2.2B THE EFFECTS OF PULSE RATE, POWER, WIDTH AND CODING ON SIGNAL DETECTABILITY

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When deciding upon radar and signal processing parameters for MST radars, the quantity that one attempts to maximize within existing constraints is the signal detectability. For Doppler spectral analysis the detectability can be defined (see BALSLEY, 1978 or GAGE and BALSLEY, 1978) as the ratio of the amplitude of the largest spectral peak of the received signal to the noise level fluctuation, $D = S_r/\Delta S_N$ (see Figure 1).

This paper will examine the effects on the detectability of varying the pulse repetition rate (PRF), peak pulse power (P_{pk}) and pulse width (τ_p). Both coded and uncoded pulses will be considered. During this discussion the following quantities will be assumed to be constant: antenna area, echo reflectivity, Doppler shift, spectral width, spectral resolution, effective sampling rate, and total incoherent spectral averaging time. The detectability will be computed for two types of targets: 1) discrete target (i.e., a single echoing region smaller than the smallest pulse width).

First let us examine the effects of coded pulses. The received signal from a coded pulse is decoded by convolving the received voltage with the code. The phase of the received signal from the echoing region will be the mirror image of the transmitted code. Since the autocorrelation function of a code of length L_c has a peak value of L_c, the decoding process enhances the echo signal power by a factor of L_c^2. For white noise which is uncorrelated between each bit of the code, the convolution will add the power incoherently and thus the noise power will be increased by a factor of L_c. The above is true regardless of the type of code used. For multicode processing (using complementary codes or pseudo-random codes) the signals from successive codes must be added coherently to obtain the desired autocorrelation sidelobe response. However, as long as the number of codes used is less than the normal number of coherent averages, multi-code processing will not have any additional effect on the signal detectability. Of course, the sidelobes of the code autocorrelation functions will affect range contamination of signals and influence the choice among various codes.

Now we will determine the signal detectability for coded and uncoded pulses as a function of PRF, transmitter power, pulse width, and code length. Let us define the fundamental bit length or resolution pulse width of a coded pulse to be τ_o and the total pulse length to be $\tau_p = L_c\tau_o$. The same symbols can be used for uncoded pulses by letting $L_c = 1$ and $\tau_p = \tau_o$. The returned signal power, P_s, is proportional to peak transmitter power for discrete targets and to peak power and pulse width for diffuse targets. Specifically, for both coded and uncoded pulses,

$$P_s \propto P_{pk} L_c^2 \quad \text{(discrete targets)}$$

$$P_s \propto P_{pk} \tau_o L_c^2 \quad \text{(diffuse targets).}$$

The noise power can be written as

$$P_N \propto B\, L_c/m \propto L_c/\tau_o m$$

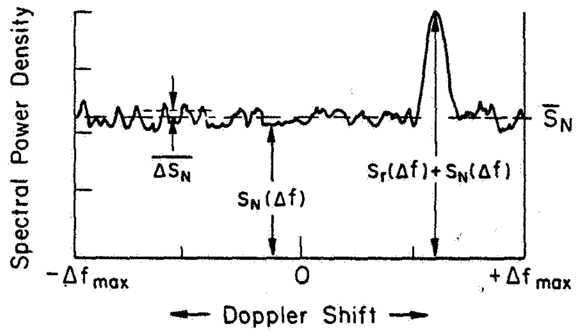

Figure 1. Typical Doppler spectrum (after BALSLEY, 1978a).

where $B \propto 1/\tau_o$ is the receiver bandwidth and m is the number of coherent averages. The signal-to-noise ratio, $SNR = P_s/P_N$, then becomes

$$SNR \propto P_{pk}\, m\tau_o\, L_c \quad \text{(discrete)}$$

$$SNR \propto P_{pk}\, m\tau_o^2 L_c \quad \text{(diffuse)}$$

Following the derivation of signal detectability, D, used by BALSLEY (1978) and eliminating dependencies on the fixed quantities given previously, it can be shown that

$$D \propto P_{pk}\ (PRF)\ \tau_o L_c = P_{AV} \quad \text{(discrete)}$$

$$D \propto P_{pk}\ (PRF)\tau_o^2 L_c = P_{AV}\ \tau_o \quad \text{(diffuse)}.$$

Table 1 summarizes these results for 4 cases with varying values of PRF, P_{pk}, τ_o, and L_c for a constant average transmitter power ($P_{AV} = P_{pk}$ (PRF) $L_c\tau_o$) The "reference" values of each quantity are PRF = f, $P_{pk} = P$, $\tau_o = \tau$, and $L_c = 1$. Each quantity is multiplied in turn by an integer constant N, keeping the average power constant in each case. The receiver bandwidth, B, is set to $1/\tau_o$ and the number of coherent averages n_c is adjusted to maintain a constant effective sampling rate, PRF/n_c. The resulting dependencies of the signal-to-noise ratio and detectability are shown in the last two columns.

Table 1 can be simplified by writing the first 3 independent variables in terms of dimensionless quantities PRF/f, P_{pk}/p, τ_o/τ and thus showing only the dependency on N. This has been done in Table 2. For the discrete targets we can see that all 4 cases have the same signal-to-noise ratio and the same detectability. For the diffuse case, because of the dependence on the resolution pulse width, the long uncoded pulse has a detectability which is a factor of N greater than the other 3 cases.

In those diffuse cases where high resolution is obtained, we note that using coded pulses gives the same signal detectability as using short uncoded pulses with either higher PRF or higher peak power. Pulse coding becomes desirable, then, when high resolution is needed and when the peak power cannot be increased due to transmitter limitations and the PRF cannot be increased, perhaps because of range aliasing problems.

Note that, given a set of Doppler power spectra obtained with any of the high resolution systems (cases 5, 6 or 8 in Table 2) the detectability can be increased by $N^{1/2}$, at the expense of range resolution, by averaging the spectra across N range gates. This effect occurs because the spectral noise power

Table 1. Signal detectability for constant average transmitter power

	PRF	P_{pk}	Res PW τ_o	Code Size L_c	Tot PW τ_p	P_{AV}	B	n_c	P_s	P_N	SNR	D
Discrete Targets												
1) High PRF	Nf	P	τ	1	τ	$NfP\tau$	$1/\tau$	Nm	$\propto P$	$\propto 1/N\tau m$	$\propto NPm\tau$	$\propto NPf\tau$
2) High Peak Power	f	NP	τ	1	τ	$NfP\tau$	$1/\tau$	m	$\propto NP$	$\propto 1/\tau m$	$\propto NPm\tau$	$\propto NPf\tau$
3) Long Pulse	f	P	$N\tau$	1	$N\tau$	$NfP\tau$	$1/N\tau$	m	$\propto P$	$\propto 1/N\tau m$	$\propto NPm\tau$	$\propto NPf\tau$
4) Coded Pulse	f	P	τ	N	$N\tau$	$NfP\tau$	$1/\tau$	m	$\propto N^2P$	$\propto N/\tau m$	$\propto NPm\tau$	$\propto NPf\tau$
Diffuse Targets												
1) High PRF	Nf	P	τ	1	τ	$NfP\tau$	$1/\tau$	Nm	$\propto P\tau$	$\propto 1/N\tau m$	$\propto NPm\tau^2$	$\propto NPf\tau^2$
2) High Peak Power	f	NP	τ	1	τ	$NfP\tau$	$1/\tau$	m	$\propto NP\tau$	$\propto 1/\tau m$	$\propto NPm\tau^2$	$\propto NPf\tau^2$
3) Long Pulse	f	P	$N\tau$	1	$N\tau$	$NfP\tau$	$1/N\tau$	m	$\propto NP\tau$	$\propto 1/N\tau m$	$\propto N^2Pm\tau^2$	$\propto N^2Pf\tau^2$
4) Coded Pulse	f	P	τ	N	$N\tau$	$NfP\tau$	$1/\tau$	m	$\propto N^2P\tau$	$\propto N/\tau M$	$\propto NPm\tau^2$	$\propto NPf\tau^2$

PRF = Pulse repetition frequency

P_{pk} = Peak transmitted power

τ_o = Resolution pulse width

L_c = Number of bits in code

τ_p = Total pulse width = $L_c \tau_o$

P_{AV} = Average transmitted = $(PRF) P_{pk} \tau_p$

B = Required receiver bandwidth $\propto 1/\tau_o$

n_c = Number of coherent averages

P_s = Returned signal power

$\propto P_{pk} L_c^2$ (discrete)

$\propto P_{pk} \tau_o L_c^2$ (diffuse)

P_N = Noise power $\propto L_c B/n_c$

SNR = Signal-to-noise ratio = P_s/P_N

D = Signal detectability

Table 2. Relative signal detectability for constant average transmitter power

	PRF	P_{pk}	Res PW τ_o	Code Size L_c	Tot PW τ_p	P_{AV}	B	n_c	P_s	P_N	SNR	D
Discrete Targets												
1) High PRF	N	1	1	1	1	N	1	N	1	1/N	N	N
2) High Peak Power	1	N	1	1	1	N	1	1	N	1	N	N
3) Long Pulse	1	1	N	1	N	N	1/N	1	1	1/N	N	N
4) Coded Pulse	1	1	1	N	N	N	1	1	N^2	N	N	N
Diffuse Targets												
5) High PRF	N	1	1	1	1	N	1	N	1	1/N	N	N
6) High Peak Power	1	N	1	1	1	N	1	1	N	1	N	N
7) Long Pulse	1	1	N	1	N	N	1/N	1	N	1/N	N^2	N^2
8) Coded Pulse	1	1	1	N	N	N	1	1	N^2	N	N	N

PRF = Pulse repetition frequency

P_{pk} = Peak transmitted power

τ_o = Resolution pulse width

L_c = Number of bits in code

τ_p = Total pulse width = $L_c\ \tau_o$

P_{AV} = Average transmitted = $(PRF)P_{pk}\ \tau_p$

B = Required receiver bandwidth $\propto 1/\tau_o$

n_c = Number of coherent averages

P_s = Returned signal power

$\propto P_{pk}\ L_c^2$ (discrete)

$\propto P_{pk}\ \tau_o\ L_c^2$ (diffuse)

P_N = Noise power $\propto L_c\ B/n_c$

SNR = Signal-to-noise ratio = P_s/P_N

D = Signal detectability

fluctuations are proportional to $N^{-1/2}$. Thus by using two types of post-processing, these high resolution systems can give range resolution improved by a factor of N in regions of good SNR and, in regions of low SNR, a detectability degraded only by $N^{1/2}$, compared to the long pulse case.

REFERENCES

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