N84 27280

THE DUAL-FREQUENCY SCATTEROMETER RE-EXAMINED

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ABSTRACT

We demonstrate here that the utility of dual-frequency scatterometers in measuring ocean wave directional spectra can be increased by adding a third frequency to the system. We show that the background which effectively limits signal detectability in dual-frequency operation can be made a part of the signal through the addition of this third frequency. Thus signal detectability is limited only by system thermal noise and space-based operation becomes more feasible.

1. INTRODUCTION

The operation of the dual-frequency scatterometer when receiving backscatter from the ocean surface is now well understood. It has been shown, both experimentally and theoretically, that the power spectrum of the received signal consists of a sharp line which obeys an ocean gravity wave dispersion relation superimposed on a broad background spectrum resulting from the convolution of the doppler spectra from the two transmitted frequencies (Plant 1977). This is illustrated in Figure 1.

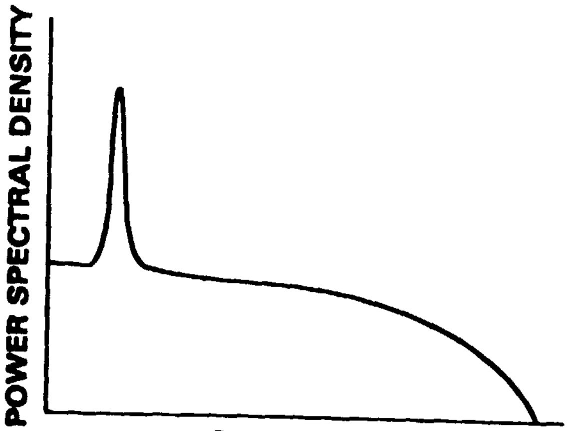

Figure 1 Power spectrum of the output of a dual-frequency scatterometer.

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The intensity of the sharp line is a measure of the amplitude of the ocean wave whose frequency corresponds to that of the line. The background spectrum, however, is essentially independent of this wave amplitude. Thus the proper measure of the detectability of ocean waves by the dual-frequency scatterometer is the ratio of the integral of the sharp line to the integral of the background spectrum. This ratio χ is well described in deep water by the following equation:

$$\chi = \frac{2\pi^2 |m|^2 S}{A} \quad (1)$$

where m is the modulation transfer function, S is the wave slope spectral density evaluated at constant wavenumber and A is the illuminated area (Plant and Schuler, 1980). Alpers and Hasselmann (1978) have shown that even if the background spectrum is low-pass filtered to increase χ by the ratio of the natural bandwidth to the filtered bandwidth, a typical value for χ from satellite altitudes would be 1.8. Thus the detectability of the wave is quite low under these conditions.

In this paper we will show that if the number of transmitted frequencies is increased to three and if proper processing of the received signal is performed then this constraint on wave detectability can be removed. We will show that if the three transmitted frequencies are equally spaced, then the background signal is also proportional to the amplitude of the detected ocean wave. Under these conditions the proper measure of wave detectability is the ratio of the intensity of the sharp line plus the background intensity to the system thermal noise. Thus ocean waves may be much more easily measured using such a three-frequency system.

2. THEORY

The concept is most easily approached by considering four transmitted frequencies as shown in Figure 2.

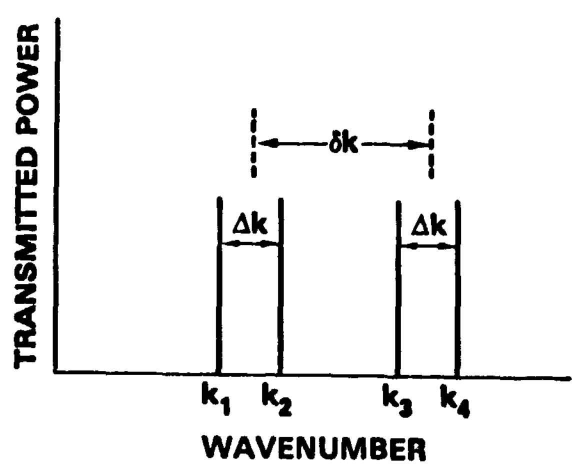

Figure 2 Transmitted signals used to generalize dual-frequency operation.

Transmitted wave numbers will be k_1, $k_2=k_1+\Delta k$, $k_3=k_1+\delta k$, $k_4=k_2+\delta k$. Back-scatter from the ocean may be most conveniently described in terms of a two-scale model and a complex reflectivity. Thus the field E_i received due to any one of our transmitted frequencies (whose wavenumber is $\underline{k}_i$ and whose Poynting vector is unity at the surface) may be represented by,

$$E_i(t) = \int r(\underline{x},t)\, e^{i2\underline{k}_i \bullet \underline{x}}\, d\underline{x} \tag{2}$$

where $\underline{x}$ is a horizontal position vector, t is time, and $r(\underline{x},t)$ is the complex reflectivity. The standard assumption, which is supported experimentally, is that the covariance function of this complex reflectivity may be represented by

$$\begin{aligned} \langle r(\underline{x},t)\; r^*(\underline{x}+\zeta,t+\tau)\rangle &= \sigma_o(\underline{x},t)R(\underline{x},t,\tau)\delta(\zeta) \\ \langle r(\underline{x},t)\; r(\underline{x}+\underline{\zeta},t+\tau)\rangle &= 0 \end{aligned} \tag{3}$$

where σ_o is the normalized radar cross section, R is the autocorrelation function so that $R(x,t,0) = 1$, $\delta(\zeta)$ is the δ-function and $\langle\rangle$ represents an ensemble average. Using (2) and (3), we have,

$$\langle E_i(t)\; E_j^*(t+\tau)\rangle = \int \sigma_o(\underline{x},t)\; R(\underline{x},t,\tau)\; e^{i2(\underline{k}_i - \underline{k}_j).\underline{x}}\, d\underline{x}. \tag{4}$$

Experiment shows that R falls to zero rapidly with τ. Since R=1 at $\tau=0$, a good first order approximation for our purpose here is to ignore the dependence of R on x and t. Then (4) may be written

$$\langle E_i(t)E_j^*(t+\tau)\rangle = R(\tau)\,\sigma_o(\underline{K},t) \tag{5}$$

where $\underline{K}=(2(k_i-k_j)\cos\theta,\ 0)$ and θ is the grazing angle. If $\tau=0$ and i=j, Equation (4) is simply a definition of the radar cross-section.

In processing dual-frequency scatterometer return, the two received signals are multiplied together and a power spectrum of the product is computed. Formally, the power spectrum is obtained by multiplying the product by a time-delayed version of itself, averaging over time and transforming with respect to the lag variable. Here we generalize by letting the time-lagged signal be the product of the third and fourth transmitted signals. That is we cross-correlate the products $E_1E_2^*$ and $E_3E_4^*$ rather than autocorrelate $E_1E_2^*$. Gaussian statistics hold adequately for short time intervals so we may write,

$$\langle E_1 E_2^*(t) E_3^* E_4(t+\tau)\rangle = \langle E_1 E_2^*\rangle\langle E_3^* E_4\rangle + \langle E_1 E_3^*\rangle\langle E_2^* E_4\rangle + \langle E_1 E_4\rangle\langle E_2^* E_3^*\rangle. \tag{6}$$

The last term is zero by virtue of (3) while the other terms yield,

$$\langle E_1 E_2^*(t) E_3^* E_4(t+\tau)\rangle = \tilde{\sigma}_o(\Delta \underline{k},t)\tilde{\sigma}_o(\Delta \underline{k}, t+\tau) + R^2(\tau)\tilde{\sigma}_o^{\,2}(\delta \underline{k},t) \tag{7}$$

Here $\Delta \underline{k} = (2\Delta k\cos\theta, 0)$, $\delta \underline{k} = (2\delta k\cos\theta, 0)$.

Averaging over t and transforming with respect to τ yields the cross-power spectrum,

$$P(f) = \int \overline{\tilde{\sigma}_o(\Delta \underline{k},t)\tilde{\sigma}_o(\Delta \underline{k},t+\tau)}\, e^{i2\pi f\tau}\, d\tau + \tilde{R}^2(f)\, \overline{\tilde{\sigma}_o^{\,2}(\delta \underline{k},t)} \tag{8}$$

where overbars are time averages, and $\tilde{R}^2(f)$ indicates the Fourier Transform of $R^2(\tau)$. Since the pattern of σ_o moves along with the surface wave, the first term yields the same sharp line as the standard dual-frequency method. Simil_rly, the second term yields the broad background term; if $\delta k=0$, it is identical to the standard term. In general, however, $\delta k \neq 0$ and the second term is not proportional to the mean square value of $\sigma_o(t)$ but to the power spectral density of $\sigma_o(\underline{x},t)$ evaluated at $\delta \underline{k}$. Figure 3 illustrates this difference.

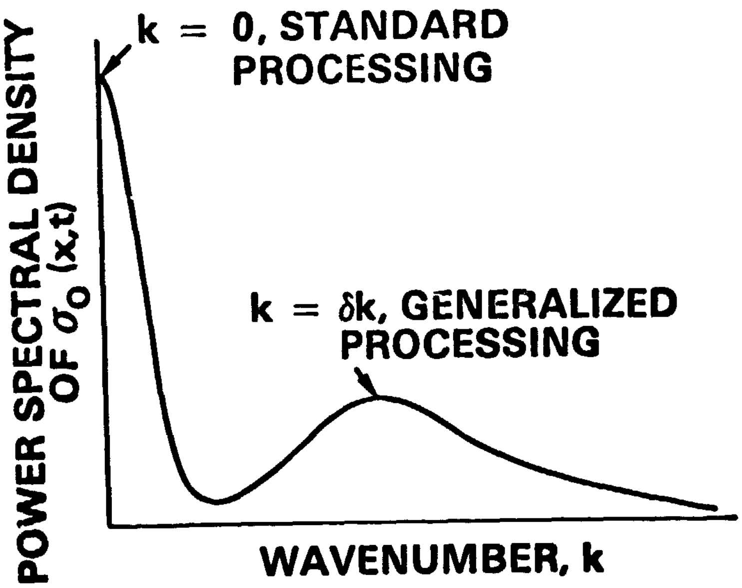

Figure 3 Wavenumber spectrum of $\sigma_o(\underline{x},t)$ showing wavenumbers at which the background signal is evaluated using standard processing and generalized processing.

If we integrate Equation (8) over frequency, we get,

$$P = \overline{\tilde{\sigma}_o^{\,2}(\Delta \underline{k},t)} + \overline{\tilde{\sigma}_o^{\,2}(\delta \underline{k},t)} \tag{9}$$

In particular, if $\Delta \underline{k} = \delta \underline{k}$ so that three equally-spaced signals are transmitted, then

$$P = 2\,\overline{\tilde{\sigma}_o^{\,2}(\Delta \underline{k},t)}. \tag{10}$$

Since this spectral density of σ_o evaluated at $\Delta \underline{k}$ is proportional to the wave slope spectral density through the transfer function m, the entire received signal is proportional to the intensity of the ocean wave of wave number $\Delta \underline{k}$ and the background signal has become part of the desired signal.

3. EXPERIMENT

We have experimentally evaluated the ratio $x_a^{-1} = \overline{\sigma_o^{\,2}(\delta \underline{k},t)}/\overline{\sigma_o^{\,2}(\Delta \underline{k},t)}$ for a fixed $\Delta \underline{k}$ using measurements taken on the Chesapeake Bay. The results of this study are presented in Figure 4 along with the standard dual-frequency

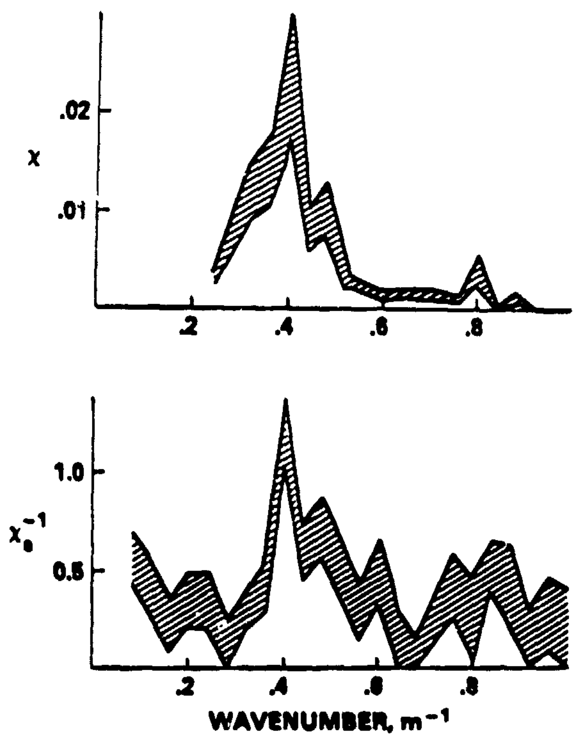

Figure 4 Normalized wavenumber spectrum of $\sigma_o(x,t)$ obtained on April 15, 1981 on the Chesapeake Bay. The top graph was obtained from the intensity of the standard dual-frequency resonance line. The lower graph was produced by integrating the background obtained with the generalized processing scheme. x and x_a^{-1} are defined in the text.

output $x = \overline{\tilde{\sigma}_o^2(\Delta k,t)}/\overline{\tilde{\sigma}_o^2(0,t)}$ as a function of Δk. Both the integrated background signal in the generalized method (x_a^{-1}) and the integral of the sharp peak in the standard method (x) indicate that a 15 meter dominant wave was present on the surface. This result shows that the background in the generalized method is proportional to surface wave amplitude, in agreement with the above analysis. A paper describing these measurements in detail is currently in preparation.

4. CONCLUSION

The three-frequency scatterometer described here is perhaps the most promising technique for the measurement of ocean wave directional spectra from space. Unlike dual-frequency or short-pulse spectrometers, signal detectability is limited only by thermal noise. Unlike synthetic aperture techniques, the ocean wave spectrum can be obtained from the system output through a linear transfer function. We suggest that a program to check system performance by operating a three-frequency scatterometer from an aircraft should be undertaken as soon as possible.

5. REFERENCES

Alpers, W. and K. Hasselmann (1978), The Two Frequency Microwave Technique for Measuring Ocean-Wave Spectra from an Airplane or Satellite, Boundary Layer Meteorol., 13, 215-230.

Plant, W. J. (1977), Studies of Backscattered Sea Return with a CW, Dual-Frequency, X-Band Radar, IEEE Trans. Antennas Propagat., AP-25, 28-36.

Plant, W. J. and D. L. Schuler (1980), Remote Sensing of the Sea Surface Using One- and Two-Frequency Microwave Techniques, Radio Science, 15, No. 3, 605-615.