2.1E IS VHF FRESNEL REFLECTIVITY DUE TO LOW FREQUENCY BUOYANCY WAVES?

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VHF radar echoes are greatly enhanced near the zenith relative to other directions. This enhancement must be due to reflection from horizontally stratified laminae of refractive index. In this paper, we suggest that the refractivity laminae are due to the displacements of low frequency buoyancy (internal gravity) waves acting on the background vertical gradient of refractivity.

The radar cross section $\sigma(\underset{\sim}{k})$ is given by (OTTERSTEN, 1969)

$$\sigma(\underset{\sim}{k}) = \frac{\pi}{8} k^4 \quad \Phi\mu(\underset{\sim}{k})$$

where $k = 4\pi/\lambda_{radar}$, μ is the potential radio refractive index, and $\Phi\mu(\underset{\sim}{k})$ is the spatial power spectral density of μ. If the fluctuations of μ are due to a spectrum of vertical displacement acting on $\overline{M} = \langle d\mu/dz \rangle$, the background gradient of μ, then

$$\Phi\mu(\underset{\sim}{k}) = \overline{M}^2 E_\zeta \; (\underset{\sim}{k})$$

where $E_\zeta \; (\underset{\sim}{k})$ is the spatial power spectrum of vertical displacement. In order for $\sigma(\underset{\sim}{k})$ to be strongly enhanced near the zenith, $E_\zeta \; (\underset{\sim}{k})$ must also be strongly enhanced.

VANZANDT (1982) has shown that the observed spectra of mesoscale wind fluctuations in the troposphere and lower stratosphere can be modeled by a universal spectrum of buoyancy (internal gravity) waves that is a slight modification of the GARRETT and MUNK (1975) model of oceanic internal gravity waves. Since the observed frequency spectrum is red, the buoyancy wave model of the vertical displacement spectrum is strongly enhanced near the zenith. In other terms, the resulting refractivity irregularities are strongly stratified. The model spectrum is

$$E_\zeta \; (\underset{\sim}{k}) = \frac{1}{\pi k^4} \left[\frac{9\pi}{16} \left(\frac{n}{k}\right)^{1/2} \frac{E_*}{u_*^{3/2}} \; \tilde{F}(\theta) \right]$$

where n is the buoyancy (Brunt-Vaisala) frequency, E_* is the normalized energy per unit mass ($E_* n \sim 10 \; (m/s)^2$), and u_* is a scale phase velocity ($\sim$6 m/s).

$$\tilde{F}(\theta) = \frac{4}{9\pi\tilde{f}^2} \frac{(\theta/\tilde{f})^2}{((\theta/\tilde{f})^2 + 1)^{7/3}}$$

describes the angular variation normalized so that $\int_0^{2\pi} \int_0^{\infty} \tilde{F}(\theta) \sin\theta \, d\theta \, d\phi = 1$,

where θ is the zenith angle of $\underset{\sim}{k}$, and $\tilde{f} = f/n$, where f is the inertial frequency. $\tilde{F}(\theta)/(4/9\pi\tilde{f}^2) = (\theta/\tilde{f})^2/((\theta/\tilde{f})^2 + 1)^{7/3}$ is plotted in the figure.

The maximum lies at $\theta_{max}(°) = (\sqrt{3}/2)\tilde{f} = (\sqrt{3}/2)(\sin(\text{latitude})/120n(\text{rad/s}))$.

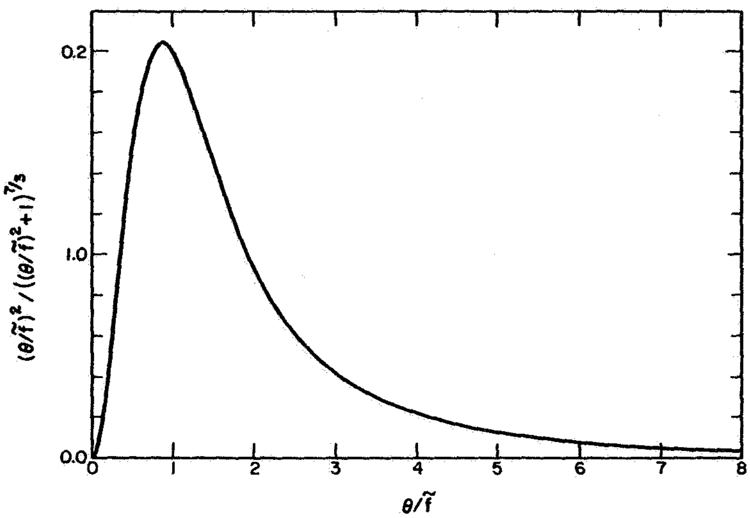

Figure 1. The angular dependence of $\sigma(\underset{\sim}{k})$.

For $n = 1 \times 10^{-2}$ (rad/s) (troposphere) at Arecibo (18.3°), Sunset (40°), and Poker Flat (65°), θ_{max} = 0.23°, 0.46°, and 0.50°, respectively.

With this form of $E_\zeta(\underset{\sim}{k})$

$$\sigma(\underset{\sim}{k}) = \frac{\overline{M}^2}{8} [\ ] = \frac{9\pi}{128} \overline{M}^2 \left(\frac{n}{k}\right)^{1/2} \frac{E_*}{u_*^{3/2}} \tilde{F}(\theta)$$

and the power reflectivity is

$$|\rho|^2 = 4 \iint \sigma(\underset{\sim}{k})) \, g^2(\theta',\phi') \sin\theta \, d\theta \, d\phi$$

where $g(\theta',\phi')$ is the normalized antenna gain function. If g is broad compared with F, as it usually is, then near the zenith the integral = 1.

This model can be tested by comparison with measurements of $\sigma(\underset{\sim}{k})$ or $|\rho|^2$ as functions of the parameters. Two reservations must be kept in mind, however. First, it has been assumed implicitly that E_* is a universal constant, independent of latitude, altitude, etc. In fact, E_* could depend on latitude through f and on altitude through the atmospheric density ρ, as it should theoretically. Observations so far have been inadequate to describe such variations. Nevertheless, in order to avoid this uncertainty, tests of the model should be made at a given latitude and over a small altitude range.

Second, although the buoyancy wave model fits observed spectra rather well down to vertical scales as small as ~20 m, the limit of the observed spectra, at some scale not much smaller than 20 m the spectrum should start to become increasingly attenuated by K-H instability and eventually by viscosity. Thus, the model can be quantitatively correct only for radar frequencies smaller than about 7.5 MHz (corresponding to 20 m). Unfortunately, the only suitable observations of Fresnel reflectivity are in the attenuated range at frequencies between 40 and 54 MHz ($\lambda_{radar}/2$ between 3.7 and 2.8 m). In this range the observed reflectivity should be much smaller than the model reflectivity and the dependence of k should be much stronger than $k^{-1/2}$. The dependence on n, $\tilde{f}$, and θ may also differ from the model.

Nevertheless, comparisons at frequencies between 40 and 54 MHz are of interest. Results so far are: (1) The width of the model angular spectrum convolved with the radar beam is roughly consistent with the observed widths (GREEN et al., 1981; ROTTGER et al., 1981). (2) The model angular width varies inversely with the Brunt-Vaisala frequency, so that the width in the stratosphere should be about 1/2 the width in the troposphere, consistent with observations (ROTTGER et al., 1981). (3) The model dependence on n is given by $\overline{M}^2 \; n^{1/2} \propto n^{4.5}$, roughly consistent with observations (GAGE et al., 1981). (4) The model $|\rho|^2$ for $\lambda_{radar} = 7.4$ m is much larger (by a factor of 100) than the reflectivity reported by GAGE et al. (1981), as is to be expected.

These comparisons are satisfactory at the present level of development of the model and the experiments, but further, more critical, tests are clearly needed. Observations with radar half-wavelengths $\gtrsim$ 20 m (f $\lesssim$ 7.5 MHz) are clearly needed. The angular variation might be measured by means of an interferometer. In spite of the limitations of the model for half-wavelengths in the lower VHF range, further comparisons should also be made there, since the attenuation of the spectrum may not have first-order effects on the dependence on n, $\tilde{f}$, and θ.

It should be noted that if the model is substantiated, then radar observations in the attenuated range yield information about the spectrum of buoyancy waves at small scales that is very difficult to obtain by other means. The buoyancy wave spectrum in this regime has implications on the cascade of energy in the buoyancy wave field, on the turbulent energy dissipation rate, and on turbulent mixing by the K-H breakdown of buoyancy waves.

All of the proposed mechanisms for the generation of the Fresnel refractivity irregularities, including the present one, depend upon speculative assumptions. The present model is distinguished from the others by being quantitative and therefore testable, at least under some conditions.

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