between the point in question and some neighbor, or
   B2. Use a CELAS2 between the point in question and (a) some neighbor or (b) to ground.

The reason that the grounded elastic scalar works is that the amount of force developed in a soft scalar, from its connnection to ground is so small that the rigid body motion is effectively not interfered with. The value of the stiffness should be soft enough that it does not cause a disruption to the elastic behavior, but not so small that it causes matrix ill-conditioning.

There is a very special set of conditions in which this analyst got trapped. The situation involves a BAR element, aligned with a coordinate axis, making a perpendicular connection with a plate. See figure 2. The BAR rides in a sleeve bearing at the connection to the plate which makes it impossible for the BAR to transmit torsion to the GP. One sets a pin flag in element dof 4 of the BAR. Now comes the trap. Since the plate doesn't involve moments about the normal and since the moment from the BAR about that same axis is pinned, the situation seems ripe for producing a singularity. Constraining the rotational dof about the normal to the plate seems logical to avoid a singularity. This situation was a part of a much larger model which failed to produce a rigid body mode in rotation about the vertical when the constraint was

applied, but did exhibit the rigid body mode when the constraint was removed. It forces one to rethink the situation. What's wrong? What does the grid point see? What does the constraint do? What is happening internally in the BAR ? A small pilot problem, depicted in figure 2, was run to test the condition.

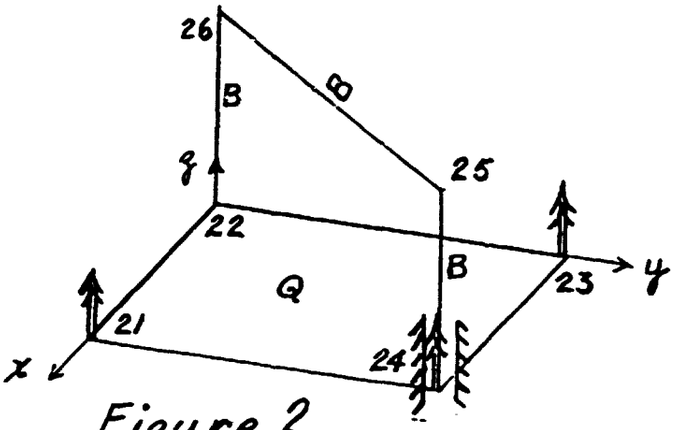

Figure 2

The grid point sees elastic paths in 5 dof coming from the plates. But, does it also see only 5 elastic paths coming from the BAR? Yes. This is confirmed by glancing at the stiffness matrix of the sample problem above, which was modeled as intended, for dof 6 of GP 24. This is column 24 of KGG. Note: col 24 of the matrix is null.

```
     COLUMN     23      ROWS      3 THRU     29     (GP 24 DOF 5)
5.44896E+00 -4.84359E+03  1.60435E+03  0.00000E+00  0.00000E+00
0.00000E+00  1.20968E+03  1.33302E+04  4.45448E+03  0.00000E+00
0.00000E+00  0.00000E+00 -7.76474E+03  4.84359E+03  2.10148E+04
0.00000E+00  6.00000E+05  0.00000E+00  6.54961E+03 -1.17895E+03
4.03848E+06  0.00000E+00 -6.00000E+05  0.00000E+00  0.00000E+00
0.00000E+00  2.00000E+06
```

```
     COLUMNS      24 THRU      24 ARE NULL.     (GP 24 DOF 6)   (No contributions)
                                                                 (from plate or )
                                                                 (bar)
     COLUMN     25      ROWS     19 THRU     36     (GP 25 DOF 1)
-1.20000E+05  0.00000E+00  0.00000E+00  0.00000E+00 -6.00000E+05
 0.00000E+00  4.94767E+05  3.32340E+05  0.00000E+00  0.00000E+00
-6.00000E+05  2.12132E+05 -3.74767E+05 -3.32340E+05  0.00000E+00
 0.00000E+00  0.00000E+00  2.12132E+05
```

For a similar orientation, but with a full 6 dof connection, the stiffness matrix shows that for column 12 corresponding to dof 6 @ GP 22 the matrix is non-null.

COLUMN	11	ROWS	3 THRU	35	(GP 22 DOF 5)
7.76474E+03	4.84359E+03	2.10148E+04	0.00000E+00	6.00000E+05	
0.00000E+00	-6.54961E+03	-1.17895E+03	4.03848E+06	0.00000E+00	
0.00000E+00	0.00000E+00	-5.44896E+00	-4.84359E+03	1.60435E+03	
0.00000E+00	0.00000E+00	0.00000E+00	-1.20968E+03	1.33302E+04	
4.45448E+03	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	
0.00000E+00	0.00000E+00	0.00000E+00	-6.00000E+05	0.00000E+00	
0.00000E+00	0.00000E+00	2.00000E+06			

COLUMN	12	ROWS	12 THRU	36	(GP 22 DOF 6)==Non null.
3.75940E+05	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	-3.75940E+05	

COLUMN	13	ROWS	2 THRU	20	(GP 23 DOF 1)
4.66418E+05	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	
-4.69925E+05	-3.50688E+03	0.00000E+00	0.00000E+00	0.00000E+00	
0.00000E+00	1.87269E+06	-4.66418E+05	0.00000E+00	0.00000E+00	
0.00000E+00	0.00000E+00	-1.40276E+06	3.50688E+03		

If this pilot model is run without dof 6 of GP 24 being constrained, it aborts in the decomposition of K00. This shows that there is a singularity and that a constraint in dof 6 is needed. The original suspicion is confirmed; the model is run through eigenvalues with the constraint in dof 6 of GP 24 in place. All 6 rigid body modes appear so one can say that this is a model of correct behavior. The question still remains, what happened to the large model to cause the rigid body modes to be interfered with? To investigate this problem, modifications were made to the pilot model. If there is a mistake in the model such that the z coordinate of one of its points, say GP 23 is above the plane of the other three points by a small amount, say Δ=.08", the mean plane of the quad is not perpendicular to the z-axis. This anomoly was run with and without the constraint on dof 6 of GP 24 in place. Figure 3 shows the postion of the mean quad in dotted lines.

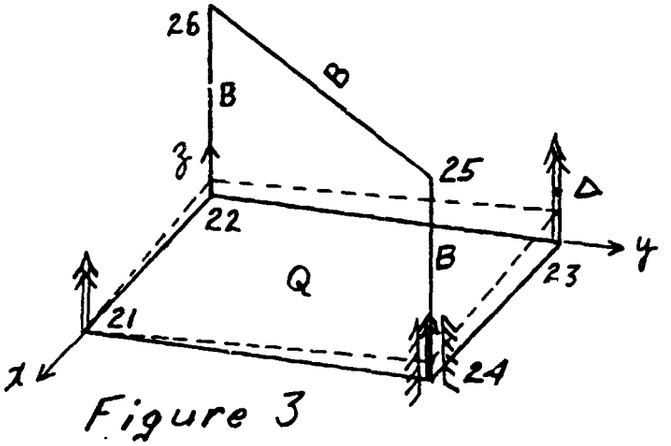

Figure 3

The results of the three runs are tabulated.

MODEL	EIGENVALUE RESULTS	SPCF COMPARISONS						
		MODE 1	MODE 2	MODE 3	MODE 4	MODE 5	MODE 6	MODE 7
AS INTENDED	6 FN = 0	------	NO ---	SPCF --	- IN --	- ANY	-- MODE	------
23(6) WARPED	4 FN = 0	21(6)=0	==	==	==	==	==	==
24(6) SPC	#5 = .02	23(6)=0	==	==	==	==	==	==
	#6 = .14	24(6)	=2.-6	=3.-6	=3.-6	=.028	=7.25	=21.39
	#7 = 106.	=.004						
23(6) WARPED	4 FN = 0	21(6)=0	==	==	==	==	==	==
24(6) NO-SPC	#5 = .014	23(6)=0	==	==	==	==	==	==
	#6 = .036	24(6)=0	==	==	==	==	==	==
	#7 = 2.67							

FREQUENCIES OF NEXT SIX ELASTIC MODES		
AS INTENDED	23(6) WARPED & 24(6) SPC	23(6) WARPED & 24(6) NOT SPC
1.068969E+02	1.064876E+02	1.066318E+02
1.892820E+02	1.893139E+02	1.893587E+02
2.989922E+02	2.866197E+02	3.050661E+02
3.107471E+02	3.076186E+02	3.719919E+02
3.214566E+02	6.107463E+02	2.653362E+03
3.018417E+03	3.073824E+03	3.209373E+03

The constraints @ GP's 21,23, & 24 provide interference with components of the elastic action coming from the plate and sabotage the rigid body mode. Note that the two first elastic modes are almost independent of the mistake in modeling, but after that differences separate their behavior. This turns out to be a particular case of the problem in figure 1 above with the canted plate, except that it appears to shift the blame to the BAR instead of pointing to the mislocated grid point. The BAR acted as a decoy. This is a classic case of the analyst swearing that "he knows that everything is correct, but the computer..", or "but NASTRAN....." is giving these crazy results." In the data that follows it is remarkable that a small glitch can cause g-r-e-a-t disruption. We can call this case the "out-of-plane" type of SPC difficulty.

MECHANISMS-MPC

Multipoint constraints (MPC's) are less conspicuous offenders. One's intuition seems comforted by the idea that if two dof's are required to move together, this is a kind of a rigid body motion and should be compatible with the execution of general rigid motion. The answer is sometimes yes and sometimes no. An example, figure 4, that is easy to visualize, is a pair of points that are constrained so that motion transverse to the line between the points is always to be the same.

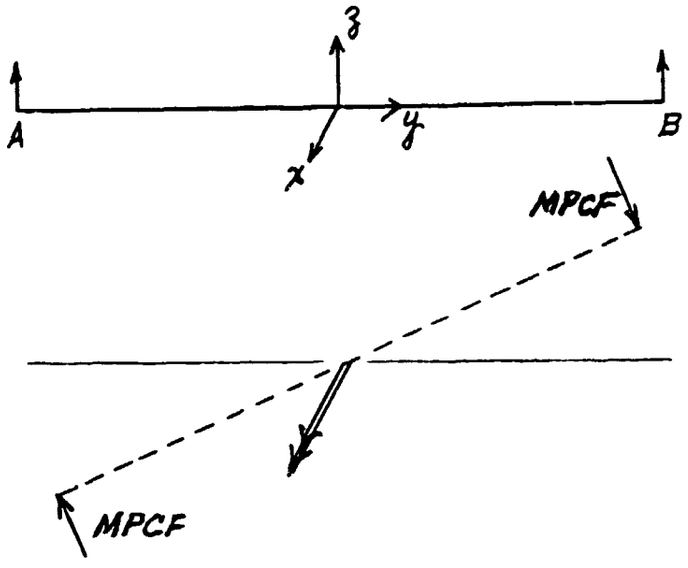

Figure 4

NASTRAN is a faithful servant in carrying out such an assignment. If the analyst says that these two points shall always move up and down together no matter what, NASTRAN will see to it. Thus, any attempt by the body to perform a rotation about an axis perpendicular to the paper is sternly opposed. External forces are set up to create couples to balance the elastic moments produced by neighboring elements tending to rotate the line between A & B. These are called MPC forces, and they interfere with free rigid body movement. In this case the MPC would interfere with the rotation about X. Does this rule out the use of MPC's? Not at all. Does this mean that all rigid elements are off limits? Not at all. If the analyst refers to eqn (56) in section 3.5.6.3 of the theoretical manual, he will see that the formulation of the rigid elements includes rotations so that the relative distance from A to B is maintained invariant for all small displacements, and causes no interference with rigid body motion. If the analyst writes his own MPC'S for two points, he must also ensure that there is no resistance to rigid body motion.

$$\begin{Bmatrix} u_{A_1} \\ u_{A_2} \\ u_{A_3} \\ u_{A_4} \\ u_{A_5} \\ u_{A_6} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & (z_B-z_A) & -(y_B-y_A) \\ 0 & 1 & 0 & -(z_B-z_A) & 0 & (x_B-x_A) \\ 0 & 0 & 1 & (y_B-y_A) & -(x_B-x_A) & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_{B_1} \\ u_{R_2} \\ u_{B_3} \\ u_{B_4} \\ u_{B_5} \\ u_{B_6} \end{Bmatrix} , \qquad (56)$$

MECHANISMS-SPRINGS

Non-small scalar spring elements are also less conspicuous agents for opposing free body motion if they are connected to ground. Since any grounding opposes free body motion, forces develop in the elements when displacements are imposed in the freedoms which are contained in the scalar springs. These scalar springs to ground must be removed. because they operate contrary to a model of a free body. The purpose for the springs being there in the first place has to be re-examined.

DETECTION

If a point is overconstrained with GRID SPC's that were unintended, NASTRAN doesn't give a fatal message. It assumes that the analyst knows what he is doing and dutifully puts the SPC's into operation. The unwanted behavior is ultimately seen in the eigenvalue results. Even these could be overlooked, if the analyst bravely presumes nothing would go wrong and specifies a frequency range over the elastic spectrum of interest only without including the zero part of the spectrum. This would be foolhardy. When doing a free body analysis it is prudent to check to see that the body is truly free by ensuring that all rigid body modes are present. As a minimum the analyst should (1) use 0.0 as the specification for the lower frequency bound on the EIGR card so that he will get a report on how many modes were zero frequency modes. (2) The EIGR request for eigenvectors should embrace 6 in addition to the expected number in the elastic range in order to recover all calculated rigid body mode shapes. If fewer than 6 natural frequencies are zero, there is something in the model opposing some free motion. Inspecting the eigenvectors (both plotted and printed) for the missing actions will give a hint as to the kind of unintended constraints that are inhibiting the free movement. In addition to these two minimum requirements the analyst is advised to request (3) SPCFORCES = ALL and (4) MPCFORCES = ALL. He will not get page after page of zeroes for having specified all, he will get a report of only those points where single point constraints are applied via an elastic path to ground in each mode. SPCF should be zero for every rigid body mode and for every non-zero frequency mode. If some of the 6 rigid body modes are not present, the low frequency modes should be examined for single point constraint forces; these modes could be candidates to be repaired and for converting into rigid body modes. The associated dof's of these forces will give evidence as to the place where the free motion is being interfered with. In the case of multi-point constraints, they should be able to act without developing opposing forces during rigid body deformation. So, by calling for MPCFORCES = ALL, there will be a report at only those points where forces arise in attempting to perform one of the six rigid body motions. If MPCFORCES are not zero, for any of the first six modes, this argues for reformulating the MPC so as to be free of oppostion to rigid body motion.

The aforementioned four measures for detecting errors in modeling of free bodies are expensive and time consuming in that they are displayed only after an eigenvalue analysis has been performed. They should always be used, but not as the primary means of checking. There are ways, well upstream of the DPD module, to discover offenders against free motion. One way is to run through the GPWG (grid point weight generator) and exit in order to find the center of gravity (C.G.). Apply SPC's in the 3 displacement dof's at the C.G. and do static analyses with three subcases. The loading in each subcase is a one-g "GRAV" load in one of the 3 coordinate directions. Call for SPCF = ALL and MPCF = ALL, and call for displacements on opposite side of the C.G. Nonzero SPCF and MPCF at other than the C.G. will give direct evidence as to offenders. Displacement outputs will exhibit any tendency to rotate about the C.G. due to externals other than inertia effects. This is a much cheaper and much quicker way than running an eigenvalue analysis.

A useful but not too specific check is to look for a report of the epsilon sub E (ϵ_E) value that is reported immediately after condensation to the A-set. No report is issued if the residue is small. When the energy check computes to a value that is not small, NASTRAN reports its value. The message appears after the user information message 3023 about the value of parameters for the decomposition of KOO.

There is a rigid body check in NASTRAN based on use of the R-set and deriving a rigid body transformation matrix based on the stiffness matrix.

$$DM = (-)[K_{11}]^{-1}[K_{1r}]$$

and calculates a net error check $\epsilon = \frac{\| X \|}{\|K_{rr}\|}$

which does not pinpoint trouble. Matrix [X] is not output and would be difficult to use for locating trouble. This has useful applications especially when it is important to prescribe the rigid body coordinate directions. It is a good idea to get clean rigid body modes without the R-set first; then introduce SUPORT cards in a final run.

If a static run precedes the eigenvalue analysis, a useful check occurs in module SSG3 if the analyst sets the parameter IRES. The residual matrix RULV is output. It gives diagnostics for every dof in the L-set.

An even quicker and cheaper approach to diagnosing is to impose a rigid body test to the "K" (stiffness) matrix immediately after it has been generated. This method was called to the author's attention by Dr. Cheng Lin of the Aerospace Corp. By definition, a rigid body transformation "D" is one that is stress free. This implies that relative displacements are zero. If relative displacements are zero then the work computed by the product of stiffness into rigid body displacement

(K x D) is zero. If it is other than zero, local impediments to free body motion will cause local perturbations in displacements which in turn result in non-zero work. The beauty of this diagnostic tool is that it operates at the level of G-set so that its non-zero terms are direct locators. One needs to be sure to run with DIAG 21 & 22 turned on in order to correlate the location from internal sequence to external sequence. The method passed on by Cheng Lin needed implementing in NASTRAN. The rigid body transformation matrix "DM" in the normal NASTRAN static solution stream comes after the stiffness matrix is decomposed; i.e. DM = $(-)[K_{LL}]^{-1} \times [K_{LR}]$. The use of the "DM" matrix from this source is like sending the fox to guard the chicken coop. What is needed is a "D" matrix based solely on geometric relations rather than stiffness relations. New modules dealing with seismic analysis were delivered to COSMIC by the author during 1984. One of these named "RBTM" produces a "D" matrix based soley on geometry called "HAICH".

The output 'HAICH' from the RBTM module is the item around which a DMAP ALTER is written. The DMAP ALTER that follows operates under the conditional jumps associated with the three succeeding matrix partitions once the G-sized "K" and "D" matrices are generated. It calls for running a K x D check at the G-level, the N-level, the F-level and the A-level. The check on the G-level should catch any overconstrained GRID SPC's and grounded scalars. The check on the N-level should catch any improper MPC's. The check on the F-level should catch any GRID and general SPC's. The check on the A-level should provide a net check over all levels. The sequence in which the non-zero values appear in any of the partitioned products will tend to isolate what kind of constraint is causing interfence. It is advisable to call for DIAG 21 & 22 so as to locate the external dof where non-zero terms in the KD product appear in the partitions for N, F, and A. Experience on the VAX computer has shown that all values in the KD product should be $< 10\text{-}8$ to ensure no impediment to free motion. Whenever this criterion is met, all six frequencies of the rigid body modes compute to values $< 10\text{-}4$. The ALTER packet for RF 3 of the APR 84 release of NASTRAN is included below. Note the module encircled. RBTM must be obtained from COSMIC to implement the DMAP packet.

```
ALTER 41
(RBTM)    BGPDT,CSTM,EQEXIN, ,USET/HAICH, ,HIRY6, /500/+2/*DIRECT* $
MPYAD     KGG,HAICH, /BALANCE/0/+1/0/0 $ FOR STATIC CHECK
MATPRN    BALANCE,,,,// $ CHECK G-SIZE
ALTER  53
VEC       USET/NVEC/*G*/*N*/*COMP* $
PARTN     HAICH,,NVEC/NRIGID,,,/+1/0 $
MPYAD     KNN,NRIGID, /NBALNC/0/+1/0/0 $
MATPRN    NBALNC,,,,// $ CHECK N-SIZE
ALTER  57
VEC       USET/FVEC/*G*/*F*/*COMP* $
PARTN     HAICH,,FVEC/FRIGID,,,/+1/0 $
MPYAD     KFF,FRIGID, /FBALNC/0/+1/0/0 $
MATPRN    FBALNC,,,,// $ CHECK F-SIZE
ALTER  70
```

```
VEC      USET/AVEC/*G*/*A*/*COMP* $
PARTN    HAICH,,AVEC/ARIGID,,,/+1/0 $
MPYAD    KAA,ARIGID,/ABALNC/0/+1/0/0 $
MATPRN   ABALNC,,,,// $ CHECK A-SIZE
ENDALTER
```

The reason that it is useful to run a counter check on the A-sized matrices is two-fold. First, the effects on the "K" matrix of removing overconstraints and non-rigid constraints is reflected. It acts as a double check on the two previous partitions. Secondly, the A-sized matrices are passed to systems analysts who combine components of several structures into a comprehensive analysis. It serves as a public certification that the transmitted matrices truly satisfy rigid body requirements so that combined matrices can be free of cross-product terms between structures.

The KD diagnostic was applied to the 3 cases of the pilot model. Their results are tabulated.

	LARGEST VALUE OF			
MODEL	KDG	KDN	KDF	KDA
AS INTENDED	1.4-10		1.4-10	3.7-10
23(6) WARPED 24(6) SPC	1313.		3275.	199.0
23(6) WARPED 24(6) NO-SPC	1313.		2659.	88.0

In cases of high condensation the KDA results can be many orders higher in magnitude than KDG, KDN, or KDF, because, depending on the nature of the error and the nature of the condensation, the errors can be swept together and intensified. The advantages of the KD check are (A) that one can checkpoint and exit after each of the G, N, F, and A partitions then proceed economically without high investment in computer time; and (B) the trouble is located and cleared up well before entering eigenvalue analysis.

SUMMARY

This paper has come to the follwing conclusions that deal with problems in analysing free bodies.

1. Mechanisms that interfere with rigid body motion are:
   Single point constraints (SPC'S)
     A. Overconstrained, B. Canted, C. Out-of-plane
   Multi-point constraints (MPC'S)
   CELAS'S that are grounded.

2. Remedies for recovering rigid body motion are:
   Eliminate SPC'S from an elastic path
   Formulate MPC's so as to generate pure rigid body motion
   Eliminate grounded scalars.

3. Detection of interference to rigid body motion from any of:
   The appearance of a non-zero frequency for any of the first six modes.
   Deformation that departs from rigid body motion in the candidate low frequency mode shapes.
   Non-zero SPCFORCES for any mode.
   Non-zero MPCFORCES for any of the first six modes
   Non-zero SPCFORCES outside of the C.G. under static acceleration
   Non-zero MPCFORCES outside of the C.G. under static acceleration
   Epsilon elastic L-R rigid body check
   Existence of an Epsilon sub E report following decomposition of KOO for condesation.
   Non-zero values for the KD products in partitions G thru A.

It is recommended that the ALTER packet for the KD product be made standard practice when performing free body eigenvalue analyses. It is simple, effective, and the least expensive of all the checks that can be applied. Taking an exit after each partition of the four KD checks until each has a clean bill of health would prevent wasteful and costly condensations and eigenvalue analyses of defective models. When his model has passed the KD check, an analyst can have high expectations of recovering 6 clean rigid body modes from the eigenvalue analysis.

Even though the KD check was designed as a tool for diagnosing a model for free eigenvalue analysis, it can serve any other type of analysis if there is concern for overconstraints.