# PILOT MODELING, MODAL ANALYSIS, AND CONTROL
# OF LARGE FLEXIBLE AIRCRAFT

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## INTRODUCTION

The issues to be addressed in this presentation are threefold. The first deals with the question of whether dynamic aeroelastic effects can significantly impact piloted flight dynamics. If so, when and how does this come about, and is there a potential design problem? For example, if one were to explore this problem experimentally, what mathematical model would be appropriate to use in the simulation? What modes, for example, should be included in the simulation, or what linear model should be used in the control synthesis? The second question deals with the appropriate design criteria or design objectives. In the case of active control, for example, what should be the design objectives for the control synthesis if aeroelastic effects are a problem. Finally, if unacceptable characteristics are to be eliminated through active control, what is the achievable performance improvement for practical systems? (See fig. 1.)

The outline of the topics to be presented includes a description of a model analysis methodology aimed at answering the question of the significance of higher order dynamics. Secondly, a pilot vehicle analysis of some experimental data will be presented that addresses the question of "What's important in the task?" The experimental data will be presented briefly, followed by the results of an open-loop modal analysis of the generic vehicle configurations in question. Finally, one of the vehicles will be augmented via active control and the results presented.

## ISSUES

● CAN DYNAMIC AEROELASTIC EFFECTS SIGNIFICANTLY

  IMPACT (PILOTED) AIRCRAFT FLIGHT DYNAMICS?

  WHEN - HOW?

  IS THERE A POTENTIAL PROBLEM?

● WHAT ARE APPROPRIATE DESIGN CRITERIA OR OBJECTIVES?

● WHAT IS THE ACHIEVABLE PERFORMANCE IMPROVEMENT VIA

  ACTIVE CONTROL?

Figure 1

## WHAT AFFECTS VEHICLE TIME RESPONSE

Linear system theory tells us that a system's response to a particular input may be represented mathematically as the summation of contributions from each of the system's modes. Each mode's contribution, furthermore, may be analytically thought of as a term in the partial-fraction expansion of the transform of the system's response. The significance of the eigenvalues of the system to its response is well known. However, equally significant is the residue associated with each system eigenvalue. For real vehicles, the response theoretically includes an infinite summation over all of the system's modes. However, practically speaking, only a finite number of these modes contributed significantly to the vehicle response. Furthermore, for a conventional aircraft and considering a short period approximation, for example, only one mode is used to approximate the vehicle's response. Furthermore, stating handling qualities specifications in terms of the modal damping and frequency was sufficient in this case to specify acceptable and unacceptable time responses. It is clear then that when the higher order modes of the system (or the eigenvalues and residues of those modes) are such that they significantly contribute to the time response of the system, those modal contributions must be considered to accurately reflect the system's dynamics. (See fig. 2.)

## CLASSICAL EXAMPLE:

### RIGID BODY $\dot{\theta}(s)/\alpha_g(s)$ (E.G. GUST PULSE)

$$\dot{\theta}(s) = \frac{R_1}{s + \lambda_1} + \frac{R_2}{s + \lambda_2} + \left[ \sum_{i=i}^{\infty} \frac{R_i}{s + \lambda_i} \right]$$

$R_i$'s FUNCTIONS OF $\lambda$'s AND ZEROS (EIGENVECTORS)

AND FOR CONVENTIONAL VEHICLES THE EFFECTS OF

$\lambda$'s ON $R_i$'s WAS ENOUGH TO ALLOW STATING HANDLING

QUAL. SPECS. ON $\lambda$'s ONLY

Figure 2

Now consider the same example response, that is, pitch rate to a gust pulse, represented mathematically in the time domain with the usual state equations. Transforming these equations into modal coordinates and expressing the partial-fraction expansion of the transform of the response, we see that the expansion may be determined directly from the parameters in the system's time-domain modal state representation. Specifically, in fact, the residues associated with each mode are simply a product of the elements obtained from the modal observability matrix and the elements obtained from the modal disturbability or controllability matrix, depending upon which input is being considered. All these results are developed for the input represented mathematically as an impulse, or the residues are the system's impulse-response residues for the input selected. (See fig. 3.)

$$\text{LET} \quad \frac{\dot{\theta}(s)}{\alpha_g(s)} = K \ N(s^m)/D(s^n)$$

$$\text{or} \quad \dot{\bar{x}} = A \ \bar{x} + B \ \alpha_g \qquad \dot{\theta}(t) = [1, \ \dots \ 0] \ \bar{x}$$

## IN MODAL COORDINATES

$$\dot{\bar{q}} = \Lambda \ \bar{q} + D \ \alpha_g \qquad \dot{\theta}(t) = Y(t) = C\bar{q}$$

$$D = T^{-1}B \qquad C = [1, \ 0 \ \dots \ 0]T \qquad T = [\bar{v}_1 \ | \ \bar{v}_2 \dots \ v_n$$

$$\text{Now} \quad \dot{\theta}(s) = Y(s) = C[sI - \Lambda]^{-1}D = \sum_{i=1}^{n} \left[ \frac{c_i d_i}{s - \lambda_i} \right]$$

$$\text{AND} \quad \dot{\theta}(t) = \sum_{i=1}^{n} c_i d_i \ e^{-\lambda_i t}$$

$$= \sum_{i=1}^{n} R_i \ e^{-\lambda_i t} \quad \text{where} \quad \underline{R_i = c_i d_i}$$

Figure 3

## VEHICLE TIME RESPONSE (CONCLUDED)

Furthermore, the residues associated with the system's step response are easily obtained in terms of the previously determined impulse residues and the eigenvalues for the particular mode. Finally, this system's step response and each mode's contribution to that response may alternatively be considered as the area under the impulse response, and the contribution of each mode to that term is shown in figure 4.

● RESIDUES FOR THE SYSTEM'S <u>STEP</u> RESPONSE ARE

$$R_S^i = R_I^i / \lambda_i \qquad i = 1, \ldots, n$$

$\lambda_i$ = eigenvalue

$R_I^i$ = impulse residue

● AREA UNDER IMPULSE RESPONSE DUE TO MODE I IS

$$A_i(t) = R_I^i / \lambda_i \ (e^{\lambda_i t} - 1)$$

Figure 4

To apply the technique presented, one must determine the appropriate system inputs that are important in the application, as well as determine the significant physical response variables of the system that are important in the piloted task. Inputs in question include the pilot stick input and atmospheric turbulence. Important vehicle responses might include rigid-body attitude and rate, sensed attitude and rate including elastic deformation effects, flight path angle, acceleration at various locations on the vehicle, and so forth. Furthermore, the inputs just cited may not be well modeled by white noise, for example. Therefore, evaluating the pure impulse response of the aircraft is not as meaningful as the response evaluated with appropriate input characteristics included. The input characteristics that are significant are the limited bandwidth properties of the pilot's input as well as the atmospheric gust spectrum. These input characteristics may be incorporated with the vehicle math model to form what might be referred to as an integrated dynamic model. Modal analysis is then performed on this model such that controllability, disturbability, observability, and, in particular, modal residues may be assessed. (See fig. 5.)
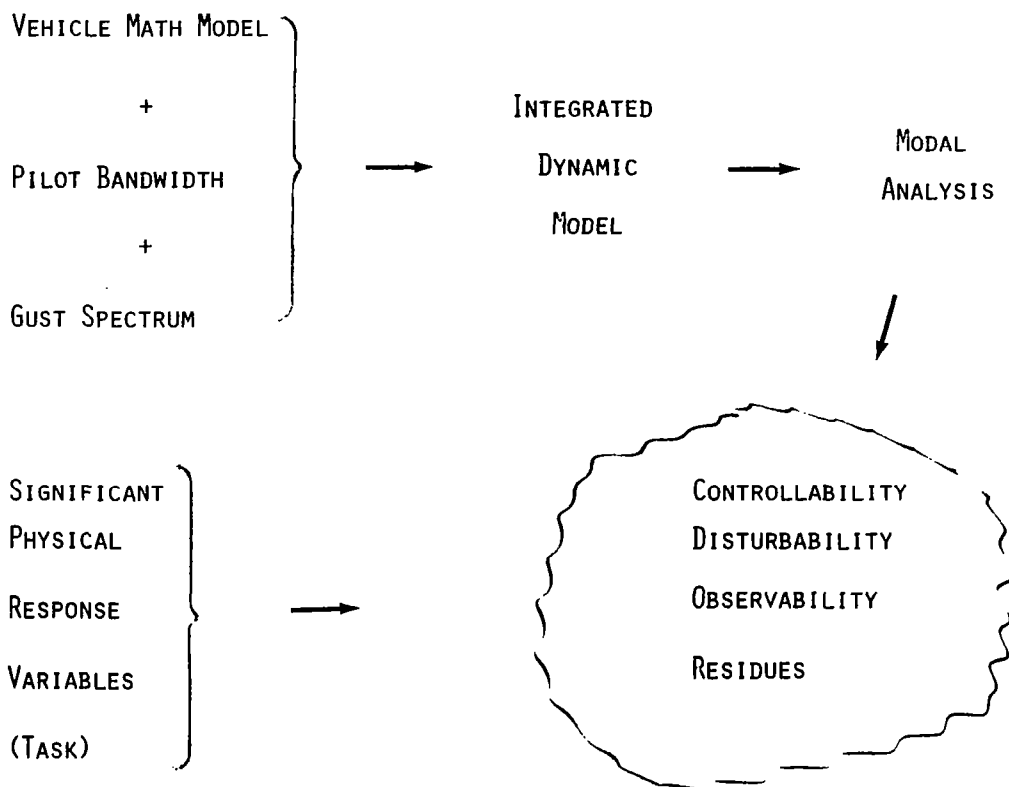
VEHICLE MATH MODEL

+

PILOT BANDWIDTH ⟶ INTEGRATED DYNAMIC MODEL ⟶ MODAL ANALYSIS

+

GUST SPECTRUM

SIGNIFICANT PHYSICAL RESPONSE VARIABLES (TASK) ⟶

CONTROLLABILITY
DISTURBABILITY
OBSERVABILITY
RESIDUES

Figure 5

## SYSTEM RESPONSES OF IMPORTANCE

The question now turns to what vehicle responses are significant in this problem. As shown in figure 6, both rigid-body attitude as well as indicated attitude, which includes the local elastic deformation of the vehicle, may be considered significant response variables. Determining the important vehicle responses in a longitudinal task will now be considered.
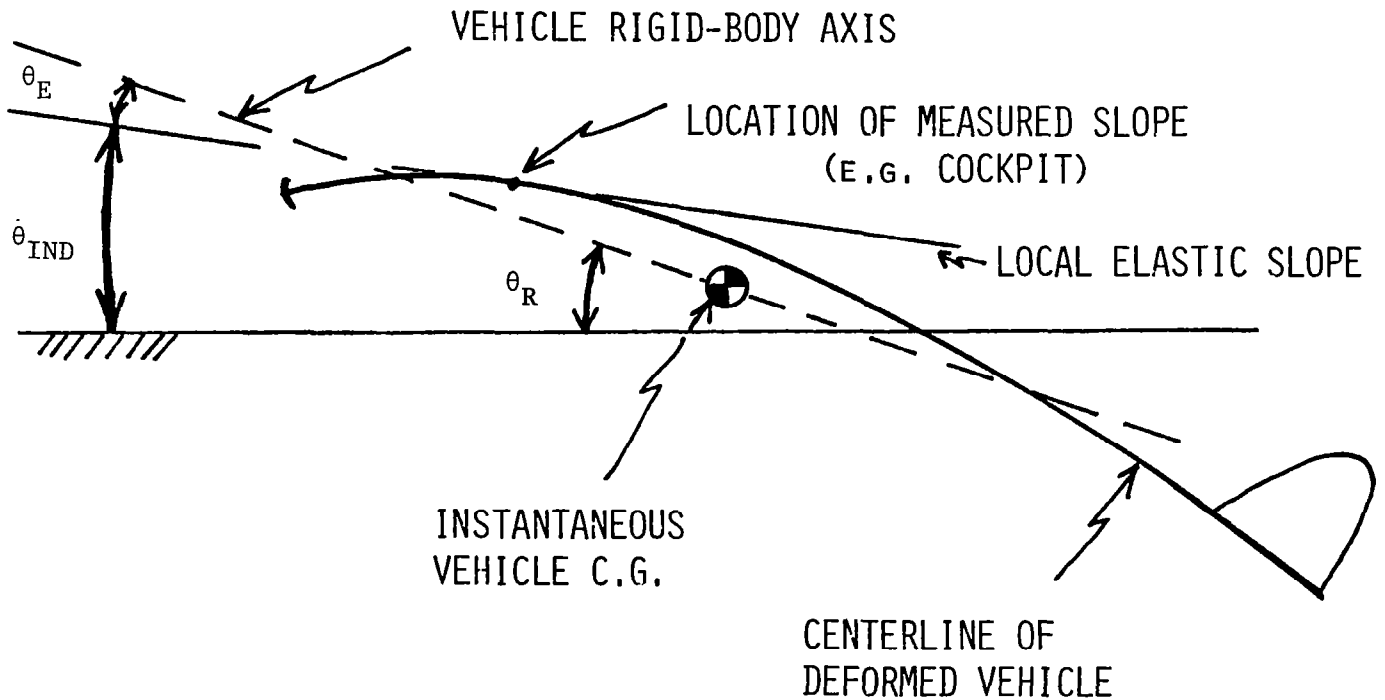


Figure 6

To better understand the important vehicle responses in the longitudinal axis, a pilot vehicle analysis was performed on a set of generic vehicle dynamics. An optimal-control pilot model was used in this evaluation with essentially "standard" model parameters. Details of this analysis may be found in reference 1. The pilots in the experimental setting were to perform a pitch-attitude tracking task, and this same task was evaluated analytically as well. The observations available to the pilots were both the indicated attitude of the vehicle, as measured at the cockpit, as well as the commanded attitude that the subjects were to follow. The issue is the selection of the appropriate pilot objective, or the appropriate vehicle response that the pilot was attempting to control. Was the pilot attempting to minimize indicated attitude, which included the elastic deformation of the vehicle, or is rigid-body attitude the response that he is attempting to control ? The geometry of the basic vehicle may be considered to be as shown in figure 7. Seven different sets of generic vehicle dynamics were evaluated, where the first configuration represents a vehicle similar to the B-1. The remaining configurations may be thought of as having the same geometric characteristics, but the material properties of the structure are changed such that the in-vacuo mode frequencies of the structure are modified from Configuration 1, or the baseline. The resulting eigenvalues of the dynamic configurations are represented in Table 1.
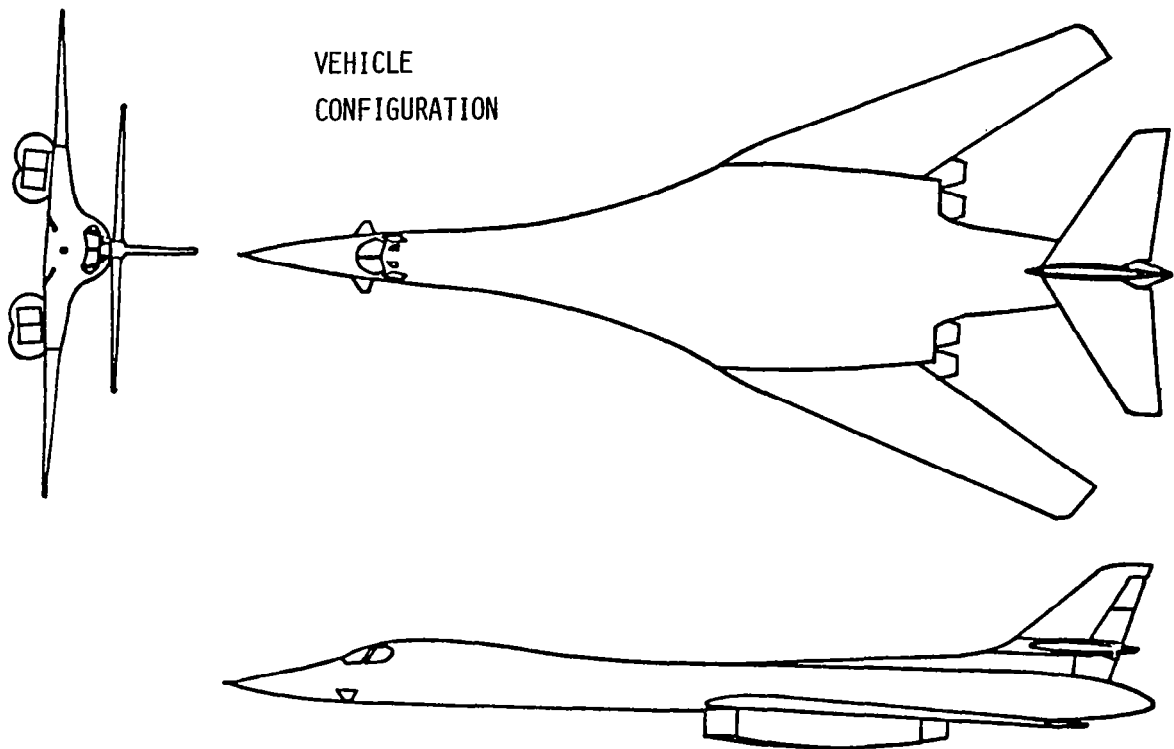
VEHICLE
CONFIGURATION



Figure 7

## CONFIGURATION SUMMARY

Shown in Table 1 are the in-vacuo mode frequencies of the first two elastic modes included in the vehicle models. Mode 1 represents the first fuselage bending mode, while mode 2 has a mode shape that would correspond to the second fuselage bending mode. These mode frequencies were varied parametrically, and the resulting aeroelastic vehicle model was obtained in each case. The eigenvalues of the vehicle are listed in the table as well. Each of the modes was identified from its eigenvector or mode shape. Note, in particular, the first four configurations. Configurations 1-3 arise from a monotonic reduction in vibration frequency of the first fuselage mode. Configuration 4 (although perhaps unrealistic physically) has a reduced mode frequency associated with the other elastic mode. Furthermore, comparing Configurations 3 and 4 in terms of their eigenvalues indicates that both have unstable phugoid modes and approximately equivalent short period eigenvalues, and also they exhibit roughly similar aeroelastic mode eigenvalues.

Table 1

| CONFIGURATION | IN-VACUO FREQUENCIES | | PHUGOID MODE* | SHORT PERIOD MODE* | ELASTIC MODES | |
| --- | --- | --- | --- | --- | --- | --- |
| | MODE 1 | MODE 2 | | | FIRST* | SECOND* |
| 1(BASELINE) | 13.7 | 21.2 | (.02, .08) | (.53, 2.8) | (.05, 13.3) | (.02,21.4) |
| 2 | 9.2 | 21.2 | (0., .06) | (.52, 2.6) | (.09, 8.8) | (.02,21.4) |
| 3 | 6.2 | 21.2 | (+.09)(-.08) | (.52, 1.8) | (.20, 5.9) | (.02,21.4) |
| 4 | 13.7 | 4.8 | (+.15)(-.13) | (.69, 1.6) | (.05, 13.3) | (.11, 6.0) |
| 5 | 10.7 | 9.3 | (+.05, -.03) | (.55, 2.4) | (.11, 10.3) | (0.0, 9.8) |
| 6 | 11.7 | 11.7 | (0., .05) | (.54, 2.6) | (.08, 11.7) | (0.0,11.6) |
| 7 | 6.9 | 6.9 | (+.18)(-.15) | (.70, 1.4) | (.19, 7.3) | (0.0, 6.9) |

*MODAL PARAMETER NOTATION, COMPLEX $(\zeta, \omega_N)$, REAL (-P), ALL FREQUENCIES IN RAD/SEC

## SUMMARY OF EXPERIMENTAL RESULTS

Shown in Table 2 is the summary of tracking scores and subjective rating associated with the seven dynamic configurations. Of significant importance is the pilot comment associated with Configuration 5. He specifically stated that he was attempting to ignore the oscillation that he observed in the display, and he attempts to control the **rigid-body** attitude. Note in the results a monotonic degradation in tracking performance and subjective rating as the elastic mode frequencies in Configurations 1-3 are reduced.

### Table 2

| Configuration | RMS Error (deg) (mean ± 1σ) | Cooper-Harper Rating (mean ± 1σ) | Comments |
|---|---|---|---|
| 1 | 1.2 ± 0.6 | 1.6 ± 0.4 | Very nice; No problem. |
| 2 | 1.0 ± 0.5 | 2.0 ± 0.3 | Little oscillation; More difficult than C1; Slight control response lag. |
| 3 | 5.7 ± 1.1 | 5.9 ± 1.9 | Difficult; Required high concentration; PIO problem; Extreme response lag. |
| 4 | 1.9 ± 0.3 | 3.1 ± 1.1 | Little more difficult than C1; Slightly sluggish attitude response. |
| 5 | 1.2 ± 0.5 | 1.9 ± 0.4 | Not difficult, little more oscillation, but could ignore it and fly rigid-body; Like config. 2. |
| 6 | 1.5 ± 0.7 | 2.0 ± 0.5 | Pretty good; Same as 2. |
| 7 | 7.6 ± 2.8 | 6.7 ± 1.6 | With severe oscillations, virtually uncontrollable. Abrupt control inputs led to disaster. |

# COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS

Shown in figure 8 is the comparison of the tracking performance obtained analytically with the pilot/vehicle analysis and the performance obtained experimentally, shown in the previous table. Note in the case of Configuration 3, for example, the low tracking score predicted from the model under the assumption that the subjects were attempting to control the displayed error. This error, as you recall, included the elastic contribution to the displayed attitude. On the other hand, modeling the task as a rigid-body attitude control task results in the analytical tracking errors as shown in the figure.
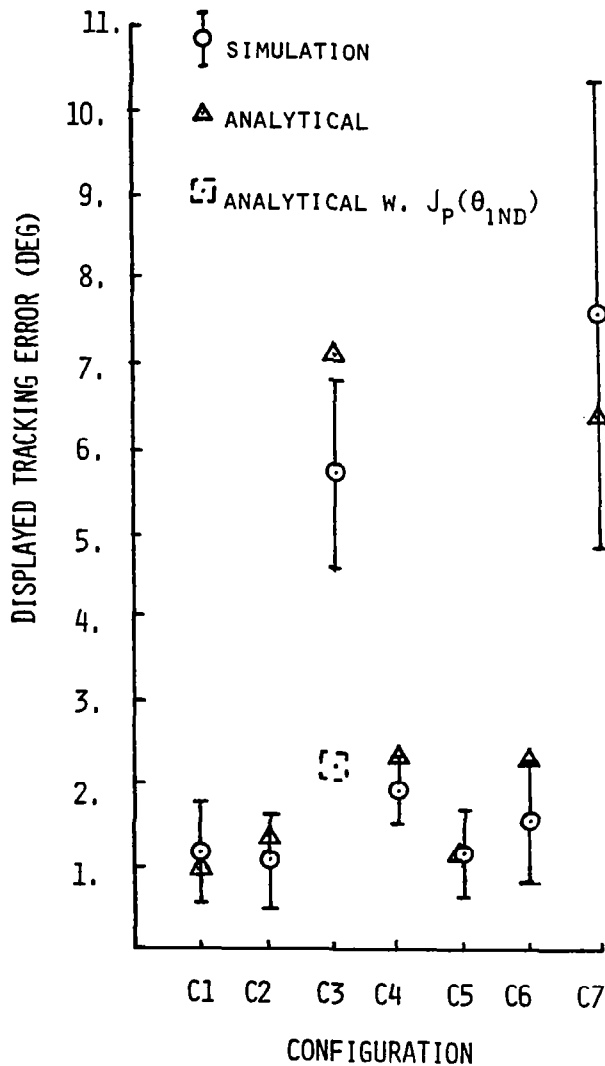


Figure 8

## COMPARISON OF ANALYTICAL AND EXPERIMENTAL SUBJECTIVE RATINGS

Shown in figure 9 is the excellent correlation between experimental subjective ratings and the ratings obtained analytically from a model-based metric. The metric used was simply the magnitude of the quadratic cost function obtained naturally in the modeling process. These results, and the results of the previous figure, indicate that rigid-body attitude is the primary control variable in the closed-loop pitch tracking task. Also, experimental and analytical results indicate clearly that as elastic mode frequencies coalesce with the rigid-body modes, the tracking performance is significantly degraded.
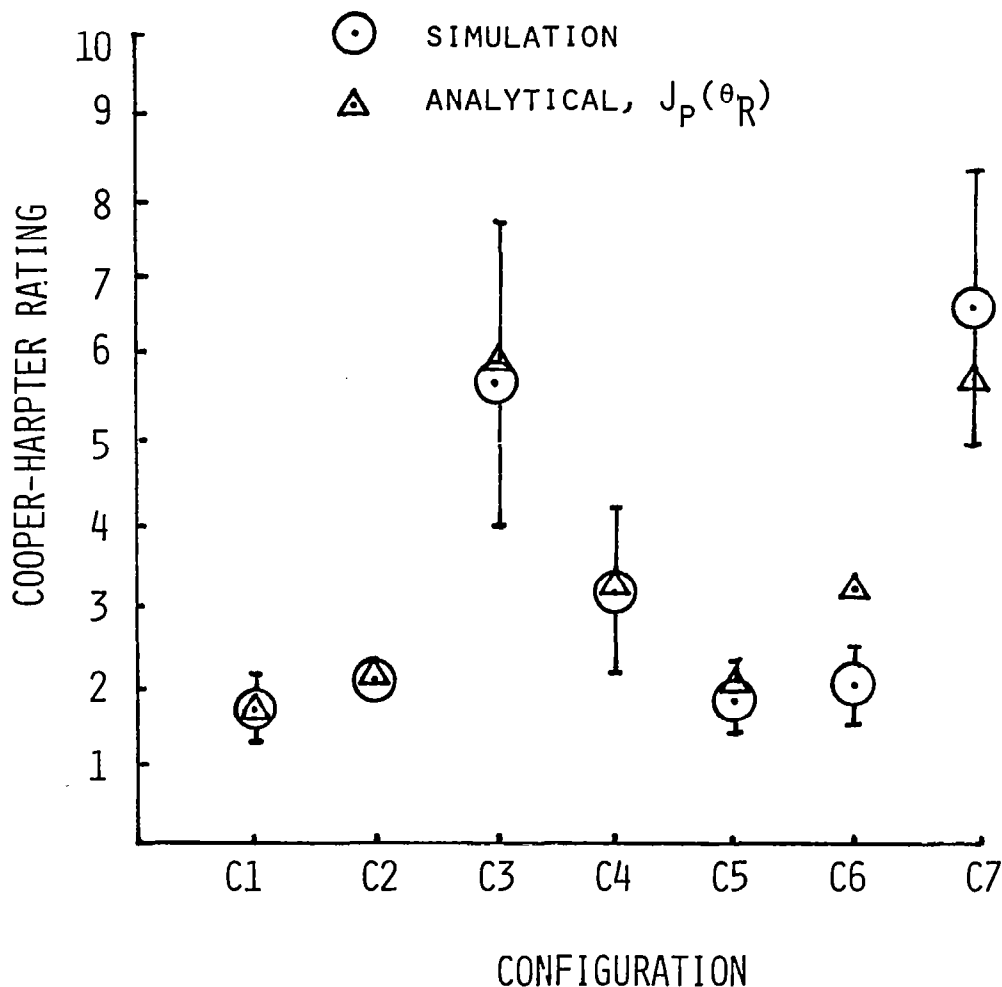


Figure 9

Performing the modal analysis outlined previously on the seven configurations results in the modal residues shown in figures 10-13 for Configurations 1-4. (See Ref. 2 for complete results.) Along with other system's responses, these results indicate clearly the contribution of the first aeroelastic mode to the rigid-body pitch rate ($\dot{\theta}_R$) pilot impulse response. We refer to these residues as pilot impulse residues because they include the important characteristic of limited pilot bandwidth. This is modeled simply as an impulse passed through a first-order lag with time constant representative of that of the pilot. Note the monotonic increase in the residue in the rigid-body pitch rate associated with the first aeroelastic mode, as the frequency of this mode is reduced (Conf. 1-3). This residue in the case of Configuration 3 is actually larger than the residue associated with the "rigid-body" short period mode. Reiterating, in the case of Configuration 3, the rigid-body pitch rate response is dominated by the first aeroelastic mode, where dominance is defined in terms of residue magnitude.
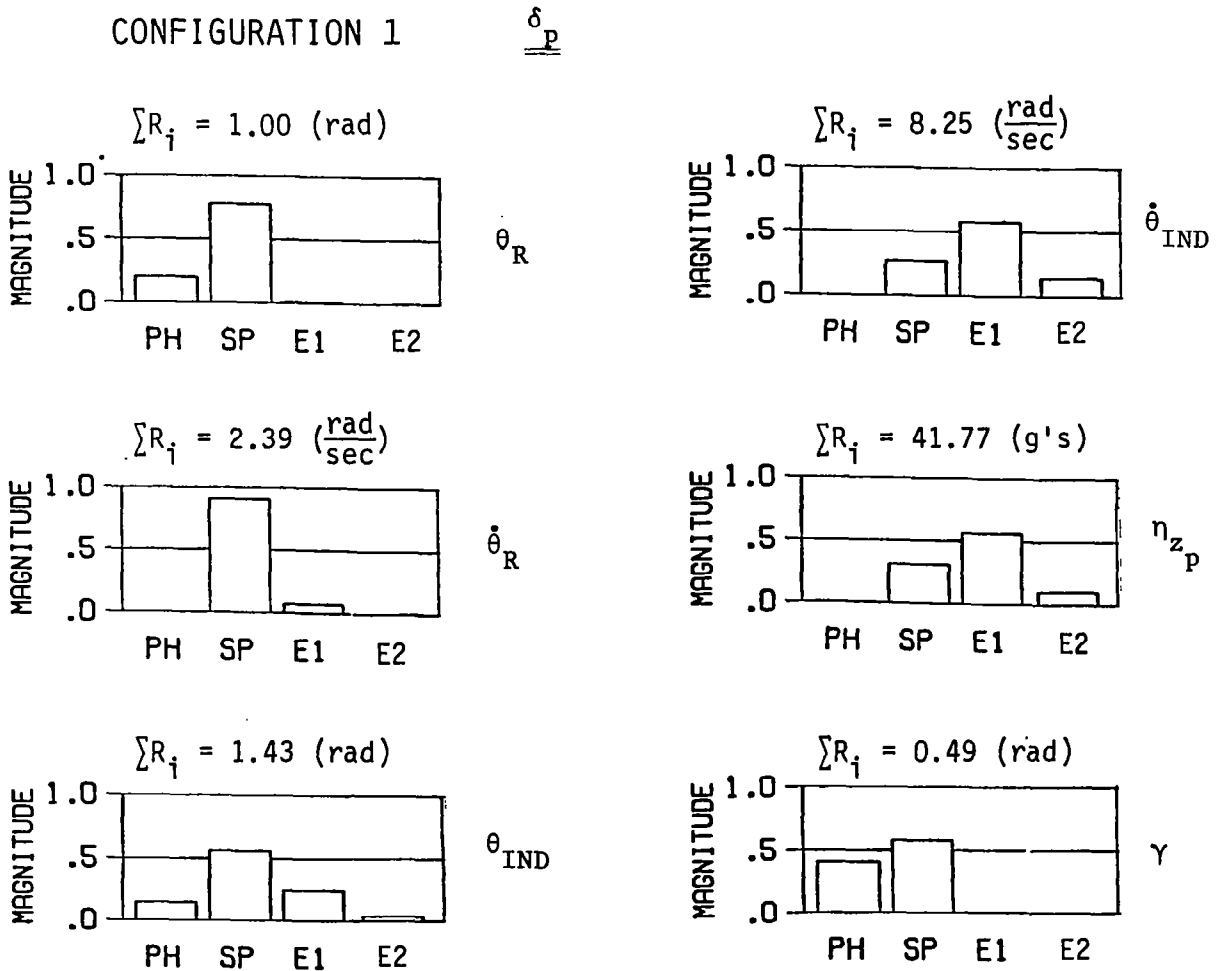
## PILOT IMPULSE RESIDUES

CONFIGURATION 1    $\dfrac{\delta_p}{}$



Figure 10

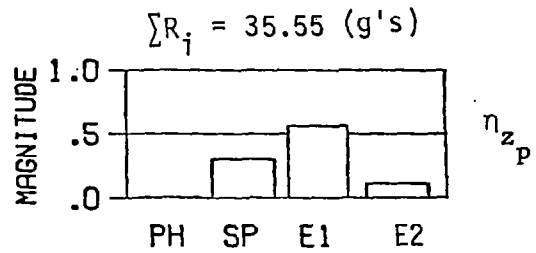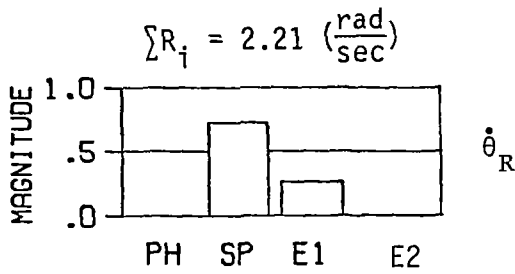# PILOT IMPULSE RESIDUES

## CONFIGURATION 2 $\quad \underline{\delta_p}$

$\sum R_i = 0.88$ (rad)

$\theta_R$

$\sum R_i = 9.49 \left(\frac{rad}{sec}\right)$

$\dot{\theta}_{IND}$

$\sum R_i = 2.21 \left(\frac{rad}{sec}\right)$

$\dot{\theta}_R$

$\sum R_i = 35.55$ (g's)

$\eta_{z_p}$

$\sum R_i = 1.59$ (rad)

$\theta_{IND}$

$\sum R_i = 0.44$ (rad)

$\gamma$

(Each plot: MAGNITUDE axis 1.0, .5, .0; horizontal axis labels PH SP E1 E2)
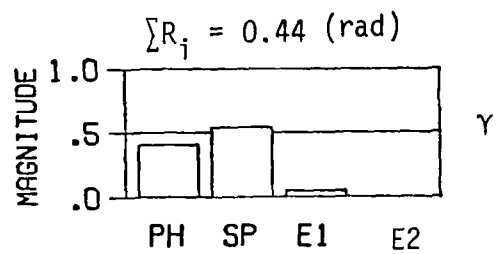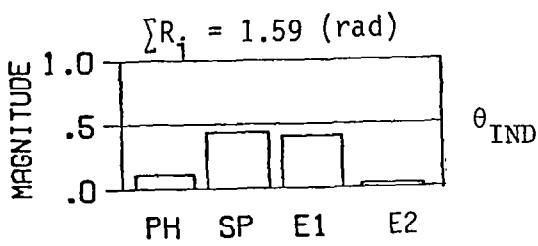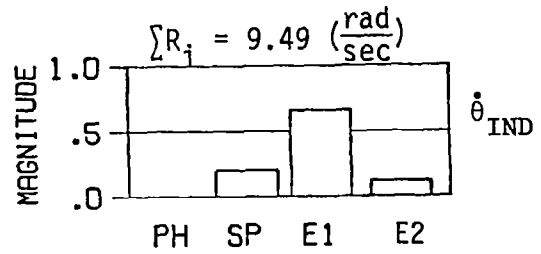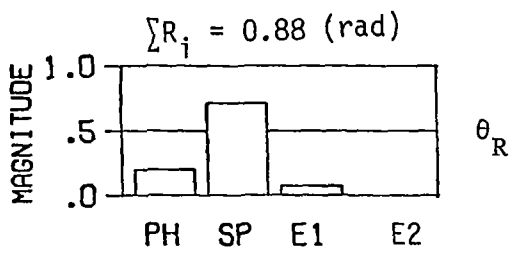
Figure 11

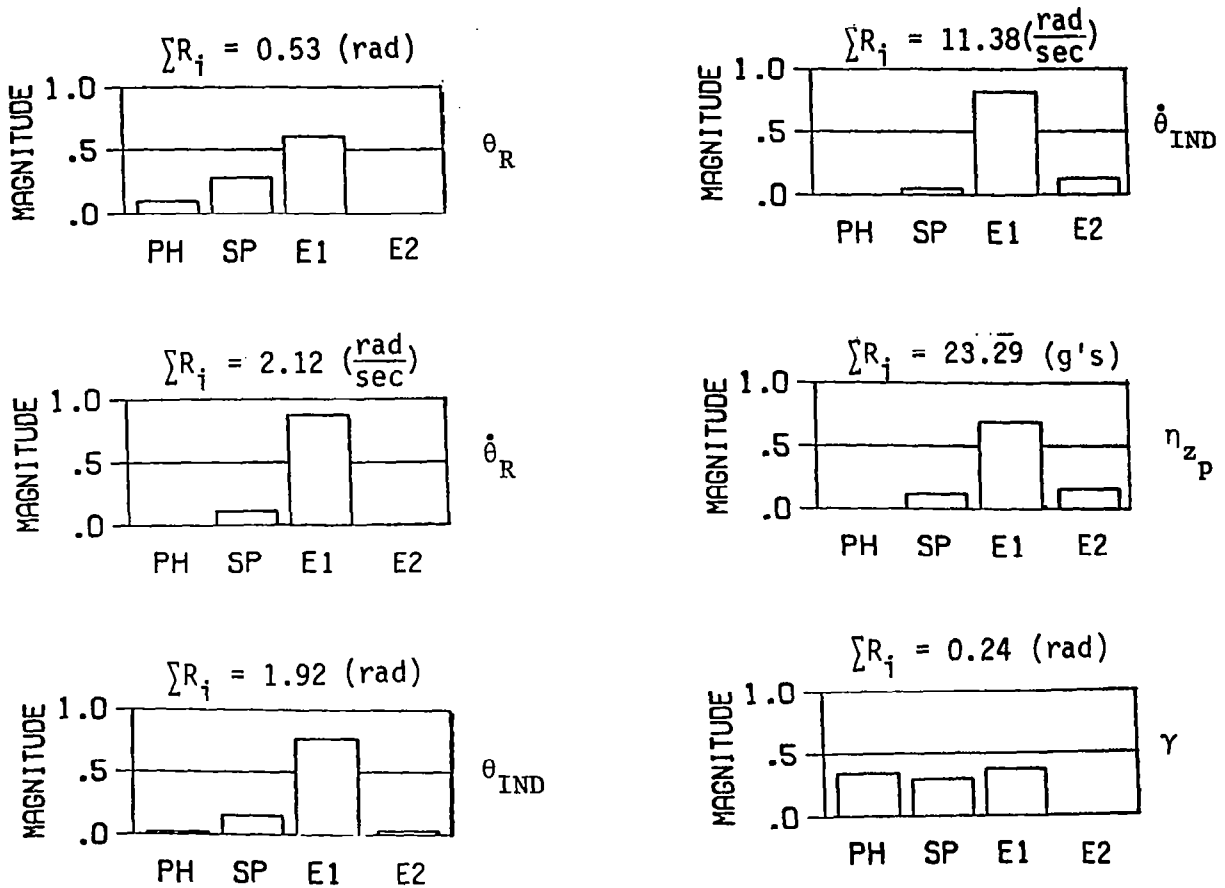# PILOT IMPULSE RESIDUES

CONFIGURATION 3 $\qquad$ $\underline{\delta}_p$



$\sum R_i = 0.53$ (rad) $\qquad$ $\theta_R$

$\sum R_i = 11.38(\frac{rad}{sec})$ $\qquad$ $\dot{\theta}_{IND}$

$\sum R_i = 2.12$ $(\frac{rad}{sec})$ $\qquad$ $\dot{\theta}_R$

$\sum R_i = 23.29$ (g's) $\qquad$ $\eta_{z_p}$

$\sum R_i = 1.92$ (rad) $\qquad$ $\theta_{IND}$

$\sum R_i = 0.24$ (rad) $\qquad$ $\gamma$

Figure 12

Note, on the other hand, the residues for rigid-body pitch rate for Configuration 4. Although the contribution of this elastic mode to the response is measurable, the response is still dominated by the short-period mode. This result, along with the results for Configuration 3, explains why the tracking performance and subjective rating for Configuration 3 were so drastically inferior to those of Configuration 4. This was true in spite of the fact that these two configurations had roughly comparable eigenvalues. Clearly, the eigenvalues alone do not completely explain the results obtained experimentally.

## PILOT IMPULSE RESIDUES

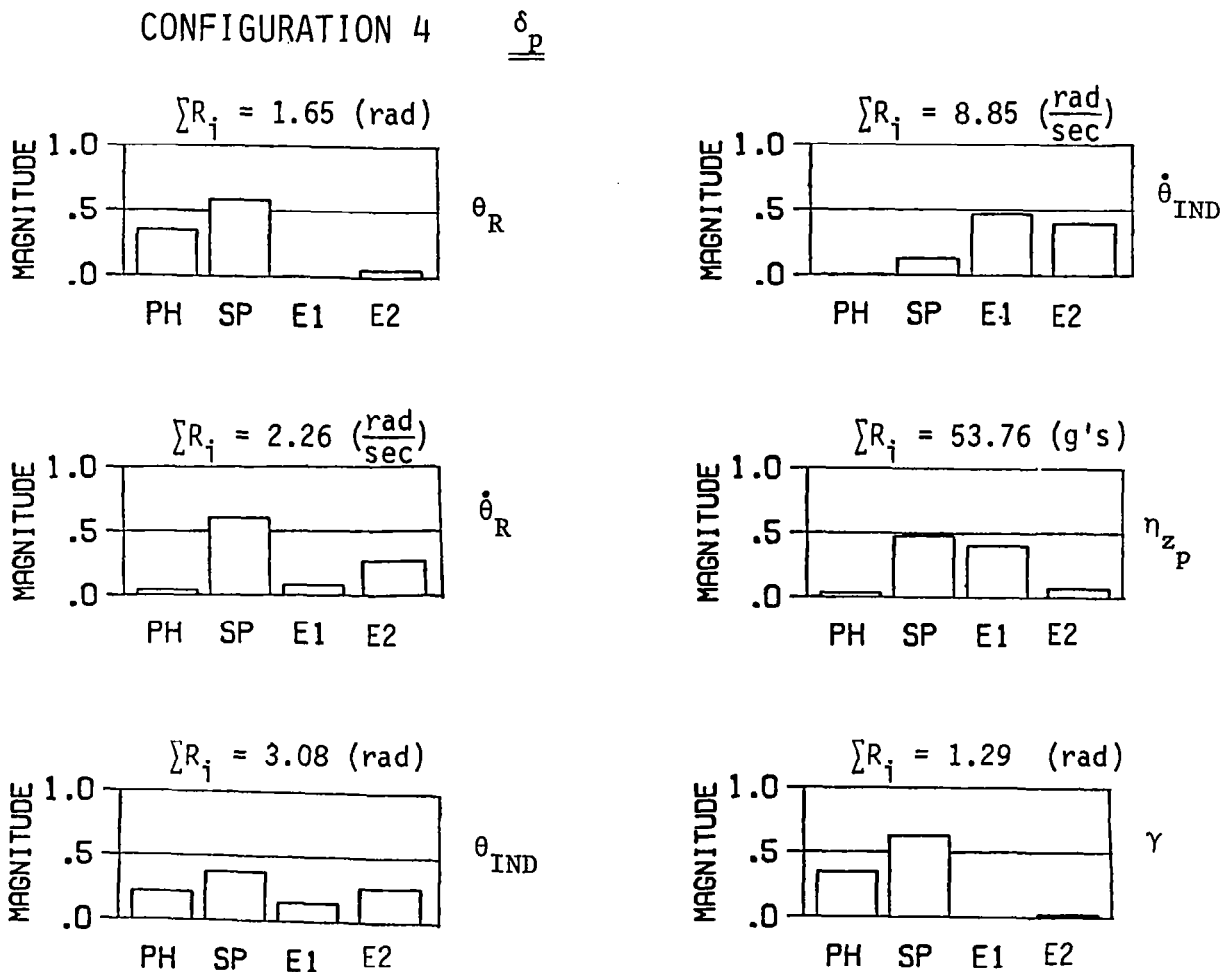CONFIGURATION 4  $\underline{\delta_p}$



Figure 13

## MODIFICATION THROUGH MODAL CONTROL

Given a linear system's dynamics, a control law can be determined such that the closed-loop eigenvalues and eigenvectors are modified to exhibit more desirable characteristics (fig. 14). The number of closed-loop or augmented system modes that may be "placed" is equal to the number of measurements available for feedback, and the freedom to specify the mode shapes, or eigenvectors associated with these modes, depends on the rank of the control vector. (See Refs. 3, 4). Control laws based on this theoretical concept may be implemented with constant gain measurement-feedback architecture, or they may be synthesized with linear quadratic Gaussian (LQG) optimal control (Ref. 5). In the case of measurement feedback, if insufficient measurements are available, the unspecified system modes may be unstable. Conversely, control synthesis using LQG invokes the asymptotic properties of such controllers and theoretically guarantees augmented system stability. To explore the achievable performance that might be obtained through such modal control concepts, we have augmented one of the seven vehicle configurations considered previously (Configuration 2) with a constant gain feedback controller with gains determined directly from the eigenspace assignment goal.

## EIGENSPACE ASSIGNMENT

### (MODAL CONTROL)

GIVEN THE LINEAR SYSTEM

DYNAMICS  $\dot{x} = Ax + Bu$

MEASUREMENTS  $Z = Cx$; DIM $Z = M$

CONTROLS  $U = Gz$; DIM $U = R$

A GAIN G MAY BE FOUND SUCH THAT IF

$$(A + BGC)\bar{v}_i = \lambda_i \bar{v}_i$$

$\lambda_i$ AND $\bar{v}_i$ ; $i = 1. . .M$

HAVE DESIRABLE CHARACTERISTICS

Figure 14

Shown in figure 15 are the mode shapes associated with three of the four modes of interest for Configuration 2 discussed previously. Recall that this configuration differs from the baseline in that the first fuselage bending mode frequency is reduced to approximately 9 radians per second. The baseline on the other hand had a first elastic mode of about 13 radians per second. It is evident in the figure that the "short period" mode actually includes a significant amount of elastic deformation. In contrast, the first aeroelastic mode also reflects the presence of rigid-body attitude in its mode shape. It is due to these modal characteristics that the rigid body response was degraded and the elastic mode's contribution was significant in the rigid-body pitch rate.
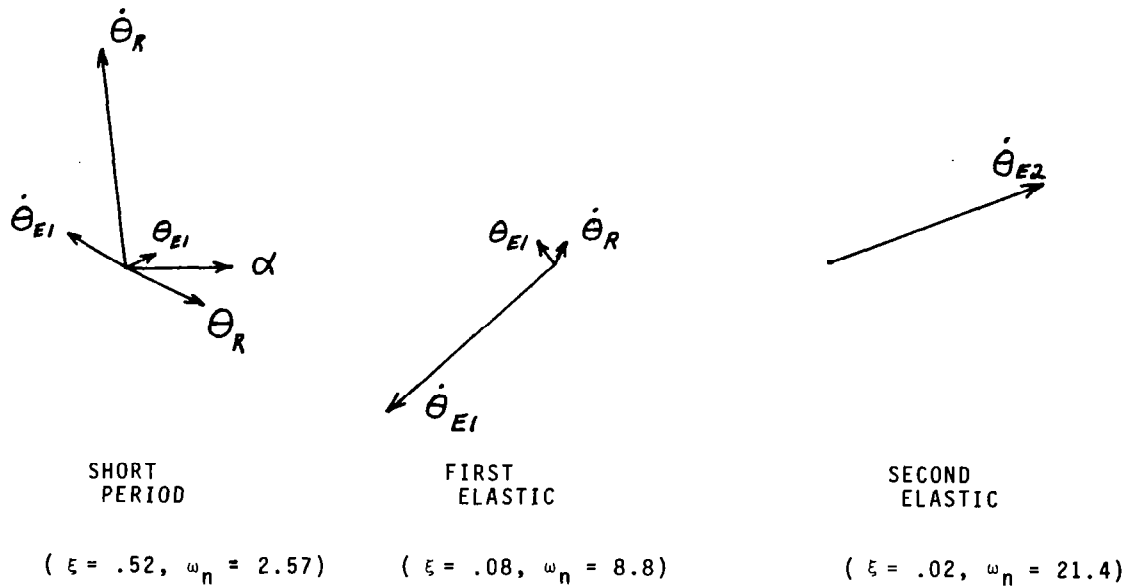


SHORT
PERIOD

FIRST
ELASTIC

SECOND
ELASTIC

( $\xi$ = .52, $\omega_n$ = 2.57)    ( $\xi$ = .08, $\omega_n$ = 8.8)    ( $\xi$ = .02, $\omega_n$ = 21.4)

Figure 15

# UNAUGMENTED VEHICLE STEP RESPONSE

Shown in figure 16 is the rigid-body pitch rate step response for Configuration 2. The contribution of the first aeroelastic mode in this time response is clearly evident. The eigenspace assignment goal used for augmenting these dynamics included increasing the short period frequency slightly and increasing the damping of the first elastic mode from 0.08 to 0.20. In addition, the eigenvectors associated with these two modes were modified. The short-period eigenvector was to represent pure rigid-body response, while the first elastic eigenvector was selected for purely elastic deformation.
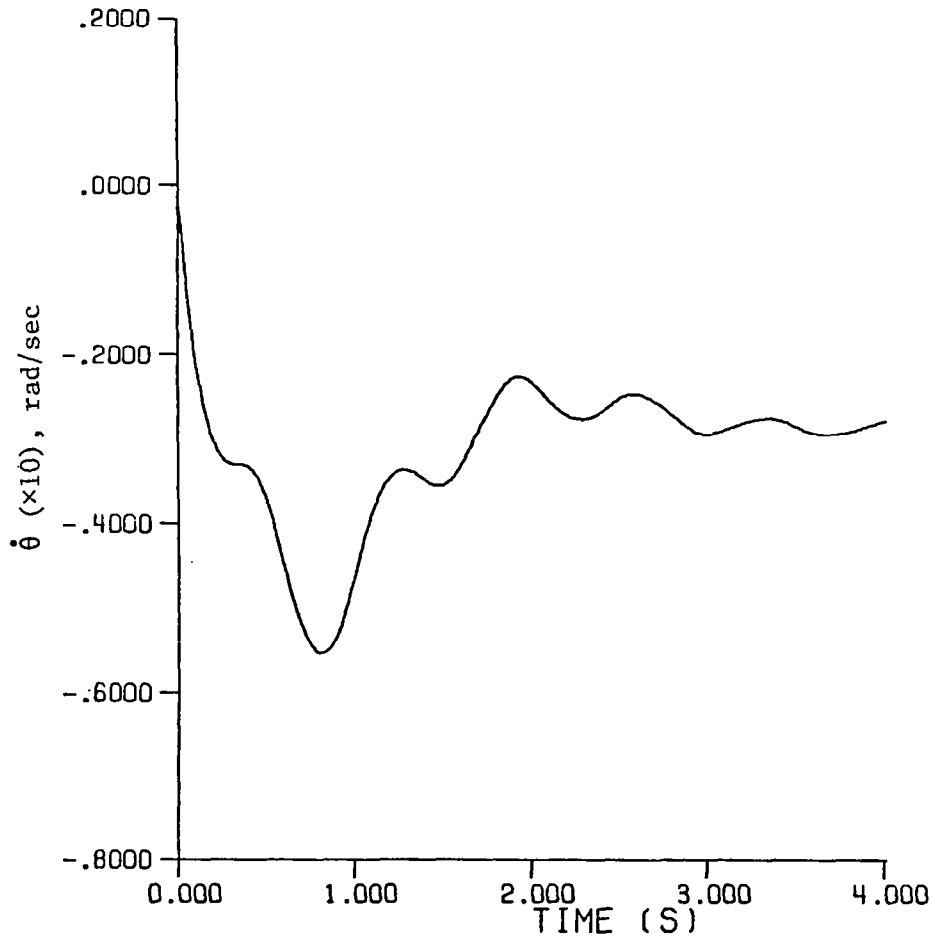


Figure 16

## AUGMENTED VEHICLE MODE SHAPES

Shown in figure 17 are the eigenvectors of the augmented vehicle modified through modal control. Clearly, the short period mode approaches that of a "rigid" vehicle, while the first elastic mode is purified as well. In this example, only 4 measurements were selected for feedback (i.e., two accelerometers, appropriately positioned in the fuselage, along with pitch rate and pitch attitude gyros). Consequently, only 4 eigenvalues (or two modes) were specified The phugoid mode and the second aeroelastic mode were not placed in this case but could be if more measurements are made available. In addition, a control law with limited bandwidth should be selected such that the second elastic mode would be attenuated.
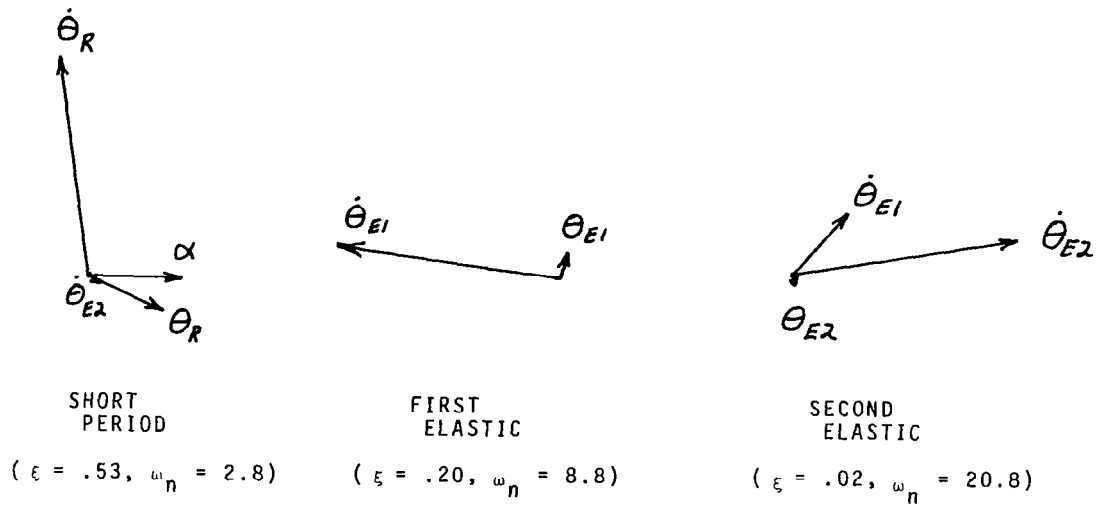
SHORT
PERIOD

$( \xi = .53, \omega_n = 2.8)$

FIRST
ELASTIC

$( \xi = .20, \omega_n = 8.8)$

SECOND
ELASTIC

$( \xi = .02, \omega_n = 20.8)$

Figure 17

# AUGMENTED VEHICLE STEP RESPONSE

Shown in figure 18 is the step response of the augmented vehicle. When compared to that in Configuration 2 the reduction of the contribution of the elastic mode to this rigid-body pitch rate response is evident. The long period divergence of this response is due to the phugoid mode instability. This demonstrates one of the shortcomings of implementing modal control through measurement feedback alone as cited previously. Additional measurements or equalization are required to stabilize the phugoid mode.
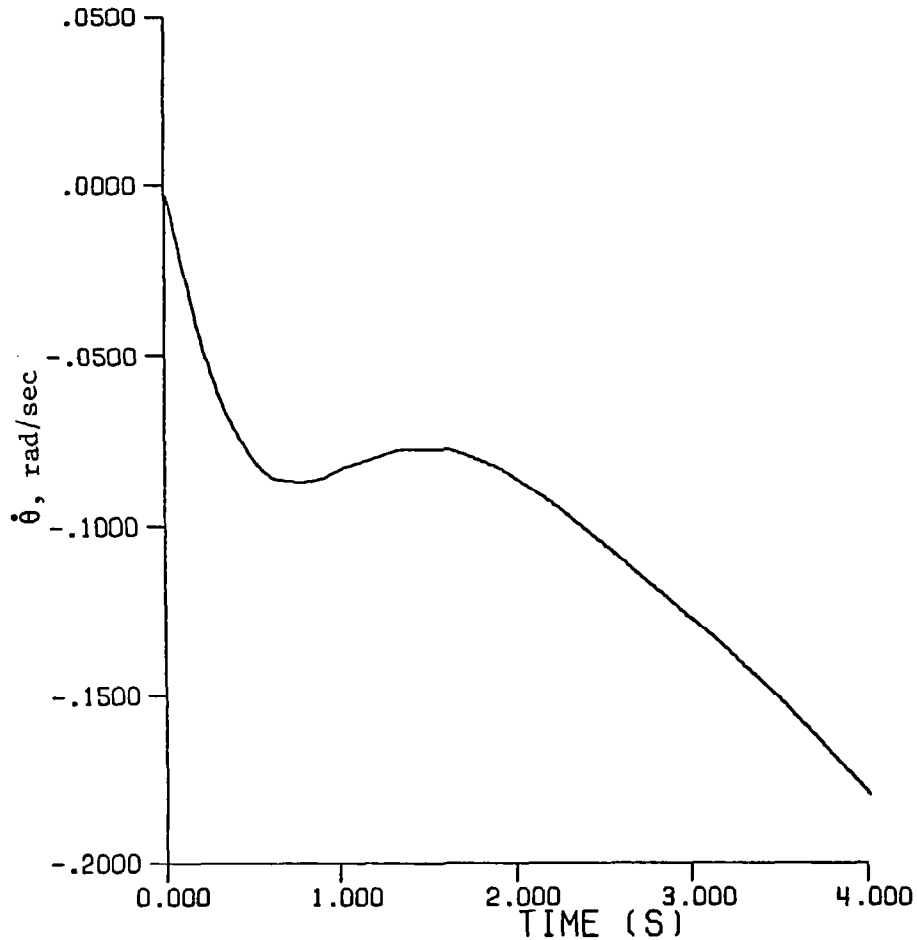


Figure 18

Shown in figure 19, finally, is the effect of the augmentation on residues for the pitch rate impulse response. These residues are comparable with those shown previously for the seven configurations. Compared to the unaugmented vehicle (Configuration 2), the results for the augmented vehicle clearly indicate the dominance of the short-period mode in this response.

In conclusion then, we see that handling characteristics, as measured by tracking performance and subjective rating in the tracking task, were significantly degraded due to the presence of dynamic aeroelastic effects. The rigid-body attitude angle was shown to be fundamental in the vehicle's response in this task. Furthermore, this response may be dominated by "aeroelastic" modes in severe cases. Clearly, from these results, a rigid-body mathematical model of the vehicle is inappropriate. Finally, the ability to modify the modal characteristics of the vehicle through modal control or eigenspace assignment appears to have merit in this application. Multiple control surfaces and an appropriate sensor complement will be required to implement practical modal controllers. Appropriate design criteria for flexible vehicles might be expressed in terms of allowable residue magnitudes of the higher order mode.
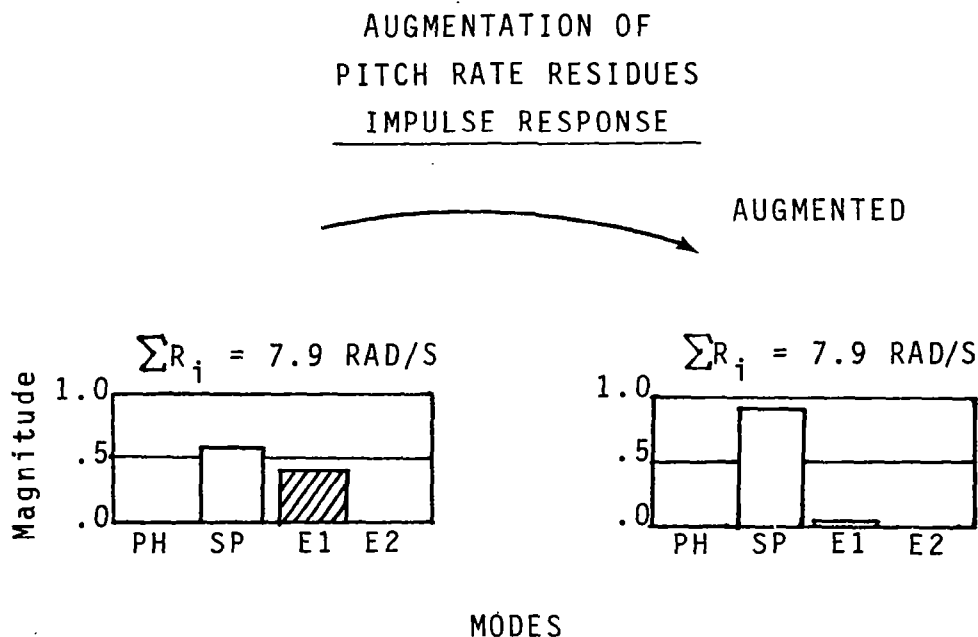
AUGMENTATION OF

PITCH RATE RESIDUES

IMPULSE RESPONSE

AUGMENTED

$\sum R_i$ = 7.9 RAD/S

$\sum R_i$ = 7.9 RAD/S

MODES

Figure 19

# REFERENCES

1.  Schmidt, K.K., "Pilot Modeling and Closed-Loop Analysis of Flexible Aircraft In the Pitch Tracking Task", AIAA Paper No. 83-2231, Guid. and Control Conference, August 1983.

2.  Schmidt, D.K., "A Modal Analysis of Flexible Aircraft Dynamics With Handling Qualities Implications," AIAA Paper No. 83-2074, Atm. Flight Mech. Conference, August 1983.

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