PRELIMINARY INVESTIGATION OF THE ELECTRODYNAMICS OF A CONDUCTING TETHER

W. B. Thompson
University of California at San Diego*
La Jolla, California 92093

An introductory study of the properties of an electrically conducting tether flown from the shuttle is presented. Only a single configuration is considered: a vertical conductor moving normally across the Earth's field, connecting the shuttle to a large conducting balloon that passively extracts electrons from the ionosphere. The rather surprising nature of the distortions in the plasma at maximum current collection are described, as are the local and distant wakes. Numerical values are crude approximations, the emphasis being on the nature of the process, not on engineering values, but it is hoped that this discussion will prove useful.

ELECTRODYNAMIC PROPERTIES OF A CONDUCTING TETHER

There is a developing interest in the possible uses of tethers in space for such purposes as transferring energy and momentum between bodies, effecting rendezvous and probing neighboring parts of space, including regions such as the upper atmosphere otherwise inaccessible to a spacecraft. Tether lengths of up to 100 km have been considered as possible, and major experiments are being designed.

It has been conjectured, perhaps first by Alfven, that if such a tether were made conducting, then since in the neighborhood of the Earth it moves across a magnetic field and hence experiences an emf between the ends, it could draw a current from the surrounding plasma which could provide power for use on a spacecraft (ref. 1). In this paper some aspects of this conjecture are considered in its earliest and perhaps simplest mode: a conducting tether extending up from the shuttle in low Earth orbit to some suitable collector.

Possible Uses of a Conducting Tether (refs. 2, 3)

Power. - In low Earth orbit, $h = 200$ km, $V = 8$ km/sec, and the electric field along the tether $E = V \times B \simeq 0.24$ V/m. The exact level of the voltage drop along the wire is $\Delta\Phi = -\Delta L \cdot [\underline{V} \pm \underline{V}_E) \cdot \underline{B}]$, V_E being the rotational speed speed of the Earth; hence it depends on position and orientation of the tether, field, and velocity: the given figures are representative. If the tether contained an 18-gauge copper wire (0.1-cm diameter, 7.3-kg/km weight, and 20-Ω/km resistance), the maximum current that could be drawn would be ~12 A. Fifteen-gauge wire has half the resistance and double the weight, but would permit a

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maximum current of 24 A to be drawn. At this current level the power dissipated would be 2.4 kW/km, or 240 kW for a 100-km tether. With 18-gauge wire about one-quarter of this could be used in the spacecraft; with 15-gauge wire, about one-half.

The energy, of course, is not free: it is obtained by increasing the drag on the spacecraft and, if steady operation is required, this drag must be compensated for by using rocket fuel. However, as has been pointed out, there is an energetic advantage in using fuel this way rather than directly for onboard power. The energy obtained by using the fuel for thrust includes its potential, which is comparable to the chemical energy, the only fraction used if it is burnt for onboard power.

<u>Thrust and drag</u>. - If the tether carries a current I, it experiences a force $I \times B$/unit length; hence the system (spacecraft + tether + collector) experiences a total force $I \times BL_t$, where L_t is the vertical length of the tether. For shuttle heights and a current of 10 A, this is about 0.2 N/km ≈ 20 N for a 100-km tether. It is essentially a drag, doing work at the rate $I \times B \cdot VL_t = -I\Phi$, although it may be important that there is an out-of-plane component to the force. Moreover, this component of the drag is readily controlled simply by modulating the current that is drawn.

The tether may also be used actively. If the collector can also operate as an electron emitter, a current can be driven through the tether, in which case a force of $\underline{I}_D \times \underline{B}L_t$ acts on the shuttle. The potential drop needed to drive this current is $\Delta\Phi = I_D R + \underline{V} \times \underline{B} \cdot L_t + \Delta\phi$, where $\Delta\phi$ is the potential drop needed to collect the current from the plasma. The efficiency is then

$$\eta = \frac{I_D \times BL_t \cdot V}{I_D R + V \times B \cdot L_t + \Delta\phi}$$

Introduce the saturation current

$$I_0 = \frac{V \times B \cdot L_t}{R}$$

and a fictitious "collection current" $\left(\Delta\phi/R = I_C\right)$, which we shall later show is usually small, then

$$\eta = \frac{1}{1 + \dfrac{I_0 + I_C}{I_D}}$$

Thus although the thrust is small (like most electromagnetically driven thrusters), the efficiency can be quite high. Adequate current collection may cause problems. Moreover, the thrust need not be in the orbital plane, and depends on the orientation of the tether. This in turn may be modified by modulating the current, and many electromechanical games may be played.

<u>Communications</u>. - The tether constitutes a very long antenna immersed in a magnetoactive plasma; it carries a current that can be passively modulated just by activating a switch and may be useful for communications at very low

frequencies. Because of the peculiar transmission properties of the magnetized plasma, there may be important advantages in using an antenna of this type. Since these transmission properties are so complex, a good deal of theoretical investigation is required before this will have been adequately explored.

Even if the current is not modulated, a significant disturbance is produced by the passage of the tether: the wake it leaves behind in the magnetosphere. The collector also leaves a wake, and these two have rather different and by no means obvious character. Some fraction of the signals produced are attenuated only weakly as the collector-tether system goes by and have characteristics signatures.

<u>Exploration</u>. - Many of the signals that are, or could be, produced by the tether have propagation characteristics that depend in some detail on the nature of the plasma through which they propagate. Hence they may be useful in large-scale synoptic exploration of parts of the magnetosphere. These explorations are of practical importance since they also constitute explorations of possible communications channels.

Questions

<u>Current collection</u>. - The electrodynamic use of a tether requires that it be able to collect the necessary current. It is by no means obvious that this can be done. A small positive probe placed in a plasma collects a current of the order of $ne\tilde{v}$/area, where $\tilde{v} = \sqrt{kT/2m\pi}$. At shuttle heights, where $n \approx 2 \times 10^5/\mathrm{cm}^3$ and the temperature is approximately 0.2 eV, this is of the order of 2.5 $\mathrm{mA/m}^2$, and a collector area of approximately 2000 m^2 should be enough to collect about 5 A (refs. 4, 5). On the other hand, it is not clear that the same current can be drawn by a large probe. Suppose the collector has an area $\sim D^2$ and hence a length across the magnetic field of $\sim D$. Then current is collected from any field line for a time D/V, and the number of electrons extracted per unit area is approximately equal to $\tilde{n}v\ (D/V)$. To collect this, all of the electrons must be removed from a column of length $L_e \approx (\tilde{v}/V)\ D$. For electrons at 0.2 eV and a collector speed of 8 km/sec, this is $\sim 10\ D \approx$ 100 m for a D of 10 m. How does this happen? And how much energy is needed to collect this current?

<u>Wakes</u>. - Some well-known work has been done on the production of Alfven waves (ref. 6) by large structures moving through the magnetosphere. We have instead a structure that draws an electric current, extracting negative charge from one region and ejecting it in another, while seriously modifying the local magnetic field. What is the effect of these modifications on the local and distant properties of the magnetosphere? What happens to the charges? How does the current close? Where does the momentum come from? What is the energy balance?

<u>Communications</u>. - The whole question of low-frequency propagation through the magnetosphere is one that can be answered in principle (i.e., the response function is well known and can easily be written down for a plane wave). How is this connected to the signal produced by a source of a given configuration moving through a nonuniform plasma? (or for that matter, moving through a uniform plasma?) Attempts to deal with these problems require handling the dispersion relation for a warm magnetoplasma, a mathematical object with quite horrendous properties.

Are there significant model experiments that could be carried out on a laboratory scale and would give significant information here? And if so, how should such information be interpreted?

The Present Investigation

To obtain significant results, the present investigation concentrated on two questions for a particular configuration. The two questions were: how is the current collection affected? and what are the most important features of the wake?

The configuration considered is a 100-km tether drawing a saturation current of 5 A from a sphere of radius 25 m, a balloon, with a surface conductivity of a 1-mil layer of copper. The magnetosphere is considered as containing a density of 2×10^5 electrons/cm^3, neutralized by an equal number of singly charged positive O^{16} ions, both at a temperature of 0.2 eV (≈2200°) (ref. 11).

Current collection. - If the electrons were collected through an inverse Langmuir Childs sheath from a virtual cathode that moved out from the collector to a distance of ~100 m, the required potential drop

$$\phi \sim \frac{kT}{e} \cdot \frac{4}{3} \left(\frac{x}{\lambda_D} \right)^{4/3}$$

would be of order of 10^5 V, and essentially such a current could not be collected. However, this assumes that the positive ions play no role and that the field of the collector is screened by the electrons that it is collecting. We have considered the role of the ions and have come to a much more optimistic conclusion.

Consider the effect near the leading edge of the collector. The parallel electric field that it produces is effectively wiped out by the motion of electrons along the magnetic lines of force ahead of the collector. However, as soon as the collector crosses a line of force, the electrons on that line are drained off, and the electric field begins to appear. As a consequence, the potential on that field line begins to rise. Now, although electrons are confined to magnetic field lines, their gyroradius being ~0.15 cm, this is not true for the much heavier ions that have energies of $1/2\ MV^2$ in the collector frame. This is about 6 eV, and until the potential reaches this value, the ions are free to move across the field; however, when this value is exceeded, the ions are reflected. Since the ions have a thermal spread, not all are reflected at this potential; a somewhat higher potential of ~12 eV is needed to reflect most of the ions.

The electrons are accelerated by the parallel electric field and gain a velocity given by $\sqrt{2e\phi/m}$. Since the electron flux remains $\sim n\tilde{v}$, the electron density is reduced by a factor of $\sqrt{kT/e\phi} \approx 1/8$. Since both the electron and ion densities are reduced, the electrons by parallel acceleration and the ions by partial reflection, there exists a potential $\phi_0 \approx 12$ eV at which the electron and ion densities are equal. If the collector sits at this potential with respect to the undisturbed plasma, it can collect the saturation current, even though the accelerating fields sit not near the collector, but on a narrow sheath that extends out at an angle $\tilde{v}/V$ along the magnetic field and reaches a distance $L_S \approx \tilde{V}/V \cdot D \sim 150$ m from the collector. The voltage drop needed to collect this current is then only of the order of 12 V, which is equivalent electrically to increasing the tether length by ~50 m. Hence it is negligible for kilometer-length tethers.

Because this extended sheath reflects the ions while it extracts the electrons, everywhere behind the extended sheath the plasma density is very low. As soon as the collector has passed, electrons are no longer extracted and the potential rapidly drops. As it does so, electrons rush out with approximately the thermal speed, leaving only a small negative charge, so that the plasma although at low density is again neutral at a distance of the order of D, the collector width, behind the collector. Behind this, the low-pressure region fills in at a rate given by the ion thermal speed, ~1.5 km/sec. Hence the plasma does not recover its original density for $2L_S\,(V/v_\theta^+) \approx 1$ km behind the collector.

The reflected ions have velocity in the plasma frame of 2 V and hence move in gyro orbits with a radius $r_L = 2V/\Omega_+ \approx 90$ m, about a center 90 m below the collector. They produce a net positive charge of the order of $D/4r_L \approx 2.5$ percent of the total ion charge density. This is neutralized by an influx of electrons along the field lines in a distance again of the order of D. The total charge does not disappear but spreads out along the field line, with the electron thermal speed essentially halving in the distance D. After 100 m the charge density is reduced to ~0.1 electrons/cm^2, and its dynamic effect essentially disappears. The excess charge drifts down and across the magnetic field lines and eventually is balanced in the ionosphere.

The wake. - The local effects of the collector are as follows: a volume of height D (≈25 m extending out along the field lines a distance $(\tilde{V}/V)\,D$ (≈250 m) on either side of the collector and tapering back to vanish at a distance

$$\frac{V}{v_\theta^+}\,\frac{v_\theta^-}{V}\,D = \frac{v_\theta^-}{v_\theta^+}\,D = 5 \text{ km}$$

behind the collector is evacuated of plasma; while a volume of height $\approx 4r_L \approx 180$ m, of width 150 m, and of length ~D acquires a charge of $D/4r_L \approx 3.5$ percent $n_e e$/unit volume.

The positive current flows out of the collector, with a current density of ~2.5 mA/m^2, carried by the electrons; flows forward and down across the field lines, with a density of 3 mA/m^2, carried by the ions; then flows along the field lines with a density of ~0.3 mA/m^2, carried by the electrons. The associated magnetic perturbations are ~$5\text{x}10^{-15}$ g for the first current (a magnetic twist), $3\text{x}10^{-6}$ g for the second (an increase in field strength), and $1.2\text{x}10^{-6}$ g for the third (a further magnetic twist).

The major modification in the background is produced by the magnetic field in the tether. For ~2 cm about the tether this field exceeds the Earth's field and the field lines close about the tether, while beyond this the disturbance falls off as 1/r. It is important to note, however, that the tether bends and compresses the field lines but does not introduce a twist. This is important when we consider the nature of the wakes and is a result of our assumption of a horizontal field.

The MHD wake. - The low-frequency disturbances produced by the passing of the tether can be described in the simplest approximation by considering the plasma as an ideally conducting fluid. For our case the velocity of sound is much less than the Alfven speed, $C_S \approx 2$ km/sec, $C_A \approx 130$ km/sec, and there are three possible waves (ref. 10): an Alfven wave, which propagates along the magnetic field with the Alfven speed; a channeled sound wave, which again propagates along the magnetic field, but now with the speed of sound; and an isotropic magnetoacoustic wave, which propagates isotropically with the Alfven speed.

Since the Alfven speed is much greater than the speed of the tether (8 km/sec), the magnetoacoustic wake does not appear as a dynamic wave. Instead it is static in the tether frame and is modified slightly by the "Lorentz" contraction $z \rightarrow z\left(\sqrt{1 - V^2/C_A^2}\right)$, which for our case is ~0.1 percent.

The magnetic twists ($\sim 2\times10^{-4}$ g) propagate along the magnetic field lines with the Alfven speed but are slowly attenuated as a result of collisions, with a scale length substantially greater than the electron mean free path: in fact of the order of

$$L \approx \frac{D^2}{d_0^2} \frac{C_A^2}{v_e - V} \frac{L_S}{D} \lambda_f$$

where d_0 is the collisionless screening length, $c/\omega_{p^-} \approx 4$ m for our case and hence $L \approx (C_A/V)^2 \lambda_f \approx 800$ km. If the magnetic field has a vertical component or if the tether is not perpendicular to the field, then a very much larger signal can be propagated as an Alfven wave, and it is here that communications possibilities may exist (frequencies must be substantially less than the ion gyrofrequency, when Doppler shifted, $\Omega_+ \approx 180$, $\nu \approx 28$ cycles/sec).

The magnetoacoustic mode produces a modification in the magnetic field strength that is almost indistinguishable from the vacuum field. The azimuthal field has the vacuum form, but there is superimposed on this a small radial field

$$B_r \sim I \frac{V^2}{c^2} \frac{\sin 2\theta}{r}$$

and an associated current density

$$j = I \frac{V^2}{c^2} \frac{\cos 2\theta}{r^2}$$

This can be quite large near the tether but is only correctly described for values of $r \gg 2$ cm, where nonlinear modifications become unimportant (hence $j \ll I$).

The magnetosonic wave that carries the pressure pulse represents a superposition of the channeled sound wave, and the isotropic magnetoacoustic wave.

The pressure pulse propagates along the field line and, as it propagates, produces a pressure disturbance in its neighborhood that falls off as $1/\rho$, where ρ is the perpendicular distance to the field line. The disturbance rapidly becomes small, and in addition, in a more rigorous treatment, is damped.

High-frequency components. - If the plasma is treated by the more rigorous and appropriate method of collisionless kinetic theory, the low-frequency MHD results are reproduced, providing that the conditions $k_{\parallel} v_{\theta}^{-}/k \cdot V << 1$ and $k_{\parallel} v_{\theta}^{+}/k \cdot V >> 1$ are satisfied. (Note that this invalidates the slow sound wave, which becomes strongly damped.) There are, however, other modes of oscillation possible, and many of these waves have peculiar transmitting properties.

If we work only to lowest order in the ion gyroradius, using the cold ion approximation, then the tether field gives rise, in addition to the quasi-static response, to an oscillating field whose amplitude depends essentially on the distance along the field line from the point of observation to the tether, and only slowly on the distance along the direction of motion. The frequency, however, varies with this distance and, as a result, a signal modulated in both amplitude and frequency is produced as the tether passes.

The charge density can also excite a narrow band of Langmuir waves, providing that $k \cdot V/k_{\parallel} v_{\theta}^{-} >> 1$. These combine again to give a modulated wave as the collector passes. The signal occurs in planes above and below the collector, and this time has a constant amplitude depending only on the distance projected along the field line and is again a slowly modulated sine wave in the region of its appearance.

CONCLUSIONS

This has been an introductory essay on the subject of conducting tethers. We have presented arguments suggesting that current collection can be reasonably efficient even with a fairly simple system, although the local modification in the plasma is both unexpected and dramatic. We have also discussed some features of the wake and have explored several components: the Alfven wave, magnetoacoustics, and high-frequency elements.

This is by no means a complete study; it is suggestive rather than demonstrative. We have not given a complete and consistent theory of the extended sheath (the high field region needs to be analyzed), nor have we discussed its overall stability.

The discussion of the wake is also incomplete. The pressure mode (the slow magnetoacoustic wave) is probably strongly damped, and the connections to the source have not been evaluated in full detail. Some of the results, the lack of Alfven waves from the tether, for example, depend on the orientation chosen and would be modified as the system moved.

The treatment of the high-frequency components is also incomplete. We have used a greatly oversimplified representation of the dielectric response, neglecting the effects of ion thermal motion, including the higher cyclotron resonances.

More importantly, we have restricted the analysis to the static wake. If the current is modulated, new phenomena must be expected, some of which can be analyzed by the methods we have described, but some of which call for more complex investigations.

We have not presented a careful analysis of motion around the tether. This is important since the coupling to the linear waves requires a treatment of the local nonlinear region.

Finally, we have not attempted to consider the possible effect of electrodynamic forces on the motion of the tether.

Much remains to be done, but at least a start has been made on a detailed and rigorous analysis of the electrodynamics of a tether in space.

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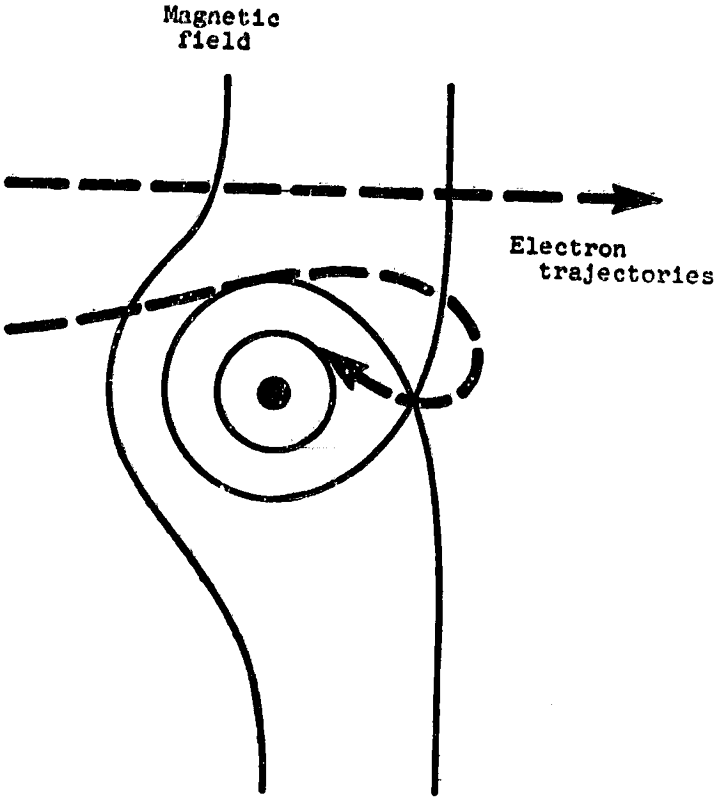


Figure 1. - Field and motion near tether.

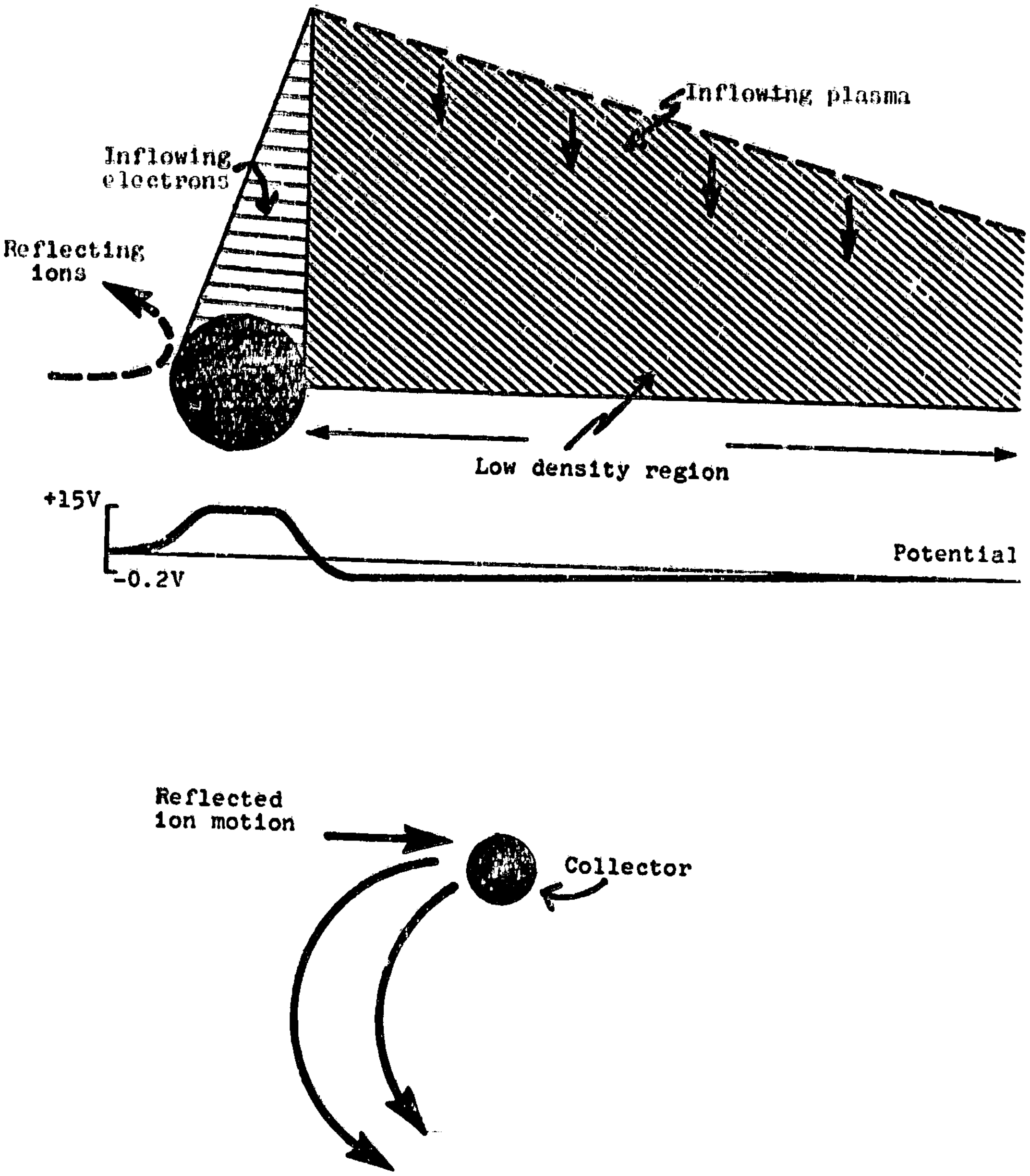


Figure 2. - Current collection - local effects.

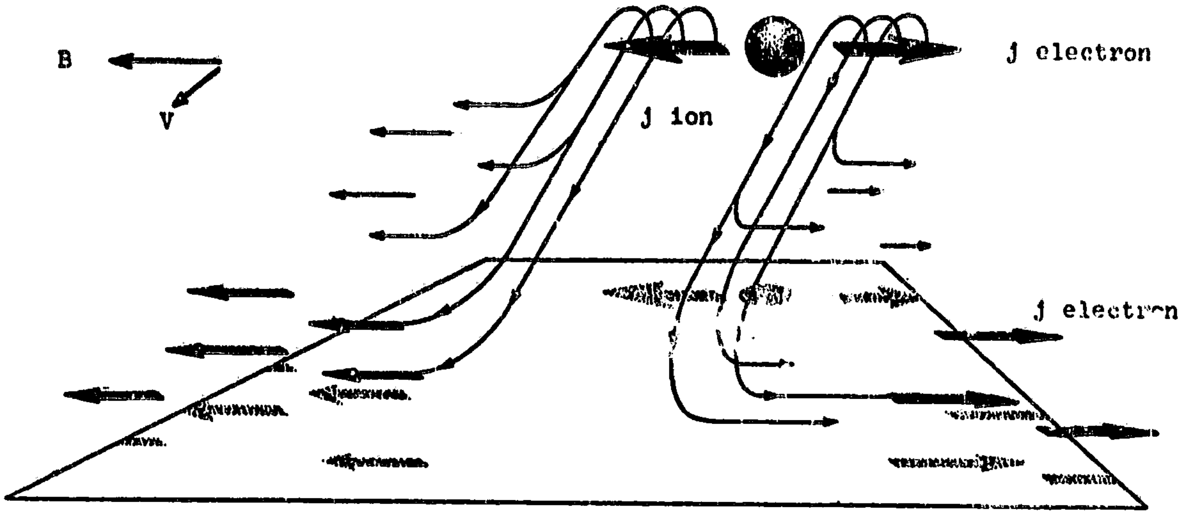
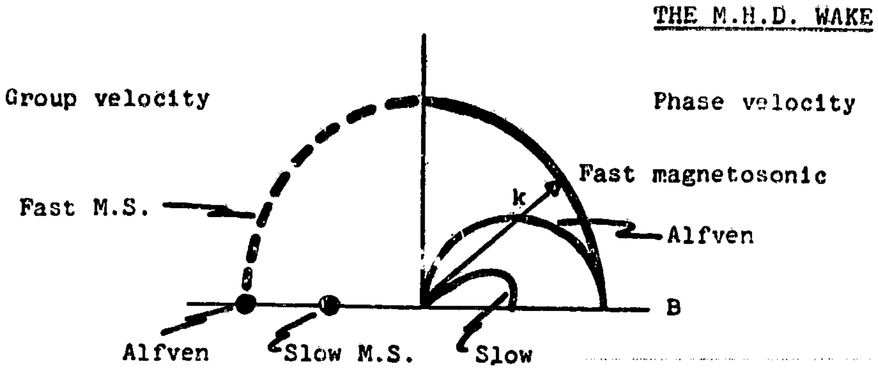
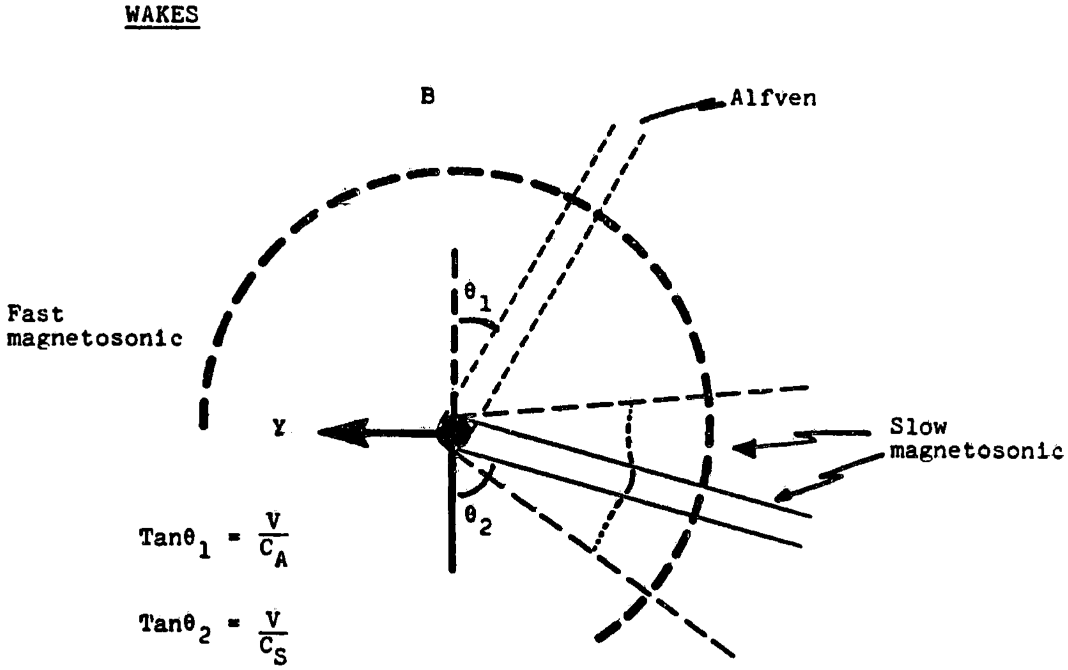


Figure 3. - Current flow near collector.





$$\tan\theta_1 = \frac{V}{C_A}$$

$$\tan\theta_2 = \frac{V}{C_S}$$

Figure 4. - Wakes.

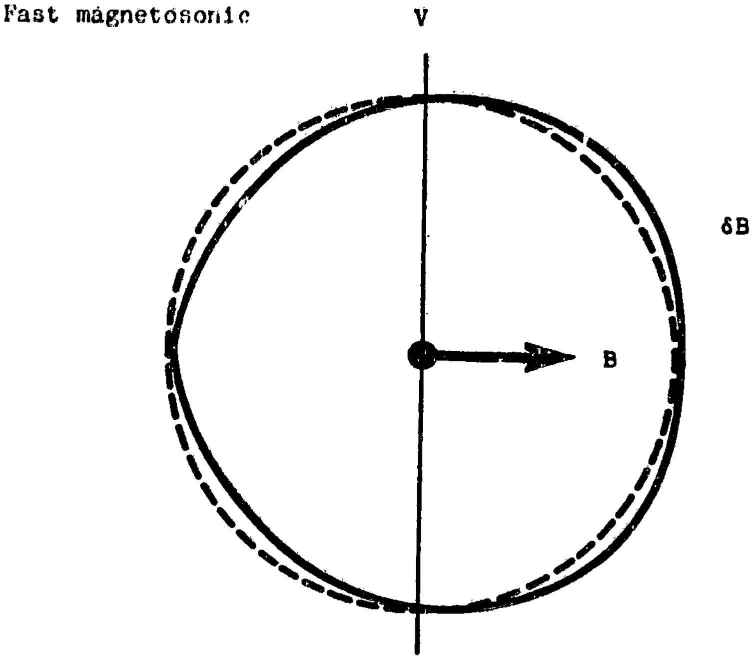


Figure 5. - Tether wake.

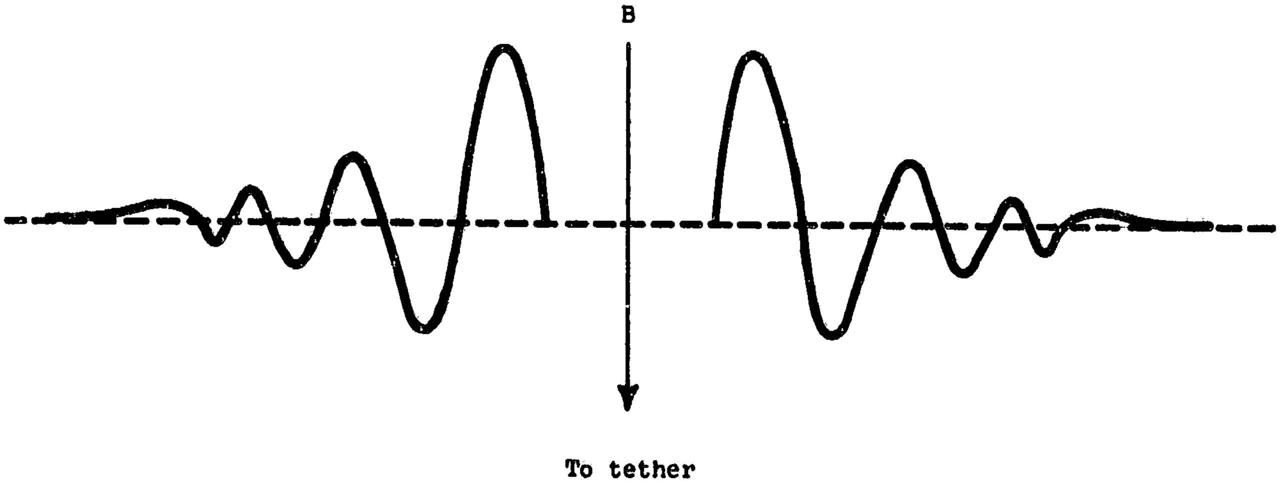


Figure 6. - High-frequency tether wake.