N85-25879

ON THE INTERCONNECTION OF INCOMPATIBLE SOLID FINITE ELEMENT MESHES USING MULTIPOINT CONSTRAINTS

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SUMMARY

Incompatible meshes, i.e. meshes that physically must have a common boundary, but do not necessarily have coincident grid points, can arise in the course of a finite element analysis. For example, two substructures may have been developed at different times for different purposes and it becomes necessary to interconnect the two models. A technique that uses only multipoint constraints, i.e. MPC cards (or MPCS cards in substructuring), is presented. Since the method uses only MPC's, the procedure may apply at any stage in an analysis; no prior planning or special data is necessary.

INTRODUCTION

When two separate finite element solid meshes are combined with a common boundary, the degrees of freedom on the common surface of one of the models must be eliminated. The elimination process must preserve the compatibility of the joined surface when the model is subjected to external loads which cause the interface surface to deform. The use of MPC's can provide this capability to first order, i.e. linear in terms of grid point displacements.

The MPC equations presented are based on a linear surface spline. A three dimensional surface spline is a mathematical tool used to find a function, f(u,v,w) for all points (x,y,z) when f(u(x,y,z), v(x,y,z), w(x,y,z) is known for a discrete set of points, (x_i,y_i,z_i) and (u,v,w) are displacements in the (x,y,z) directions. The three dimensional surface spline given here is based on the shape function of the linear displacement tetrahedra. This shape function will produce compatible surface deformations for models using linear elements.

The example presented implements this method by assuming that one surface is composed of a set of imaginary triangles formed by grid points on one of the mating surfaces. The vertices of a triangle form a unique surface described by (x_i,y_i,z_i), where i = 1,2,3. Grid points of the surface to be interconnected that lie within the boundary of the triangle are MPC'ed to the displacement of the vertices, (u_i,v_i,w_i), based on the spline functions f($u(x_i,y_i,z_i)$, $v(x_i,y_i,z_i)$, $w(x_i,y_i,z_i)$). Results of a practical example in substructure analysis are presented. A computer program that automatically writes the MPC cards is included in the Appendix.

DEFINITION OF THE PROBLEM

Consider a planar triangle in space defined by three points; A, B and C, as shown in Figure 1.

The vector {A} represents the distance from the origin to point A; similarly for points B, C and O. The subject of this paper is the determination of the relationship between displacements of points A, B, C and O when

$$\begin{aligned} \{dA\} &= \{A' - A\} = (u^a, v^a, w^a) \\ \{dB\} &= \{B' - B\} = (u^b, v^b, w^b) \\ \{dC\} &= \{C' - C\} = (u^c, v^c, w^c) \\ \{dO\} &= \{O' - O\} = (u^o, v^o, w^o) \end{aligned} \qquad (1)$$

such that the point O' lies in the plane described by the triangle in space defined by the three points A', B' and C'.

In the application described, the points A, B and C lie on the surface of a region described by a group of finite elements; the point "O" is a grid (or node) in the same or in a different group of finite elements defining a region that has a physically congruent undeformed boundary that the points A, B, C and O lie on.

In the notation used here, {] is a row vector and { } is a scalar quantity. A matrix multiplication, {] [ ] [} results in a scalar, as indicated by the left and right elements; similarly, [} {] would indicate a matrix result. In the text a scalar will often be written without the braces.

SELECTION OF THE SPLINE FUNCTION

Some rule must be selected that relates the displacement of any point in the plane of the triangle with the motion of the three reference points A, B and C, i.e. a spline function. The shape function of a linear displacement tetrahedra is used for this purpose. However, in order to use this shape function, a tetrahedra must be constructed from the reference triangle. This is easily accomplished by forming a cross product of the vectors $\{V^{ab}\}$ and $\{V^{ac}\}$;

$$\{V^{ap}\} = \{V^{ab}\} \times \{V^{ac}\} \qquad (2)$$

where

$$\{V^{ij}\} = \{V^j\} - \{V^i\}$$

The result of equation (2) is the formation of a reference point, P, which forms the fourth vertex of the reference tetrahedra. Clearly, if $\{V^{ab}\}$ and $\{V^{ac}\}$ are unique, then point P will be unique.

The shape function for a linear displacement tetrahedra is [Reference 1, Section 5.12]

$$\begin{Bmatrix} u^a \\ u^b \\ u^c \\ u^p \end{Bmatrix} = \begin{bmatrix} 1 & x^a & y^a & z^a \\ 1 & x^b & y^b & z^b \\ 1 & x^c & y^c & z^c \\ 1 & x^p & y^p & z^p \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix} \qquad (3)$$

or

$$\{u^i\} = [T]\{D\}$$

where

u^i = the displacement of the i^{th} vertex in the x coordinate direction

Two additional sets of equations, similar to equation (3), relate v and w, the y and z displacement, to the undetermined coefficients, {D}. The displacement of a point "O" is, in the x direction

$$\{u^o\} = D_1 + D_2X^o + D_3Y^o + D_4Z^o \qquad (4)$$

or from the solution of equation (3)

$$\{u^o\} = \{1 \; x^o \; y^o \; z^o\}[S]\{u\}$$

where

$$[S] = [T]^{-1}$$

Similar equations exist for v and w.

It is useful to write equation (4) in the following form for the displacements in the three coordinate directions:

$$\begin{aligned} u^o(x^o, y^o, z^o) &= \mathbf{A}\, u^a + \mathbf{B}\, u^b + \mathbf{C}\, u^c + \mathbf{P}\, u^p \\ v^o(x^o, y^o, z^o) &= \mathbf{A}\, v^a + \mathbf{B}\, v^b + \mathbf{C}\, v^c + \mathbf{P}\, v^p \\ w^o(x^o, y^o, z^o) &= \mathbf{A}\, w^a + \mathbf{B}\, w^b + \mathbf{C}\, w^c + \mathbf{P}\, w^p \end{aligned} \tag{5}$$

where

$$\begin{aligned} \mathbf{A} &= S_{11} + S_{21}x^o + S_{31}y^o + S_{41}z^o \\ \mathbf{B} &= S_{12} + S_{22}x^o + S_{32}y^o + S_{42}z^o \\ \mathbf{C} &= S_{13} + S_{23}x^o + S_{33}y^o + S_{43}z^o \\ \mathbf{P} &= S_{14} + S_{24}x^o + S_{34}y^o + S_{44}z^o \end{aligned}$$

and

u^i = displacement of point (x^i, y^i, z^i) in X direction
v^i = displacement of point (x^i, y^i, z^i) in Y direction
w^i = displacement of point (x^i, y^i, z^i) in Z direction

Now equation (5) is in the form of a multipoint constraint (MPC) relationship [Reference 1, Section 3.5]. The coefficients **A, B, C** and **P** are constants that depend only on the constrained geometry. Equation (5) cannot be used directly, however, since point **P** does not actually exist. Point **P** must be removed from the set of equations.

The motion of point **P**, in terms of the motion of point **A** and the rotation vector, $\{\alpha, \beta, \gamma\}$, of the triangle is [Reference 1, Section 3.5]:

$$\begin{Bmatrix} u^p \\ v^p \\ w^p \end{Bmatrix} = \begin{Bmatrix} u^a \\ v^a \\ w^a \end{Bmatrix} + \begin{bmatrix} 0 & z^{ap} & -y^{ap} \\ -z^{ap} & 0 & x^{ap} \\ y^{ap} & -x^{ap} & 0 \end{bmatrix} \begin{Bmatrix} \alpha \\ \beta \\ \gamma \end{Bmatrix} \tag{6}$$

or

$$\{u^p\} = \{u^a\} + [H]\{\alpha\}$$

where

α, β, and γ = rotation about x, y, z axes.

CALCULATION OF THE ROTATION VECTOR

It is necessary to determine the relative angular displacement of the unstrained and strained triangles in terms of the displacements of points **A, B** and **C**. Equation (6) can be written in matrix form for the three points **A, B** and **C** and the rotation vector (α, β, γ) as:

$$
\begin{bmatrix}
0 & z^{ab} & -y^{ab} \\
-z^{ab} & 0 & x^{ab} \\
y^{ab} & -x^{ab} & 0 \\
\hline
0 & z^{bc} & -y^{bc} \\
-z^{bc} & 0 & x^{bc} \\
y^{bc} & -x^{bc} & 0 \\
\hline
0 & z^{ca} & -y^{ca} \\
-z^{ca} & 0 & x^{ca} \\
y^{ca} & -x^{ca} & 0
\end{bmatrix}
\begin{Bmatrix} \alpha \\ \beta \\ \gamma \end{Bmatrix}
=
\begin{Bmatrix}
u^{ab} \\ v^{ab} \\ w^{ab} \\ \hline u^{bc} \\ v^{bc} \\ w^{bc} \\ \hline u^{ca} \\ v^{ca} \\ w^{ca}
\end{Bmatrix}
\qquad (7)
$$

or, symbolically

$$[R]\{\alpha\} = \{\Delta u\}$$

where

$$\begin{Bmatrix} u^{ar} \\ v^{ar} \\ w^{ar} \end{Bmatrix} = \begin{Bmatrix} u^{r} - u^{a} \\ v^{r} - v^{a} \\ w^{r} - w^{a} \end{Bmatrix}$$

Now a unique solution to equation (7) exists so long as the triangles (A, B, C) and (A', B', C') are unique. Physically this must be the case, otherwise a zero strain would result. The solution is [Reference 2]:

$$\{\alpha\} = ([R]^T [R])^{-1} [R]^T \{\Delta u\} \qquad (8)$$

Equation (8) gives the rotation vector, for small angles, when the displacements are known at the triangle vertices.

Equation (6), when $\{\alpha\}$ is determined from equation (8) becomes:

$$\{u^p\} = \{u^a\} + [Q]\{\Delta u\} \qquad (9)$$

where

$$[Q] = [H](R^T R)^{-1} [R]^T$$

Equation (9), when combined with equation (5) provides the MPC relationship that is a function of the displacements of the original four points of the tetrahedra, **A, B, C** and **P.**

Equation (9) written in detail is:

$$
\begin{Bmatrix} u^p \\ v^p \\ w^p \end{Bmatrix} = \begin{Bmatrix} u^a \\ v^a \\ w^a \end{Bmatrix} + \begin{bmatrix} Q_{11} & Q_{12} & \cdots & Q_{19} \\ \vdots & & & \\ Q_{31} & Q_{32} & \cdots & Q_{39} \end{bmatrix}
\begin{Bmatrix}
u^b - u^a \\ v^b - v^a \\ w^b - w^a \\ u^c - u^b \\ v^c - v^b \\ w^c - w^b \\ u^a - u^c \\ v^a - v^c \\ w^a - w^c
\end{Bmatrix}
$$

The displacement of point **P** can be determined from the displacements u^a, u^b and u^c in accordance with equation (9), i.e.

$$
\begin{aligned}
u^p = {} & (1 + Q_{17} - Q_{11})u^a + (Q_{11} - Q_{14})u^b + (Q_{14} - Q_{17})u^c \\
+ {} & (Q_{18} - Q_{12})v^a + (Q_{12} - Q_{15})v^b + (Q_{15} - Q_{18})v^c \\
+ {} & (Q_{19} - Q_{13})w^a + (Q_{13} - Q_{16})w^b + (Q_{16} - Q_{19})w^c
\end{aligned}
\tag{10}
$$

and similarly for v^p and w^p.

SPLINE EQUATION FINAL FORM

Substituting equation (10) into the MPC equation (5) gives the final form of the spline equations:

$u(x^o, y^o, z^o) =$

$$
\begin{aligned}
& [A + P(1+Q_{17} - Q_{11})]u^a + [B+P(Q_{11} - Q_{14})]\ u^b + [C+P(Q_{14} - Q_{17})]\ u^c \\
& + P(Q_{18} - Q_{12})\ v^a + P(Q_{12} - Q_{15})\ v^b + P(Q_{15} - Q_{18})\ v^c \\
& + P(Q_{19} - Q_{13})\ w^a + P(Q_{13} - Q_{16})\ w^b + P(Q_{16} - Q_{19})\ w^c
\end{aligned}
$$

$v(x^o, y^o, z^o) =$

$$
\begin{aligned}
& P(Q_{27} - Q_{21})\ u^a + P(Q_{21} - Q_{24})\ u^b + P(Q_{24} - Q_{27})\ u^c \\
& +[A + P(1+Q_{28} - Q_{22})]v^a + [B+P(Q_{22} - Q_{25})]\ v^b + [C+P(Q_{25} - Q_{28})]\ v^c \\
& P(Q_{29} - Q_{23})\ w^a + P(Q_{23} - Q_{26})\ w^b + P(Q_{26} - Q_{29})\ w^c
\end{aligned}
$$

$w(x^o, y^o, z^o) =$

$$
\begin{aligned}
& P(Q_{37} - Q_{31})\ u^a + P(Q_{31} - Q_{34})\ u^b + P(Q_{34} - Q_{37})\ u^c \\
& + P(Q_{38} - Q_{32})\ v^a + P(Q_{32} - Q_{35})\ v^b + P(Q_{35} - Q_{38})\ v^c \\
& +[A + P(1+Q_{39} - Q_{33})]w^a + [B+P(Q_{33} - Q_{36})]\ w^b + [C+P(Q_{36} - Q_{39})]\ w^c
\end{aligned}
$$

A SIMPLE NUMERICAL EXAMPLE

Consider a triangle with vertices at A=(1,0,0), B=(0,1,0) and C=(0,0,1). The point, O, to be removed is at O=(1/3, 1/3, 1/3). This point lies in the plane of the triangle ABC with the distance to the origin of $1/\sqrt{3}$. Values of the various scalars, vectors, and matrices used in calculating the MPC coefficients are given below.

$$\{v^{ap}\} = (1.0,\ 1.0,\ 1.0)$$

$$
[T] = \begin{bmatrix} 1.0 & 1.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 1.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 1.0 \\ 1.0 & 2.0 & 1.0 & 1.0 \end{bmatrix}
$$

$$[S] = \begin{bmatrix} .6667 & .3333 & .3333 & -.3333 \\ .3333 & -.3333 & -.3333 & .3333 \\ -.6667 & .6667 & -.3333 & .3333 \\ -.6667 & -.3333 & .6667 & .3333 \end{bmatrix}$$

A = 1/3
B = 1/3
C = 1/3
P = 0

$$[R] = \left[\begin{array}{ccc} 0 & 0 & -1 \\ 0 & 0 & -1 \\ 1 & 1 & 0 \\ \hline 0 & 1 & 1 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \\ \hline 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$[R]^T[R] = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix}$$

$$([R^T][R])^{-1} = \begin{bmatrix} .2778 & -.0556 & -.0556 \\ -.0556 & .2778 & -.0556 \\ -.0556 & -.0556 & .2778 \end{bmatrix}$$

$([R]^T[R])^{-1}\ [R]^T =$

$$\begin{bmatrix} .0556 & .0556 & .222 & -.111 & -.2778 & -.2778 & .0556 & .2222 & .0556 \\ .0556 & .0556 & .222 & .222 & .0556 & .0556 & -.2778 & -.1111 & -.2778 \\ -.2778 & -.2778 & -.111 & .222 & .0556 & .0556 & .0556 & .2222 & .0556 \end{bmatrix}$$

$[Q] =$

$$\begin{bmatrix} .3333 & .3333 & .3333 & .0000 & .0000 & .0000 & -.3333 & -.3333 & -.3333 \\ -.3333 & -.3333 & -.3333 & .3333 & .3333 & .3333 & .0000 & .0000 & .0000 \\ .0000 & .0000 & .0000 & -.3333 & -.3333 & -.3333 & .3333 & .3333 & .3333 \end{bmatrix}$$

and finally,

$$u(1/3,\ 1/3,\ 1/3) =$$

$$\begin{aligned} & .3333u^a + .3333u^b + .3333u^c \\ + & .0 \quad v^a + .0 \quad v^b + .0 \quad v^c \\ + & .0 \quad w^a + .0 \quad w^b + .0 \quad w^c \end{aligned}$$

$$v(1/3,\ 1/3,\ 1/3) =$$

$$\begin{aligned} & .0 \quad u^a + .0 \quad u^b + .0 \quad u^c \\ + & .3333v^a + .3333v^b + .3333v^c \\ + & .0 \quad w^a + .0 \quad w^b + .0 \quad w^c \end{aligned}$$

$$w(1/3,\ 1/3,\ 1/3) =$$

$$\begin{aligned} & .0 \quad u^a + .0 \quad u^b + .0 \quad u^c \\ + & .0 \quad v^a + .0 \quad v^b + .0 \quad v^c \\ + & .3333w^a + .3333w^b + .3333w^c \end{aligned}$$

If the displacements of points **A, B,** and **C** each move along the x, y and z axes, respectively, as shown in Figure 2, then

$$u^o = .3333, \quad v^o = .3333, \quad w^o = .3333$$

and point 0' = (.6667, .6667, .6667). It is useful to note that since **P** = 0 the MPC equations (11) reduce to:

$$u(x^o, y^o, z^o) = \mathbf{A}\ u^a + \mathbf{B}\ u^b + \mathbf{C}\ u^c$$

$$v(x^o, y^o, z^o) = \mathbf{A}\ v^a + \mathbf{B}\ v^b + \mathbf{C}\ v^c$$

This situation apparently results when the point to be removed, 0, lies in the plane of the triangle formed by points **A, B** and **C.** The equations derived are such that the strain perpendicular to the triangle **ABC** are null. Points lying in the plane of the triangle have non-null strains only in the plane of the triangle.

A PRACTICAL APPLICATION

The technique presented in this paper was developed during the course of a finite element analysis of a bolted, flange-type coupling used in marine risers [3,4]. Figure 3 shows the coupling components in the bolt-up condition. The coupling is modeled as three separate substructures; the Pin, Box, and Bolt, as shown in Figures 4, 5, and 6. Advantage is taken of the cyclic symmetry of the coupling, hence a 22.5° pie section is modeled with the appropriate boundary conditions. The finite element mesh of the Pin in the area of the countersink used very small elements in order to properly calculate stress concentrations. The corresponding area of the Bolt, at the Pin-Bolt interface, need not be modeled with a mesh this fine.

A considerable savings in computer run time is achieved if the interface of the Pin and Bolt can be made consistent with the boundary conditions but allow a lower density mesh on the Bolt. Grid points on the Pin that were not coincident with grid points on the Bolt were removed. The results of a stress analysis of the Pin are shown in Figure 7. These results show that, in the region of the grids that were removed by MPCS cards, a smooth and consistent stress pattern results.

REFERENCES

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2. Mood, A.M. and Graybill, F.A., Introduction to the Theory of Statistics, McGraw-Hill, 1963, pp 343-346.

3. Fox, Gary L. and Geminder, Robert, "Analysis of the HMF Riser Connector", Proceedings of Oceans '84, September, 1984, IEEE.

4. Fox, Gary L., "Nonlinear Analysis of a Bolted Marine Riser Coupling Using NASTRAN® Substructuring", Twelfth NASTRAN® Users' Colloquium, May, 1984, NASA Conference Publication 2328.

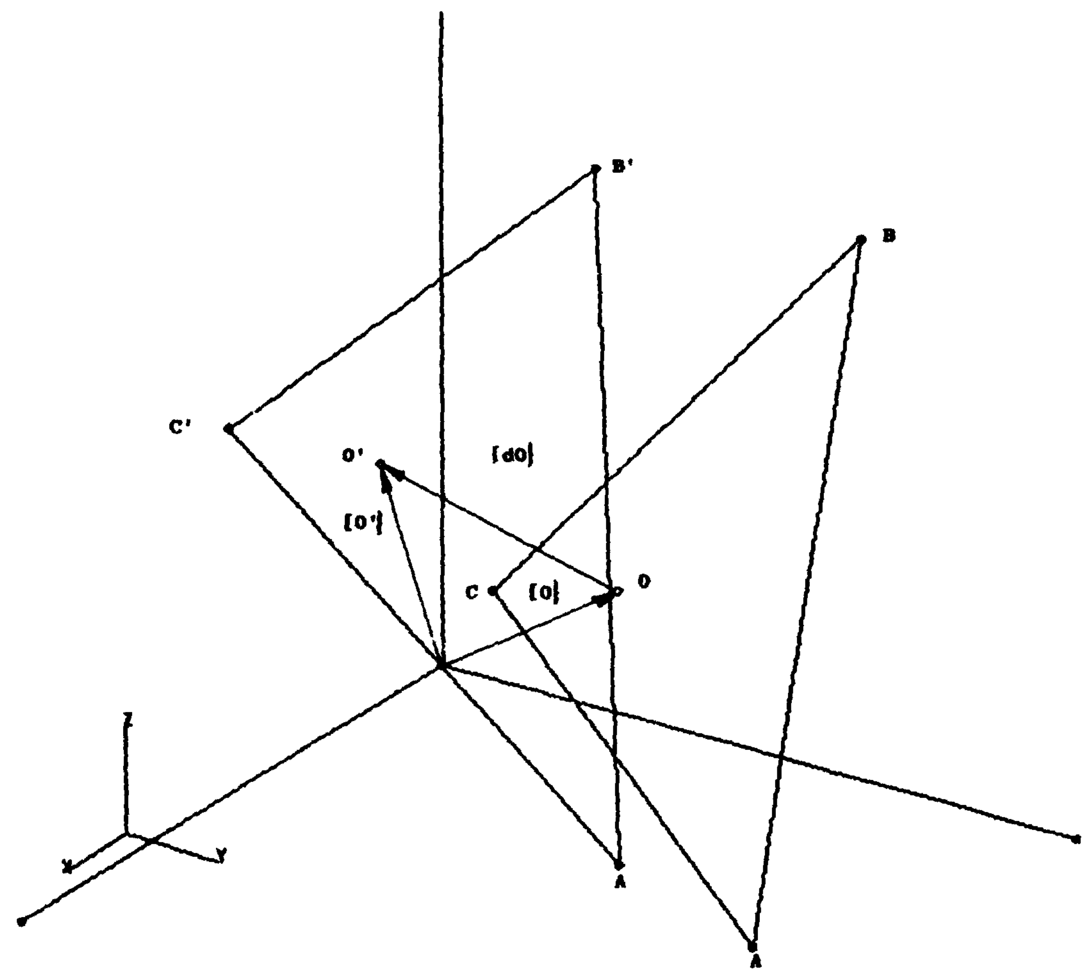

FIGURE 1 - SCHEMATIC REPRESENTATION OF AN UNSTRAINED TRIANGLE (A,B,C)
AND A STRAINED TRIANGLE (A',B',C')

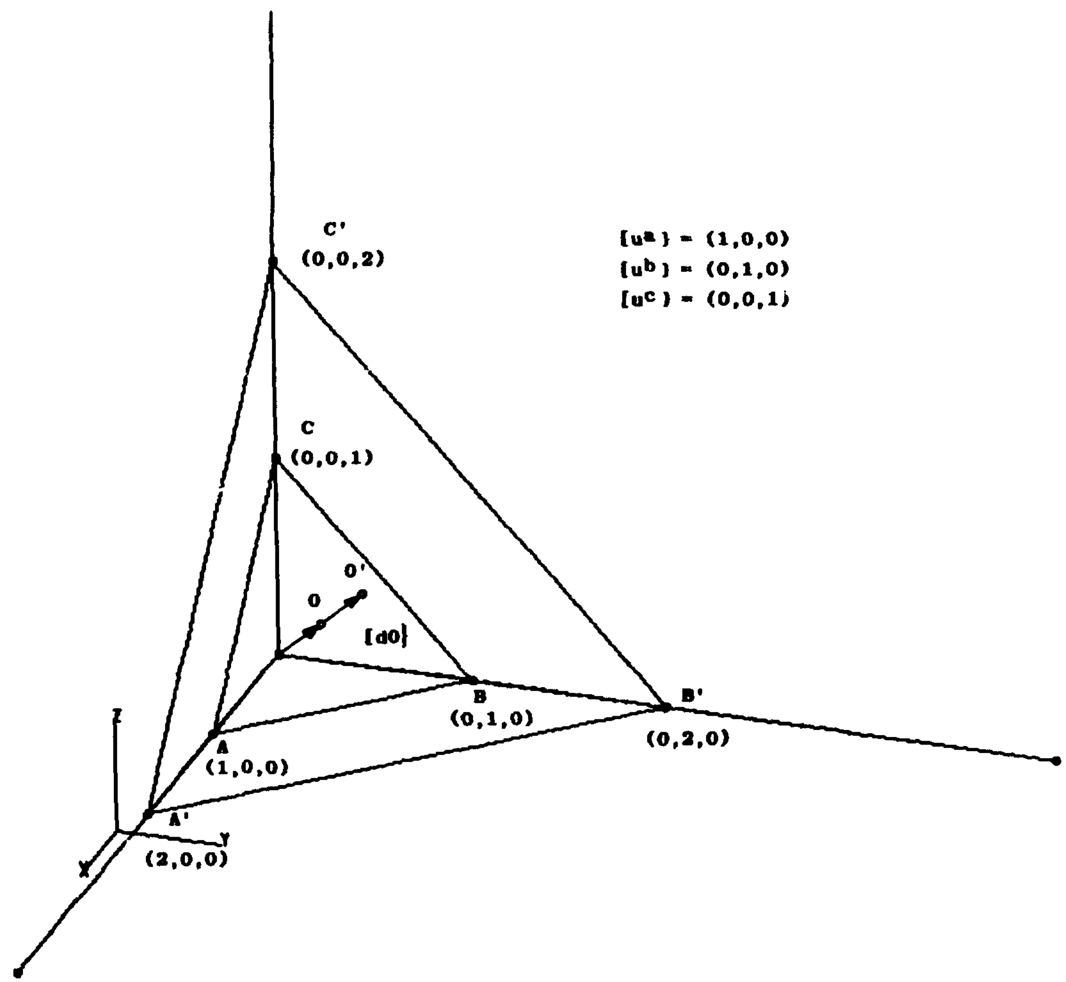

FIGURE 2 - EXAMPLE PROBLEM

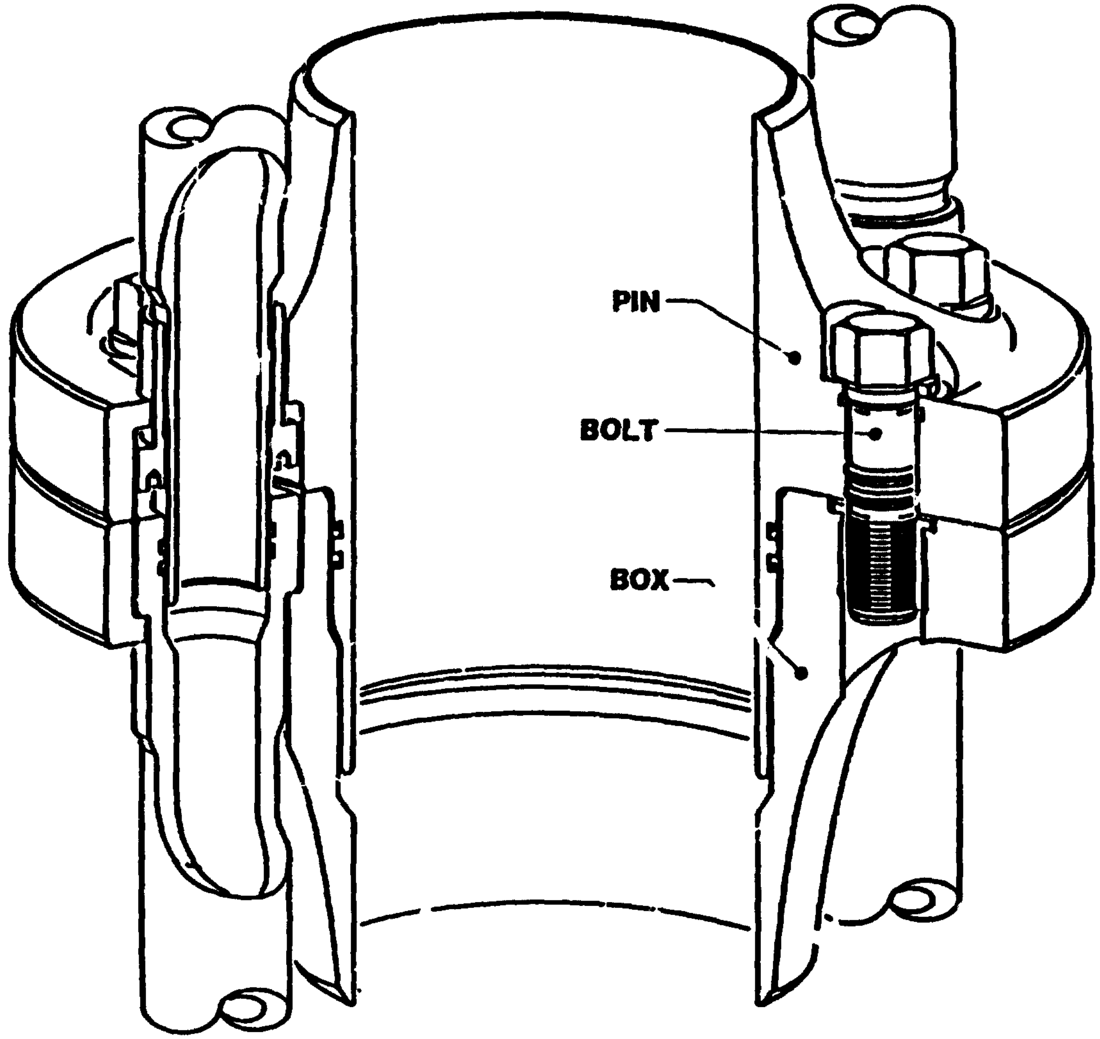

FIGURE 3 - HMF RISER CONNECTOR

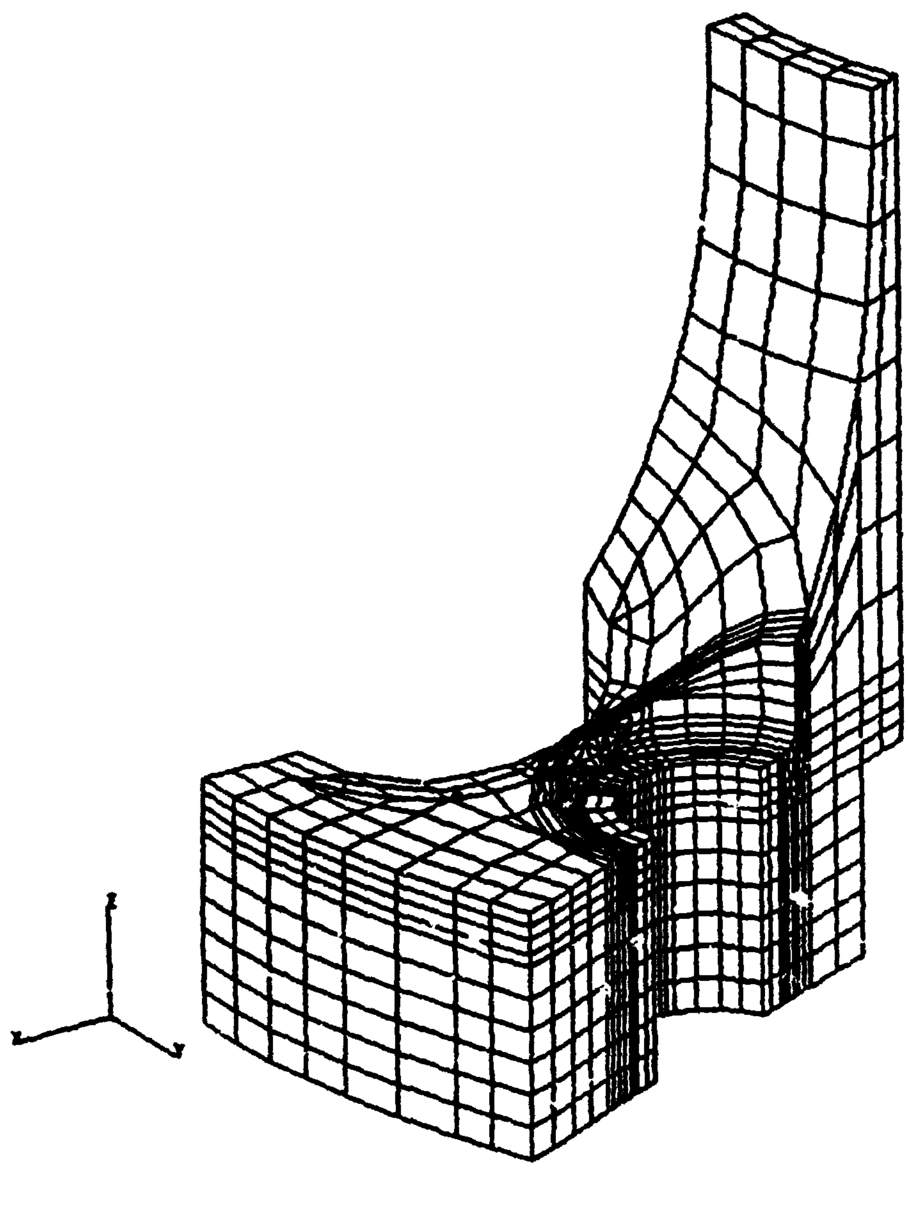

FIGURE 4 - HIDDEN LINE VIEW OF HMF PIN, 22.5° PIE SECTION

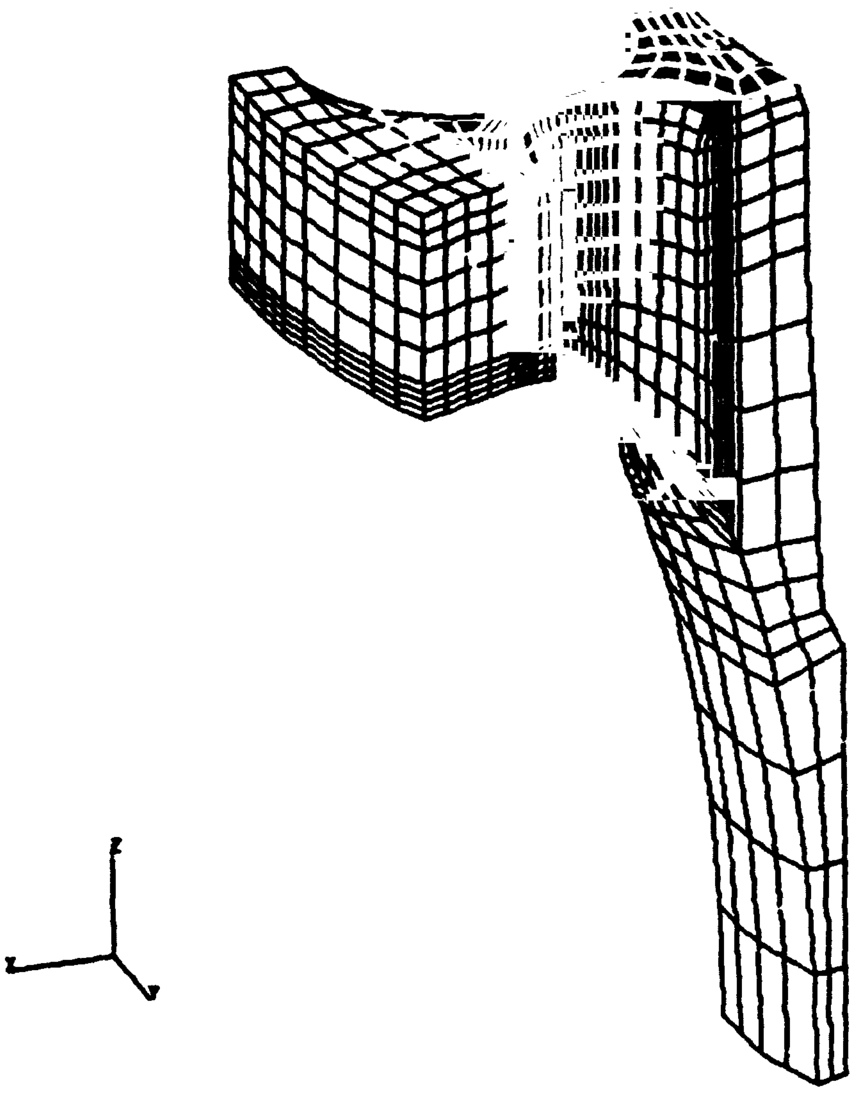

FIGURE 5 - HIDDEN LINE VIEW OF HMF BOX, 22.5° PIE SECTION

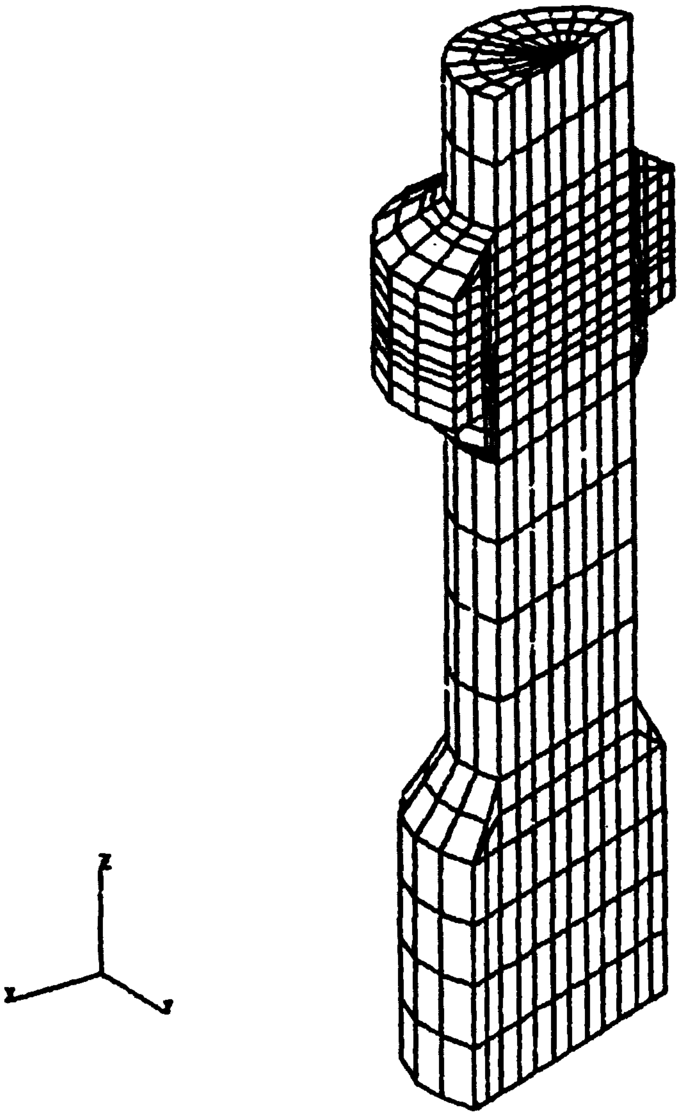

FIGURE 6 - HIDDEN LINE VIEW OF HMF BOLT, 22.5° PIE SECTION

FIGURE 7 - VON MISES EQUIVALENT STRESS CONTOURS MAPPED ONTO A HIDDEN LINE PLOT OF HMF PIN

APPENDIX

This Appendix gives the source code of a FORTRAN computer program that reads a control file (BDYMPC.INP), the NASTRAN grid card file (BDYMPC.BDF), and produces MPC cards for inclusion to the NASTRAN Bulk Data File. Figure A-1 shows the control file for the sample problem. The first line gives the number of points, NG, to be MPC'ed and the three GRID ID's. The next NG lines contain the GRID ID's. The group may be repeated as many times as necessary. Figure A-2 shows the GRID cards and Figure A-3 shows a listing of MPC cards generated for the sample problem. The source listing is given in Figure A-4.

1 21 22 23
25

FIGURE A-1 SAMPLE PROBLEM CONTROL FILE

GRID	21	Ø	1.Ø	Ø.Ø	Ø.Ø
GRID	22	Ø	Ø.Ø	1.Ø	Ø.Ø
GRID	23	Ø	Ø.Ø	Ø.Ø	1.Ø
GRID	25	Ø	Ø.33333	Ø.33333	Ø.33333

FIGURE A-2 SAMPLE PROBLEM GRID CARD INPUT

```
$DEBUG
$NOFLOATCALLS
      PROGRAM BDYMPC
C
C
C     UNIT 11  *.INP  CONTROL INPUT DATA, CORNER GRID ID'S
C                     NO. AND ID'S OF INTERNAL GRIDS
C
C     UNIT 12  *.BDF  NASTRAN GRID CARDS OF FEM
C
C     UNIT 13  *.MPC  OUTPUT OF NASTRAN MPC CARDS
C
C     UNIT 14  *.DIA  DIAGNOSTIC AND CHECK OUTPUT
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION AP(3),Q(3,9),R(9,3),T(4,4),S(4,4),RT(3,9),RTR(3,3)
     &,RTRI(3,3),H(3,3),RTRIRT(3,9)
      CHARACTER DUM*8
      COMMON /XMPCR/XMPC(10),IDG(3),LINE,X(3)
      OPEN(11,FILE='BDYMPC.INP',STATUS='OLD')
      OPEN(12,FILE='BDYMPC.BDF',STATUS='OLD')
      OPEN(13,FILE='BDYMPC.MPC',STATUS='NEW')
      OPEN(14,FILE='BDYMPC.DIA',STATUS='NEW')
      NCR=0
      LINE=1
      XMPC(1)=-1.0
      DO 100 I=1,4
 100  T(I,1)=1.0
C
C     START LOOP FOR EACH REGION
C
 105  READ(11,5,END=1000)NG,IDG
   5  FORMAT(BN,4I8)
      WRITE(14,1)NG,IDG
   1  FORMAT(1X,'NO. OF INTERNAL GRIDS = ',I8
     &,'  CORNER GRID ID''S ',3I8)
      NCF=0
C
C     FIND CORNER GRID ID'S
C
 107  REWIND 12
 110  READ(12,2,END=2000) DUM,ID,IDUM,X
   2  FORMAT(BN,A8,2I8,3E8.0)
      WRITE(14,4) DUM,ID,IDUM,X
   4  FORMAT(1X,A8,2I8,3E12.5)
      IF (ID.NE.IDG(NCF+1)) GO TO 110
      WRITE(14,'(1X)')
      NCF=NCF+1
      DO 120 I=1,3
 120  T(NCF,I+1)=X(I)
      IF(NCF.EQ.3) GOTO 130
      GOTO 107
 130  CONTINUE
C
```

FIGURE A-4 COMPUTER PROGRAM SOURCE CODE LISTING

MPC	312	25	1	-1.000	21	1	.333	+M 1
+M 1		22	1	.333	23	1	.333	+M 2
+M 2		21	2	.000	22	2	.000	+M 3
+M 3		23	2	.000	21	3	.000	+M 4
+M 4		22	3	.000	23	3	.000	
MPC	312	25	2	-1.000	21	1	.000	+M 5
+M 5		22	1	.000	23	1	.000	+M 6
+M 6		21	2	.333	22	2	.333	+M 7
+M 7		23	2	.333	21	3	.000	+M 8
+M 8		22	3	.000	23	3	.000	
MPC	312	25	3	-1.000	21	1	.000	+M 9
+M 9		22	1	.000	23	1	.000	+M 10
+M 10		21	2	.000	22	2	.000	+M 11
+M 11		23	2	.000	21	3	.333	+M 12
+M 12		22	3	.333	23	3	.333	

FIGURE A 3 SAMPLE PROBLEM MPC CARD OUTPUT

```
C
C     FORM CROSS PRODUCT AB X AC FOR VECTOR AP
C
      XAB=T(2,2)-T(1,2)
      XAC=T(3,2)-T(1,2)
      XBC=T(3,2)-T(2,2)
      YAB=T(2,3)-T(1,3)
      YAC=T(3,3)-T(1,3)
      YBC=T(3,3)-T(2,3)
      ZAB=T(2,4)-T(1,4)
      ZAC=T(3,4)-T(1,4)
      ZBC=T(3,4)-T(2,4)
C
      AP(1)=YAB*ZAC-ZAB*YAC
      AP(2)=ZAB*XAC-XAB*ZAC
      AP(3)=XAB*YAC-YAB*XAC
C
C     POINT P ADDED TO T MATRIX
C
      T(4,2)=T(1,2)+AP(1)
      T(4,3)=T(1,3)+AP(2)
      T(4,4)=T(1,4)+AP(3)
      CALL MINV4(T,S)
C
C     FORM R AND H MATRICES
C     NOTE THAT ZCA=-ZAC, ETC.
C
      R(1,1)= 0.0
      R(1,2)=+ZAB
      R(1,3)=-YAB
      R(2,1)=-ZAB
      R(2,2)= 0.0
      R(2,3)=+XAB
      R(3,1)=+YAB
      R(3,2)=-XAB
      R(3,3)= 0.0
      R(4,1)= 0.0
      R(4,2)=+ZBC
      R(4,3)=-YBC
      R(5,1)=-ZBC
      R(5,2)= 0.0
      R(5,3)=+XBC
      R(6,1)=+YBC
      R(6,2)=-XBC
      R(6,3)= 0.0
      R(7,1)= 0.0
      R(7,2)=-ZAC
      R(7,3)=+YAC
      R(8,1)=+ZAC
      R(8,2)= 0.0
      R(8,3)=-XAC
      R(9,1)=-YAC
      R(9,2)=+XAC
      R(9,3)= 0.0
C
```

```
C
      H(1,1)= 0.0
      H(2,2)= 0.0
      H(3,3)= 0.0
      H(1,2)= AP(3)
      H(1,3)=-AP(2)
      H(2,3)= AP(1)
      H(2,1)=-H(1,2)
      H(3,1)=-H(1,3)
      H(3,2)=-H(2,3)
C
      DO 135 I1=1,3
      DO 135 J1=1,9
      RT(I1,J1)=R(J1,I1)
  135 CONTINUE
C
C              T
C     FORM  R   * R = RTR
C
      DO 136 I1=1,3
      DO 136 J1=1,3
      RTR(I1,J1)=0.0
      DO 136 K1=1,9
      RTR(I1,J1)=RTR(I1,J1)+RT(I1,K1)*R(K1,J1)
  136 CONTINUE
      CALL MINV3(RTR,RTRI)
C
C     FORM RTRI*RT
C
      DO 137 I1=1,3
      DO 137 J1=1,9
      RTRIRT(I1,J1)=0.0
      DO 137 K1=1,3
  137 RTRIRT(I1,J1)=RTRIRT(I1,J1)+RTRI(I1,K1)*RT(K1,J1)
C
      DO 138 I1=1,3
      DO 138 J1=1,9
      Q(I1,J1)=0.0
      DO 138 K1=1,3
  138 Q(I1,J1)=Q(I1,J1)+H(I1,K1)*RTRIRT(K1,J1)
C
      WRITE(14,3) ((R(IL,JL),JL=1,3),IL=1,9)
     &,((    T(IL,JL),JL=1,4),IL=1,4)
     &,((    RTR(IL,JL),JL=1,3),IL=1,3)
     &,((  RTRI(IL,JL),JL=1,3),IL=1,3)
     &,AP
     &,((RTRIRT(IL,JL),JL=1,9),IL=1,3)
     &,((       Q(IL,JL),JL=1,9),IL=1,3)
    3 FORMAT(1X,' MATRIX R ',/9(/3F8.4)
     &,//1X,' MATRIX T ',/4(/4F8.4)
     &,//1X,' MATRIX RTR ',/3(/3F8.4)
     &,//1X,' MATRIX RTRI ',/3(/3F8.4)
     &,//1X,' VECTOR AP ',/3F8.4
     &,//1X,' MATRIX RTRIRT ',/3(/9F8.4)
     &,//1X,' MATRIX Q ',/3(/9F8.4) )
```

```
C
C     START LOOP FOR EACH GRID TO BE MPC'ED - GRID ID IS THE KEY
C
      DO 200 ING=1,NG
      READ(11,5,END=3000) IDR
      NCR=NCR+1
      REWIND 12
 140  READ(12,2,END=2000)DUM,ID,IDUM,X
      IF(ID.NE.IDR) GOTO 140
C
C     CALC OF A,B,C,P
C     ALL ARE FUNCTIONS OF GRID COORDINATE TO BE REMOVED, X(1),X(2),X(3)
C
      A=S(1,1)
      B=S(1,2)
      C=S(1,3)
      P=S(1,4)
      DO 150 I=1,3
      A=A+S(I+1,1)*X(I)
      B=B+S(I+1,2)*X(I)
      C=C+S(I+1,3)*X(I)
      P=P+S(I+1,4)*X(I)
 150  CONTINUE
      WRITE(14,6) A,B,C,P
    6 FORMAT(1X,/1X,'A= ',F10.4,'  B= ',F10.4,'  C= ',F10.4
     &,'  P= ',F10.4,/1X )
C
C     CALC COEF FOR DIRECTION 1 AND WRTIE MPC CARDS ON UNIT 13
C
      XMPC(2)= P*(1+Q(1,7)-Q(1,1) )+A
      XMPC(3)= P*(  Q(1,1)-Q(1,4) )+B
      XMPC(4)= P*(  Q(1,4)-Q(1,7) )+C
      XMPC(5)= P*(  Q(1,8)-Q(1,2) )
      XMPC(6)= P*(  Q(1,2)-Q(1,5) )
      XMPC(7)= P*(  Q(1,5)-Q(1,8) )
      XMPC(8)= P*(  Q(1,9)-Q(1,3) )
      XMPC(9)= P*(  Q(1,3)-Q(1,6) )
      XMPC(10)=P*(  Q(1,6)-Q(1,9) )
      CALL MPCR(1,IDR)
      LINE=LINE+4
C
C     CALC COEF FOR DIRECTION 2 AND WRITE MPC CARDS ON UNIT 13
C
      XMPC(2)= P*(  Q(2,7)-Q(2,1) )
      XMPC(3)= P*(  Q(2,1)-Q(2,4) )
      XMPC(4)= P*(  Q(2,4)-Q(2,7) )
      XMPC(5)= P*(1+Q(2,8)-Q(2,2) )+A
      XMPC(6)= P*(  Q(2,2)-Q(2,5) )+B
      XMPC(7)= P*(  Q(2,5)-Q(2,8) )+C
      XMPC(8)= P*(  Q(2,9)-Q(2,3) )
      XMPC(9)= P*(  Q(2,3)-Q(2,6) )
      XMPC(10)=P*(  Q(2,6)-Q(2,9) )
      CALL MPCR(2,IDR)
      LINE=LINE+4
C
```

```
C      CALC COEF FOR DIRECTION 3 AND WRITE MPC CARDS ON UNIT 13
C
       XMPC(2)= P*(  Q(3,7)-Q(3,1) )
       XMPC(3)= P*(  Q(3,1)-Q(3,4) )
       XMPC(4)= P*(  Q(3,4)-Q(3 7) )
       XMPC(5)= P*(  Q(3,8)-Q(3,2) )
       XMPC(6)= P*(  Q(3,2)-Q(3,5) )
       XMPC(7)= P*(  Q(3,5)-Q(3,8) )
       XMPC(8)= P*(1+Q(3,9)-Q(3,3) )+A
       XMPC(9)= P*(  Q(3,3)-Q(3,6) )+B
       XMPC(10)=P*(  Q(3,6)-Q(3,9) )+C
       CALL MPCR(3,IDR)
       LINE=LINE+4
C
  200  CONTINUE
       GOTO 105
 1000  STOP
 2000  WRITE(14,*) ' ERROR NO. 2000'
       STOP
 3000  WRITE(14,*) ' ERROR NO. 3000'
       STOP
       END
C
C
C
C
       SUBROUTINE MINV4(C,A)
C
C      A=INV(C)  C IS NOT DESTROYED. ONE=A*C FOR CHECK
C
       IMPLICIT REAL*8 (A-H,O-Z)
       DIMENSION ONE(4,4)
       DIMENSION A(4,4),C(4,4),B(4,4),IPVOT(4),INDEX(4,2),PIVOT(4)
       N=4
       M=0
       DO 999 I=1,N
       DO 999 J=1,N
  999  A(I,J)=C(I,J)
       WRITE(14,1) ((A(I,J),J=1,4),I=1,4)
  1    FORMAT(1X,'INPUT MATRIX TO MINV4',/1X,4(/1X,4F12.4),/1X )
  57   DET=1.0D0
       DO 17 J=1,N
  17   IPVOT(J)=0.0D0
       DO 135 I=1,N
C
C      FOLLOWING 12 STMTS SEARCH FOR PIVOT ELEMENT
C
       T=0.0D0
       DO 9 J=1,N
       IF(IPVOT(J)-1) 13,9,13
  13   DO 23 K=1,N
       IF(IPVOT(K)-1) 43,23,81
  43   IF(ABS(T)-ABS(A(J,K)))  83,23,23
  83   IROW=J
       ICOL=K
```

```
      T=A(J,K)
 23   CONTINUE
  9   CONTINUE
      IPVOT(ICOL)=IPVOT(ICOL)+1
C
C     FOLLOWING 15 STMTS TO PUT PIVOT ELEMENT ON THE DIAGONAL
C
      IF(IROW-ICOL) 73,109,73
 73   DET=-DET
      DO 12 L=1,N
      T=A(IROW,L)
      A(IROW,L)=A(ICOL,L)
 12   A(ICOL,L)=T
      IF(M) 109,109,33
 33   DO 2 L=1,M
      T=B(IROW,L)
      B(IROW,L)=B(ICOL,L)
  2   B(ICOL,L)=T
 109  INDEX(I,1)=IROW
      INDEX(I,2)=ICOL
      PIVOT(I)=A(ICOL,ICOL)
      DET=DET*PIVOT(I)
C
C     NEXT 6 STMTS TO DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
      A(ICOL,ICOL)=1.0D0
      DO 205 L=1,N
      A(ICOL,L)=A(ICOL,L)/PIVOT(I)
 205  CONTINUE
      IF(M) 347,347,66
 66   DO 52 L=1,M
 52   B(ICOL,L)=B(ICOL,L)/PIVOT(I)
C
C     NEXT 10 STMTS REDUCE NON PIVOT ROWS
C
 347  DO 135 LI=1,N
      IF(LI-ICOL) 21,135,21
 21   T=A(LI,ICOL)
      A(LI,ICOL)=0.0D0
      DO 89 L=1,N
 89   A(LI,L)=A(LI,L)-A(ICOL,L)*T
      IF(M) 135,135,18
 18   DO 68 L=1,M
 68   B(LI,L)=B(LI,L)-B(ICOL,L)*T
 135  CONTINUE
C
C     NEXT 11 INTERCHANGE COLUMNS
C
 222  DO 3 I=1,N
      L=N-I+1
      IF(INDEX(L,1)-INDEX(L,2)) 19,3,19
 19   JROW=INDEX(L,1)
      JCOL=INDEX(L,2)
      DO 549 K=1,N
      T=A(K,JROW)
```

```
      A(K,JROW)=A(K,JCOL)
      A(K,JCOL)=T
 549  CONTINUE
 3    CONTINUE
 81   CONTINUE
      WRITE(14,992) ((A(L,M),M=1,N),L=1,N),DET
 992  FORMAT(/1X,' INVERSE OF MATRIX IN MINV4 ',/1X,4(/1X,4F8.4)
     &,/1X,' DET T = ',1PE12.4)
C
C     CHECK INVERSE AND WRITE TO UNIT 14
C
      DO 120 I=1,N
      DO 120 J=1,N
      ONE(I,J)=0.0D0
      DO 120 K=1,N
      ONE(I,J)=ONE(I,J)+A(I,K)*C(K,J)
 120  CONTINUE
      WRITE(14,991) N,ONE
 991  FORMAT(1X,//1X, ' CHECK INV  ORDER OF MATRIX = ',I4,//1X
     &,4(/1X,4F8.4))
      RETURN
      END
C
C
C
      SUBROUTINE MINV3(T,S)
C
C     S=INV(T)  T IS NOT DESTROYED. ONE=S*T FOR CHECK
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION T(3,3),S(3,3),ONE(3,3),B(3),C(3),A(4,4)
      DATA ONE/9*0.0/,B/3*0.0/,C/3*0.0/,A/16*0.0/
C
C     MAKE SURE FIRST PIVOT ELEMENT IS NOT ZERO
C
      N=4
      NN=N-1
      WRITE(14,3) ((T(L,M),M=1,NN),L=1,NN)
    3 FORMAT(1X,//,1X,' MATRIX  T ',/1X,3(/1X,3F8.4) )
      A(1,1)=1.0D0
      DO 10 I=1,NN
      DO 10 J=1,NN
   10 A(I+1,J+1)=T(I,J)
      A(1,1)=1.0D0/A(1,1)
      DO 110 M=1,NN
      K=M+1
   50 DO 60 I=1,M
      B(I)=0.0D0
      DO 60 J=1,M
   60 B(I)=B(I)+A(I,J)*A(J,K)
      D=0.0D0
      DO 70 I=1,M
   70 D=D+A(K,I)*B(I)
      D=-D+A(K,K)
        A(K,K)=1.0/D
```

```
      DO 80 I=1,M
   80 A(I,K)=-B(I)*A(K,K)
      DO 90 J=1,M
      C(J)=0.0D0
      DO 90 I=1,M
   90 C(J)=C(J)+A(K,I)*A(I.J)
      DO 100 J=1,M
  100 A(K,J)=-C(J)*A(K,K)
      DO 110 I=1,M
      DO 110 J=1.M
  110 A(I,J)=A(I,J)-B(I)*A(K,J)
      DO 111 I=1,MM
      DO 111 J=1,MM
  111 S(I,J)=A(I+1,J+1)
      WRITE(14,2) ((A(L,M),M=1,M),L=1,M),D
    2 FORMAT(/1X,' INVERSE OF AUGMENTED T ',/1X,4(/1X,4F8.4)
     &,/1X,' DET T = '.1PE12.4)
C
C     CHECK INVERSE AND WRITE TO UNIT 14
C
      DO 120 I=1,MM
      DO 120 J=1,MM
      ONE(I,J)=0.0D0
      DO 120 K=1,MM
      ONE(I,J)=ONE(I,J)+S(I,K)*T(K,J)
  120 CONTINUE
      WRITE(14,1) MM,ONE
    1 FORMAT(1X,//1X, ' CHECK INV  ORDER OF MATRIX = ',I4,//1X
     &,3(/1X,3F8.4))
      RETURN
      END
C
C
C
      SUBROUTINE MPCR(ICOR,IIDR)
      IMPLICIT REAL*8 (A-H,O-Z)
      CHARACTER MPC*8,SPACE*8,PM*2
      COMMON /XMPCR/ XMPC(10),IDG(3),LINE,X(3)
      DATA MPC,SPACE,PM/'MPC     ','        ','+M'/
      ISID=312
      I1=1
      I2=2
      I3=3
      WRITE(14,1) IIDR, ICOR, X, XMPC
    1 FORMAT(1X,'  MPC EQUATIONS: GRID REMOVED ',
     &I8,' DIRECTION ',I4,4X,/1X,4F12.4,
     1/1X,F9.4,' UA   ',F9.4,' UB   ',F9.4,' UC   ',
     2/1X,F9.4,' VA   ',F9.4,' VB   ',F9.4,' VC   ',
     3/1X,F9.4,' WA   ',F9.4,' WB   ',F9.4,' WC   ')
      L1=LINE
      L2=LINE+1
      L3=LINE+2
      L4=LINE+3
      WRITE(13,2) MPC, ISID, IIDR, ICOR, XMPC(1)
     1,IDG(1),I1,XMPC(2),SPACE,PM,L1,PM,L1,SPACE,IDG(2),I1,XMPC(3)
```

```
   2,IDG(3),I1,XMPC(4),SPACE,PM,L2,PM,L2,SPACE,IDG(1),I2,XMPC(5)
   3,IDG(2),I2,XMPC(6),SPACE,PM,L3,PM,L3,SPACE,IDG(3),I2,XMPC(7)
   4,IDG(1),I3,XMPC(8),SPACE,PM,L4,PM,L4,SPACE,IDG(2),I3,XMPC(9)
   5,IDG(3),I3,XMPC(10)
2   FORMAT(1X,A8,3I8,F8.3
   1,2I8,F8.3,A8,A2,I6,/1X,A2,I6,A8,2I8,F8.3
   2,2I8,F8.3,A8,A2,I6,/1X,A2,I6,A8,2I8,F8.3
   3,2I8,F8.3,A8,A2,I6,/1X,A2,I6,A8,2I8,F8.3
   4,2I8,F8.3,A8,A2,I6,/1X,A2,I6,A8,2I8,F8.3
   5,2I8,F8.3)
    END
```