

## NEW USES FOR SENSITIVITY ANALYSIS: HOW DIFFERENT MOVEMENT TASKS AFFECT LIMB MODEL PARAMETER SENSITIVITY

### INTRODUCTION

Sensitivity analysis techniques have traditionally been used by the systems engineer to help understand the behavior of complex systems. Although the details of the techniques seen in practice differ, the basic approach is the same: A system parameter is varied in a controlled, systematic manner, and the subsequent variations in output are measured and described quantitatively (Frank (1978), Tomovic and Vukobratovic (1972), Lehman and Stark (1982)). When the system is nonlinear, as is the usual case, numerical techniques involving computer simulation typically need to be employed. Insights gained from such techniques are of value both for systems analysis and design.

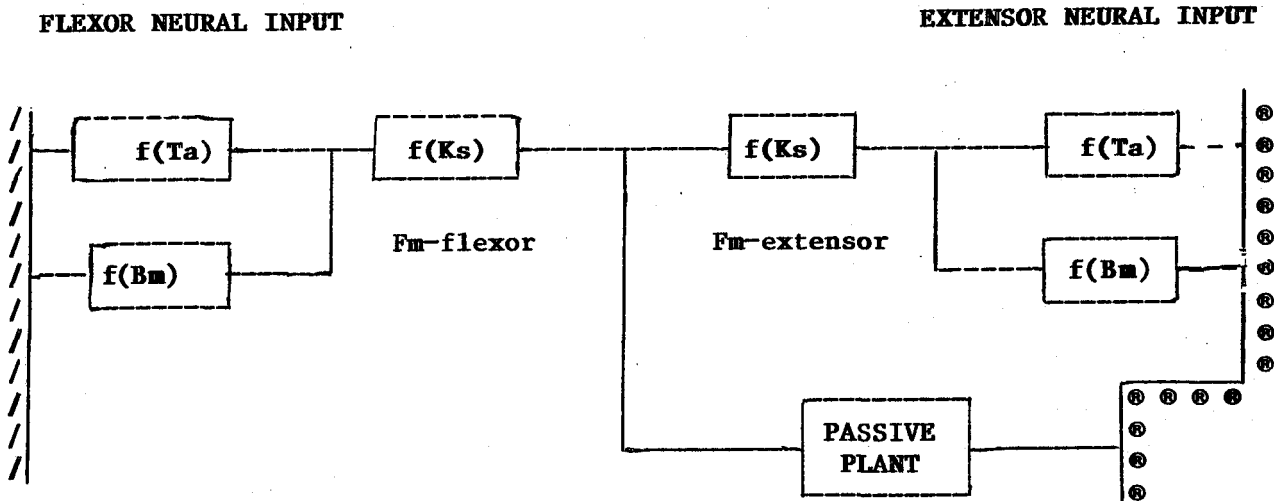
The methods presented here represent extensions of previous work (Clark and Stark (1976), Hsu et al (1976), Lehman and Stark (1979), Bahill (1980) and Zangemeister et al (1981)), only with a wider range of sensitivity tools employed. Furthermore, the model considered here is for limb flexion-extension movements, rather than for head or eye rotation. The model **structure** has also been expanded and the constitutive equations representing basic muscle properties improved so as to more accurately characterize basic neuromuscular system dynamics. Consequently, there are a larger number of internal model **parameters**. A greater number of output behaviors are also considered.

In addition to presenting this expansion of previous sensitivity analysis tools and extending these methods to a larger number of model parameters, a major role of this presentation is to show how sensitivity analysis results are a function of the model **task**. When the task under consideration is changed (i.e. the model input controller signal sequence is fundamentally different, resulting in a different type of output), the relative role of each parameter in affecting performance also changes. This fact, surprisingly neglected in the literature, is developed quantitatively here.

The result is one model that can adequately simulate any basic physiologically realizable flexion-extension task and a set of sensitivity tools that help explain the relative role of any specific parameter for any particular task - tools that can help make the goal these modeling efforts, gaining **insight** into the role of biomechanical systems in neuromotor control, a reality.

## METHODS:

**A. Model Structure:** Before presenting the sensitivity analysis protocol employed here, it will first be advantageous to develop a basic understanding of the biomechanical system being modelled. The first step to any modeling effort is to assume a basic structure. Once chosen, this structure will be the major constraint on the success of the model - too simple a structure can result in a poor approximation of actual behavior, while too complex a structure reduces insight (or results in a model with poorly defined parameters). The basic structure for the model, based on the classical muscle work of Hill (1938) and elbow flexion-extension work of Wilkie (1950) and supported by numerous more recent experimental work on muscle mechanics, is presented in Figure 1. The sixth-order structure has been found to be the lowest order structure that is capable of approximating all fundamental muscle properties needed for an antagonistic pair of lumped "equivalent" muscle actuators rotating a joint.



**FIGURE 1:** Model of System Showing the Nonlinear Blocks.  
Lumped Flexor Muscle is on the left, Extensor on right.

Experimental work is often able to approximately isolate each of these elements in the model. Fundamental to such an approach is this idea of an "equivalent muscle". This concept of an "equivalent" muscle for the lumping of a number of synergistic muscles has been previously developed, based on experimental work, for both the flexor group (Bouisset et al (1973, 1976) and the extensor group (Cnockaert and Pertouzon (1974)) and confirmed by Cnockaert (1978) and Le Bozec (1980). This idea is supported and expanded on here in the following sense: not only is it a good representation for the ideal case of elbow flexion-extension but it also should be expected to hold for more complex one degree of freedom movements such as wrist or head rotation because two lumped antagonistic muscles with the blocks described above should be structurally capable of approximating all basic muscle properties for such movements whenever muscles contract approximately synergistically. In these more involved movement systems, however, parameter identification is more difficult.

A good summary of much of the work on the material properties of muscle is found in the review by Close (1972). Once fiber type and fiber

orientation are determined, basic skeletal muscle material properties for a given muscle can be well estimated. **Geometric** data for the muscles around the elbow joint also exists (An et al (1981), Amis et al (1979)). By creating **algorithms** that combine material and geometrical information, first generation parameter values for the torque-velocity and series elastic elements can be established. These results are then combined with the wealth of experimental work on controlled **intact** limb movements, the best of which include Dern et al (1947), Wilkie (1950), Pertuzon and Bouisset (1973), Jorgensen et al (1971, 1976), Hatze (1981a,b) and Komi (1973) for torque-velocity information, Wilkie (1950), Goubel and Pertuzon (1973) and Cnockaert et al (1978) for the series elastic relation, and Boon (1973) and Hayes and Hatze (1977) for passive viscoelastic data. Limb inertial data exists in abundance. Insights from preliminary sensitivity analysis work (not presented here) are also used for fine-tuning parameters. The actual protocol followed for parameter development is beyond the scope of the present presentation and will not be described here. The resulting model parameter values for the elbow flexion / extension model, one of the five models currently under pursuit, are displayed in Table 1.

PARAMETER:	VALUE:	CONSTITUTIVE EQUATION:
<b>Passive Plant:</b>		
Jp:	0.06 Kg-m**2/rad	} $F_{kp} = K_p * x + K_{p1} * (\exp(K_{p2} * x) - 1)$
Bp:	0.15 N-m-sec/rad	
Kp:	1.4 N-m/rad	
Kp1:	0.0001 ...	
Kp2:	10.0	
<b>Series Elasticity:</b>		
Ks1-f:	4.8 N-m/rad	} $F_{ks} = K_{s1} * (\exp(K_{s2} * (x_h - x)) - 1)$
Ks2-f:	7.0	
Ks1-e:	4.5 N-m/rad	
Ks2-e:	7.2	
<b>Torque-Velocity:</b>		
Af-f:	0.34	} $B_m = \begin{cases} \frac{(1 + A_f) * F_h}{(V_h + B_h)} & V_h < 0 \\ \frac{(1 + A_f * A_{fv}) * F_h * F_{mfv}}{(V_h + B_h * b_{fv})} & V_h > 0 \end{cases}$
Bh-f:	8.0 rad/sec	
Af-e:	0.30	
Bh-e:	7.0 rad/sec	
Fm-fv:	0.3	
Af-fv:	0.6	
Bh-fv:	0.25	
(where $F_m = F_n - B_m * V_h$ )		
<b>Activation Dynamics:</b>		
Ta1:	40 ms	
Ta2:	10 ms	
Fmax-f:	60 N-m	
Fmax-e:	50 N-m	

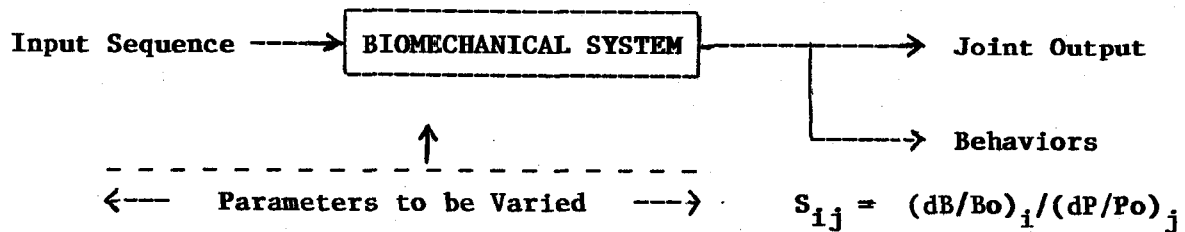
**TABLE 1:** Current Parameter Values for the Elbow Flexion-Extension Model. Constitutive Equations are for: Parallel Elasticity ( $F_{kp}$ ), Series Elasticity ( $F_{ks}$  (=  $F_m$ )) and Torque-Velocity ( $B_m$ ). (Parameter values are for 70 Kg male of average strength.)

The passive plant values represent the inertia and viscoelastic properties of the lumped joint/muscle system, with all three elements in parallel as is usual. The exponential fit for the parallel and series elasticities has become a standard representation for load-bearing collagen-elastin based soft tissue (Fung (1968), Giantz (1974) and Hatze (1981)). The "Hill's" parameters (A<sub>f</sub> and B<sub>h</sub>) for shortening muscle are the standard representation for the classic force-velocity relation of muscle, used constantly in the literature to document experimental results (see, for instance, Close (1972)). The scaling of the instantaneous torque-velocity relation by the activation level was suggested as early as 1956 by Wilkie, and has been supported by the work of Pertuzon and Bouisset (1973). The relation used for lengthening muscle (an inverted, skewed Hill's-type equation) is a new method that appears to adequately approximate past data (Joyce et al (1969a,b), Komi (1971) and Hatze (1981)), plus numerous observations that the peak eccentric torque is about 30% above the peak isometric force. Activation dynamics is simulated by two time constants, compatible with the basic neuromuscular literature (see Close (1972) or Bahler (1967) for reviews). This second-order form represents simplifications suggested by the more detailed work of Lehman (1983) and Hatze (1981). There is also numerous isometric peak torque data available in the physical education literature - the values presented here are for a "typical" human male. For reasons of clarity, the static torque-angle parameters (based on an abundance of literature) were not presented above.

As seen above, all indications are that all of these elemental building blocks are nonlinear. The function of these nonlinear properties is still poorly understood, and one of the main problems faced is to explore the sensitivity of the system to these nonlinearities. There is ample evidence, supported here, that the relative importance of various parameters is a function of the task in question. Consequently, it can be a major mistake to over-simplify this basic system if one is interested in a variety of movement tasks. Furthermore, since sensitivity methods provide just the information needed for task-specific model simplification, it is suggested that the more complex model be considered first - any model simplification is then based on a solid foundation.

**B. Computer Simulation Algorithm:** The simulation algorithm is contained within a more general set of modules that are linked to a main routine, called "JAMM" (Juiced-up Antagonistic Muscle Model). This user-friendly program will simulate second, sixth and eighth order models with degrees of nonlinearity ranging from linear to highly nonlinear. Once the biomechanical model of interest is chosen, a data base, complete with all the current numerical values of parameters for any user-desired combination of linear/nonlinear parameter defaults, is read. The user is prompted for parameter modification, for various external loading options, for the type of run (interactive, sensitivity analysis, optimization), and for the controller signal input sequence for each equivalent muscle.

The options under sensitivity analysis include: determining the parameters that are to be varied, one by one, for a given run and determining the range of the parameter variation and the number of times varied. Parameter variation is by a reciprocal format (for example, 4/5 and 5/4 of nominal). Typically results for five reciprocal pairs are obtained. Raw behavior data, behavior and parameter ratios, and linear and logarithmic sensitivity coefficients are stored for later plotting and/or printing.



**Figure 2** Schematic of Sensitivity Analysis Method.

Note that varied parameters can include system, input or disturbance values, and that input sequence depends on task.

**C. Sensitivity Analysis Protocol:** Past sensitivity analysis work on eye and head systems concentrated on the development of a "sensitivity matrix" (Hsu et al (1976), Lehman and Stark (1979), and Zangemeister et al (1981)). A schematic of this basic method is presented in Figure 2. An input sequence that will define a certain type of task is chosen, and output trajectories are measured. In general, the input and output can be scalar or vector quantities. Here, for the generalized equivalent flexor and extensor muscles, there are two inputs, one to each of the lumped equivalent muscles. The outputs of interest are the position, velocity and acceleration of the limb, plus the muscle torques. The "behaviors" of interest are a function of these output trajectories. The parameter varied is typically an internal model parameter value, but may also be an input signal parameter value, such as a pulse height or width, or an external disturbance. Traditionally, each column would indicate the sensitivity of all the different behaviors (each on a different row) to that column's parameter.

The actual value of each matrix element is called the "sensitivity coefficient",  $S_{ij}$ , of the  $i$ -th behavior to the  $j$ -th parameter. This coefficient represents the relative change in behavior divided by the relative change in parameter  $(dB/B_o)_i / (dP/P_o)_j$ , where  $P_o$  and  $B_o$  are the nominal parameter value and the resulting nominal behavior value, respectively. The range of the change in parameter for the determination of the matrix coefficients is up to the discretion of the user. Typically, the range chosen for the sensitivity matrix coefficient computation was from one half to twice the nominal parameter value. Another design consideration is the equation used to determine the coefficient. Two equations are used here:

"linear": 
$$((B_2 - B_1) / B_o) / ((P_2 - P_1) / P_o)$$

"logarithmic": 
$$(\log(B_2 / B_o) / \log(P_2 / P_o)) - (\log(B_1 / B_o) / \log(P_1 / P_o))$$

where  $P_2$  is the parameter value greater than nominal and  $B_2$  the resulting behavior value;  $P_1$  the reciprocal fraction of  $P_2$  and  $B_1$  the resulting behavior due to  $P_1$ . Because the first method gives a value proportional to the relative difference in behavior without regard to one direction maybe having a greater shift, it tends to weigh behavior changes greater than  $B_o$  disproportionately more than those below. The second method weighs ratios both below and above nominal equally. For this reason, the second method is usually preferred. Notice that, if the behavior were to change in a manner proportional to the parameter change, the sensitivity coefficient for

either method would be "1.0".

Once this "sensitivity matrix" is completed, it gives a global view of model behavior for the task under question. Table 2 presents the results for a simple "generic" run. Here, an input signal, about 30% of maximum, is applied to the flexor group for 200 ms. The extensor group is about 3% of maximum. The model is run for each of the parameters chosen for variation at values one half and twice nominal. The behaviors of interest are measured, and the sensitivity coefficients determined, here by both methods.

Each of the resulting column gives one a feel for how a given parameter effects the various behaviors, while a given row indicates what parameter(s) most influence the particular behavior. For convenience in matrix inspection, the following conventions are used: the highest value in each column is printed in italics; the highest in each row is in boldface; and the three three most influential parameters are also printed in boldfaced italics. For this example coefficients for both the "linear" and "logarithmic" descriptions are provided. Note the similarity in coefficient values. All later work uses only the logarithmic method of determination.

TABLE 2:

SENSITIVITY MATRIX FOR TASK: "Generic, Medium-Speed Movement":

	NOMINAL	<i>Jp</i>	<i>Bp</i>	<i>Kp</i>	<i>Kp1</i>	<i>Kp2</i>	<i>Ks1-f</i>	<i>Ks2-f</i>	<i>Af-f</i>	<i>Bb-f</i>	<i>Fvmax</i>	<i>Tal</i>
Magn:	114 deg	-0.012	-0.108	-0.026	-0.012	<b>-0.289</b>	-0.002	-0.004	-0.200	0.037	-0.101	-0.046
		-0.013	-0.120	-0.029	-0.013	<b>-0.398</b>	-0.002	-0.004	-0.223	0.055	-0.112	-0.050
Vmax:	565 d/s	-0.141	-0.125	0.005	-0.000	<b>-0.367</b>	-0.007	-0.019	-0.290	<b>0.542</b>	-0.059	-0.115
		-0.162	-0.138	0.006	-0.000	<b>-0.575</b>	-0.070	-0.020	-0.333	<b>0.593</b>	-0.064	-0.131
Amax:	5492 d/s/s	-0.510	-0.057	0.077	-0.002	<b>1.732</b>	-0.021	-0.039	-0.096	0.274	-0.057	<b>-0.415</b>
		-0.556	-0.062	0.082	-0.002	<b>0.924</b>	-0.023	-0.043	-0.105	0.300	-0.063	<b>-0.468</b>
Amin:	-4858 d/s/s	<b>-0.671</b>	<b>-0.231</b>	<b>0.150</b>	-0.028	<b>1.960</b>	-0.020	-0.041	<b>-1.128</b>	<b>5.573</b>	-0.002	-0.390
		<b>-0.757</b>	<b>-0.227</b>	<b>0.156</b>	-0.030	<b>1.000</b>	-0.021	-0.044	<b>-0.800</b>	<b>1.749</b>	-0.002	-0.454
Fm-f:	9.1 N-m	0.154	0.005	-0.015	-0.001	<b>0.295</b>	0.005	0.022	-0.063	0.185	0.033	-0.123
		0.166	0.006	-0.017	-0.001	<b>0.264</b>	0.006	0.040	-0.069	0.202	0.050	-0.134
Fb-f:	14.4 N-m	-0.100	-0.056	0.011	-0.000	<b>-0.364</b>	-0.001	-0.004	0.106	-0.170	-0.029	-0.138
		-0.110	-0.061	0.012	-0.000	<b>-0.569</b>	-0.020	-0.005	0.114	-0.191	-0.032	-0.160
Tmagn:	363 ms	<b>0.298</b>	0.097	-0.064	<b>-0.041</b>	0.000	-0.009	0.017	0.000	0.233	0.017	0.252
		<b>0.302</b>	0.104	-0.071	<b>-0.044</b>	0.000	-0.010	0.080	0.001	0.241	0.018	0.252
Tvmax:	209 ms	<b>0.131</b>	-0.013	-0.035	0.000	0.077	0.000	0.003	-0.061	0.042	-0.003	0.057
		<b>0.147</b>	-0.014	-0.038	0.001	0.078	0.001	0.003	-0.068	0.045	-0.003	0.061
Tamax:	59 ms	0.260	-0.023	-0.011	-0.011	-0.158	-0.068	-0.192	-0.034	0.068	0.045	<b>0.294</b>
		0.278	-0.024	-0.012	-0.012	-0.192	-0.073	0.207	-0.036	0.071	0.048	<b>0.320</b>
Tamin:	278 ms	0.103	-0.132	-0.007	-0.031	<b>-0.528</b>	-0.005	-0.017	-0.120	0.029	<b>-0.134</b>	0.089
		0.111	-0.135	-0.080	-0.033	<b>-1.182</b>	-0.005	-0.018	-0.125	0.032	<b>-0.136</b>	0.096
Tfm-f:	54 ms	0.284	0.025	-0.037	0.000	<b>0.877</b>	<b>-0.086</b>	<b>-0.247</b>	-0.037	0.099	0.037	0.247
		0.390	0.027	-0.040	0.010	<b>0.605</b>	<b>-0.095</b>	<b>-0.260</b>	-0.040	0.105	0.040	0.265
Tfb-f:	273 ms	0.023	0.000	-0.011	0.000	0.010	0.000	0.000	0.000	0.000	0.000	<b>0.036</b>
		0.050	0.000	-0.012	0.010	0.011	0.010	0.001	0.000	0.000	0.001	<b>0.040</b>

Use of a coefficient determined by two values only gives the linear slope over the operating range of the two parameters. For a nonlinear system, this may give misleading information (discussed later). For this reason, the parameter ratio range for a given sensitivity matrix is an important design variable. Thus, if one is interested in a deeper understanding of the role of a certain parameter, a simple column of coefficients is not enough.

The tools described here for a more in-depth examination of the role of a specific parameter will be called "sensitivity graphs" and "sensitivity trajectories". They are best used as the next step after a preliminary sensitivity matrix has been developed for the task in question. The "sensitivity trajectory" is simply the set of output versus time plots that result from a range of parameter variation, in superimposed plots (Figure 3, left panel). Inspection of these output plots can be a surprisingly effective way of coming to an understanding of the role of the parameter, making use of human talents for visualizing information, and putting the column of sensitivity coefficients in proper perspective (these coefficients can be occasionally misleading (discussed later). This simple step should be used on all parameters with significant sensitivity columns.

"Sensitivity graphs" further expand ones insight into model sensitivity to a certain parameter of interest, and also bring together sensitivity columns and trajectory information. This method consists of graphing behavior ratios versus parameter ratios for a wide range, as in Figure 3 (right panel). Notice that each "graph" is basically a graphical extension of each coefficient, basically showing the five possible coefficients (slopes) that could be placed in the particular location. Typically logarithmic scales are employed. Visual inspection of this graph provides information on how linearly the behavior changes with parameter variation. It also suggests the useful operating range of the parameter of interest. This is possibly the most important sensitivity tool from a design perspective.

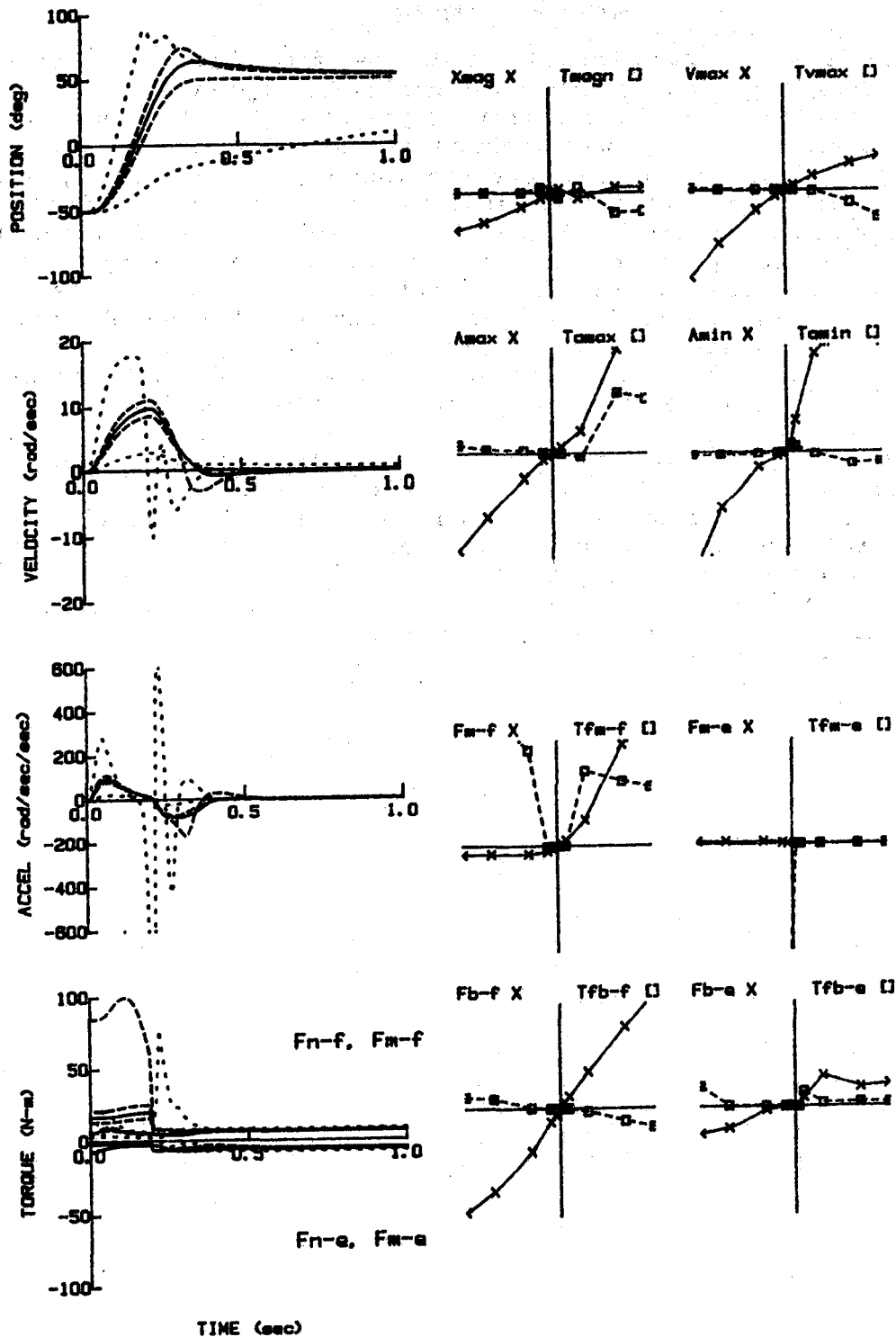


Figure 3: Sensitivity Trajectories (left panel) and Sensitivity Graphs (right panel) for the controller parameter PH-f, the pulse height for the agonist pulse, Task #1. Large dash is for 4/5 and 5/4 times nominal. For Sensitivity Graphs, range is 1/10 to 10, on logarithmic scales, for both the parameter (abscissa) and the behavior (ordinate).



## RESULTS:

**Task #1: "Generic" Run:** Some simple results, for a "typical", moderate-speed run, were displayed in Table 2 (sensitivity matrix) and Figure 3 (sensitivity trajectories and sensitivity graphs). From the sensitivity matrix, it is seen that the most important system parameters are the plant inertia ( $J_p$ ), especially for peak accelerations and for the timing of all peak values; Hill's parameter  $Bh-f$ , especially for peak magnitudes; the time constant of activation ( $Ta_1$ ), and the parallel elastic fit parameter  $Kp_2$ . From the nature of the equation for the parallel elasticity (Table 1) we see that the location of the parallel elastic concavity will automatically define the position operating range. The model output is also particularly sensitive to controller signal pulse parameters such as PH1 and PH2 - not an unexpected finding since a well-designed tracking system is usually sensitive to its own input for tracking tasks.

Based on these sensitivity matrix results, the parameters mentioned above appear to be of particular interest for this task. In Figure 3, variation in the agonist pulse height parameter (PH1) is displayed using sensitivity trajectories and sensitivity graphs. These results show more explicitly the effect of varying the agonist pulse height. For reference, an average adult male can contract the flexor group to about 60 N-m. Similar plots, not presented here, are then produced for the other highly sensitive parameters.

**Task #2: Unloaded Fast Voluntary Elbow Flexion:** The effects of a 60 N-m, 100 ms agonist pulse are displayed in Figure 4. The corresponding antagonist pulse during this time is only 1 N-m. Notice that the peak angular velocity is about 14 rad/sec. (For a longer pulse width, one finds a peak velocity of about 19 rad/sec both for the model and for the average intact adult human (Dern (1947), Wilkie (1950), Pertuzon and Bouisset (1973)). Zero degrees is defined here as the rest elbow position (where the lumped parallel elasticity is zero), of 100 degrees between the humerus and the ulna. Notice that movements of about 80 deg are possible in each direction, with the nonlinear parallel elastic element automatically keeping the joint position within this physiological operating range.

In Table 3 a more complete sensitivity matrix for internal system parameters, including flexor and extensor parameters for both series elastic and torque-velocity (shortening and lengthening) properties, is presented. Notice that, while the general trends in this table are similar to those in the previous table, the details are quite different. Also notice the general insensitivity of the "extensor" muscle parameters. This shows that it is the flexor parameters that are of primary importance for this particular task and furthermore shows the relative significance of the various flexor parameters to each other for each behavior.

In Figure 5 sensitivity trajectories and sensitivity graphs are obtained for a few of the more important internal system parameters such as Bh, Jp and Ta1. Notice that, by combining all three techniques, a remarkably clear picture of the role of each of these parameters emerges. We see, by all three methods, that the inertial term affects mainly acceleration information and the timing of peak values. The latter two methods both show that the system is more sensitive to increases in inertia - something common in everyday life and sporting events. All three methods also show that the torque-velocity parameter Bh (Figure 5b) mainly influences magnitude information, with less effect on timing. The activation time constant parameter Ta1, which basically filters the neuromuscular signal before "passing" it, effects system behavior as might be expected. Notice that the sensitivity increases proportionally more when the parameter increases in value than when it decreases (best seen by the sensitivity graph).

TABLE 3:

SENSITIVITY MATRIX FOR TASK: "Simple, Fast, Unloaded Movement":

NOMINAL	Jp	Bp	Kp	Kp1	Kp2	Ks1-f	Ks2-f	Ks1-e	Ks2-e	Af-f	Af-e	Bh-f	Bh-e	Fv-fv	Af-fv	Bh-fv	Ta1	Ta2
Magn: 122.6 deg	0.050	-0.065	-0.036	-0.015	-0.200	-0.002	-0.006	0.001	0.002	-0.173	-0.017	0.335	0.023	-0.073	-0.017	0.022	-0.001	-0.003
Vmax: 805 d/s	-0.247	-0.090	0.024	-0.004	-0.130	-0.017	-0.055	-0.000	-0.003	-0.323	-0.005	0.565	0.005	-0.031	-0.005	0.005	-0.226	-0.038
Amax: 16407 d/s/s	-0.527	-0.051	0.037	-0.004	-0.170	-0.015	-0.011	-0.005	-0.012	-0.166	-0.004	0.387	-0.008	-0.026	-0.004	-0.008	-0.447	-0.112
Amin: 6081 d/s/s	-0.766	-0.005	0.080	-0.002	-0.046	-0.037	-0.095	0.000	0.000	-0.905	0.022	1.932	-0.026	0.137	0.022	-0.026	-0.528	-0.051
Tmag: 320 ms	0.131	0.750	-0.101	-0.069	-0.755	0.022	0.057	0.000	0.000	0.000	0.000	0.087	0.000	-0.017	0.000	0.000	0.333	0.049
Tvmax: 125 ms	0.160	-0.029	-0.015	0.000	0.036	-0.012	-0.029	0.002	0.009	-0.094	0.022	-0.136	0.002	0.770	0.022	0.023	0.137	-0.038
Tamax: 51 ms	0.281	-0.028	0.000	0.000	0.170	-0.015	-0.287	0.000	0.000	0.000	0.000	-0.014	0.000	0.042	0.000	0.000	0.252	-0.112
Tamin: 189 ms	0.234	-0.061	0.008	0.0038	0.078	-0.004	-0.011	0.000	0.000	-0.284	-0.008	0.134	0.000	0.038	-0.008	0.000	0.190	-0.011
Fv-f: 15.6 N-m	0.283	0.009	-0.013	-0.004	-0.069	0.003	0.040	-0.004	-0.010	-0.145	0.006	0.343	-0.017	0.036	0.006	-0.017	-0.213	-0.060
Fv-e: 8.4 N-m	0.080	0.009	0.078	0.019	1.569	0.000	-0.002	-0.000	-0.010	0.026	0.070	-0.032	-0.043	0.183	0.037	-0.043	-0.070	0.002
Fb-f: 37.6 N-m	-0.219	-0.028	0.000	-0.001	-0.063	0.016	0.059	0.001	0.001	0.101	-0.003	-0.136	0.004	-0.019	-0.030	0.004	-0.424	-0.108
Fb-e: 1.9 N-m	0.101	-0.027	-0.008	0.001	0.741	-0.004	-0.011	0.000	0.010	-0.338	0.156	0.712	-0.167	0.971	0.156	-0.167	-0.118	-0.010
Fv-fe: 11.4 N-m	0.500	0.018	-0.086	-0.034	-0.110	-0.029	-0.057	-0.005	-0.011	-0.202	-0.040	0.468	-0.007	-0.027	-0.004	-0.008	-0.488	-0.112
Tfa-f: 50 ms	0.287	0.015	-0.015	0.000	0.000	-0.102	-0.310	0.000	0.000	0.000	0.000	0.014	0.000	0.029	0.000	0.000	0.240	0.171
Tfa-e: 207 ms	-	-0.028	-	-	-	0.003	0.003	0.000	-0.003	0.094	0.000	-	-0.021	-	0.000	-0.021	0.279	0.082
Tfb-f: 104 ms	0.007	-0.007	0.000	0.000	-0.022	-0.007	-0.007	0.000	0.000	0.007	0.000	0.000	0.000	0.000	0.000	0.000	0.041	0.041
Tfb-e: 193 ms	0.151	-0.008	-0.019	0.000	-4.492	0.000	-0.004	0.000	-0.004	-0.328	-0.004	0.159	-0.049	-0.494	-0.004	-0.049	0.215	0.077
Tv-fe: 55 ms	0.249	0.026	0.000	0.013	-	-0.080	-0.292	0.001	0.000	-0.026	0.000	0.013	-0.013	0.039	0.000	-0.013	0.287	0.193

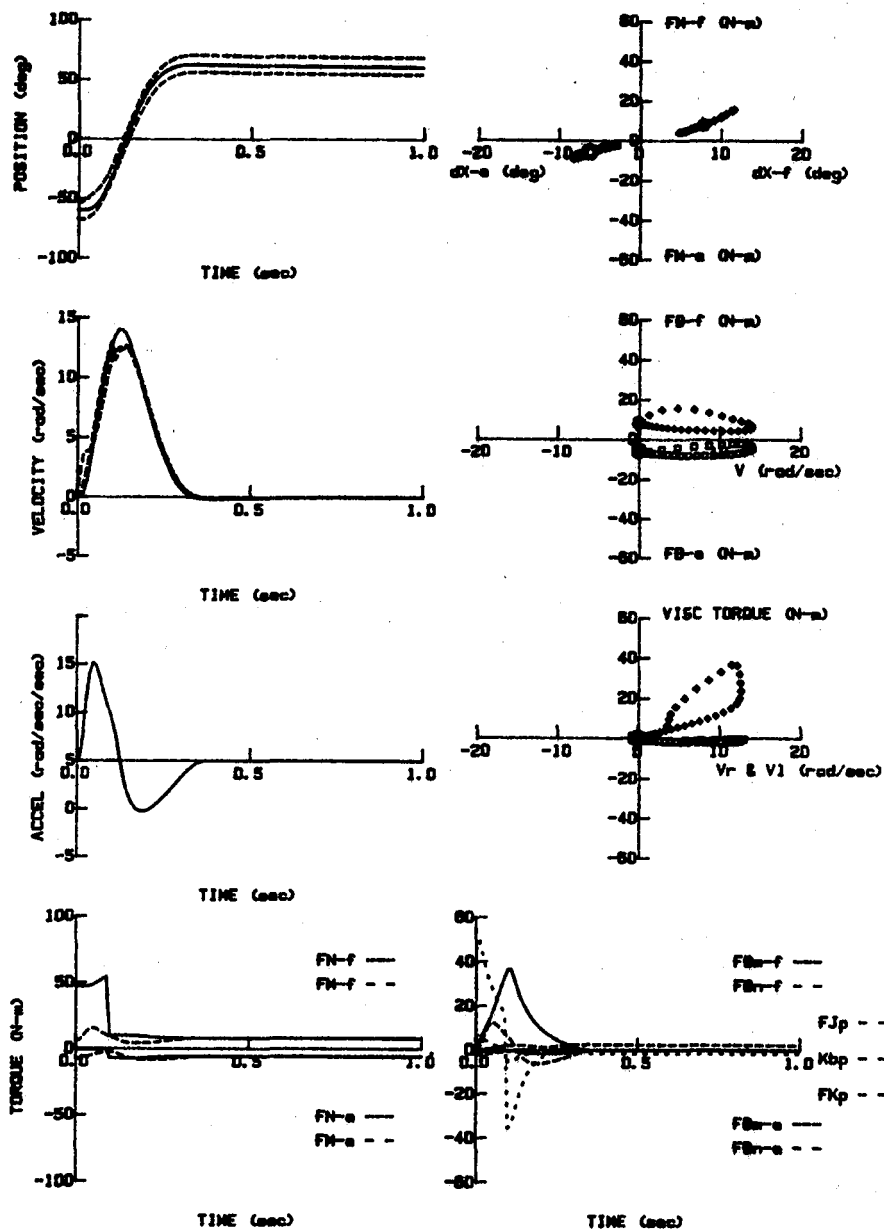


Figure 4: Plots of input, output, and internal variables for the "normal" run of Task #2. Lower left contains the model input variables (FN-f & FN-e) and the resulting muscle output torques (FM-f & FM-e). Upper and middle left contain kinematic output information as well as the positions and velocities of flexor and extensor internal nodes (dashed). On the right, from top to bottom, are internal variable plots for the series elastic element, the instantaneous externally seen torque-velocity behavior, the viscous muscle torque versus node velocity, and the torque propagation for various model elements. For the 3 top right plots, time is an implicit parameter, with a point being produced every 10 ms.

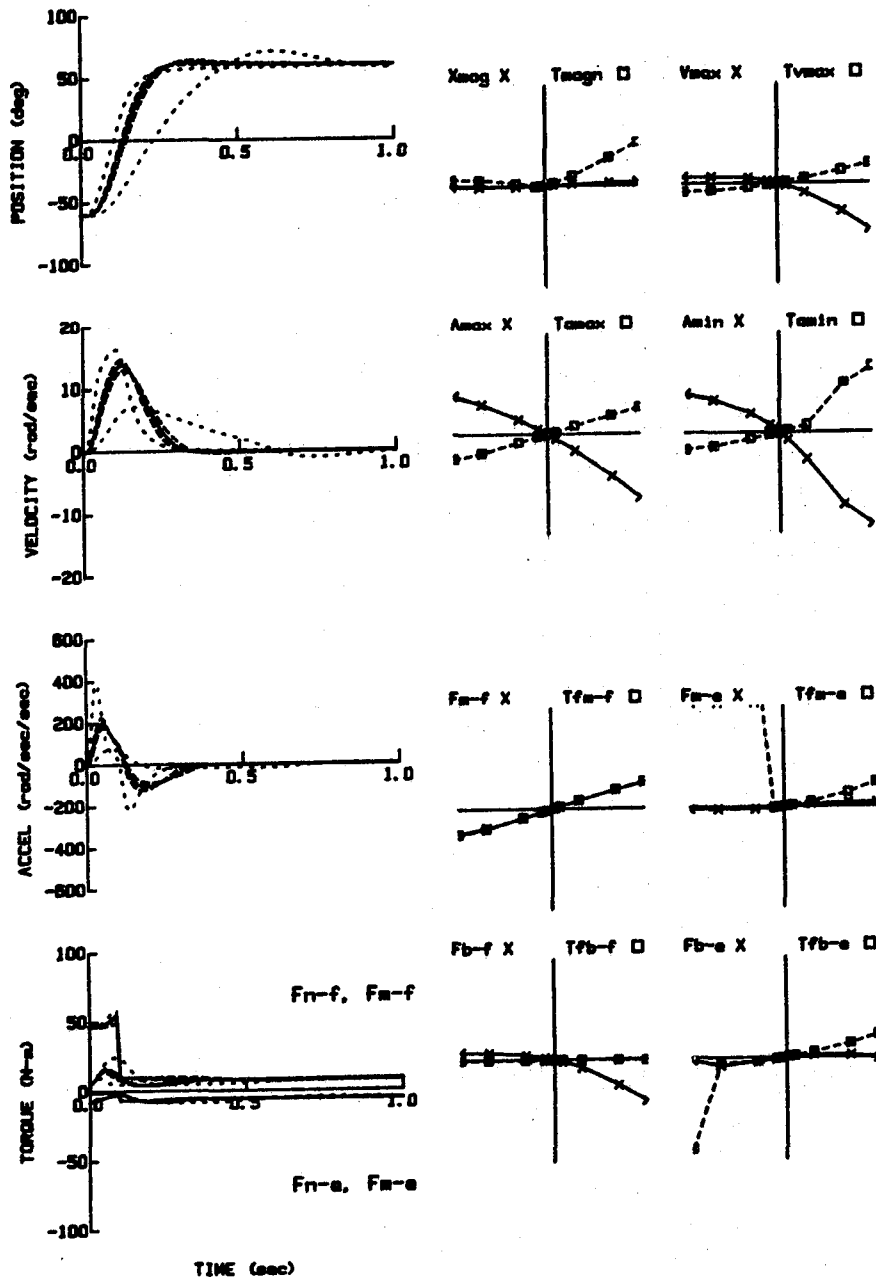


Figure 5a: Sensitivity Trajectories (left panel) and Sensitivity Graphs (right panel) for parameter  $J_p$  (plant inertia), Task #2. Large dash is for  $4/5$  and  $5/4$  times nominal, smaller dash is for  $1/5$  and  $5$  times nominal. For Sensitivity Graphs, range is  $1/10$  to  $10$ , on logarithmic scales, for both the parameter (abscissa) and behavior (ordinate).

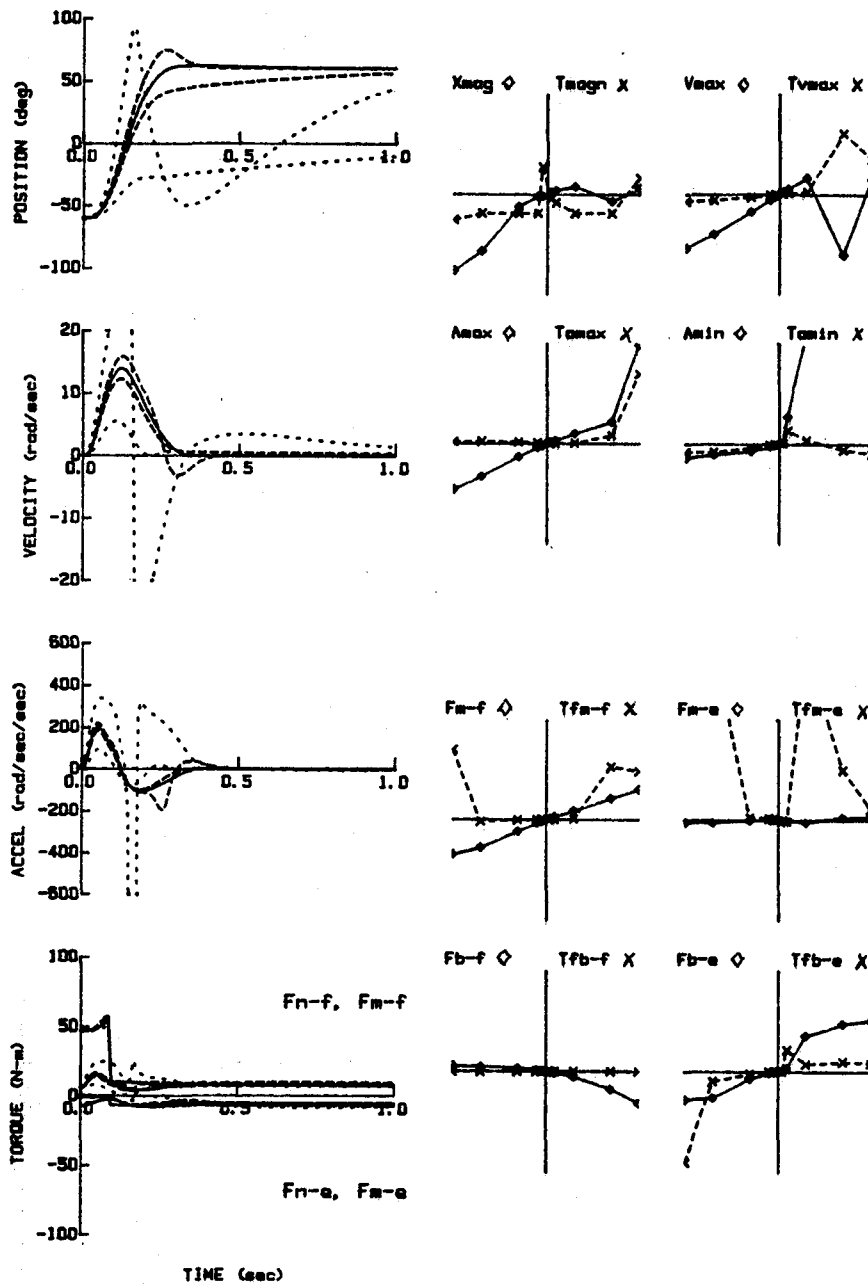


Figure 5b: Sensitivity Trajectories (left panel) and Sensitivity Graphs (right panel) for parameter  $B_h$  (one of the "Hill's" constants for shortening muscle). Large trajectory dashes: 4/5 and 5/4 of nominal, smaller dashes: 1/5 to 5 times nominal. Sensitivity Graph range: 1/10 to 10, logarithmic units, for both parameter (abscissa) and behavior (ordinate).

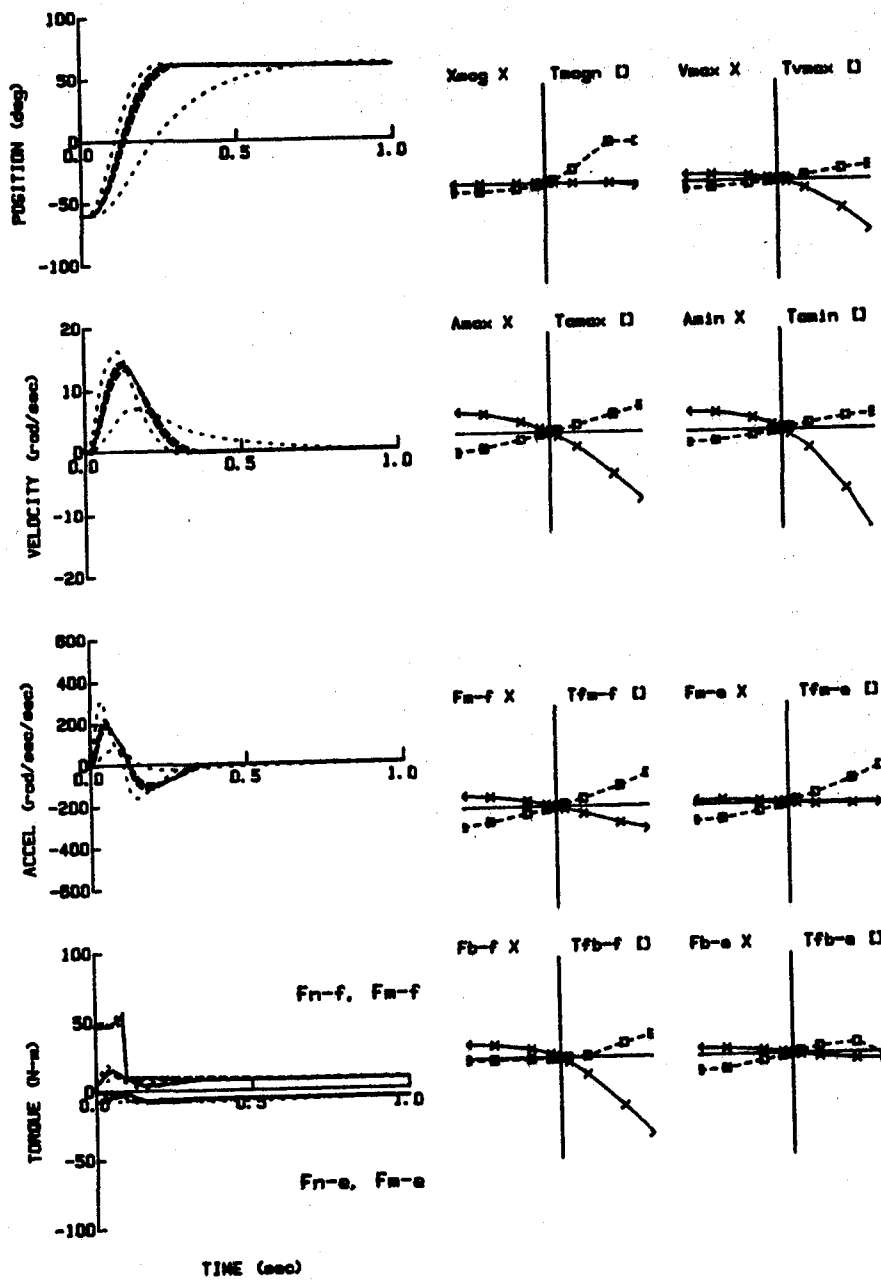


Figure 5c: Sensitivity Trajectories (left panel) and Sensitivity Graphs (right panel) for parameter  $Ta_1$  (one of the two time constants representing muscle activation). Large trajectory dashes: 4/5 and 5/4 times nominal, smaller dashes: 1/5 and 5 times nominal. Sensitivity Graph range: 1/10 to 10, logarithmic units, for both parameter (abscissa) and behavior (ordinate).

**Task #3: Maximum Voluntary Flexion with an External Load:** This task represents an extension of the previous task, only with the addition of an external "isotonic" load of 12 N-m and a 0.1 kg-m\*\*2 added inertia (a light bar being grasped). The task is based on some of the classical experimental data of Dern et al (1947), and the results are consistent with this work. The time of contraction is now 250 msec. Run dynamics are displayed in Figure 6a. An inspection of Table 4 shows that, once again, while many of the parameters are very similar, others differ sharply. Such observations guide further analysis of this task (not presented here).

In Figure 6b we see the sensitivity to a new type of parameter - a disturbance (i.e. the external load). Here the "base" external load of 12 N-m is varied as any other parameter would be. The sensitivity of the system to such an external load becomes well understood via inspection when all three methods are used. Notice that the factor of ten parameter change (to 120 N-m) is not attempted since it would injure a normal limb.

In Figure 6c and 6d we see the sensitivity to another new type of parameter: an initial condition. In this instance it is an initial against-movement velocity of 2 and 2- rad/sec. Notice that, in the absence of any initial inertial dynamics, the lengthening torque-velocity properties are able to easily compensate for the initial condition of Figure 6d, with the effect being negligible by the time of peak velocity, position or negative acceleration - only the peak positive acceleration is significantly affected, and then only for initial velocities of greater than 4 rad/sec. The sensitivity for the "with" initial velocity is also compensated for fairly well. These findings are in contrast to previous results for the fast, low inertia eye movement system (Winters et al (1983).

TABLE 4:

SENSITIVITY MATRIX FOR: Maximal Contraction Against External Load																	
	NOMINAL	Jp	Bp	Kp	Kpl	Ka1-f	Ka2-f	Af-f	Bb-f	Pmfv	Ta1	Ta2	PH-f	PH-e	EXT LOAD	Vo-	Vo+
Magn:	114 deg	0.011	-0.032	-0.013	-0.398	-0.001	-0.006	0.095	-0.268	-0.031	-0.018	-0.003	0.135	-0.175	-0.365	-0.021	0.021
Vmax:	565 d/s	-0.131	-0.059	-0.000	-0.575	-0.003	-0.017	-0.433	0.788	-0.015	-0.096	-0.016	0.536	-0.259	-0.401	-0.007	0.004
Amax:	5492 d/s/s	-0.574	-0.024	0.002	0.924	-0.006	-0.009	-0.129	0.368	-0.101	-0.419	-0.081	0.759	-0.184	-0.475	0.306	-0.210
Amin:	-4858 d/s/s	0.228	-0.183	0.028	1.000	-0.299	-1.201	-0.512	1.839	-0.907	1.480	0.159	1.814	-0.539	-1.507	-0.068	0.384
Tmag:	363 ms	0.268	0.056	-0.040	0.000	-0.005	0.019	0.000	-0.065	0.024	0.184	0.025	0.088	0.740	-0.196	0.038	-0.057
Tvmax:	209 ms	0.086	-0.003	0.000	0.078	0.000	0.028	-0.031	0.017	-0.003	0.052	0.039	-0.021	-0.010	-0.006	0.003	-0.006
Tamax:	59 ms	0.293	-0.010	-0.011	-0.192	-0.057	-0.274	-0.011	0.022	0.023	0.269	0.154	-0.123	-0.012	-0.069	-0.180	0.157
Tamin:	278 ms	0.285	-0.111	0.033	-	-0.020	-0.007	-0.061	-0.051	-0.113	0.141	0.031	-0.014	-0.108	-0.037	-0.097	-0.040
Pm-f:	9.1 N-m	0.157	0.002	-0.001	0.264	0.003	0.023	-0.064	0.294	0.017	-0.123	-0.028	0.536	0.036	0.170	0.133	-0.086
Pm-e:	7.1 N-m	0.029	0.000	0.019	3.388	0.001	0.000	0.006	0.060	0.079	0.000	0.000	-0.043	0.058	0.000	0.000	0.112
Pb-f:	14.4 N-m	-0.092	-0.026	-0.000	-0.569	-0.000	-0.004	0.066	-0.120	-0.007	-0.120	-0.020	1.260	-0.122	-0.317	0.009	0.000
Pb-e:	1.5 N-m	0.231	-0.048	-0.001	3.269	-0.002	-0.018	-0.051	0.838	0.392	-0.098	0.070	0.542	-0.003	-0.406	0.020	0.314
Pm-fe:	5.0 N-m	0.229	0.004	-0.012	3.614	-0.005	-0.018	-0.075	0.377	-0.006	-0.224	-0.041	1.220	-0.237	0.144	0.147	-0.078
Tfm-f:	54 ms	0.307	0.000	0.000	0.605	-0.060	-0.311	-0.012	1.196	0.024	0.235	0.151	-0.338	0.079	-1.380	-0.145	0.165
Tfm-e:	-	-	0.000	0.000	0.000	-	-	-	-	-	-	-	-	-	0.000	0.000	-0.085
Tfb-f:	201 ms	0.041	0.000	0.000	0.011	0.000	0.000	0.000	-0.041	0.000	0.056	0.003	-0.055	0.000	0.017	0.000	-0.017
Tfb-e:	273 ms	0.084	-0.275	0.000	-1.724	-0.275	-0.271	-0.193	0.150	-0.308	0.134	0.045	0.048	-0.284	0.698	-	-
Tfm-fe:	65 ms	0.279	0.010	0.000	-	0.010	-0.284	0.022	1.120	0.022	0.288	0.180	0.026	-0.011	-1.267	-0.236	0.151

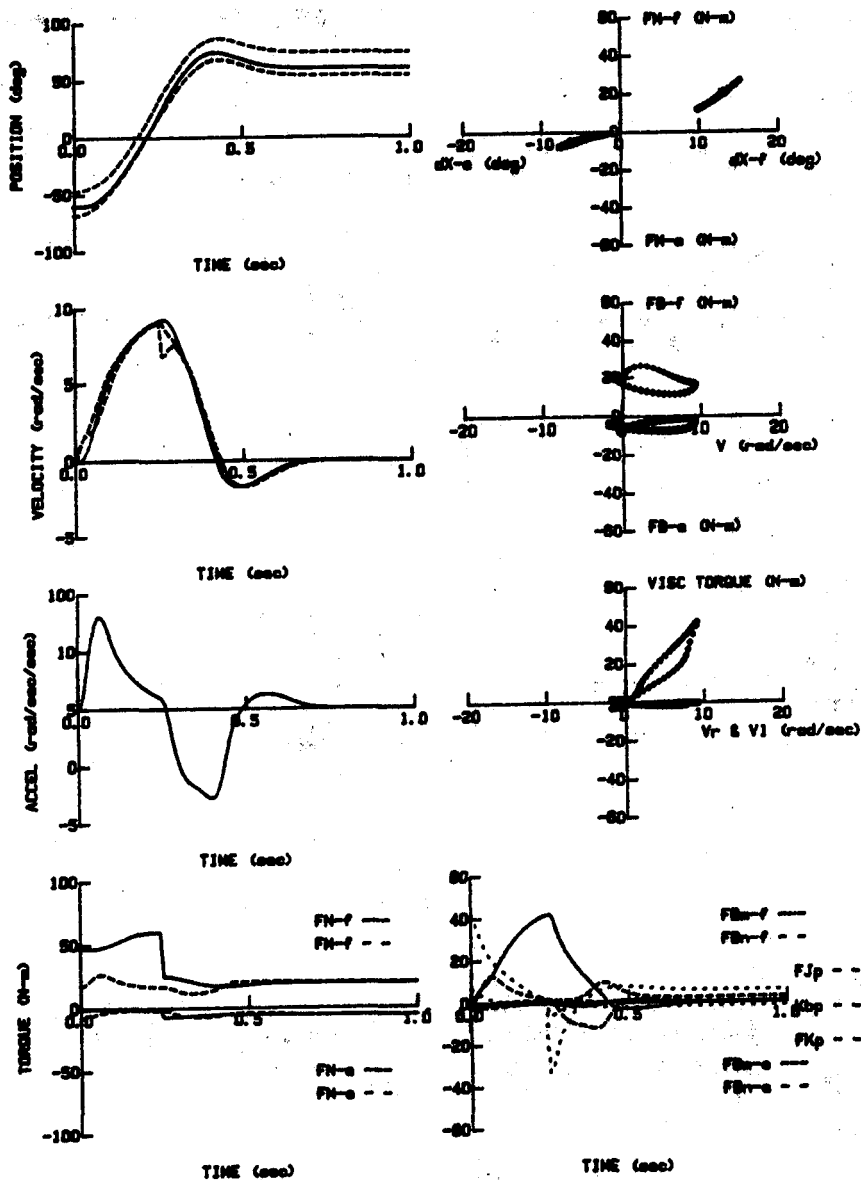


Figure 6a: Plots of input, output, and internal variables for the "normal" run of Task #3. Variables plotted are the same as for Figure 4. The input signal is a 250 ms pulse of magnitude 60 N-m for the flexor. An external load of 12 N-m (not graphed) exists throughout the movement. The steady-state input signals take this fact, plus the static torque-angle relations, into account. Notice the large torque lost to muscle viscosity.



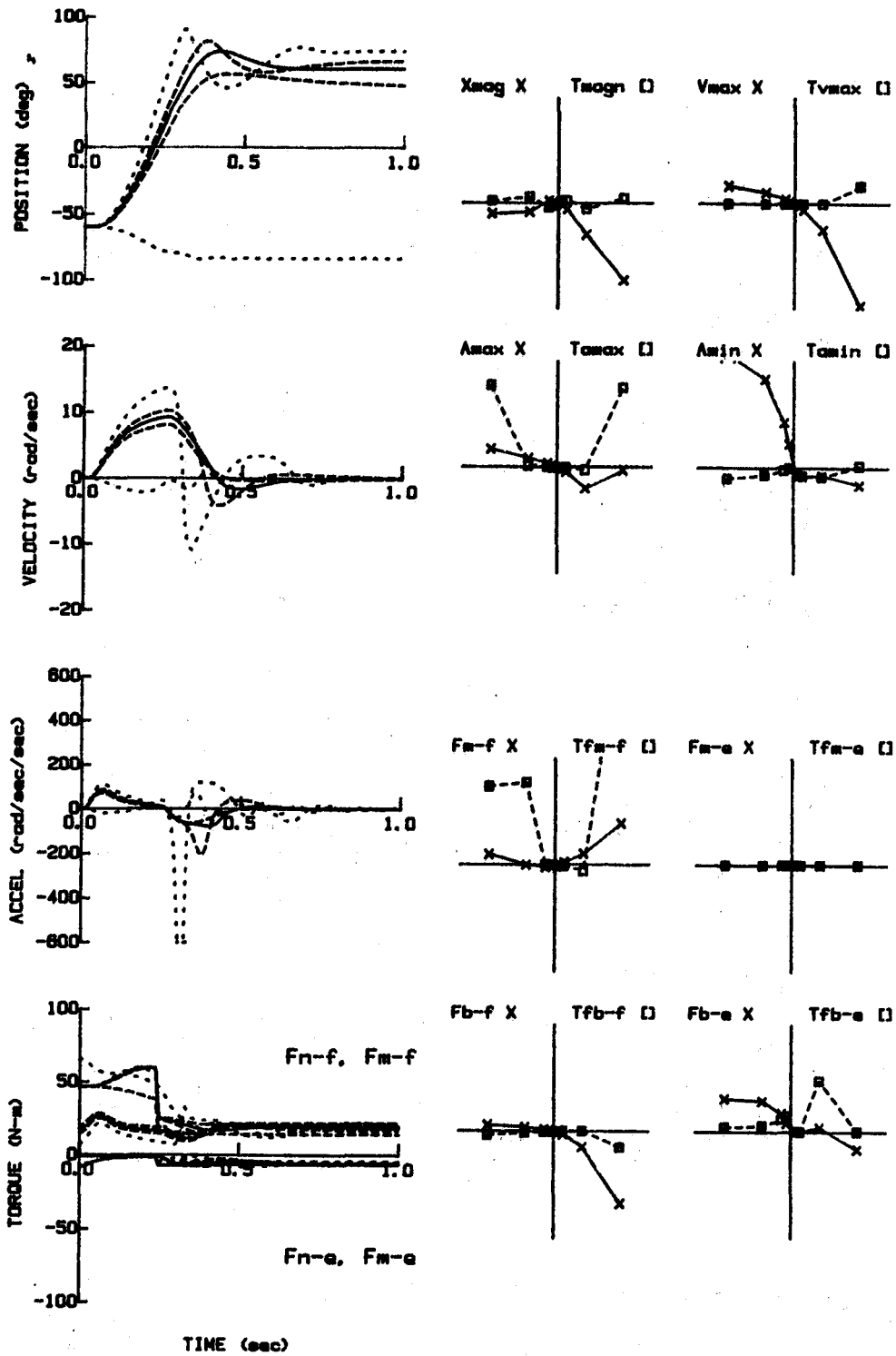


Figure 6b: Sensitivity Trajectories (left panel) and Sensitivity Graphs (right panel) for parameter "EXT LOAD", an external torque with a nominal value of 12 N-m. Variables plotted are the same as in Figure 5.

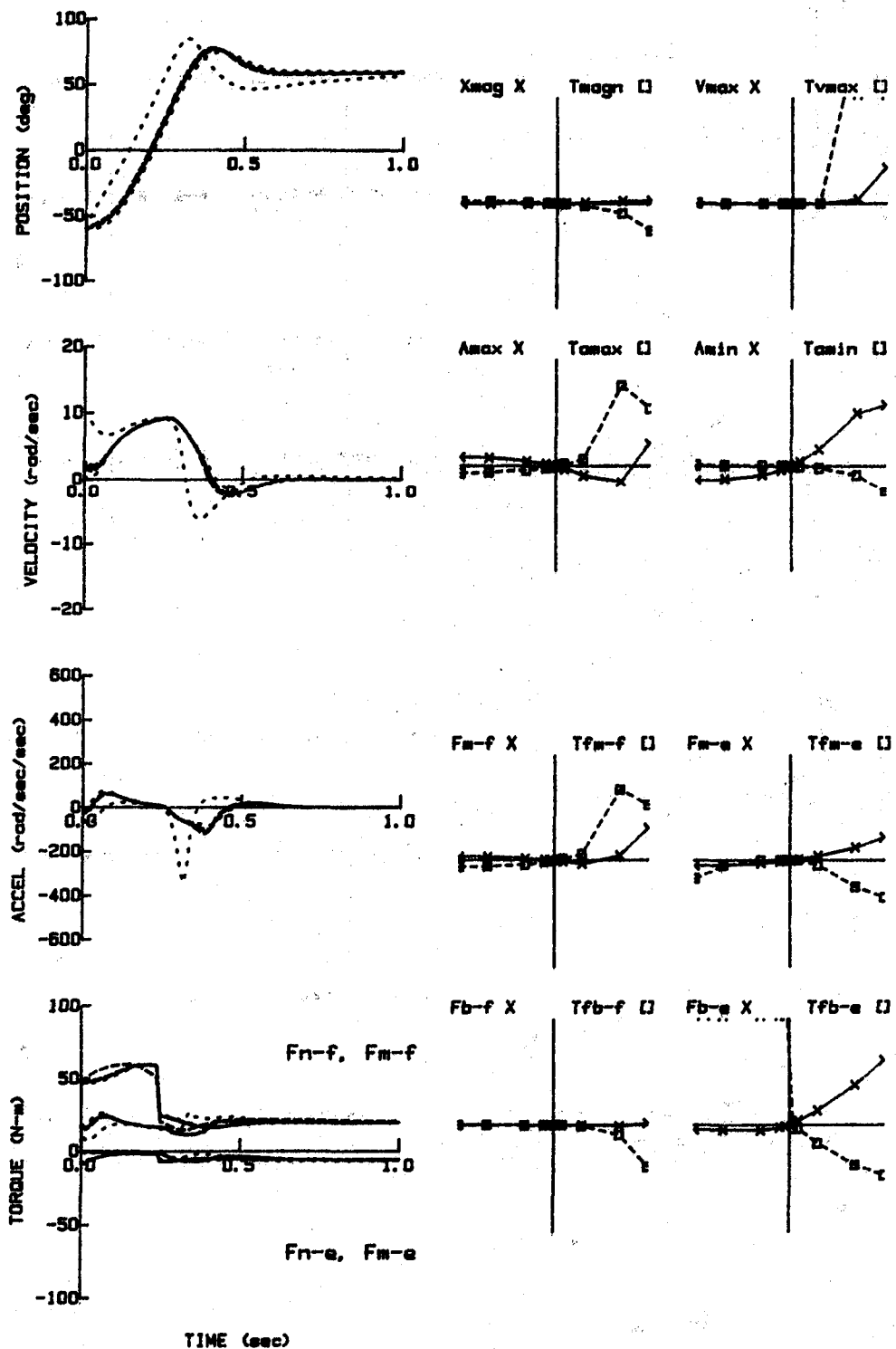


Figure 6c. Sensitivity Trajectories (left panel) and Sensitivity Graphs (right panel) for an initial velocity of 2 rad/sec (nominal). Axes ranges and behaviors plotted are the same as in Figure 5.

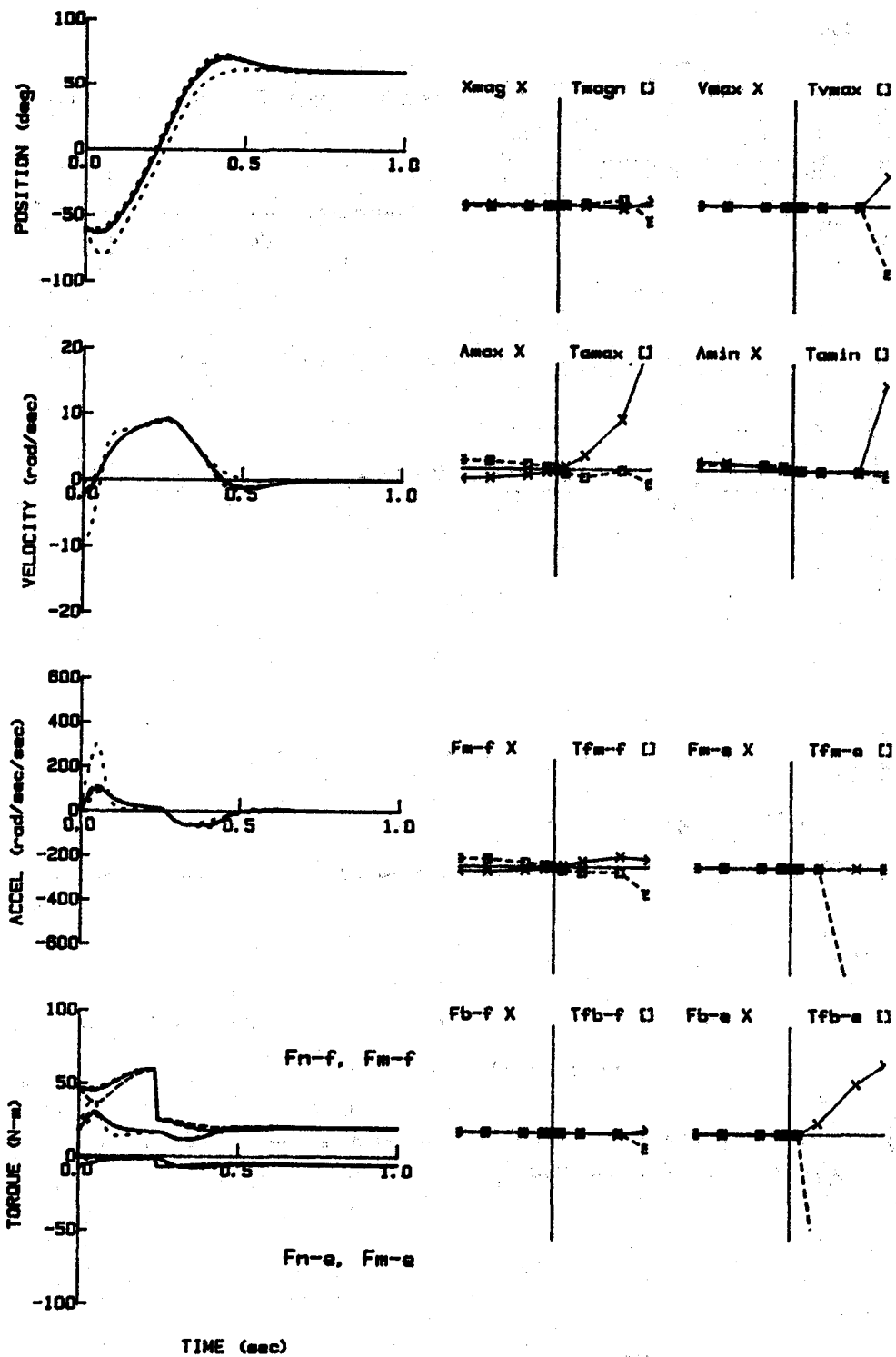


Figure 6d. Sensitivity Trajectories (left panel) and Sensitivity Graphs (right panel) for an initial velocity of -2 rad/sec (nominal). Axes ranges and behaviors plotted are the same as in Figure 5.

**Task #4: Maximal Isometric Contraction:** This is a fundamentally different type of contraction than those considered above. A maximal contraction is resisted by an equal and opposite external force. An interesting question is whether or not a different set of parameters become most important. The sensitivity matrix of Table 5 shows some sharp differences both in what parameters and in what behaviors are most important. First of all, since there is no movement, there is no change in overall position, velocity or acceleration, leaving only force propagation. As expected, the passive plant parameters  $J_p$ ,  $B_p$  and  $K_p$  have no effect. We now see that the series elasticity, as well as Hill's constants and the time constant of activation/deactivation, play a role. Also, because, for this task of contraction, the hypothetical internal muscle node is shortening, the lengthening muscle parameters like  $F_m-f_v$  also do not play a role until after 500 msec.

Inspection of the sensitivity matrix indicates that only the viscous torque behavior seems to be effected. This result is misleading, as can be seen from the sensitivity trajectory for  $K_{s2}-f$  (Figure 7). This example shows one of the major limitations of sensitivity coefficients and graphs: in the process of extracting useful information, other information is invariably lost, and, furthermore, the information extracted can sometimes be misleading. It turns out that these observations are particularly true for the series elastic parameters, which tend to primarily effect movement with high frequency components, such as movement initiation or voluntary or involuntary limb oscillation. Thus, subtle information on trajectory shapes can be lost when sensitivity coefficients and graphs are restricted to behaviors such as peak output values and the corresponding time of peak values. Although such phenomena are difficult to define by the "behaviors" presented here, they are possibly describable by other types of behavior definitions, such as oscillation frequency. In any case, sensitivity trajectories must be plotted.

TABLE 5:

SENSITIVITY MATRIX FOR TASK: "Maximal Voluntary Isometric Contraction":

		$J_p$	$B_p$	$K_p$	$K_{s1}-f$	$K_{s2}-f$	$A_f-f$	$B_h-f$	$F_m-f_v$	$T_{a1}$	$T_{a2}$
$F_m-f$ :	60.0 N-m	0	0	0	0.000	0.000	0.000	0.000	0	-0.002	0.000
$F_b-f$ :	13.2 N-m	0	0	0	-0.184	-0.727	0.278	-0.727	0	-0.610	-0.135
$T_{fm}-f$ :	500 ms	0	0	0	0.000	-0.001	0.000	-0.001	0	0.289	0.000
$T_{fb}-f$ :	43 ms	0	0	0	-0.017	-0.360	0.132	-0.360	0	0.334	0.292

**Task #5: A Simple External Load, With No Neural Control:** Here we have a steady neural input signal of 6 N-m for both the flexor and extensor groups. An external load of 12 N-m is applied for 200 msec, and no effort is made to resist this disturbance via neural feedback. Inspection of the resulting sensitivity matrix (Table 6) shows again that the relative sensitivity of the various parameters and behaviors is a function of the task.

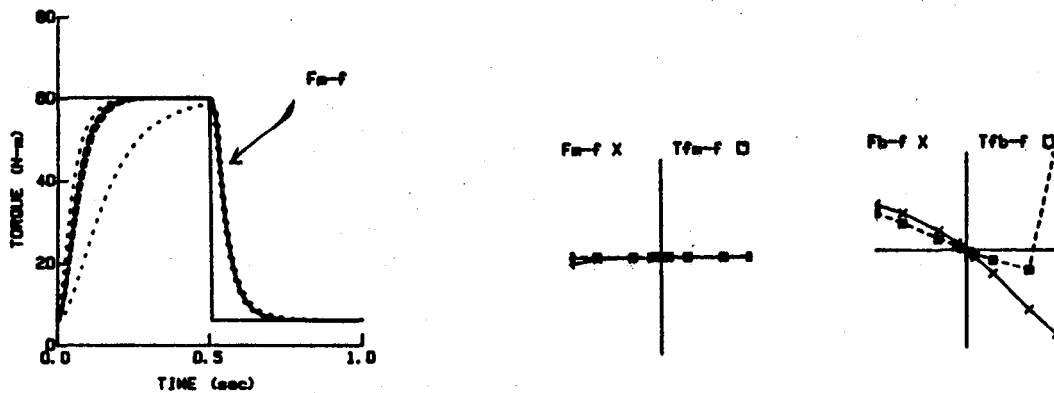


Figure 7: Sensitivity Trajectory for parameter  $K_{s2}$ , a series elastic fit parameter (left panel). Small dashed is for 1/5 and 5 times nominal - note lack of symmetry. Sensitivity Graphs (right panel) are for the same parameter. Ranges are from 1/10 to 10, in logarithmic units, for both the parameter (abscissa) and the behaviors (ordinate). Note the lack of information for  $F_{m-f}$ .

TABLE 6:

SENSITIVITY ANALYSIS FOR: External Load with no Active Resistance

	NOMINAL	$J_p$	$B_p$	$K_p$	$K_{p2}$	$K_{s1-f}$	$K_{s2-f}$	$A_{f-f}$	$B_{h-f}$	$F_{mfv}$	$T_{al}$
Magn:	49.4 deg	-0.172	-0.154	-0.142	-0.093	-0.015	-0.032	-0.164	<b>0.394</b>	-0.375	-0.010
Vmax:	267.5 d/s	-0.181	<b>-0.176</b>	-0.147	-0.024	-0.008	-0.018	<b>-0.188</b>	<b>0.437</b>	-0.405	-0.013
Amax:	5730 d/s/s	<b>-1.000</b>	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.001	0.000
Amin:	-5742 d/s/s	<b>-1.108</b>	0.018	0.078	0.816	-0.004	-0.010	0.018	-0.032	0.019	0.012
Tmagn:	266 ms	<b>0.277</b>	-0.030	-0.101	0.000	0.014	0.022	-0.049	0.100	-0.095	-0.008
Tvmax:	196 ms	<b>0.468</b>	-0.131	<b>-0.275</b>	<b>-0.223</b>	<b>0.033</b>	0.076	-0.158	0.335	-0.264	-0.057
Tamax:	1 ms	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	<b>0.001</b>	0.000
Tamin:	200 ms	<b>0.004</b>	0.000	0.000	0.003	0.001	0.000	0.000	0.000	0.000	0.000
$F_{m-f}$ :	6.00 N-m	0.001	0.000	0.030	<b>0.115</b>	0.000	0.000	0.005	-0.033	0.041	0.000
$F_{m-e}$ :	7.47 N-m	-0.018	-0.006	-0.030	-0.000	-0.001	-0.003	-0.006	0.017	<b>0.183</b>	0.003
$F_{b-f}$ :	2.87 N-m	-0.135	-0.094	-0.062	-0.004	-0.003	-0.008	0.165	<b>-0.402</b>	-0.234	0.011
$F_{b-e}$ :	1.51 N-m	-0.064	-0.039	-0.023	0.385	-0.004	-0.009	-0.041	0.101	<b>0.711</b>	0.006
$F_{m-fe}$ :	4.38 N-m	-0.110	-0.074	-0.048	-0.002	-0.003	-0.008	0.096	<b>-0.227</b>	0.173	0.009
$T_{fm-f}$ :	2 ms	-	0.000	-	-	0.000	0.000	-	-	-	0.000
$T_{fm-e}$ :	101 ms	<b>0.516</b>	-0.014	-0.051	-0.007	0.029	0.065	-0.051	0.115	0.098	<b>0.107</b>
$T_{fb-f}$ :	154 ms	<b>0.518</b>	-0.071	-0.182	-0.048	0.019	0.038	-0.087	0.226	-0.136	0.047
$T_{fb-e}$ :	135 ms	<b>0.591</b>	-0.043	-0.133	<b>0.418</b>	0.032	<b>0.077</b>	-0.072	0.165	<b>-0.858</b>	0.069
$T_{fm-fe}$ :	150 ms	<b>0.530</b>	-0.064	-0.169	-0.045	0.019	0.044	-0.074	0.196	-0.139	0.048

In Figure 8 the sensitivity trajectories and graphs are obtained for an interesting parameter that is not well understood. This is " $F_{m-fv}$ ", a torque-velocity parameter, discussed earlier, that influences only lengthening muscle (by giving the lengthening muscle torque eccentric torque saturation value, nominally 30% above isometric for any given activation level). The fact that this parameter is significant shows that, for this task, the constitutive relation used for the lengthening extensor muscle group is important.

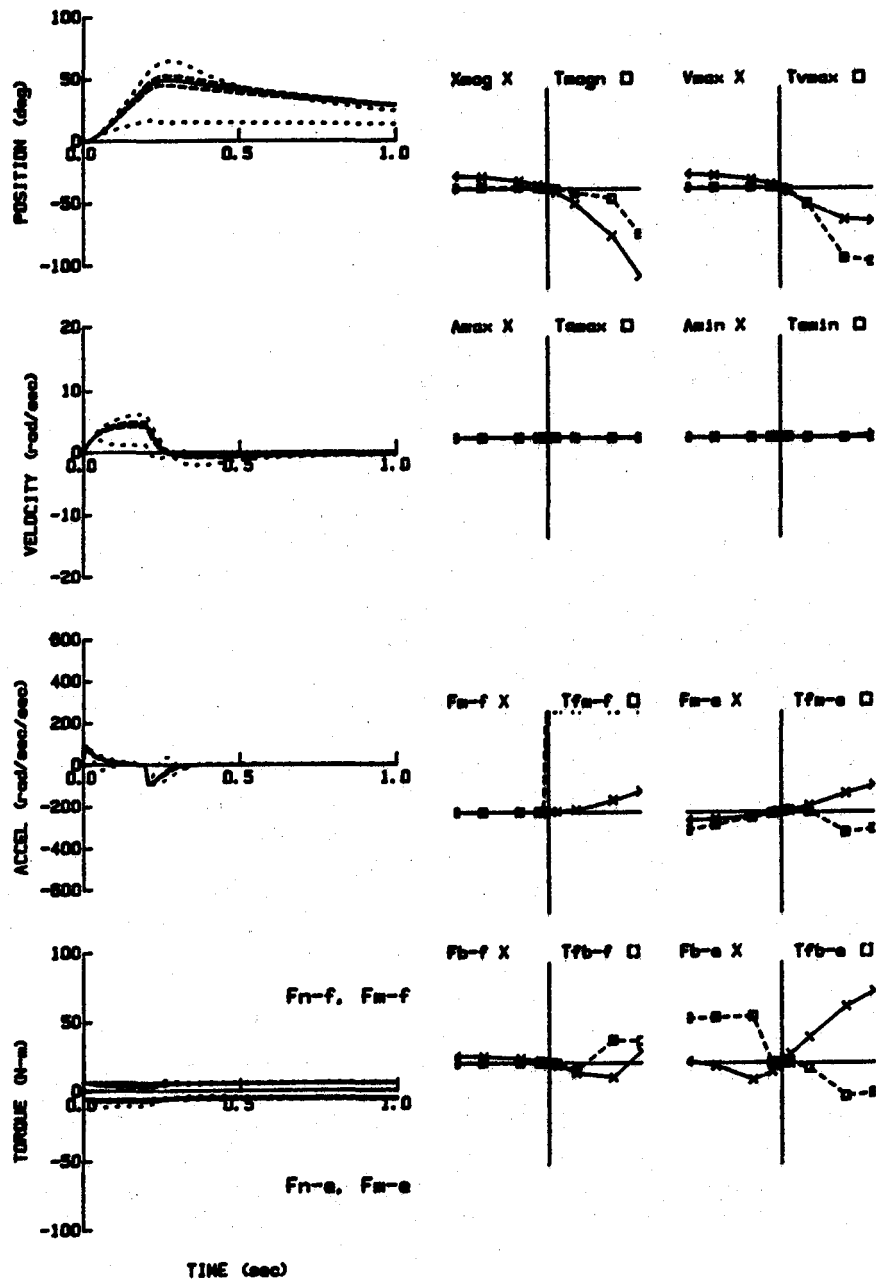


Figure 8: Sensitivity Trajectories (left panel) and Sensitivity Graphs (right panel) for parameter  $F_{m-fv}$ , a torque-velocity parameter for lengthening muscle. This parameter gives the percentage above isometric that a muscle can contract when lengthening at medium-to-high velocity, i.e. the saturating torque for lengthening muscle. Trajectory ranges, axes labeling, and axes ranges are as in Figure 5.

## DISCUSSION:

The five simple tasks considered above barely scratch the surface of all the potential movements of the elbow joint. These movements seemed representative of the range of possibilities. Furthermore, the neural inputs were purposely kept simple for the sake of clarity of presentation.

In reality, however, neural inputs - as well as system output, are more complex. Over the last few decades a large amount of research has been done on upper limb movement, including elbow movements performed both in isolation and in conjunction with movements of other joints. Much of this data is for athletic performance. Components of such data can be simulated using JAMM. This is possible because the model contains all of the parameters that are needed to describe all basic nonlinear muscle properties - the model is purposely constructed to be able to simulate a the full range of types of tasks seen in the literature.

By using sensitivity analysis the parameters of primary importance can be determined for any particular task. This provides insight into the movement task under analysis. It also suggests ways for task-specific model reduction, if desired.

Another area of interest is the sensitivity analysis protocol. Once a task for analysis is chosen, the following steps were found to be represent an optimal protocol: First, a **sensitivity matrix** is constructed, the size of which depends on the parameters and behaviors of interest. The coefficients in the matrix are usually best found using the "logarithmic" method. This gives one a global view of model performance and furthermore guides one to the areas of interest for more detailed work. Second, **sensitivity trajectories** are used to help visualize the effect of a given parameter. Sensitivity Trajectories are also a good way to scan for problems in coefficient values. Finally, sensitivity graphs are of considerable use in getting a feel for the model behavior as the parameter is varied over a wider operating range. Such information often described the potential tolerable range of the parameter and also the linearity of the change in behavior with change in parameter. As such it also can show the extent of the sensitivity matrix's sensitivity to range used for coefficient determination.

There are a couple of observations worth noting: First, all three methods have weaknesses. Second, one of the main advantages of using such a variety of methods is that the weaknesses of any particular method are exposed by the other methods. Consequently, each method gains strength when combined with the others. Third, the work presented here was only for elbow flexion-extension. These methods are currently being found equally valuable for knee flexion-extension, wrist flexion-extension, and eye, head and wrist rotation.

### CONCLUSION:

A number of sensitivity analysis tools have been used on a highly developed model of elbow flexion-extension. It has been found that maximum insight into both model performance and parameter sensitivity appears to require a systematic protocol that employs a variety of sensitivity tools. Each of the methods is strengthened when used in conjunction with the other sensitivity tools. Furthermore, it has been found that the relative sensitivity of the model parameters is a function of the task being studied. Finally, it is suggested that sensitivity analysis should be the cornerstone for task-specific model reduction.



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