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## SYMMETRIC COMPOSITE LAMINATE STRESS ANALYSIS

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## SUMMARY

In this paper it is demonstrated that COSMIC/NASTRAN may be used to analyze plate and shell structures made of symmetric composite laminates. Although general composite laminates cannot be analyzed using NASTRAN, the theoretical development presented herein indicates that the integrated constitutive laws of a symmetric composite laminate resemble those of a homogeneous anisotropic plate, which can be analyzed using NASTRAN. A detailed analysis procedure is presented, as well as an illustrative example.

## INTRODUCTION

Composite laminates have been used in many engineering structures recently, and researchers in this field have developed many finite element programs [1]. To date, the composite laminate theory has not been incorporated into any NASTRAN plate element formulation [2].

The governing equilibrium equations for symmetric laminates resemble those for anisotropic plates [3]. Since NASTRAN possesses the capability to analyze anisotropic plates, symmetric laminates can be analyzed using NASTRAN if the equivalent anisotropic plate material properties can be obtained.

The purpose of this study is to obtain equivalent anisotropic plate material properties for symmetric laminates consistent with the governing plate theory, and thus extend the capability of NASTRAN to include analysis of symmetric laminates. This theoretical formulation is given in the next section. Subsequently, a step-by-step analysis procedure and an illustrative example problem are presented.

## THEORY

The classical laminate theory [4] will be used. A Cartesian coordinate system is assumed with the x-y plane located at the midplane of the laminate. The normal coordinate is z, and the total laminate thickness is h. The laminate consists of perfectly bonded laminae. If h is considered small compared to the x and y dimensions of the laminate, the Kirchoff hypothesis may be assumed to be applicable. The displacements can then be written as

$$\left. \begin{aligned} u &= U_0 - z \frac{\partial W_0}{\partial x} \\ v &= V_0 - z \frac{\partial W_0}{\partial y} \\ w &= W_0 \end{aligned} \right\} \quad (1)$$

where  $U_0$ ,  $V_0$ , and  $W_0$  are the midplane displacements in the  $x$ ,  $y$ , and  $z$  directions, respectively. The strains at distance  $z$  from the midplane can be written as

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \epsilon_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (2)$$

where

$$\left. \begin{aligned} \epsilon_x^0 &= \frac{\partial U_0}{\partial x}; \quad \epsilon_y^0 = \frac{\partial V_0}{\partial y}; \quad \epsilon_{xy}^0 = \frac{\partial U_0}{\partial y} + \frac{\partial V_0}{\partial x}; \\ \kappa_x &= -\frac{\partial^2 W_0}{\partial x^2}; \quad \kappa_y = \frac{\partial^2 W_0}{\partial y^2}; \quad \kappa_{xy} = -\frac{\partial^2 W_0}{\partial x \partial y} \end{aligned} \right\} \quad (3)$$

The constitutive law at any point in the laminate may be written in the form

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (4)$$

where  $(\sigma_x, \sigma_y, \tau_{xy})$  are the stresses and the  $\bar{Q}_{ij}$  are the reduced moduli.

Relations between the stress and moment resultants and the stretching and curvature strains are obtained by integrating Equation (4) through the thickness; specifically,

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (5)$$

where

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} \bar{Q}_{ij} (1, z, z^2) dz \quad (6)$$

Presently, the general form Equation (5) is not used for any plate element in NASTRAN; however, if the laminate is symmetric, then  $B_{ij}=0$ , and thus the inplane extension and out of plane bending effects decouple. Equivalent anisotropic constitutive laws can be written for each effect. For in plane extension, the equivalent stress-strain law is

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{1}{h} \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (7)$$

and for bending it is

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{12}{h^3} \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (8)$$

The coefficients in Equations (7) and (8) are denoted as the equivalent anisotropic material properties. Since NASTRAN can be used for the stress analysis of a homogeneous anisotropic plate, it can thus be used for symmetric laminate stress analysis.

#### THE ANALYSIS PROCEDURE

Incorporating the aforementioned theory into NASTRAN for symmetric laminate stress analysis is not a straightforward task. The analysis procedure may be outlined as follows:

- (1) The theoretical development presented above is used to calculate the coefficients of Equations (7) and (8). The results are inputted to NASTRAN by using MAT2 cards.
- (2) To analyze plate or shallow shell structures, plate elements CTRIA1 and/or CQUAD1 are used, with associated MAT2 cards. Output will include grid point displacements and the midplane stretching and curvature strains of each element.
- (3) The strains in each layer of the element are calculated using Equation (2) and the stresses in the layer are calculated using Equation (4).
- (4) The stresses in each layer are transformed to the principal material directions and the failure criterion [5]

$$\left(\frac{\sigma_1}{S_1}\right)^2 + \left(\frac{\sigma_2}{S_2}\right)^2 - \left(\frac{\sigma_1 \sigma_2}{S_1 S_1}\right) + \left(\frac{\sigma_{12}}{S_{12}}\right)^2 = F \quad (9)$$

is used for ply failure evaluation. If the value of F exceeds unity, that layer is failed.

#### EXAMPLE PROBLEM

A Kevlar 49\*/Epoxy cylindrical shell with  $[-60/0/60]_{3s}$  layup is used to illustrate the analysis procedure. The geometry and finite element mesh are shown in Figure 1. The mechanical properties in principal material directions 1 and 2 of a unidirectionally reinforced lamina are

$$E_{11} = 7.6 \times 10^{10} \text{ Pa}$$

$$E_{22} = 5.5 \times 10^9 \text{ Pa}$$

$$G_{12} = 2.1 \times 10^9 \text{ Pa}$$

$$\nu_{12} = 0.34$$

where the 1 and 2 directions are parallel and normal to the fiber direction, respectively. The ultimate strengths of the lamina are

$$(S_1)_T = 1.38 \times 10^9 \text{ Pa}$$

$$(S_1)_C = -2.76 \times 10^8 \text{ Pa}$$

$$(S_2)_T = 2.96 \times 10^7 \text{ Pa}$$

$$(S_2)_C = -1.38 \times 10^8 \text{ Pa}$$

$$S_{12} = 6.00 \times 10^7 \text{ Pa}$$

where T and C denote tension and compression, respectively. The shell was subjected to 6895 Pa (1 psi) uniform pressure and its periphery was pinned.

The equivalent homogeneous anisotropic plate constitutive law for in plane calculation was calculated as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} 3.225 \times 10^{10} & 1.071 \times 10^{10} & 0 \\ 1.071 \times 10^{10} & 3.225 \times 10^{10} & 0 \\ 0 & 0 & 1.080 \times 10^{10} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

while for bending it was

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} 3.178 \times 10^{10} & 1.075 \times 10^{10} & -17.25 \times 10^9 \\ 1.075 \times 10^{10} & 3.178 \times 10^{10} & -5.096 \times 10^9 \\ -1.725 \times 10^9 & -5.096 \times 10^9 & 1.093 \times 10^{10} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

The input to NASTRAN was made according to Step (2) of the procedure given above.

The output displacement contours in the x, y, and z directions are shown in Figures 2, 3, and 4, respectively. Note that these deformations are not symmetric with respect to the centerlines of the shell. This is due to the presence of the  $D_{16}$  and  $D_{26}$  terms.

Table 1 summarizes the layer stresses in principal material directions for a typical element. The small values of F indicate that no ply failures occur.

\*Kevlar 49 aramid fibers, manufactured by E. I. duPont de Nemours & Co., Inc.

## CONCLUSIONS AND SUGGESTIONS

NASTRAN has been demonstrated to be a feasible tool for the stress analysis of symmetric composite laminates, though equivalent anisotropic material properties and layer stress calculations must be performed outside the NASTRAN framework. To increase the NASTRAN analysis capability to unsymmetric laminate and to simplify the analysis procedure, it would be worthwhile to incorporate the newly developed composite laminate finite element programs into NASTRAN in order to analyze the ever-increasing number of composite laminate structures.

#### REFERENCES

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5. S. W. Tsai, "Strength Theories of Filamentary Structures", in R. T. Schwartz and H. S. Schwartz (eds.), *Fundamental Aspects of Fiber Reinforced Plastic Composites*, Wiley Interscience, New York, 1968, p. 3.

TABLE 1. LAMINATE STRESSES IN PRINCIPAL MATERIAL DIRECTIONS AND FAILURE PARAMETER (F) OF ELEMENT 85

PLY NO.	$\sigma_1$ (Pa)	$\sigma_2$ (Pa)	$\tau_{12}$ (Pa)	F
1	$8.136 \times 10^6$	$-2.275 \times 10^6$	$-2.048 \times 10^5$	0.000
2	$-2.779 \times 10^7$	$-6.102 \times 10^5$	$-8.963 \times 10^5$	0.010
3	$-2.055 \times 10^7$	$-1.027 \times 10^6$	$1.144 \times 10^6$	0.006
4	$6.405 \times 10^6$	$-2.427 \times 10^6$	$-2.930 \times 10^5$	0.000
5	$-3.075 \times 10^7$	$-6.798 \times 10^5$	$-8.481 \times 10^5$	0.012
6	$-2.075 \times 10^7$	$-1.248 \times 10^6$	$1.186 \times 10^6$	0.006
7	$4.675 \times 10^6$	$-2.579 \times 10^6$	$-3.820 \times 10^5$	0.000
8	$-3.372 \times 10^7$	$-7.722 \times 10^5$	$-7.998 \times 10^5$	0.015
9	$-2.096 \times 10^7$	$-1.475 \times 10^6$	$1.227 \times 10^6$	0.006
10	$-2.103 \times 10^7$	$-1.551 \times 10^6$	$1.234 \times 10^6$	0.006
11	$-3.668 \times 10^7$	$-8.618 \times 10^5$	$-7.515 \times 10^5$	0.017
12	$1.793 \times 10^6$	$-2.834 \times 10^6$	$-5.288 \times 10^5$	0.001
13	$-2.124 \times 10^7$	$-1.779 \times 10^6$	$1.276 \times 10^6$	0.006
14	$-3.964 \times 10^7$	$-9.515 \times 10^5$	$-7.033 \times 10^5$	0.020
15	$6.143 \times 10^6$	$-2.985 \times 10^6$	$-6.178 \times 10^5$	0.001
16	$-2.144 \times 10^7$	$-2.006 \times 10^6$	$1.317 \times 10^6$	0.006
17	$-4.261 \times 10^7$	$-1.041 \times 10^6$	$-6.536 \times 10^5$	0.023
18	$-1.668 \times 10^6$	$-3.137 \times 10^6$	$-7.033 \times 10^5$	0.001

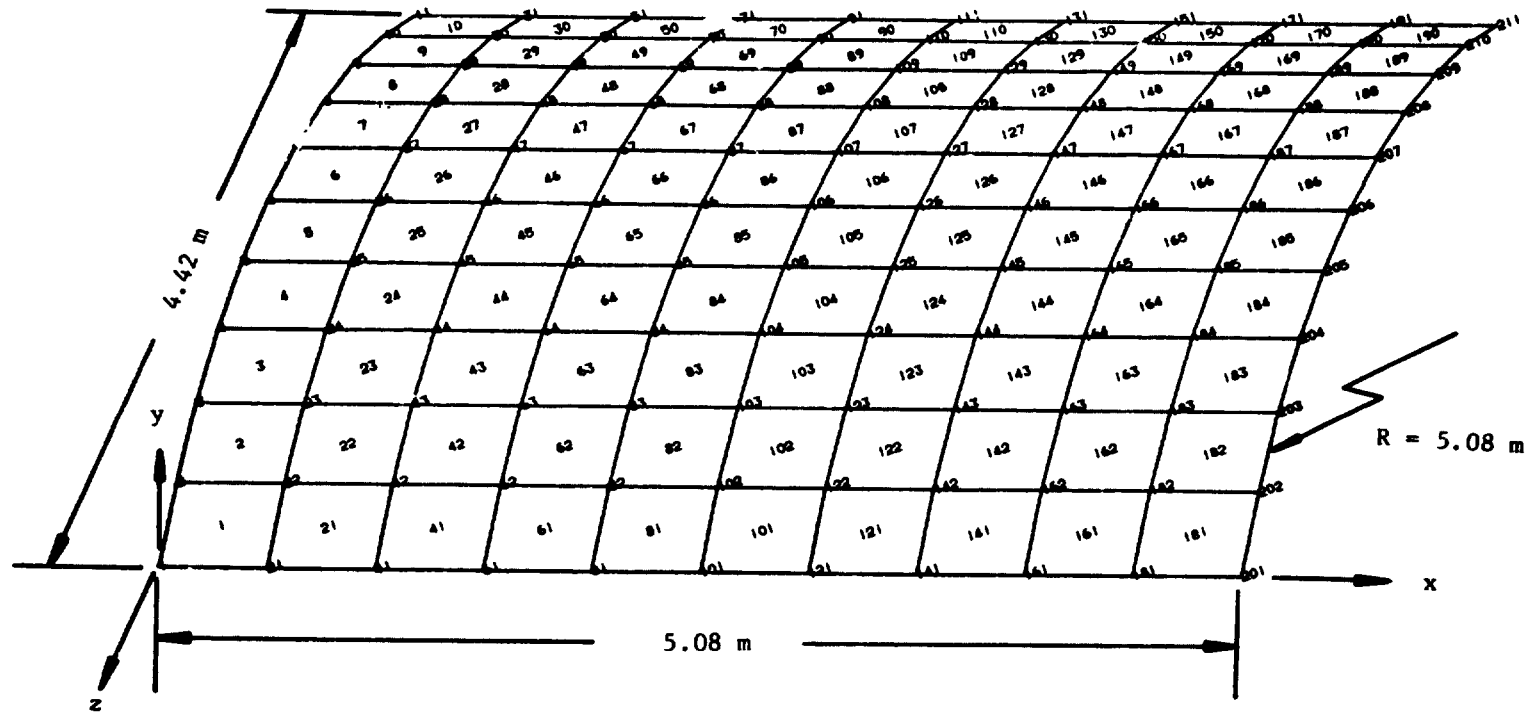
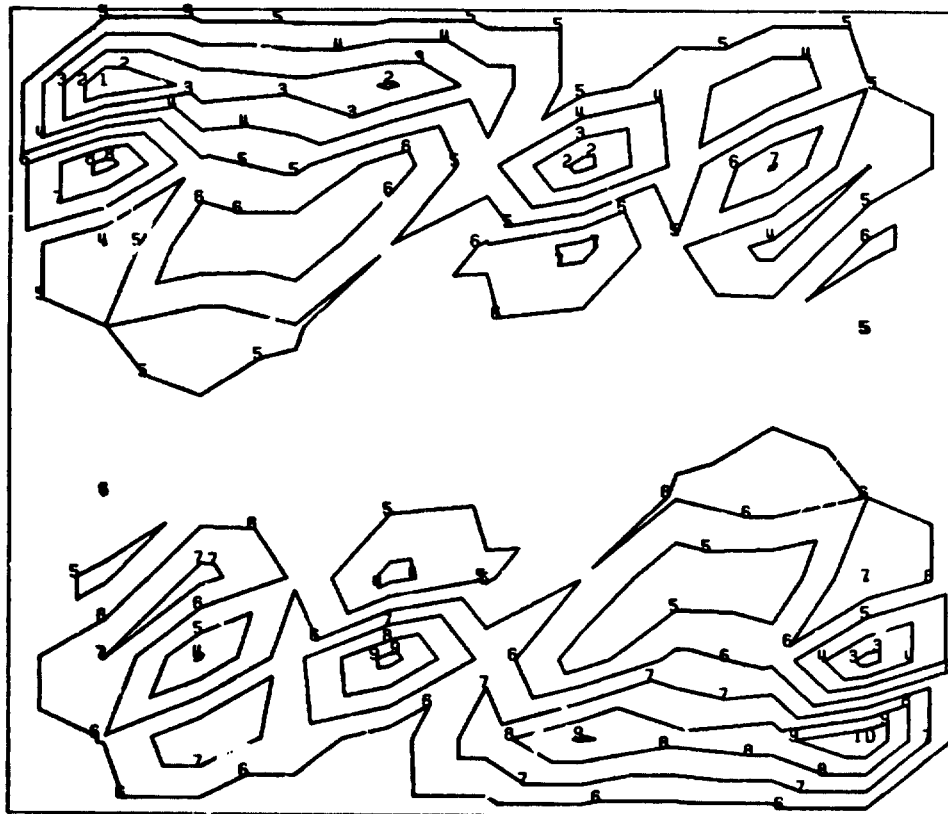


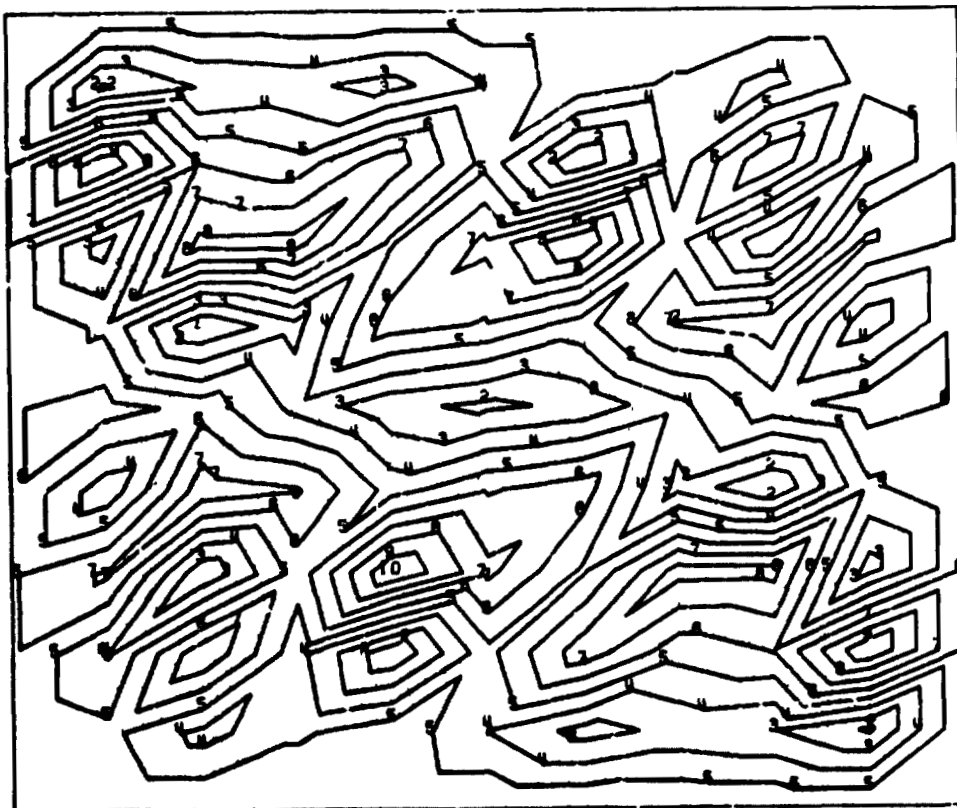
FIGURE 1 - SYMMETRIC LAMINATED COMPOSITE SHELL WITH  $[-60/0/60]_{3S}$  LAY-UP AND 0 DEGREE IS ALONG THE X-AXIS





CONTOUR NO.	DISPLACEMENT (mm)
1	-4.857
2	-3.778
3	-2.699
4	-1.619
5	-0.540
6	0.540
7	1.619
8	2.699
9	3.778
10	4.857

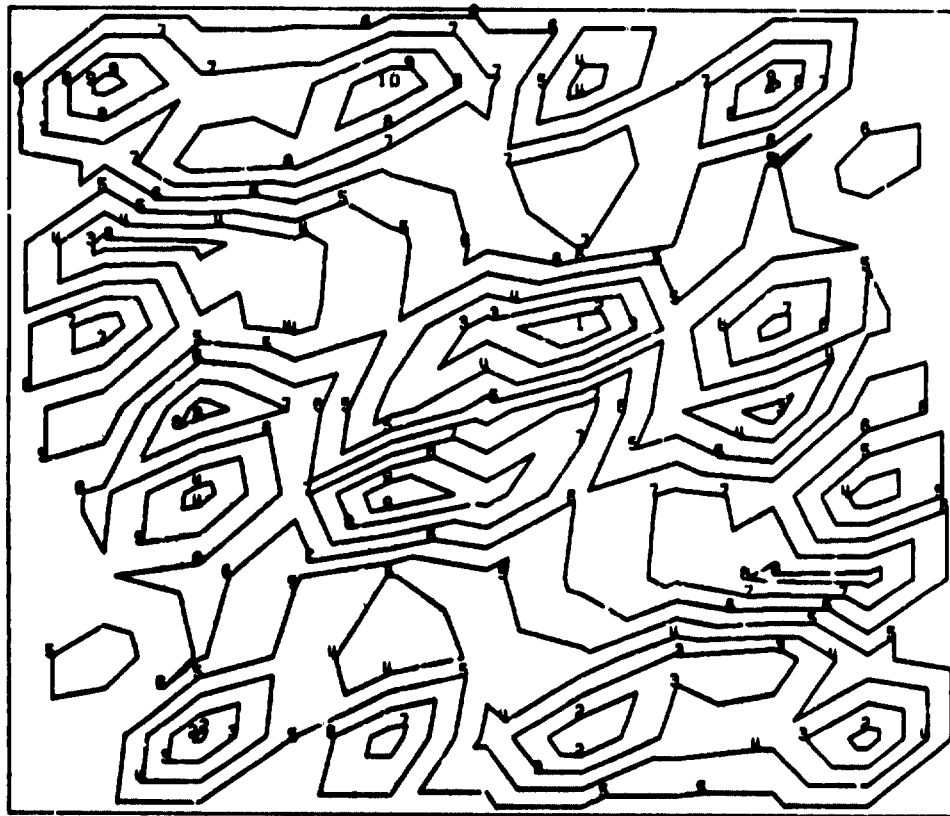
FIGURE 2 - X-DISPLACEMENT CONT' PLOT



CONTOUR NO.	DISPLACEMENT (mm)
1	-14.248
2	-11.348
3	-8.449
4	-5.549
5	-2.649
6	0.250
7	3.150
8	6.050
9	8.949
10	11.849

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FIGURE 3 - Y-DISPLACEMENT CONTOUR PLOT



CONTOUR NO.	DISPLACEMENT (mm)
1	-0.100
2	-0.080
3	-0.056
4	-0.029
5	-0.011
6	0.011
7	0.029
8	0.056
9	0.080
10	0.100

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FIGURE 4 - Z-DISPLACEMENT CONTOUR PLOT