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GRID FLEXIBILITY AND PATCHING TECHNIQUES

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The numerical determination of combustor flowfields is of great value to the combustor designer. An a priori knowledge of the flow behavior can speed the combustor design process and reduce the number of experimental test rigs required to arrive at an optimal design. Even 2-D steady incompressible isothermal flow predictions are of use; many codes of this kind are available [1], each employing different techniques to surmount the difficulties arising from the nonlinearity of the governing equations and from typically irregular combustor geometries. We will look at mapping techniques (algebraic and elliptic PDE), and at adaptive grid methods (both multi-grid and grid embedding) as applied to axisymmetric combustors.

ALGEBRAIC GRID GENERATION IN THE STARPIC CODE

The solution of the Turbulent Reynolds Equations for arbitrary geometries can be handled in several ways [1]. One popular technique is to represent a boundary with discrete 90° steps. Atlernatively, the geometry can be mapped into a square solution plane via a variety of available transformations. The simplest transformation is an algebraic one as shown in Figure 2.

On transforming the governing equations, new chain rule terms are introduced, each of which contains a dh/dx term which comes into play only at axial locations where there is a sloping or curved wall segment, thereby mathematically affecting expansion and recirculation in the flow. The complexity of extra terms is offset by the ability of the technique to handle general boundary shapes, as well as the simplification of boundary conditions in the mapped plane. For these reasons algebraic mapping shows great promise. Figures 1 through 6 show the effects of algebraic mapping on the STARPIC code.

ELLIPTIC PDE GRID GENERATION FOR A BIFURCATED COMBUSTOR INLET DIFFUSER

The purpose of a combustor inlet diffuser is to convert kinetic energy to pressure; a reliable prediction method for diffuser flows will lead to the efficacious design of those diffusers. In this work body fitted coordinate transformations are employed in the solution of turbulent flows in a bifurcated combustor inlet diffuser. The work is aimed at comparing the numerical solution for different kinds of transformations with experimental results. Furthermore, different numerical methods will also be employed for comparison. Figures 7 through 12 illustrate this work.

MULTIGRID AND GRID EMBEDDING TECHNIQUE FOR TURBULENT FLOWS IN COMBUSTOR

A multigrid method [2] applied to axisymmetric turbulent flows is useful in speeding convergence of numerical schemes. This method reduces global errors by economical relaxation of errors on coarse meshes instead of by labor intensive relaxation on one fine mesh. The multigrid equations, a summary of the multigrid algorithm, and relationships between coarse and fine meshes for a staggered mesh are depicted in Figures 13 through 17.

Grid embedding [3,4] similar to the multigrid method, uses a fine mesh in steep gradient regions to refine the solution locally, where needed. It eliminates the wasteful process of using a fine grid globally, including in regions where it is not needed. Grid embedding is depicted for a staggered mesh. As indicated in the last figure, this method is still under development. Like the multigrid method, it will steadily be extended to more arbitrary combustor geometries.

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1. Lilley D. G. and D. L. Rhode, "A Computer Code for Swirling Turbulent Axisymmetric Recirculating Flows in Practical Isothermal Combustor Geometries," Grant NAG 3-74, February 1982.

2. Brandt, A., "Multi-Level Adaptive Solutions to Boundary-Value Problems," Mathematics of Computation, Volume 31, No. 138, April 1977, pp. 333-390.

3. Brown, Jeffrey J., "An Embedded-Mesh Potential Flow Analysis," AIAA Journal, Volume 22, No. 2, February 1984, pp. 174-178.

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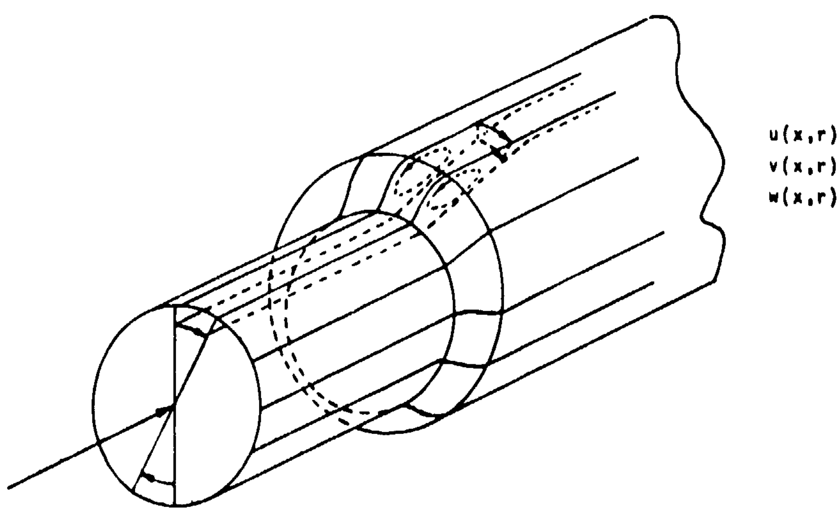
1. ALGEBRAIC GRID GENERATION IN THE STARPIC CODE - LARRY SMITH
2. ELLIPTIC PDE GRID GENERATION FOR A BIFURCATED COMBUSTOR INLET DIFFUSER - CHAIN-NAN YUNG
3. MULTIGRID AND GRID EMBEDDING TECHNIQUES FOR TURBULENT FLOWS IN COMBUSTORS - STEVE BARTHELSON

ALGEBRAIC GRID GENERATION AND STARPIC CODE



FLOW FIELD AND VARIABLES

GOVERNING EQUATIONS AND MAPPING

TURBULENT REYNOLDS EQUATIONS (IN GENERIC FORM)

$$\frac{1}{r}\left[\frac{\partial}{\partial x}\left(\rho u r \phi - \Gamma r \frac{\partial}{\partial x}\right) + \frac{\partial}{\partial r}\left(\rho v r \phi - \Gamma r \frac{\partial}{\partial r}\right)\right] = S_\phi$$

WHERE ϕ REPRESENTS:

- u
- v
- w
- 1 (Continuity)
- k
- ϵ

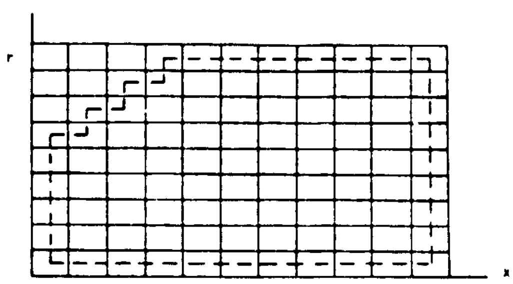
TRANSFORMATION: $\xi = x$

$\eta = r/h(x)$ where $h(x)$ is any single-valued function.

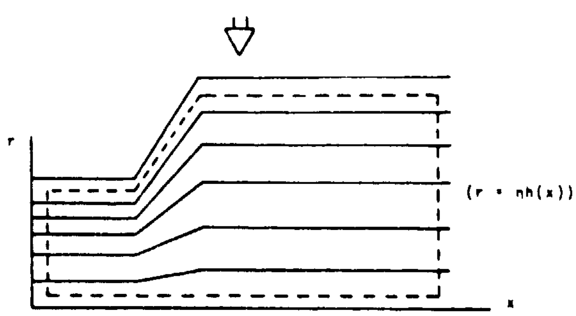
TRANSFORMATION OF DIFFERENTIAL OPERATORS:

$$\frac{\partial}{\partial x} = \frac{\partial \xi}{\partial x}\frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial x}\frac{\partial}{\partial \eta} = \boxed{\frac{\partial}{\partial \xi} - \frac{\eta}{h}\frac{dh}{dx}\frac{\partial}{\partial \eta}}$$

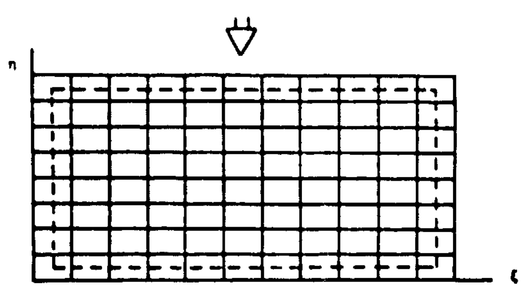
$$\frac{\partial}{\partial r} = \frac{\partial \xi}{\partial r}\frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial r}\frac{\partial}{\partial \eta} = \boxed{\frac{1}{h}\frac{\partial}{\partial \eta}}$$



STARPIC PHYSICAL AND SOLUTION PLANE



STARMAP PHYSICAL PLANE



STARMAP TRANSFORMED PLANE

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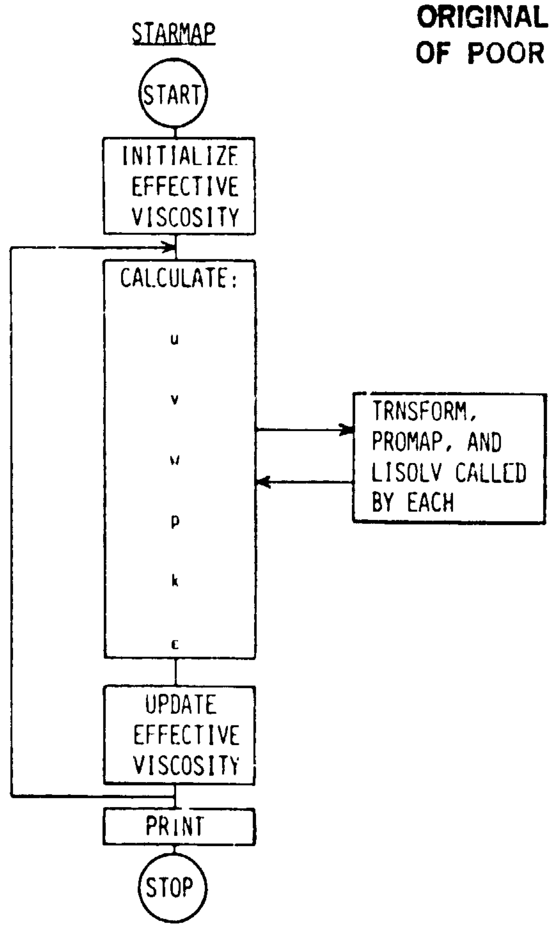


ADAPTED FROM
STARPIC

CONTRASTING STARPIC AND STARMAP

ADVANTAGES OF MAPPING:

ANY SINGLE VALUED FUNCTION VALID FOR $h(x)$.

SOLUTION OCCURS ON A SQUARE MESH IN THE $\xi - \eta$ PLANE.

BOUNDARY CONDITIONS ARE SIMPLIFIED.

DISADVANTAGES:

ADDITIONAL TERMS ARISE VIA TRANSFORMATION.

HANDLING OF "WALL FUNCTIONS" NOT OBVIOUS.

STATUS

NEARING OPERATION OF COMPLETELY MAPPED CODE.

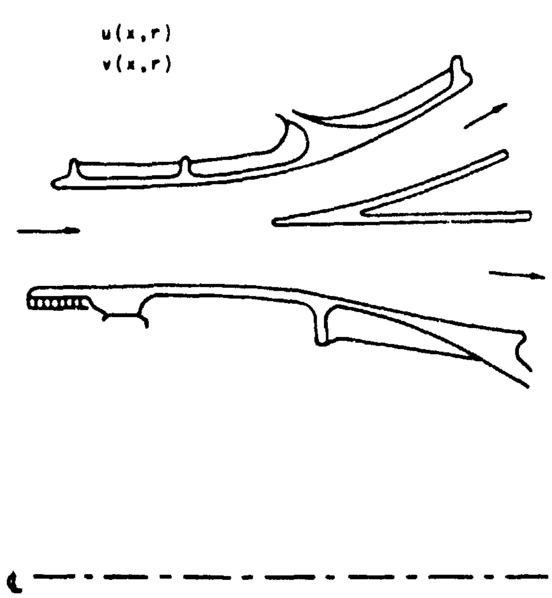
FOLLOWING FIRST SUCCESSFUL RUN, CODE WILL BE TESTED WITH A VARIETY OF GEOMETRIES IN TWO PHASES:

PHASE (I) VALIDATION OF STARMAP IN DIRECT COMPARISON WITH STARPIC RESULTS;

PHASE (II) GENERALIZATION TO OTHER GEOMETRIES NOT PRESENTLY HANDLED BY STARPIC.

ELLIPTIC PDE GRID GENERATION FOR A BIFURCATED COMBUSTOR INLET DIFFUSER

FLOWFIELD



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BODY FITTED COORDINATE TRANSFORMATION

$$\frac{\partial^2 x}{\partial \xi^2} + \frac{\partial^2 x}{\partial \eta^2} = P(\xi,\eta) \qquad \frac{\partial^2 y}{\partial \xi^2} + \frac{\partial^2 y}{\partial \eta^2} = Q(\xi,\eta)$$

WHERE $P = \sigma(\xi_x^2 + \xi_y^2)$ AND $Q = \tau(\eta_x^2 + \eta_y^2)$

$$\sigma = -\frac{x_\xi x_{\xi\xi} + y_\xi y_{\xi\xi}}{x_\xi^2 + y_\xi^2} \quad \text{on } \eta = \eta_b$$

$$\tau = -\frac{x_\eta x_{\eta\eta} + y_\eta y_{\eta\eta}}{x_\eta^2 + y_\eta^2} \quad \text{on } \xi = \xi_a$$

INTERIOR POINTS OF σ AND τ ARE CALCULATED BY LINEAR INTERPOLATION ALONG GRID LINES.

INVERTING YIELDS:

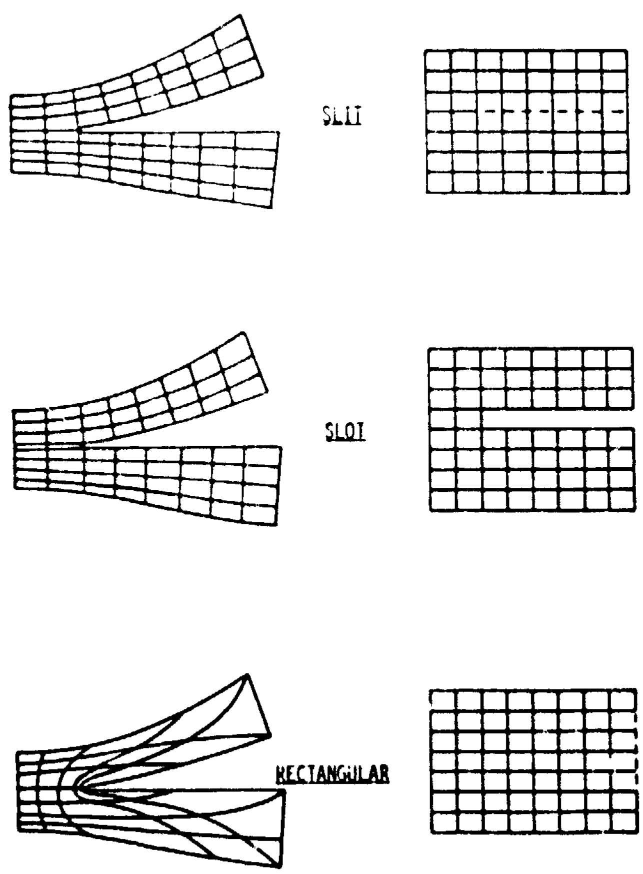
$$\alpha(x_{\xi\xi} + \sigma x_\xi) - 2\beta x_{\xi\eta} + \gamma(x_{\eta\eta} + \tau x_\eta) = 0$$

$$\alpha(y_{\xi\xi} + \sigma y_\xi) - 2\beta y_{\xi\eta} + \gamma(y_{\eta\eta} + \tau y_\eta) = 0$$

WHERE

$$\alpha = x_\eta^2 + y_\eta^2, \quad \beta = x_\xi x_\eta + y_\xi y_\eta, \quad \gamma = x_\xi^2 + y_\xi^2$$

TRANSFORMED COORDINATE SYSTEMS



GOVERNING EQUATIONS - COMPUTATIONAL DOMAIN

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$$\frac{r_\eta}{J}\frac{\partial G}{\partial \xi} - \frac{r_\xi}{J}\frac{\partial G}{\partial \eta} + \frac{x_\xi}{rJ}\frac{\partial H}{\partial \eta} - \frac{x_\eta}{rJ}\frac{\partial H}{\partial \xi} = S_\phi$$

WHERE $G = \rho u \phi - \frac{\Gamma_\phi}{J}\left(r_\eta \frac{\partial \phi}{\partial \xi} - r_\xi \frac{\partial \phi}{\partial \eta}\right)$

$H = \rho v r \phi - \frac{r\Gamma_\phi}{J}\left(x_\xi \frac{\partial \phi}{\partial \eta} - x_\eta \frac{\partial \phi}{\partial \xi}\right)$

INTEGRATION OVER A CONTROL VOLUME GIVES

$$\rho r F_1 \phi \Big|_W^E + \rho r F_2 \phi \Big|_S^N - \frac{r\Gamma_\phi}{J}\alpha \frac{\partial \phi}{\partial \xi}\Big|_W^E - \frac{\Gamma_\phi}{J}\gamma \frac{\partial \phi}{\partial \eta}\Big|_S^N$$

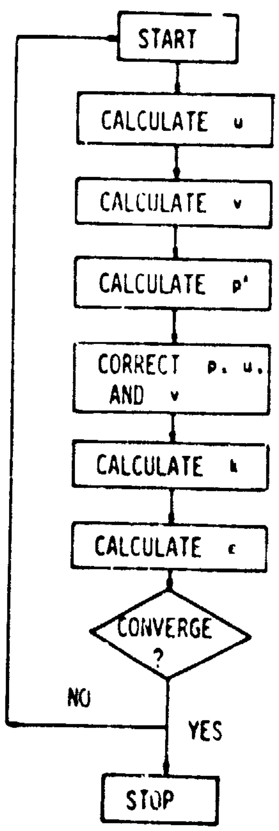
$$= S_\phi \text{ (VOLUME)} - \frac{r\Gamma_\phi}{J}\beta \frac{\partial \phi}{\partial \eta}\Big|_W^E - \frac{r\Gamma_\phi}{J}\beta \frac{\partial \phi}{\partial \xi}\Big|_S^N$$

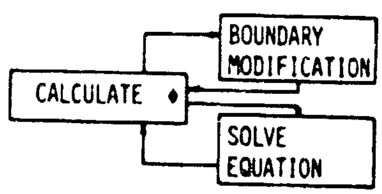
WHERE $F_1 = r_\eta U - x_\eta V$, $F_2 = x_\xi V - r_\xi U$

DISCRETIZED EQUATIONS TAKE THE FORM

$$A_P \phi_P = \sum A_i \phi_i + S_\phi \text{ (VOLUME)}$$

$$- \frac{r\Gamma_\phi}{J}\beta \frac{\partial \phi}{\partial \eta}\Big|_W^E - \frac{r\Gamma_\phi}{J}\beta \frac{\partial \phi}{\partial \xi}\Big|_S^N$$





FLOW DIAGRAM

STATUS

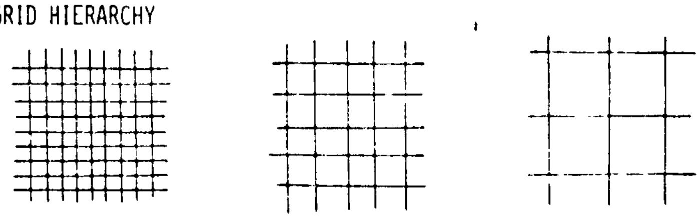
TRANSFORMATION OF THE GEOMETRY HAS BEEN ACCOMPLISHED.

TRANSFORMED GOVERNING EQUATIONS HAVE BEEN DISCRETIZED.

COMPUTER CODES TO SOLVE TRANSFORMED EQUATIONS ARE BEING PREPARED.

MULTIGRID ALGORITHM

GRID HIERARCHY



G^K G^{K-1} G^{K-2}

FUNDAMENTAL RELATIONS

LET $D\phi(r,x) - S_\phi(r,x)$ = SET OF TURBULENT REYNOLDS EQUATIONS

IN OPERATOR NOTATION THIS BECOMES

$$L(\phi) = 0$$

FINITE DIFFERENCING ON GRID G^K GIVES

$$L^k(\phi^k) = F^k$$

WHERE F^K IS MADE IDENTICALLY ZERO ONLY ON THE FINEST MESH

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CHECK

$$\frac{\left|(\phi^k)_{I+2} - (\phi^k)_{I+1}\right|_{max}}{\left|(\phi^k)_{I+1} - (\phi^k)_I\right|_{max}}$$

IF THIS RATIO $\sim$ 0.9, SWITCH TO COARSER GRID. ON GRID G^{K-1}

RELAX THE SET

$$L^{k-1}(\phi^{k-1}) = F^{k-1}$$

WHERE $$F^{k-1} = I_k^{k-1} F^k + \epsilon_k^{k-1}$$

I_k^{k-1} IS THE INJECTION OPERATOR FROM GRID G^{K-1} TO GRID G^K.

ϵ_k^{k-1} IS THE NUMERICAL ERROR AT G^{K-1} RELATIVE TO G^K AND IS ESTIMATED FROM

$$\epsilon_k^{k-1} = L^{k-1}(I_k^{k-1}\phi^k) - I_k^{k-1} L^k(\phi^k)$$

NOTE $$\epsilon_k^{k-1} = f(\phi^k)$$

$$F^{k-1} = g(\epsilon_k^{k-1})$$

THUS, F^{k-1} CHANGES UPON DIFFERENT VISITS TO G^{K-1}.

ONCE ϕ^{k-1} CONVERGES ON G^{K-1}, INTERPOLATION CAN SUPPLY MISSING VALUES IN G^K, I.E.,

$$\phi^k = I_{k-1}^k \phi^{k-1}$$

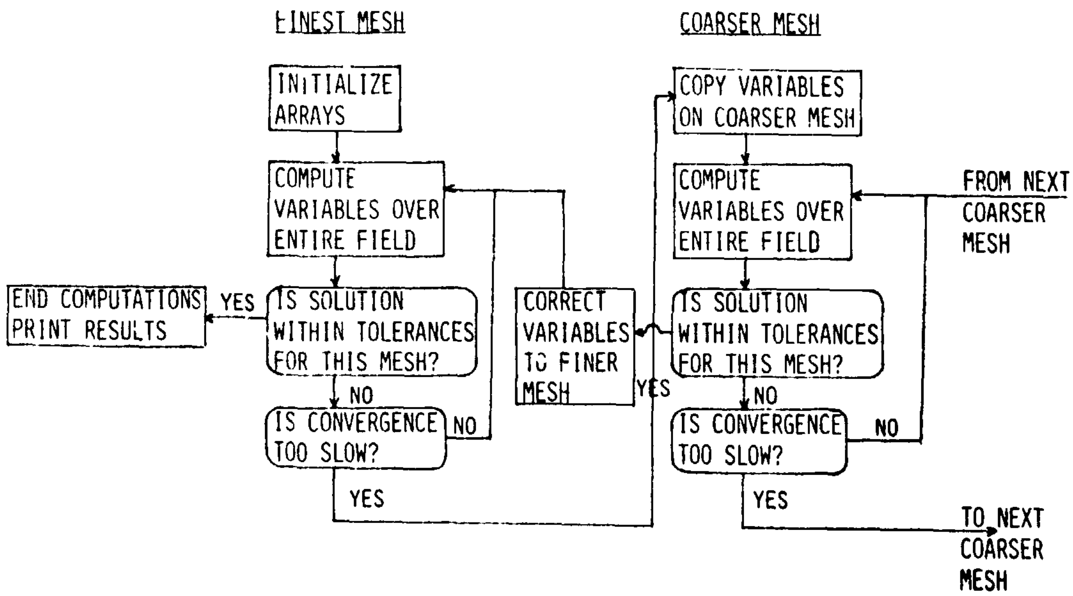
WHERE I_{k-1}^k IS AN INTERPOLATION OPERATOR FOR G^{K-1} TO G^K.

FOR SMOOTHER RESULTS, A MODIFIED VERSION IS USED

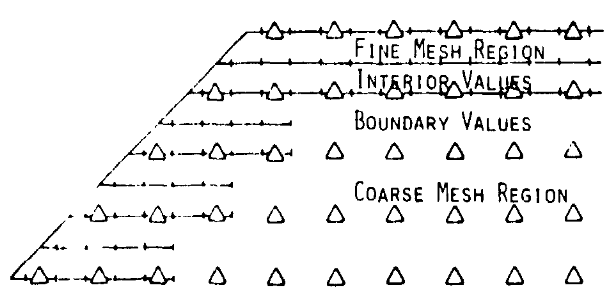
$$\phi_{new}^k = I_{k-1}^k \phi^{k-1} + \phi_{old}^k - I_{k-1}^k I_k^{k-1} \phi_{old}^k$$

$$= \phi_{old}^k + I_{k-1}^k (\phi^{k-1} - I_k^{k-1} \phi_{old}^k)$$

MULTIGRID PROCEDURE



GRID EMBEDDING USING MULTIGRID



V VELOCITIES ONLY, SHOWN FOR SIMPLICITY

FLOW DOMAIN SUBDIVIDED INTO COARSE MESH REGION AND FINE MESH REGION, WHERE STEEP GRADIENTS REQUIRE MORE RESOLUTION;

△'S ARE FINEST GRID FOR COARSE MESH REGION - IN FINE MESH REGION, FINE GRID IS USED TO REFINE SOLUTION AT △'S;

BOUNDARY VALUES ARE UPDATED FOR EACH COARSE MESH SWEEP BY INTERPOLATION BETWEEN △'S ON BOUNDARY;

$L^k \phi^k = F^k$ IN COARSE MESH REGION, AS BEFORE, AT △'S;

$L^k \phi^k = L^k(I_k^{k+1} \phi^{k+1}) + I_{k+1}^k (F^{k+1} - L^{k+1} \phi^{k+1})$ AT △'S, INTERPOLATING FROM FINE MESH.

STATUS

MULTIGRID METHOD IS BEING APPLIED TO THE STARPIC CODE

FUTURE DEVELOPMENTS

REFINE EXISTING MULTIGRID METHOD
DEVELOP EMBEDDED GRID METHOD
EXTEND BOTH METHODS TO MORE ARBITRARY GEOMETRIES