2.5A THE ΔR VS $(\Delta R)^2$ QUESTION - THE PULSE-LENGTH DEPENDENCE OF SIGNAL POWER FOR FRESNEL SCATTER

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It has been proposed that the enhanced echoes from the atmosphere observed with a vertically pointing radar are due to reflections from horizontally stratified layers. The general case in which there are many closely spaced layers at random heights has been called "Fresnel scatter". The variation of received power with transmitter pulse length is examined for various models of Fresnel backscatter. It is shown that for the model most often used in previous work, the power is proportional to the pulse-length (Δr), and not to the pulse length squared. However, for more general models a pulse-length dependence more complex than either (Δr) or $(\Delta)^2$ is found.

1. INTRODUCTION

Radar backscatter at VHF from the troposphere and stratosphere shows at times evidence of weak partial reflections from extended horizontal irregularities. These irregularities are at least a Fresnel zone in horizontal extent, and fluctuate in the vertical by less than about $\lambda/8$ over this horizontal distance. Here, λ is the radar wavelength. This type of reflection is in addition to scatter due to turbulence-induced irregularities (e.g., GAGE and GREEN, 1978; ROTTGER and LIU, 1978; ROTTGER, 1980a). GAGE et al. (1981a) have proposed that these scatterers occur at random heights in the atmosphere, and have then, using this simple assumption, proceeded to determine the expected dependence of backscattered power on the radar and atmospheric parameters. GAGE et al. (1981a) will be denoted by GBG here. The model was also discussed in GAGE and BALSLEY (1980), GREEN and GAGE (1980), GAGE et al. (1981b) and BALSLEY and GAGE (1981). The formula which GBG produced took the form

$$P_R = \frac{\alpha^2 P_t A^2}{4\lambda^2 r^2}[F(\lambda).\bar{M}]^2(\Delta r)^2 \qquad (1)$$

This formula is only relevant for the case in which the same array is used for both transmission and reception. P_R is the received power, α is the array efficiency, P_t is the peak transmitted power, A_e is the array effective area, λ is the radar wavelength, r is the range of the scatterers, M is the mean generalized refractive index gradient, and $F(\lambda)$ is a "calibration constant" which must be determined empirically for each radar. The term (Δr) represents the pulse width.

Most of equation (1) is intuitively reasonable, but the $(\Delta r)^2$ terms appears to be odd. In this paper, the procedures adopted in obtaining this $(\Delta r)^2$ dependence will be carefully re-examined. It will be shown that there were errors in this original formulation, and that a proper treatment leads to a (Δr) dependence.

This paper will primarily present the arguments for and against the $(\Delta r)^2$ formula, although some mention will be made of generalizations of the Fresnel model. A more complete discussion has been presented elsewhere (HOCKING and ROTTGER, 1983).

2. PHYSICAL PICTURE

A simplified view of the model presented in GBG is presented in Figure 1a. We will begin by discussing this simple model, and then will generalize it to gather complexity.

Imagine that a square pulse of duration ΔT is transmitted upwards into the atmosphere, and at some time $t_o/2$ the pulse is centred at a height z_o (t_o is the time for the pulse centre to go to height z and be reflected back to the ground). Consider a vertical region of length $c\Delta T/2$, centred on z_o, and assume that within this volume, there are seven reflectors, of equal reflection coefficient but at random heights. Each reflector will reflect the pulse for a time duration ΔT, and at time $t_o = 2z_o/c$ (where c is the speed of the radio waves), some part of the pulse will arrive back at the ground from each of these 7 reflectors. No signal will arrive at time t_o from reflectors outside of this region. (i.e., The received signal is a convolution between the pulse shape and the reflection coefficient profile.) The seven reflected signals will have approximately equal strengths, but because the reflectors have random heights, each signal will arrive back at the ground with random phase. The resultant signal may be described by the dark vector in the right-hand diagram of Figure 1a; that is, it is the sum of 7 vectors of equal strength but random phase. This is simply the classical two-dimensional random walk problem, as first described by RAYLEIGH (1894). It is well known that the modulus of the resultant vector of the two-dimensional random walk problem has a "Rayleigh distribution", and that the mean square length of the resultant vector is proportional to the number of contributing vectors. Thus if we double the pulse length to $\Delta T'$ and the mean number of reflectors per unit height remains the same at all heights, then we have approximately 14 reflectors in the new length $C\Delta T'/2$ in Figure 1a. As a result, an approximate doubling of the square of the resultant vector can be expected when the pulse length is doubled.

Of course in the above discussion we dealt with small numbers of randomly phased vectors, and strictly speaking the Rayleigh distribution is only relevant for large numbers of vectors. Nevertheless, even for the cases of these small numbers of vectors, the mean power is still proportional to the number of vectors, provided that the reflectors are allowed to fluctuate vertically in time (so that each reflected component has a uniform phase distribution between 0 and $2\pi^c$), and that the mean power is calculated over a long time interval. Naturally, however, the fluctuation in power about the true mean (relative to the true mean) will be smaller when larger numbers of reflectors contribute. More to the point, however, the above problem is only illustrative, and is unlikely to properly model the real atmosphere. Therefore, let us increase the complexity of the model. The above analysis at least gives one an intuitive feel that the power should be proportional to the pulse length.

A more general model is represented by Figure 1b. In this case, many reflectors are assumed to exist within one pulse length, but they are allowed to have varying reflection coefficients. This situation is analogous to that assumed in GBG. The situation is now far more complex than the classical random-walk problem. Nevertheless, HOCKING and ROTTGER (1983) showed that by dividing the reflectors into subsets of equal strength, it could be shown that the resultant vector will still be proportional to the number of contributing vectors, provided that the amplitude distribution of these vectors remains unchanged, and their phases are genuinely random. BECKMANN (1962) has considered the problem more rigorously and more generally, and has shown that the vector sum of a large set of vectors $\{\underline{s_i}\}$, which have an arbitrary amplitude distribution but random phases distributed uniformly between 0 and 2π, is a vector with a Rayleigh distribution of amplitudes. Furthermore, BECKMANN (1962) has shown that the mean squared length of the resultant vector is proportional to the number of contributing vectors, in line with the above discussion. These

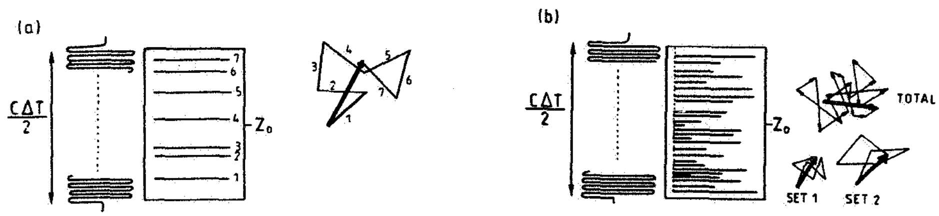


Figure 1. Pictorial description of reflection from a group of reflectors distributed randomly in height. The pulse is illustrated to the left in each figure, and the reflectors and their strengths are indicated by the horizontal lines.

results are also consistent with NORTON et al. (1955).

Therefore, it may be expected that the received power is proportional to the pulse length.

The above arguments also apply if an arbitrary form of pulse shape is used, rather than a square pulse. Provided the pulse shape is sufficiently long that many reflectors contribute to any signal, the picture is still similar to Figure 1b, but the amplitude of the pulse may change within the region $c\Delta T/2$. This simply weights the reflection coefficients, but the signals contributing to the total power at any instant are still due to signals reflected from a range of reflectors. These contributing signals are still uniformly distributed between 0 and 2π radians in phase, and the effect of the pulse is simply to modify the amplitude distribution of the component vectors. Thus the results outlined above still apply.

We may now make a general statement. If we have a sequence of vectors $\{\underline{s}_i\}$, which have an arbitrary amplitude distribution and a uniform phase distribution (0-$2\pi c$), and this sequence is multipled by an envelope function E, then the vector sum of the resultant vectors $\{\underline{E}_i\underline{s}_i\}$ obeys the relation

$$\overline{S^2} \propto W_E, \tag{2}$$

where W_E is the width of E, defined in any manner, and $\overline{S^2}$ is the mean square vector sum. This relation is true for any specific envelope shape, but cannot of course be used to compare powers between different envelopes.

It should be pointed out that if one or two of the specular reflectors are much stronger scatterers than all the others, the above statistical treatment is no longer valid. These cases require special consideration (e.g., RICE, 1944, 1945; BECKMANN, 1962), but were not considered in the model of GBG and so will not be considered here.

3. GBG TREATMENT

In this section, the treatment adopted by GBG will be briefly outlined. For a more detailed treatment, the original paper could be consulted, as the description given here will be largely qualitative. Nevertheless, the principle is so simple that the "pictorial" treatment given here actually describes the model adequately.

The approach adopted by GBG goes as follows. The reflection coefficient profile r(z) can be considered as the sum of many sinusoidal oscillations, of varying vertical scale, and varying amplitude. For example, the curves a, b and c in Figure 2 represent three of these. The amplitudes of these various scales can be found simply by Fourier transforming r(z).

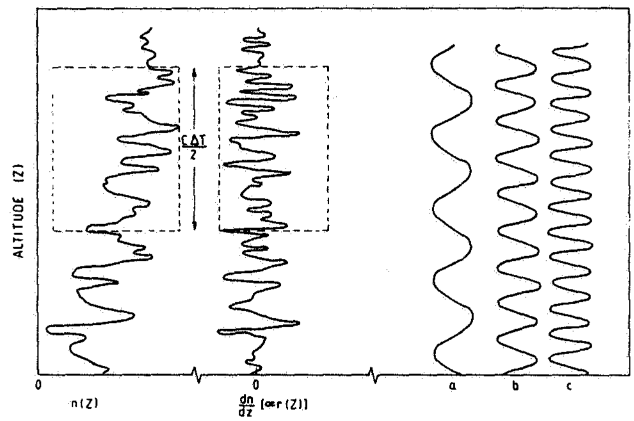


Figure 2. Typical profiles of refractive index n(z), reflection coefficient r(z), and 3 examples of Fourier components of r(z).

Then GBG state that the pulse is comprised of only one frequency, so only one of these vertical scales is important -- namely, the scale with a node-to-node distance of $\lambda/2$, λ being the radar wavelength (i.e., this is the Bragg backscatter scale). The received signal amplitude at the ground is proportional to the number of oscillations in length $c\Delta T/2$. Doubling the pulse length effectively doubles the number of oscillations of this Bragg scale, increasing the amplitude received at the receiver by a factor of 2, and therefore the power by a factor of 4. A generalization of this discussion clearly suggests that the received power is proportional to the <u>square</u> of the pulse length.

4. THE ERROR IN THE GBG ARGUMENT

The argument in the previous section contains one error, and this is in the description of the transmitted pulse. GBG stated that a pulse consists of only one frequency, but by definition a single, pure frequency must be infinite in extent. A pulse comprises a spectrum of Fourier components, centred on the central frequency. As a result, a pulse comprises a range of wavelengths, and so in the description outlined in Figure 2, a finite spectrum of Fourier scales must produce backscatter. Since r(z) is a random function of height, the phases of these contributing Fourier components are random. Each scale therefore produces a reflected signal, and each signal arrives at the ground with different phase. These signals have random phase, so a "random-walk" type problem again results.

It can be seen that a proper analysis is more complex than the simple description given by GBG. In the following section, the pulse-length dependence of the scattered power will be re-derived from the point of view of consideration of these various scales. It will be seen that the treatment given by GBG is inadequate, and the results of section 2 will be reinforced through this alternative approach.

5. QUANTITATIVE TREATMENT

Suppose that the pulse field strength at time t and height z is given by

$$z^{-1} \cdot g(t-z/c) \cdot \exp\{j\omega(t-z/c)\}\,, \tag{3}$$

where ω is the carrier frequency, and g describes the pulse envelope. In this simple description, it has been assumed that the pulse travels at a speed c(= the speed of light in a vacuum), and absorption has been ignored.

For simplicity, the z^{-1} dependence will be ignored. Let the pulse at z=0 be written as

$$g_t(t)\, e^{j\omega t} = g_z(\xi) e^{\frac{2j\omega\xi}{c}} = \underline{g}_p(\xi). \qquad (4)$$

The function g_t defines the pulse envelope, we have allowed g_p to be in general complex, and $\xi = ct/2$ is a length coordinate. We will consider only the case of g_t symmetric about its maximum, as this is almost always valid for real experiments. The following results are probably true generally, independent on this symmetry refinement, but these asymmetric cases will be ignored for simplicity. Let us also associate a phase with r(z), where the phase is determined by the height of the reflector above the ground, z. Then r(z) can be regarded as a complex profile, $\underline{r}(z)$. After backscatter from the reflection profile $\underline{r}(z)$, the signal received at time τ_o can be shown to be given approximately by

$$\underline{a}(z_o) \propto \underline{r}(z_o)^* \underline{g}_p(z_o)$$

$$\propto \int_{-\infty}^{\infty} \underline{r}(z)\, \underline{g}_p(z_o - z) dz, \qquad (5)$$

where $z_o = c\tau_o/2$.

That is to say that the received signal is a convolution between $\underline{r}(z)$ and $\underline{g}_p(z)$ (e.g., AUSTIN et al., 1969). It is convenient to work in the spatial domain, which is the reason that z_o has been used. The value z_o can be approximately regarded as the height from which most of the scattered signal received at time τ_o was reflected.

Now introduce the functions $\underline{A}$, $\underline{R}$, and $\underline{G}$, defined as the Fourier transforms of the functions $\underline{a}$, $\underline{r}$ and $\underline{g}_p$. That is,

$$\underline{a}(z) \leftrightarrow \underline{A}(\zeta)$$

$$\underline{r}(z) \leftrightarrow \underline{R}(\zeta)$$

$$\underline{g}(z) \leftrightarrow \underline{G}(\zeta), \qquad (6)$$

where ζ is the reciprocal coordinate of z. (ζ plays the same role to z as frequency does to time; the ζ coordinate will be referred to as "reciprocal space".) Then, by the convolution theorem (e.g., BRACEWELL, 1978),

$$\underline{A}(\zeta) = \underline{G}(\zeta).\ \underline{R}(\zeta). \qquad (7)$$

Thus the signal strength received at the receiver can be found in the following way. First, find $\underline{r}(z)$, and then find its Fourier transform $\underline{R}(\zeta)$. Then find the Fourier transform of the pulse, $\underline{G}(\zeta)$. If $\underline{R}$ and $\underline{G}$ are multiplied, and then reverse Fourier-transformed, the signal amplitude $\underline{a}(z)$ can be found. This description is identical to the description given in section 4, except that in this case we began by assuming a convolution in the spatial domain, whereas in section 4 we went directly to the reciprocal space domain. This shows that the treatment in section 2, and the discussion in sections 3 and 4, are in fact different ways of viewing the same problem. We must now complete the analysis in the reciprocal space domain quantitatively, to show that it does in fact produce a pulse-length dependence for power. Given that GBG chose to work in the reciprocal-space domain, the following section gives the form of analysis which they should have adopted.

Since $\underline{r}(z)$ is a random function of height, then $\underline{R}(\zeta)$ is a random function of ζ. Therefore the function $\underline{A}(\zeta)$ is a random function with an envelope described by $\underline{G}(\zeta)$. Schematic examples of $\underline{R}$ and $\underline{G}$ are shown in Figure 3, where $g_p(z)$ is taken as a Gaussian function, so $\underline{G}$ is a Gaussian function centred on $\zeta = 2/\lambda$.

In any physical experiment, it is normal to "mix" the central frequency down to 0 Hz, and for convenience we will do this in this theoretical consideration. This simply means that $\zeta = 2/\lambda$ is shifted to $\zeta = 0$. Figure 4a shows an example of $\underline{A}(\zeta)$ after such a shift has been performed, and this function is denoted by $\underline{A}_0(\zeta)$. Also shown (schematically only) in Figure 4 is the amplitude, $|\underline{a}|$, and phase, ϕ_a, of $\underline{a}(\zeta)$, which might typically result after $\underline{A}_0(\zeta)$ has been reverse Fourier transformed. Notice that no large variations in $|\underline{a}|$ or ϕ_a can occur over distances of z of less than about one pulse length of $g_p(z)$. This is because $G(\zeta)$ defines a limited frequency band of non-zero values $A_0(\zeta)$, so no frequencies outside this band can occur in $\underline{a}(\zeta)$.

In any real situation, $\underline{r}(z)$ will change as a function of time, and therefore so will $\underline{a}(z)$. The powers $|\underline{a}(z)|^2$ at any height z may be averaged to produce a mean over some time interval T. This gives the mean power at height z. Since $\underline{r}(z)$ is random, there is no "preferred" z value, and after sufficient averaging, $|\bar{a}|^2$ will be a constant, independent of z. Therefore it is only necessary to look at one height, and for convenience we choose z = 0.

By definition,

$$\underline{a}(z) = \int_{-\infty}^{\infty} \underline{A}(\zeta) e^{j2\pi z\zeta} \, d\zeta \qquad (8)$$

so for z = 0

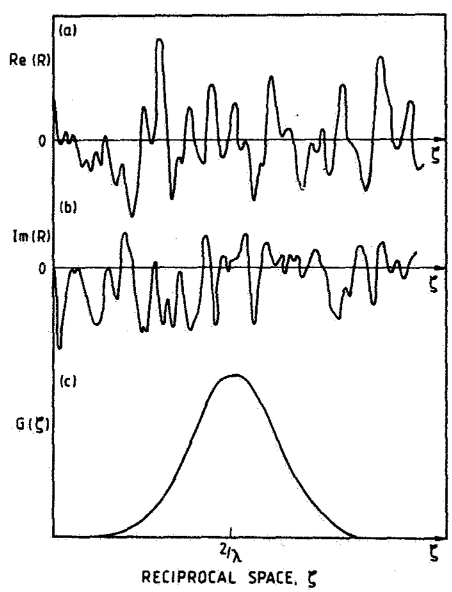


Figure 3. Schematic illustration of R(ζ), the Fourier transform of the reflection coefficient profile. (a) is the real part, (b) the imaginary component. Graph (c) shows the Fourier transform of the transmitted pulse.

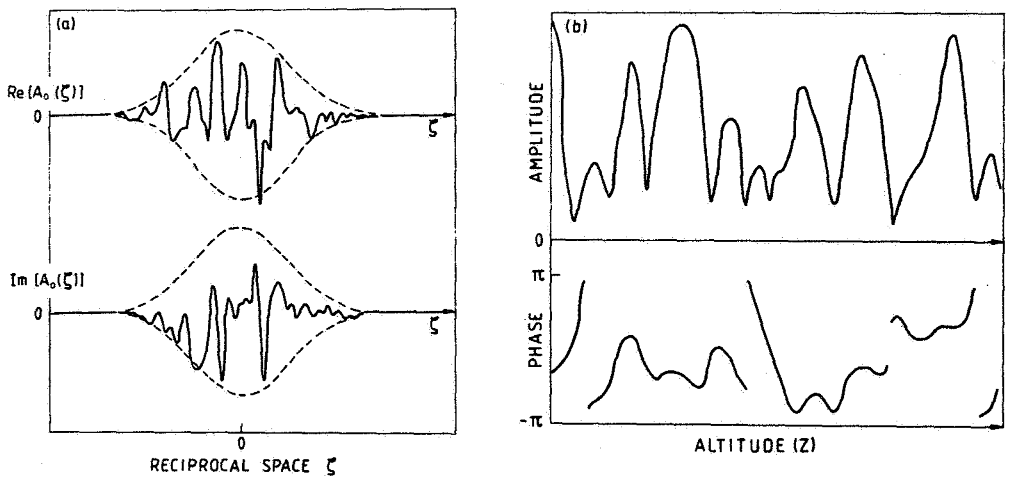


Figure 4. (a) The function $A_0(\zeta)$(see text). (b) Typical amplitude and phase which might be recorded at any instant after reflection from the atmosphere, as a function of height, z.

$$\underline{a}(0) = \int_{-\infty}^{\infty} \underline{A}_0(\zeta)d\zeta. \tag{9}$$

In other words, $\underline{a}(0)$ can be regarded as the "complex area" under $\underline{A}_0(\zeta)$. If we take $d\zeta = \Delta\zeta$ as a sufficiently small constant,

$$|\underline{a}(0)| = |\Delta\zeta \sum_{n=1}^{N} \underline{A}_0(\zeta_i)| \quad (= |\Delta\zeta \sum_{n=1}^{N} (G(\zeta_i)R(\zeta_i)|) \tag{10}$$

This amounts to simply vectorially summing a set of vectors $\{\Delta\zeta\underline{A}_{0i}\}$. It is clear that we are confronted with a very similar problem to that in section 2 -- namely, we have a random sequence of vectors $\{\Delta\zeta R(\zeta_i)\}$, which we multiply by some envelope $G(\zeta_i)$, and then we add to produce resultant. We wish to know how the modulus of the vector sum varies as we change the envelope width. The only difference compared to section 2 is that here the vectors are functions of reciprocal space, whilst in section 2 we were dealing with vectors which were functions of z. Clearly, then, the results in section 2 apply, and we see that if we hold the peak amplitude of $G(\zeta)$ fixed, and define the "width" of $\underline{G}(\zeta)$ as W_G, then the vector $\underline{a}(0)$ obeys the relation (2): i.e.,

$$\overline{|\underline{a}(0)|^2} \propto W_G. \tag{11}$$

The width W may be defined in any way (e.g., half-power width, e^{-1} width, etc), provided the definition is invariant for the chosen function.

Equation (10) deals with the width of $\underline{G}(\zeta)$. It is now necessary to determine how changing the width of the Tx pulse $\underline{g}_p(\zeta)$ affects $\underline{G}(\zeta)$. Two results from Fourier transform theory are first necessarily. Firstly, the width of g_p is inversely related to the width of G: i.e.,

$$W_{gp} \propto W_G^{-1} \tag{12}$$

and secondly,

$$\underline{g}_p(0) = \int_{-\infty}^{\infty} \underline{G}(\zeta)d\zeta. \qquad (13)$$

(conversely, $\underline{G}(0) = \int_{-\infty}^{\infty} \underline{g}_p(z)\, dz$).

We are now in a position to examine the pulse-length dependence of the received power. As seen in equation (10), we have the following "random-walk" problem. We have a sequence of random vector $\{\Delta\zeta R(\zeta_i)\}$, and we multiply them by an envelope $\{G(\zeta_i)\}$. We know that the transmitted pulse $\underline{g}_p(z)$ may change in width but must maintain constant peak amplitude $g_p(0)$. Changing the width of $g_p(z)$ affects both the width and peak value of $G(\zeta)$. The width of $G(\zeta)$ is inversely proportional to the width of $g_p(z)$ (by (12)). This fact, together with (13), means that the peak value of $G(\zeta)$ must be proportional to the width of $g_p(z)$ when $g_p(0)$ is held fixed. Thus both the width and peak value of $G(\zeta)$ change. If we consider the rescaling of the function $G(\zeta)$ and keep its width constant for now, we see that this simply increases all the vectors $\{\Delta\zeta R(\zeta_i)\ G(\zeta_i)\}$ by a factor proportional to W_{gp}. This must therefore rescale the total power by W_{gp}^2 times. Now, we must only consider the effect of changing the width of the function $G(\zeta)$. Equation (2) can be applied here, so it is clear that changing the width of G changes the power proportionally to W_G.

Combining the above effects, we have

$$\overline{|\underline{a}(0)|^2} \propto W_{gp}^{\ 2}\, W_G \qquad (14)$$

for the case of unchanging pulse peak power, and using (12),

$$\overline{|a(z)|^2} \propto \underline{W}_{gp}. \qquad (15)$$

(We have already shown that $\overline{|a(z)|^2} = \overline{|a(0)|^2}$ for all z).

This proves that the mean square received power is indeed proportional to the pulse width, even when viewed from the inverse space domain.

6. COMPUTER SIMULATION AND GENERALIZATION OF ASSUMPTION

Computer tests have been done to test equation (1), since that equation is crucial to all the arguments presented here. A Monte Carlo approach was adopted (e.g., SCHREIDER, 1967). The details of these tests will not be given here. It is suffice to say that equation (2) was completely verified by these numerical simulations.

This Monte Carlo approach also allowed a generalization of the assumptions made by GBG. In the troposphere the mean reflectivity decreases approximately exponentially with height (e.g., BALSLEY and GAGE, 1981). Therefore the situation of a pulse incident on such a reflectivity structure has been investigated. In such circumstances, varying the pulse width will vary the form of the amplitude distribution of the reflected signals, and so the pulse-length dependence for scattered power is no longer simply proportional to (Δr). The details of this simulation can be found in HOCKING and ROTTGER, (1983), but the results are summarized here with Figure 5. Suppose that the RMS reflectivity as a function of height is $\langle r^2(z)\rangle^{1/2}$, and that $\langle r^2(z)\rangle^{1/2}/z$ takes the form $\exp(-z/H)$. Suppose that a Gaussian pulse of half-power full width h is transmitted. Then the received backscattered power is a function of h/H, and follows the form indicated in Figure 5. Clearly for $h > 0.5 \times H$, the power is no longer simply proportional to the pulse width.

7. DISCUSSION

In any experiment to test the pulse-length dependence of backscattered power, various precautions are necessary, or else misleading results can ensue.

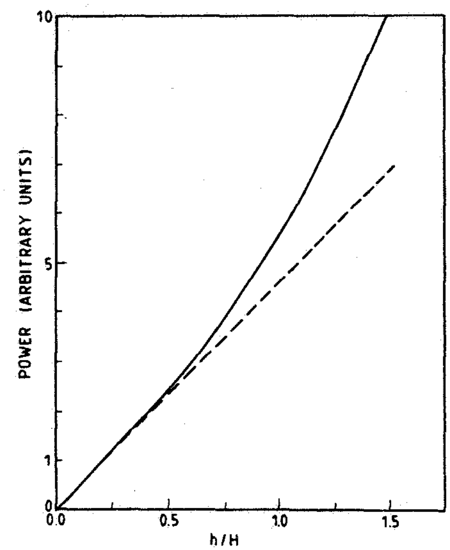


Figure 5. Plot of received power as a function of the ratio of the pulse width to the scale height of the reflection strengths, for the case of an exponential decay in reflection strength with height.

Firstly, it is important that the receiver has sufficient frequency bandwidth to accommodate the pulse. For example, imagine the situation of transmitting a Gaussian pulse, and using a receiver matched to the pulse. (This is normally done, in order to optimize the signal-to-noise ratio.) If the transmitted pulse $g_p(z)$ is now made narrower, the peak value of $G(\zeta)$ falls proportionally. But if the receiver bandwidth is not widened to accommodate the wider range of frequencies, then equation (14) becomes

$$|\underline{a}(0)|^2 \propto [W(g_p)^2].\ 1, \tag{16}$$

and the received power appears to be proportional to the square of the pulse width. In fact, in any real investigations of this reflecting process, the effective pulse is not simply the transmitted pulse but rather that pulse convolved with the impulse response of the receiver. This last point is important, and care must be taken in performing receiver matching. The receiver bandwidth must not be just equal to the bandwidth of the Fourier transform of the pulse, but considerably wider. For example, suppose that the transmitted pulse is described by $g(t)$, and the Fourier transform of $g(t)$ is $G(\omega)$, ω being the angular frequency. Let the receiver response be also $G(\omega)$ -- then the effective transmitted pulse is not $g(t)$, but rather $g(t)*g(t)$ -- or a function roughly $\sqrt{2}$ times wider than the transmitted pulse. The receiver response should be flat over all non-zero values of $G(\omega)$ in order that the effective pulse is the same as the transmitted pulse.

Secondly, if the receiver is matched to the transmitted pulse on all occasions, it is interesting to look at the signal-to-noise ratio. Doubling the pulse width doubles the received power -- but if the noise is constant as a function of frequency over the bandwidth of the receiver, and the receiver band width is halved, to match the transmitter, then the received noise power also decreases by a factor of 2. For the case of VHF radars, the main noise is cosmic noise, and this can be regarded as constant over the band width of most VHF systems. Thus the signal-to-noise ratio is proportional to the square of the pulse length. It is important in any experimental test of the preceding theory to measure absolute power, and not signal-to-noise ratios.

HOCKING and ROTTGER (1983) presented a preliminary test of the above theory, using experimental data from the SOUSY radar. These results are summarized in Figures 6a-c. Figure 6a shows the experimentally observed power profile for a 150 m pulse after averaging over a period of 50 min. Figure 6b shows the profile which would have been observed had a pulse of length 1.5 km been used, with peak power equal to that of the 150 m pulse. A factor (1500/150) has been removed from this figure for ease of comparisons. This profile 6b was produced by computer manipulations; the details are discussed in HOCKING and ROTTGER (1983).

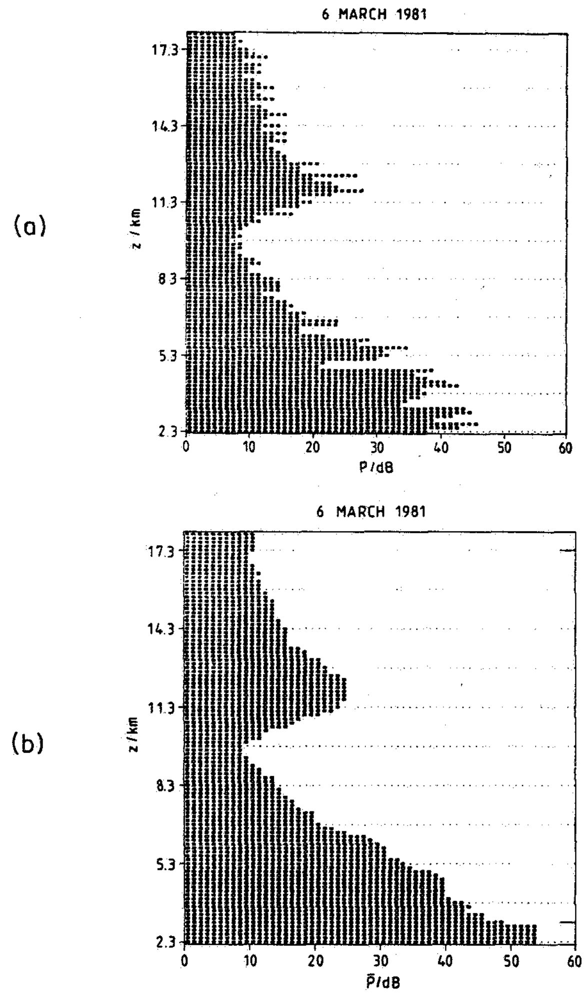
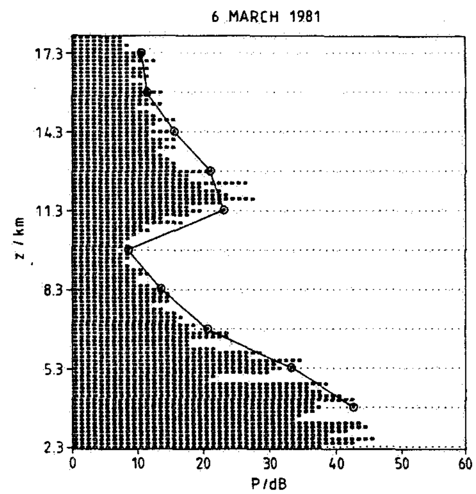


Figure 6. (a) Mean power as a function of height, recorded with the SOUSY radar on 6 March 1981. The noise has not been subtracted; the noise level was about 5-8 dB. (b) The resulting profile which would have resulted from using a pulse with a coarser resolution.

Figure 6c shows a comparison of the 2 profiles -- and it can be seen that they are in approximate agreement, now that the factor (1500/150) (i.e., the ratio of the pulse resolutions) has been removed from the low resolution profile. This is support for an approximately (Δr) power dependence. However, a great many more experimental results are necessary to properly test the theory. Also, agreement is not perfect in Figure 6c; this is discussed further in HOCKING and ROTTGER (1983).

Figure 6d shows the power as a function of height and time during this recording interval, after the mean power profile for the period has been removed. Notice the existence of certain stable, well-defined echoes. These are not consistent with the "Fresnel Scatter" model of GBG, and their existence must be borne in mind. This point is discussed further in HOCKING and ROTTGER (1983). The Fresnel scatter model may have relevance to the atmosphere, but it is not always applicable.

(c)



(d)

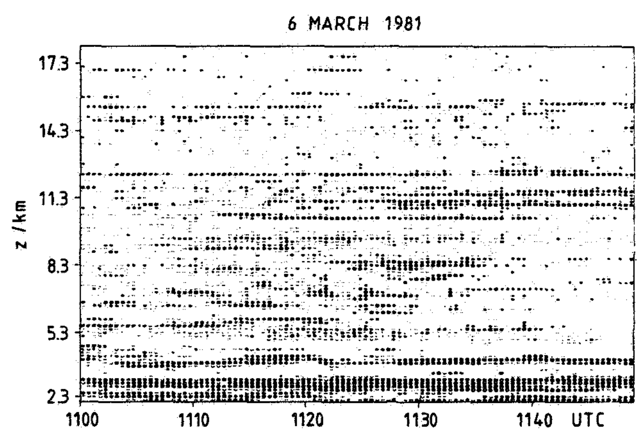


Figure 6. (c) A composite of (a) and (b). The solid line shows Figure 6(a) taken at steps of 1500 m. (d) Details of the echo strengths as a function of time during the period used to form the mean profile 6(a). Darker spots indicate greater intensity. The mean profile over the period has been subtracted, so these plots are "residual signal strengths".

CONCLUSION

The Fresnel scatter model by GAGE et al. (1981a) has been critically examined. It has been found that equation (1) is in error, and the $(\Delta r)^2$ part should simply read (Δr). Appropriate adjustment of $F(\lambda)$ is also necessary.

In the more general case of an exponential decay of $\langle r(z)^2 \rangle^{1/2}/z$ with height, a more complex proportionality results, and this has been illustrated with a numerical Monte Carlo approach.

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